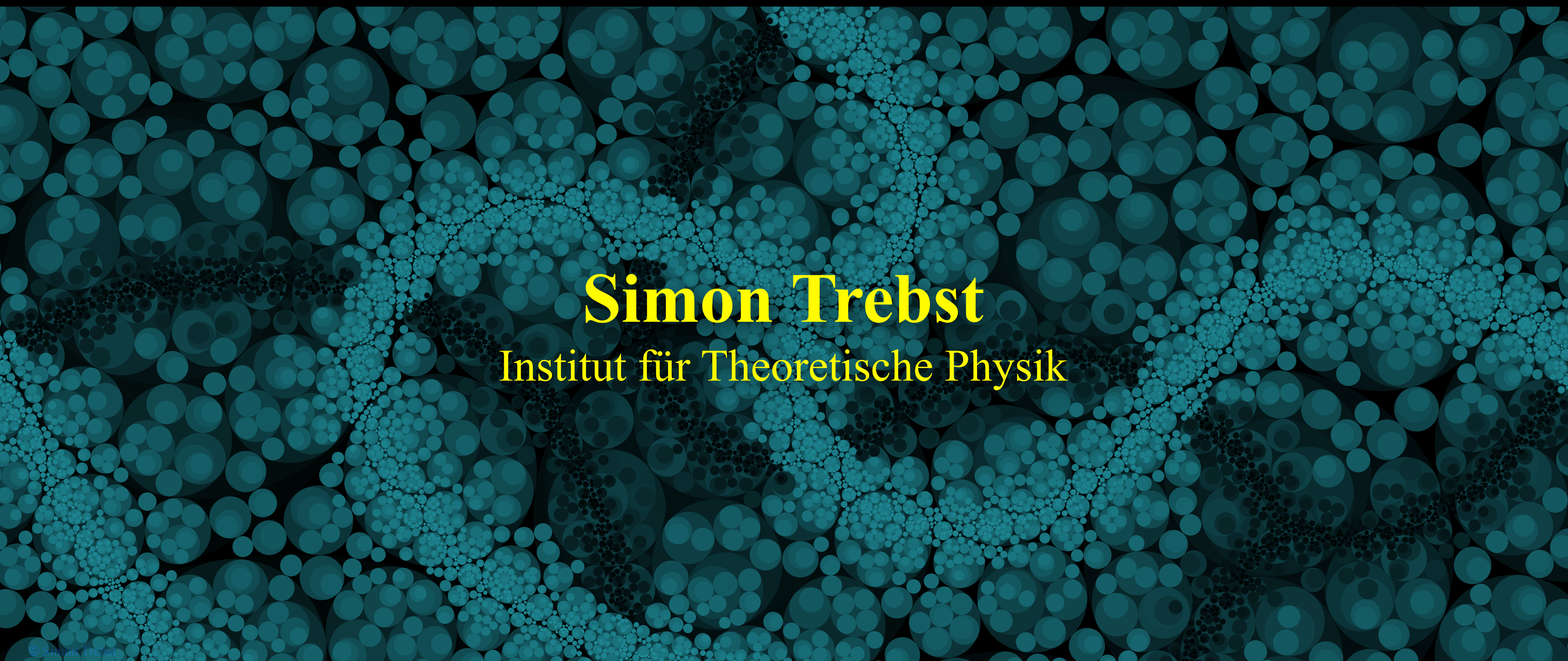
# Emergenz nahe des Temperaturnullpunktes

# Topologische Quantenflüssigkeiten

Großes Physikalisches Kolloquium – 9. Juli 2013

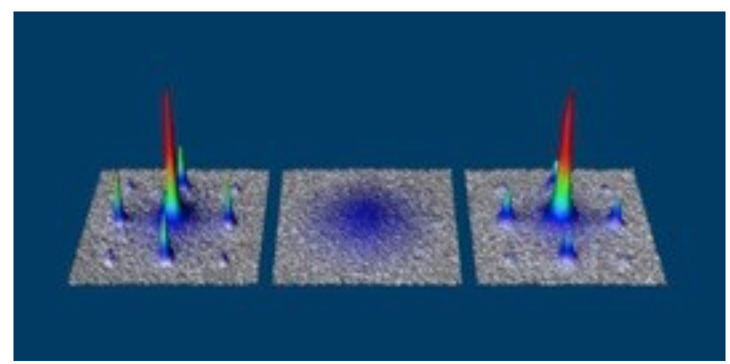


**Simon Trebst**
Institut für Theoretische Physik

# (Quantum) matter

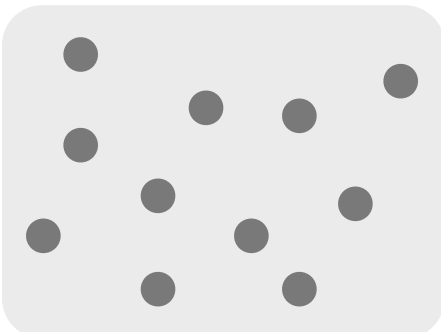
water ice

superconductor



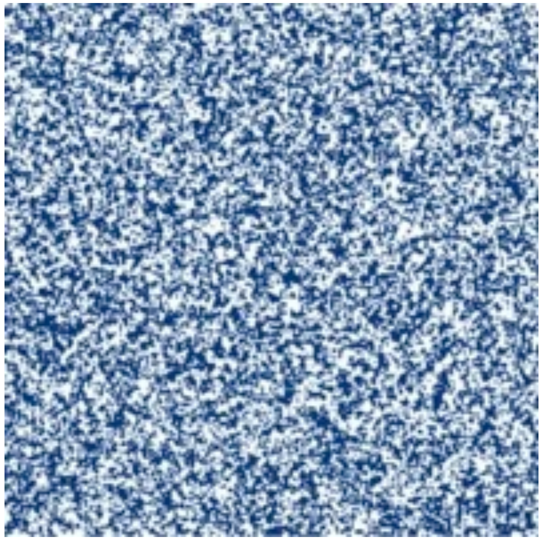
Bose-Einstein condensate

# Motivation – a paradigm



interacting
**many-body system**

$$\mathcal{H} = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$



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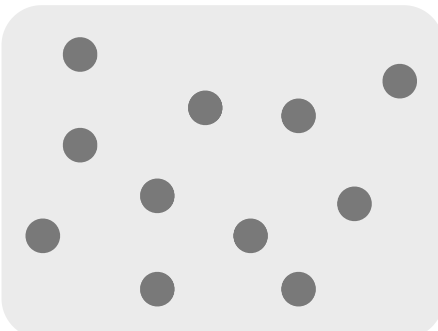

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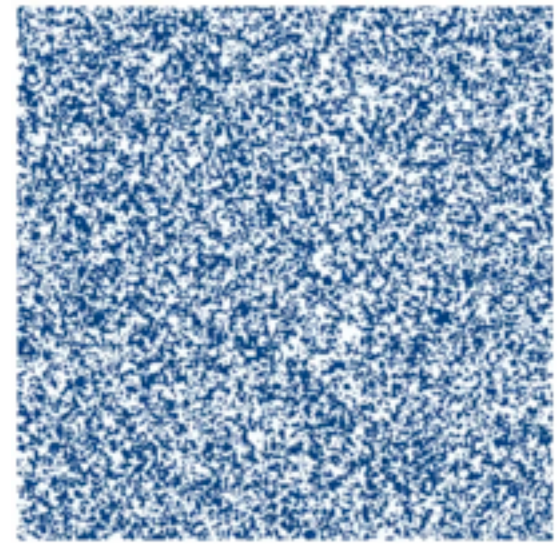

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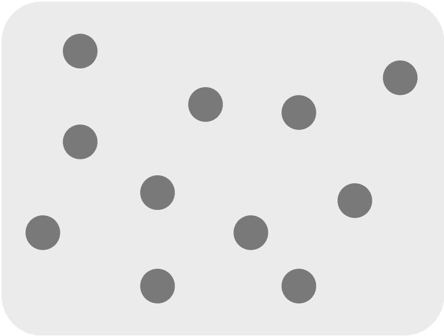

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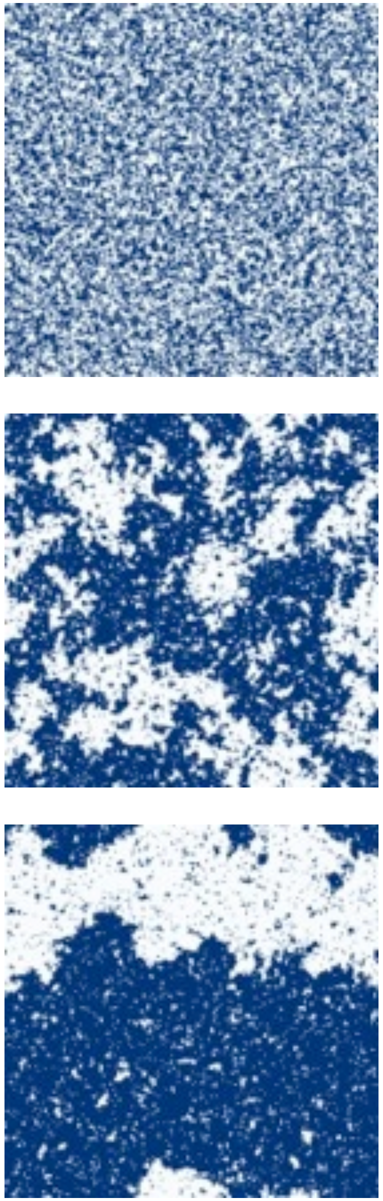
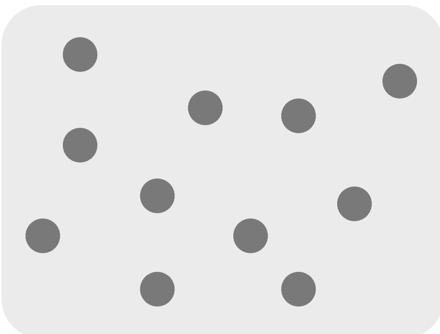


## Spontaneous symmetry breaking

- ground state has **less** symmetry than Hamiltonian
- **local** order parameter
- **phase transition** / Landau-Ginzburg-Wilson theory

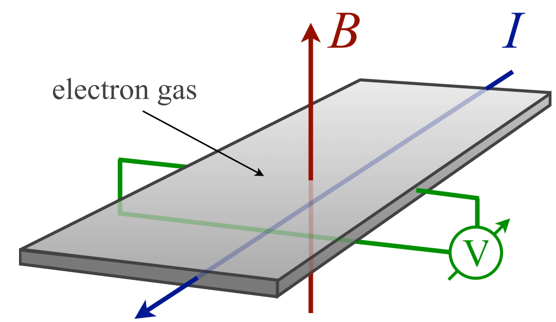
# Every rule has an exception



interacting
**many-body system**

$$\mathcal{H} = \sum_{j=1}^{N} \left( \frac{1}{2m} \left( \mathbf{p}_j - \frac{e}{c} \mathbf{A}(\mathbf{x}_j) \right)^2 + e\mathbf{A}_0(\mathbf{x}_j) \right) + \sum_{i<j} V(|\mathbf{x}_i - \mathbf{x}_j|)$$

$$\mathrm{tr}\left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$



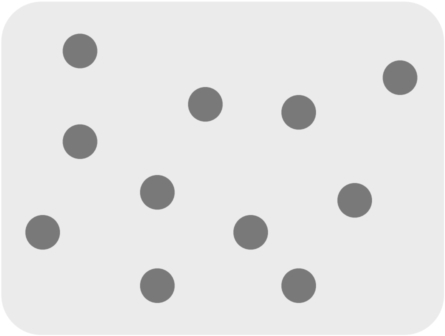


## Sometimes, the exact opposite happens

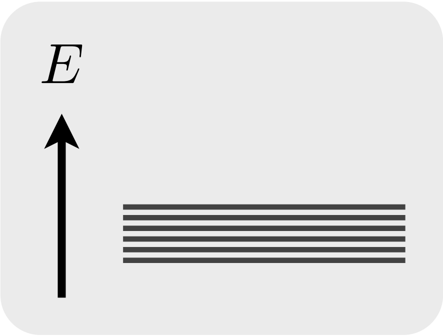
- ground state has **more** symmetry than Hamiltonian
- **non-local** order parameter
- **emergence** of degenerate ground states, exotic statistics, ...

# When does this happen?

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of 'accidental' degeneracies.



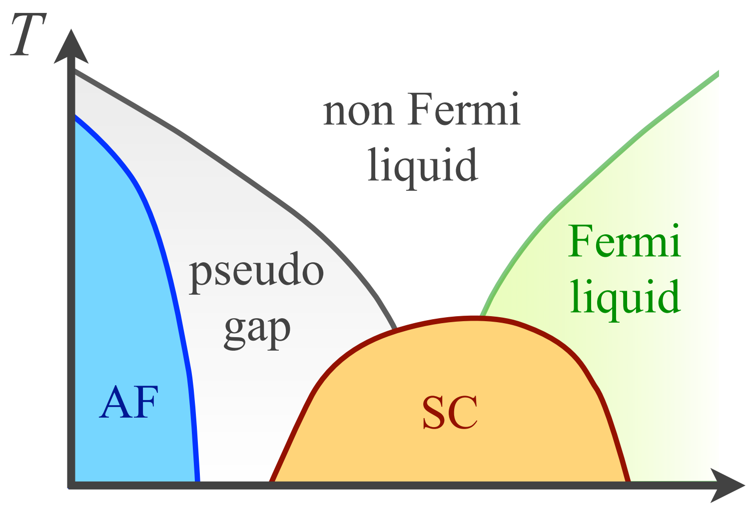
interacting
**many-body system**




'accidental'
**degeneracy**

**residual effects**
select ground state

phase diagram of
cuprate superconductors



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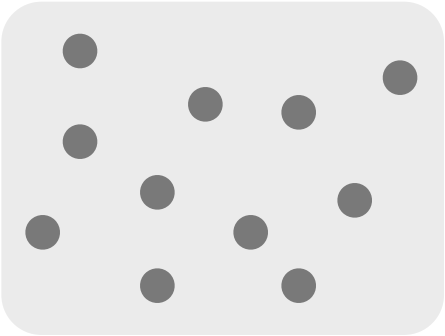

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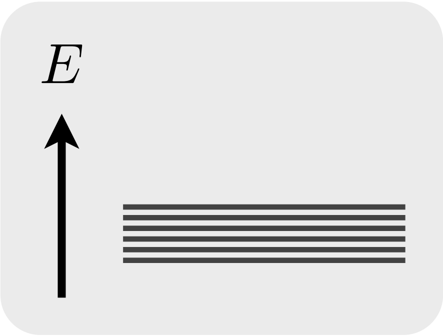



‘accidental’
**degeneracy**

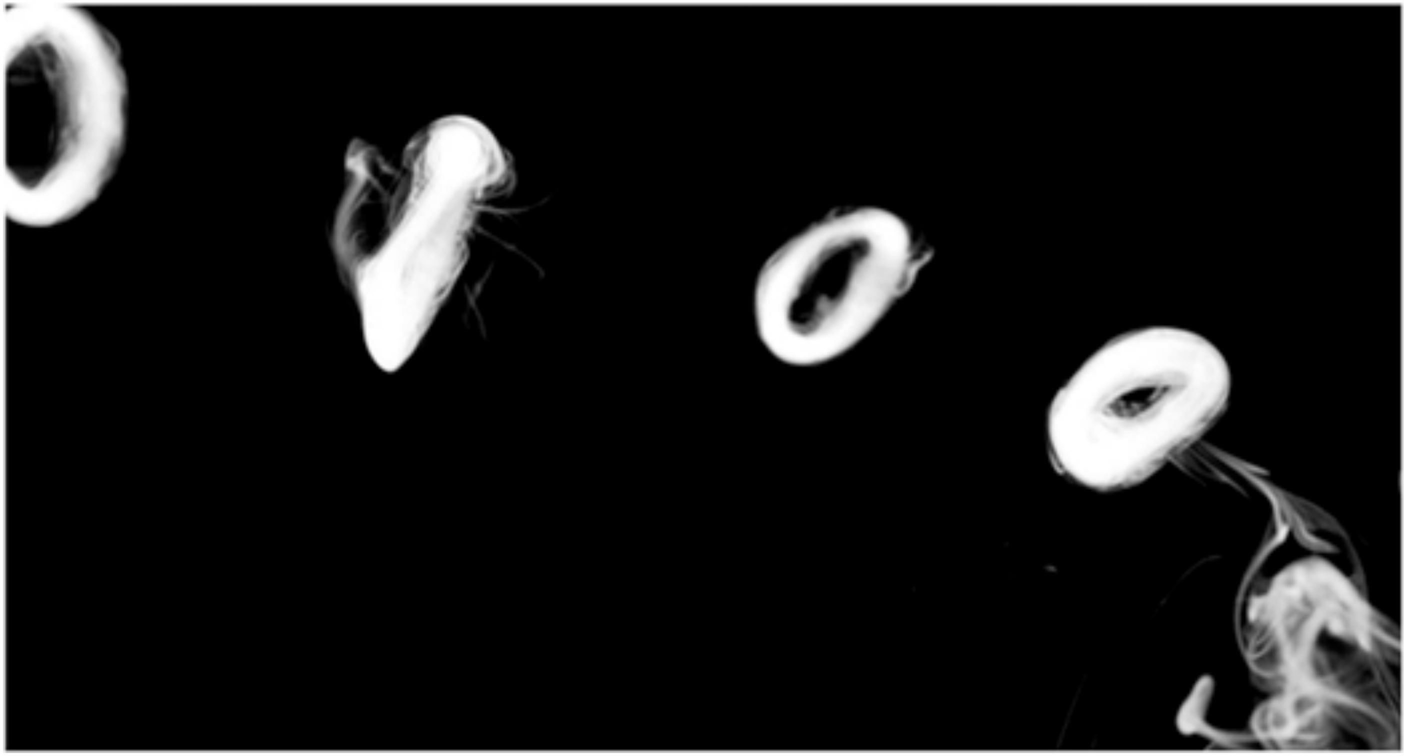
**residual effects**
select ground state

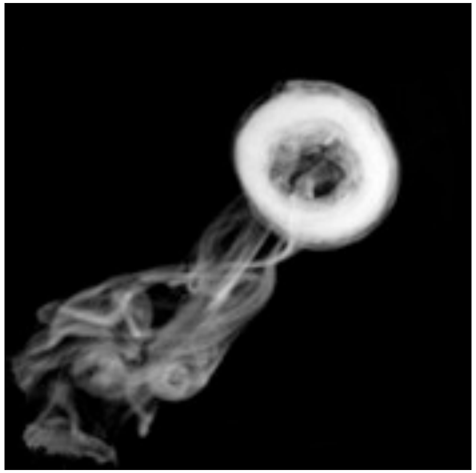
But they are also notoriously difficult to handle, due to

- multiple energy scales
- complex energy landscapes / slow equilibration
- strong coupling

# Topologische Quanten-Materie

# Topological quantum matter



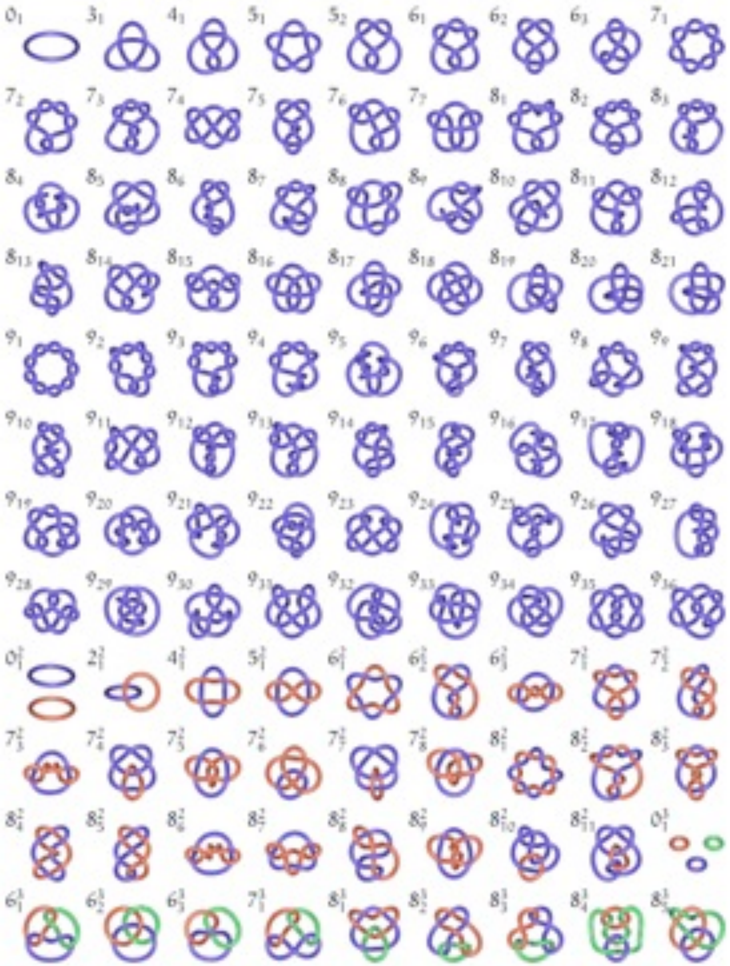


# Topological quantum matter

- 1867: **Lord Kelvin**
  atoms = knotted tubes of ether
  Knots might explain stability, variety, vibrations, ...

- **Maxwell**: “It satisfies more of the conditions than any atom hitherto imagined.”

- This inspires the mathematician **Tait** to classify knots.

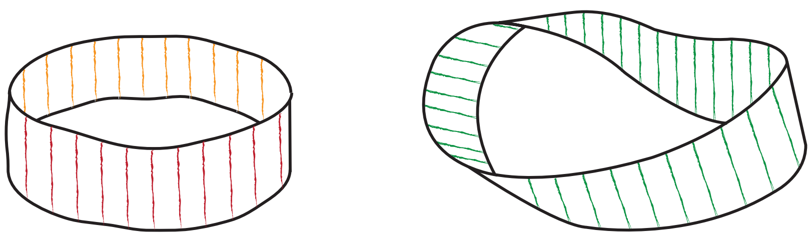


knot classification table
table of the elements ?

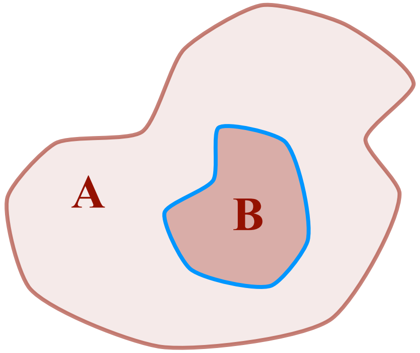
# Topological quantum matter

- **A first attempt:** A ground state of a many-body system that *cannot* be fully characterized by a *local* order parameter.

- Often characterized by a variety of non-local "*topological properties*".



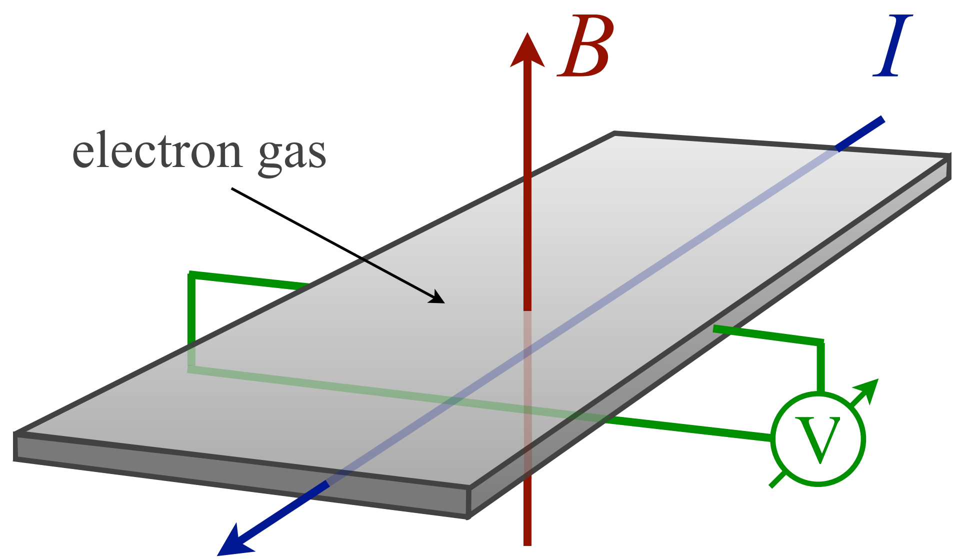
- A topological phase can be positively identified by its *entanglement properties*.

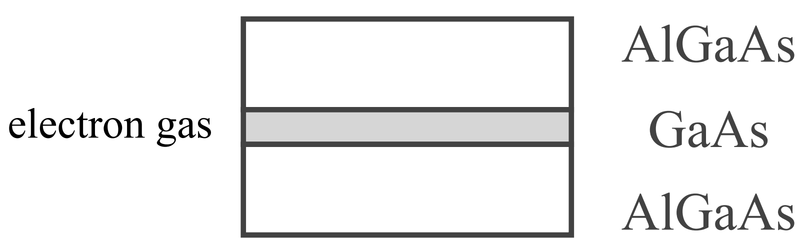


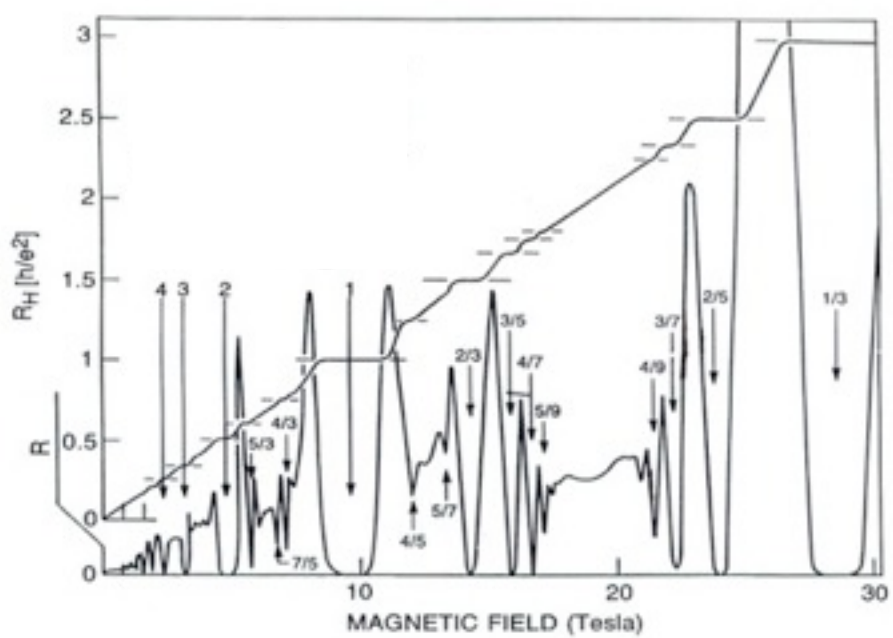
# Quantum Hall effect

**Quantization of Hall conductivity**
for a two-dimensional electron gas
at very low temperatures
in a high magnetic field.

$$\sigma = \nu \frac{e^2}{h}$$



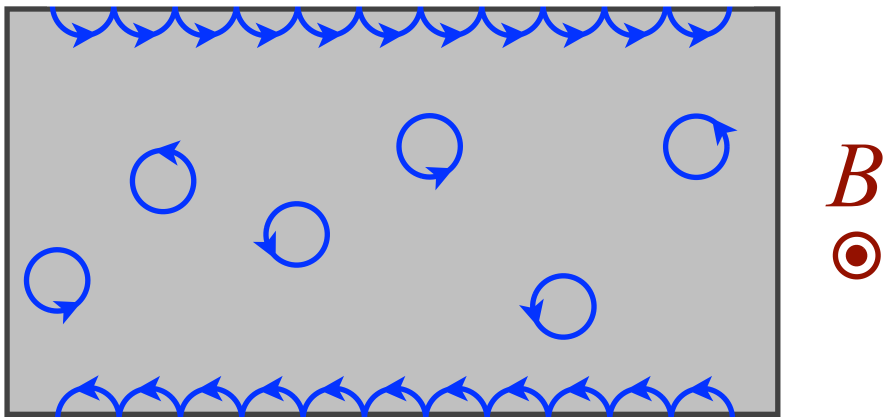




Semiconductor heterostructure confines
electron gas to two spatial dimensions.

# Quantum Hall states

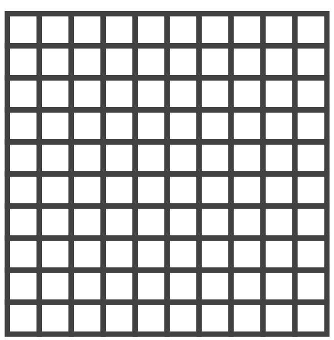
**Landau levels**

$$E_n = h\frac{eB}{m}\left(n+\frac{1}{2}\right)$$




edge states

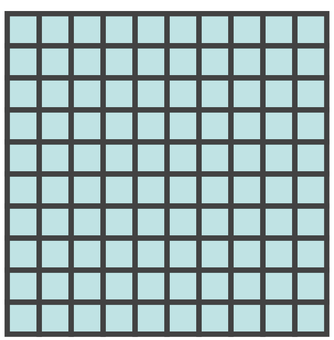
Landau level **degeneracy**



$$2\Phi/\Phi_0$$

orbital states

**integer** quantum Hall

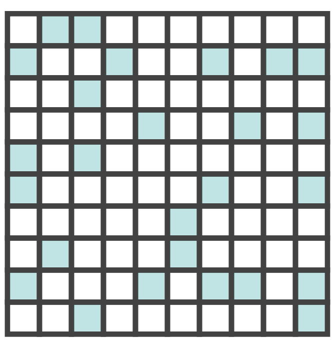


**filled** level

↓

**incompressible** liquid

**fractional** quantum Hall



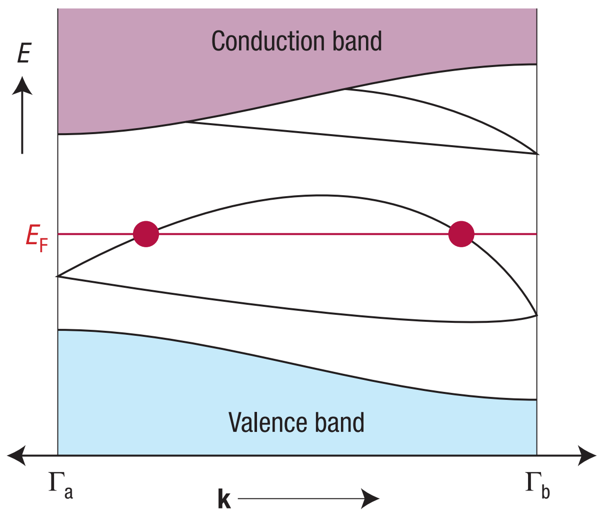
**partially filled** level

↓ Coulomb repulsion

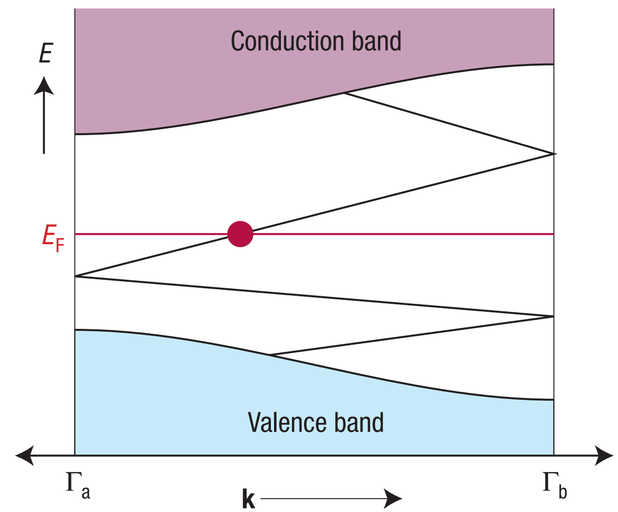
**incompressible** liquid

# Topological insulators

Spin-orbit driven band inversion of a conventional band insulator.

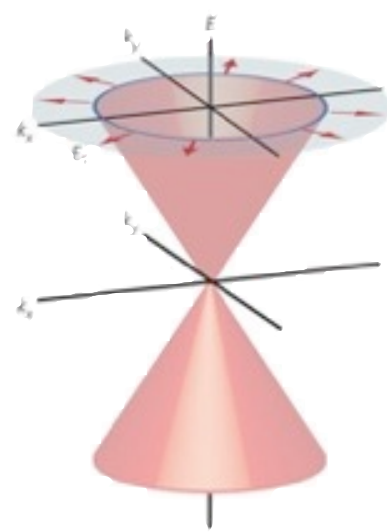


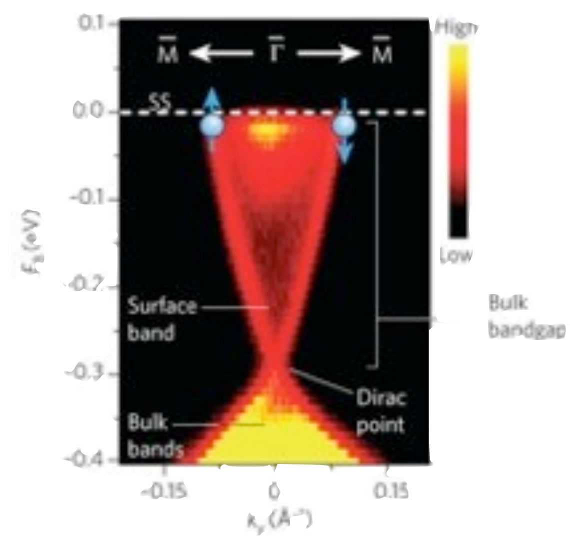

conventional band insulator




topological band insulator

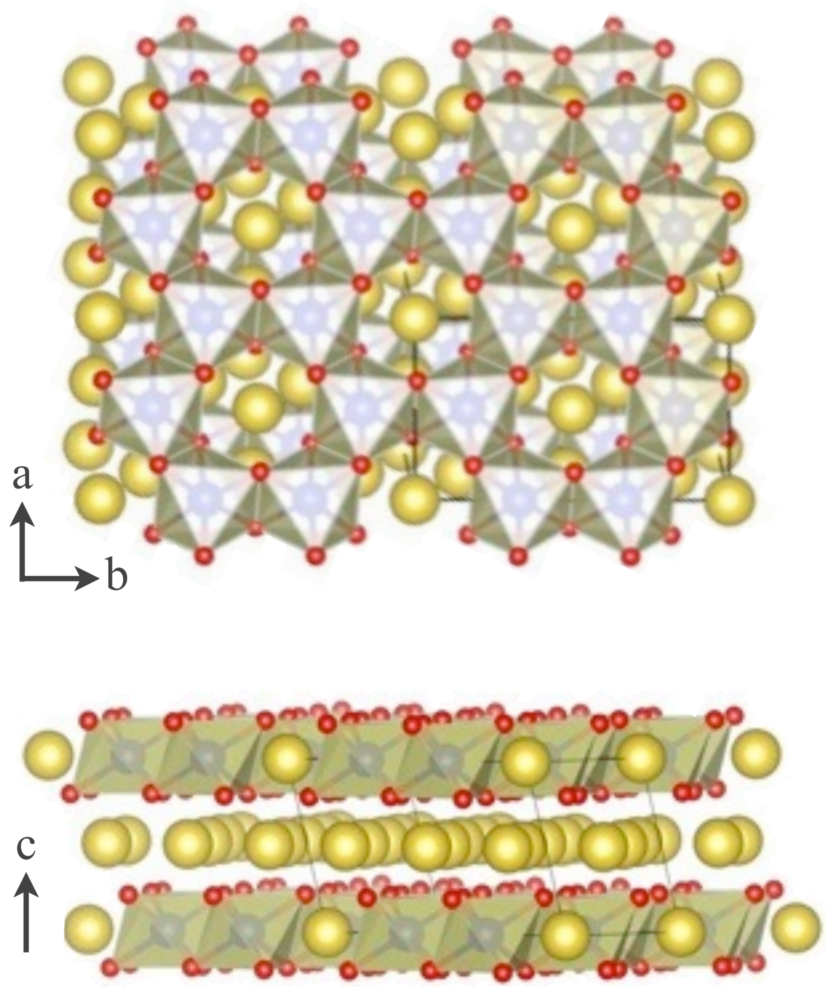
$Bi_2Se_3$

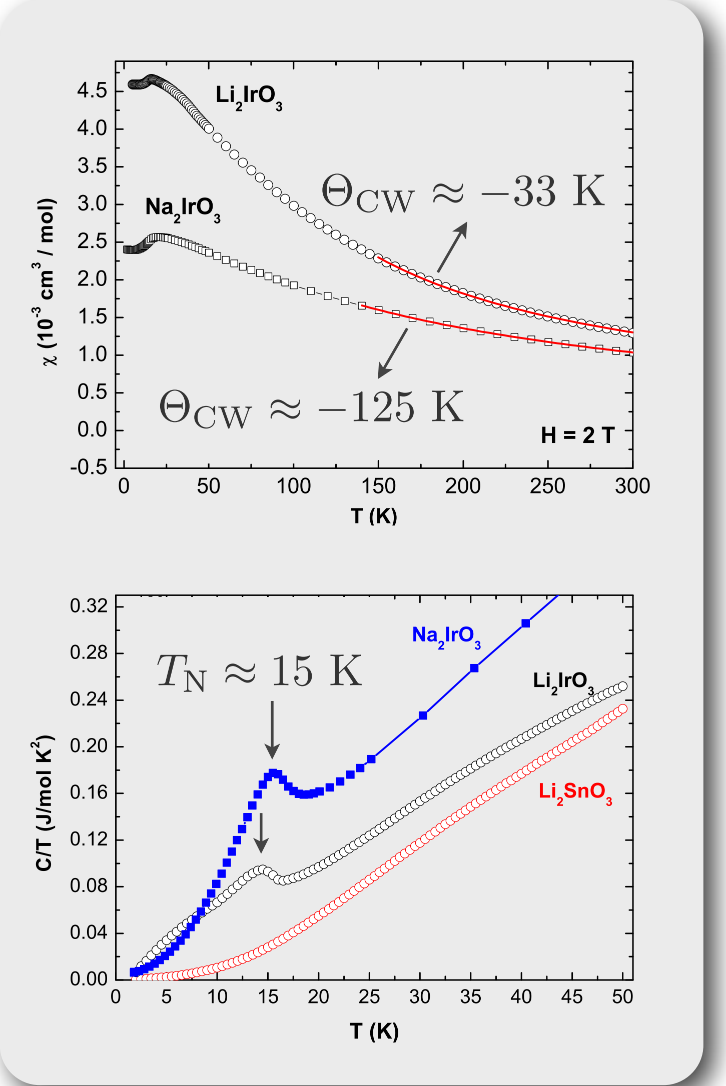




# Spin-orbit assisted Mott physics in Iridates

## $(Na,Li)_2IrO_3$



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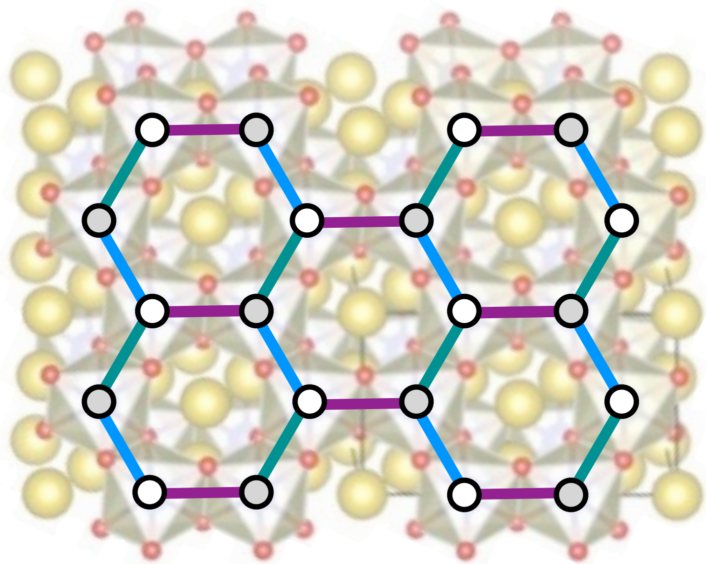
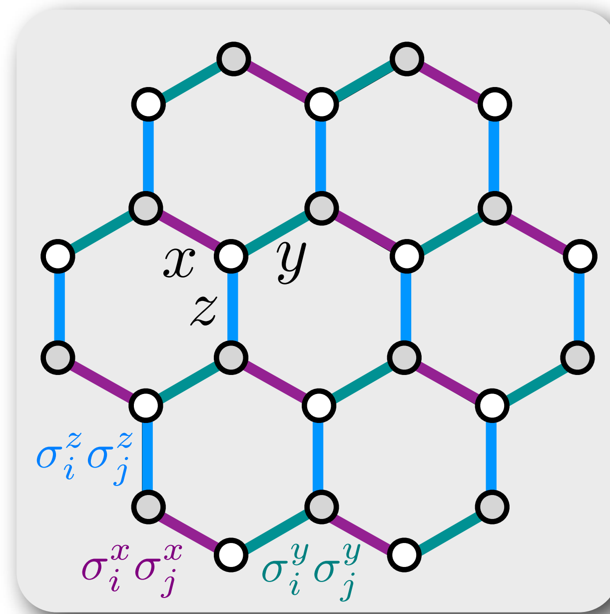
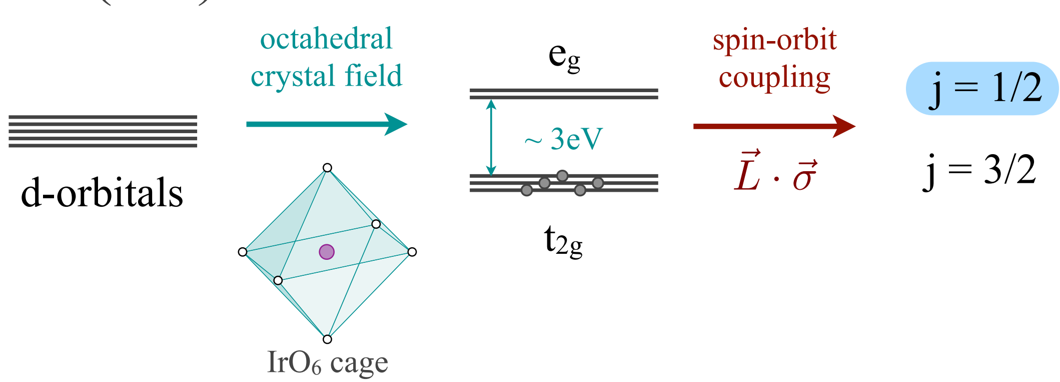





$Ir^{4+}$ $(5d^5)$




$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma \sigma_i^\gamma \sigma_j^\gamma$$

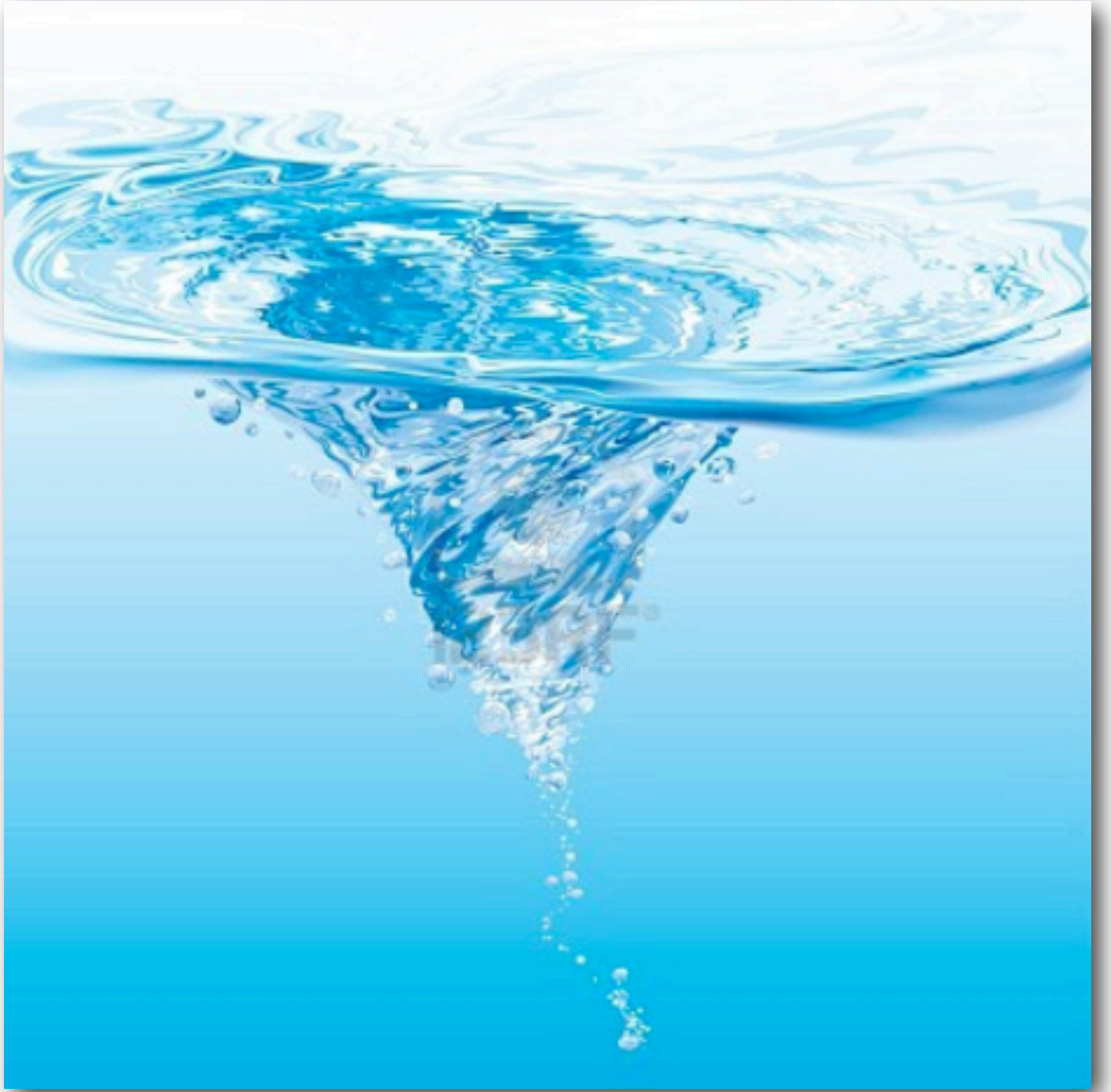
Rare combination of a model of **fundamental conceptual importance** (harboring topological phases) and an **exact analytical solution**.

Was haben

# Knoten, Quanten & Computer

miteinander zu tun?

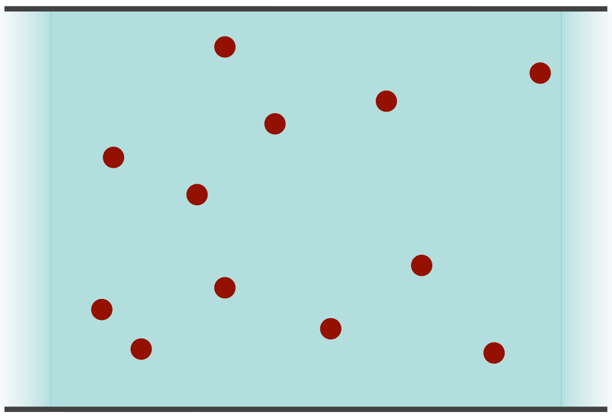
# Vortices



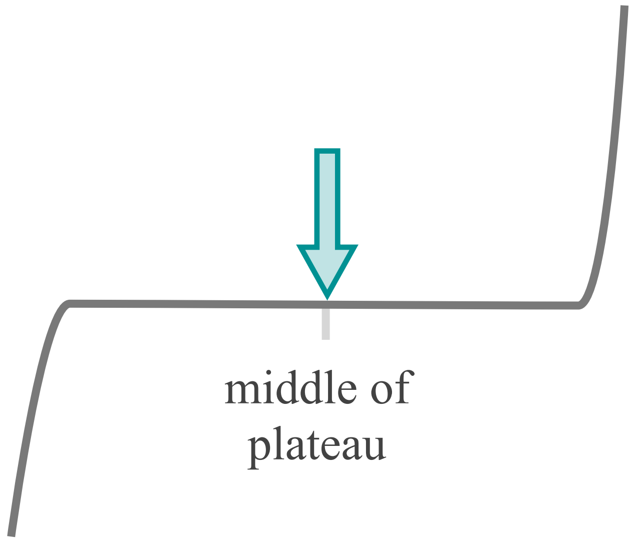
# Vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

Abelian



**single** state



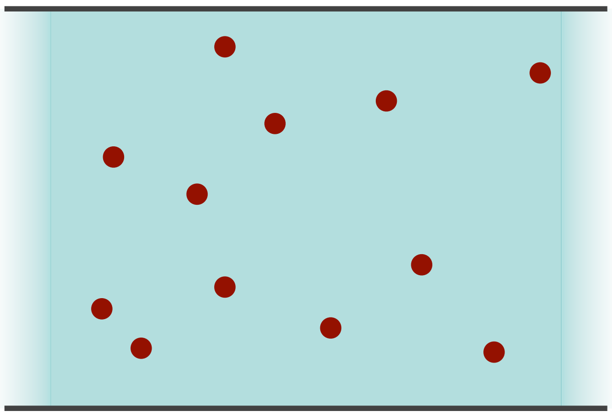

Example:

Laughlin-wavefunction + quasiholes

# Abelian vs. non-Abelian vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

## Abelian



**single** state

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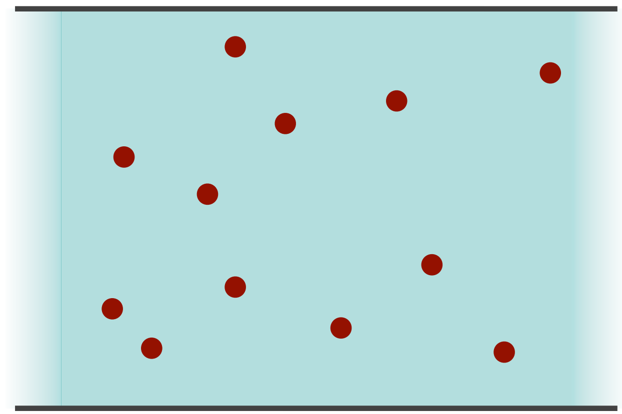
## non-Abelian



(degenerate) **manifold** of states

Manifold of states grows **exponentially** with the number of vortices.

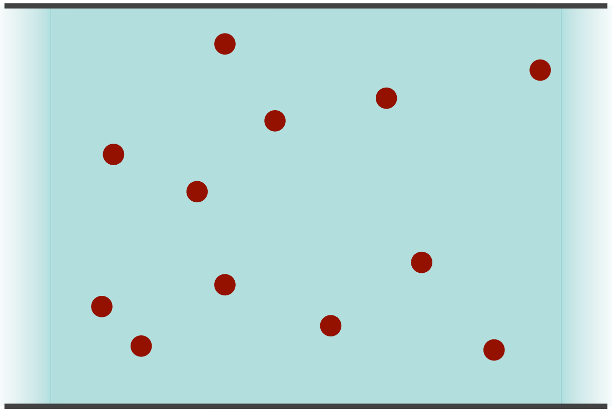
Ising anyons (Majorana fermions) $\sqrt{2}^N$

Fibonacci anyons $\phi^N$

# Abelian vs. non-Abelian vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

## Abelian

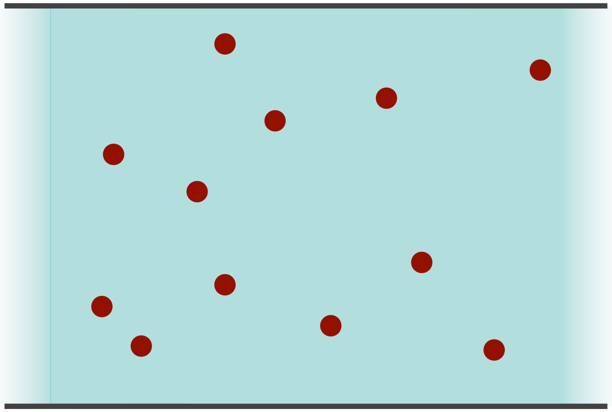


**single** state

$$\psi(x_2, x_1) = e^{i\pi\theta} \cdot \psi(x_1, x_2)$$

fractional phase

## non-Abelian



(degenerate) **manifold** of states

matrix

$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

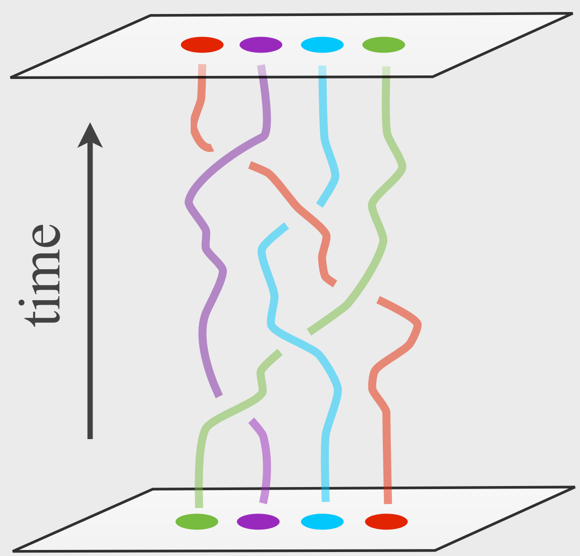
In general *M* and *N* do not commute!

# Topological quantum computing

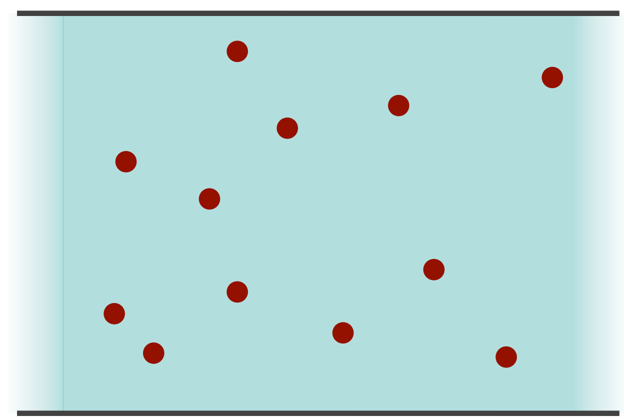
### Topological quantum computing

Degenerate manifold = qubit

Employ **braiding** of non-Abelian vortices to perform computing (unitary transformations).




non-Abelian



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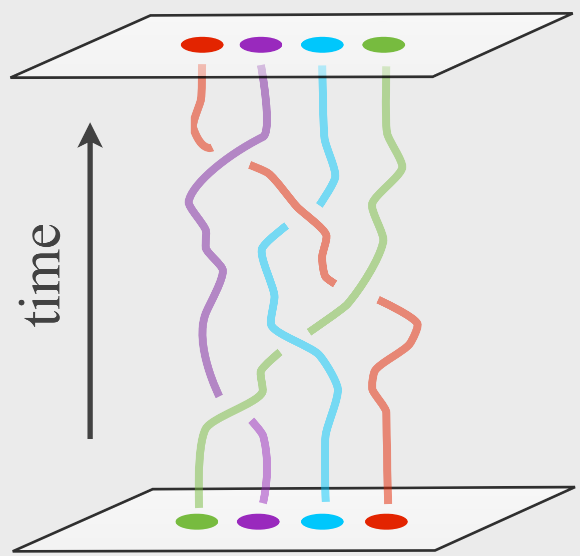



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# Will this work?

Answer depends a lot on whom you ask ...

Xiao-Gang Wen

MIT/Perimeter

Alexei Kitaev

Caltech

Mike Freedman

Microsoft Station Q

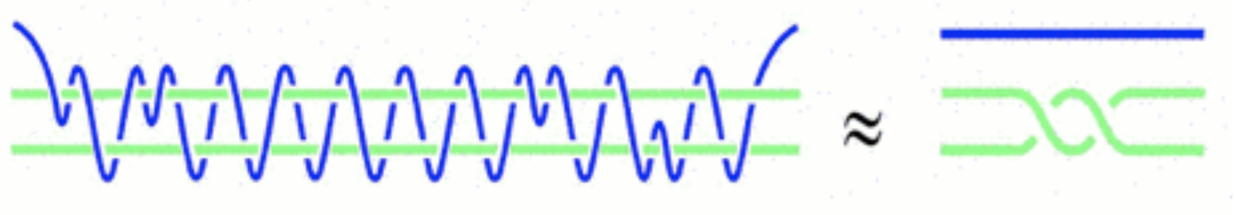
Absolutely yes!

# How will this work?

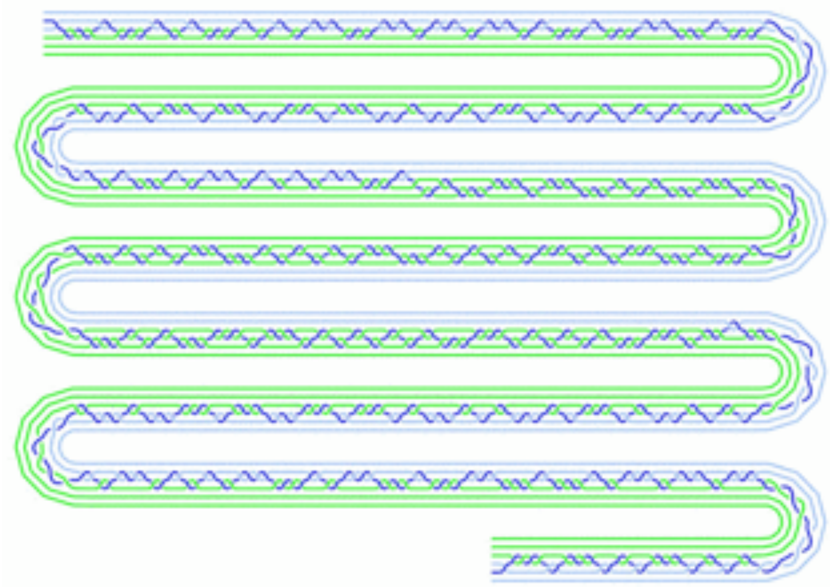
Nick Bonesteel

Florida State

Controlled – **"Knot" Gate**



A **quantum circuit**



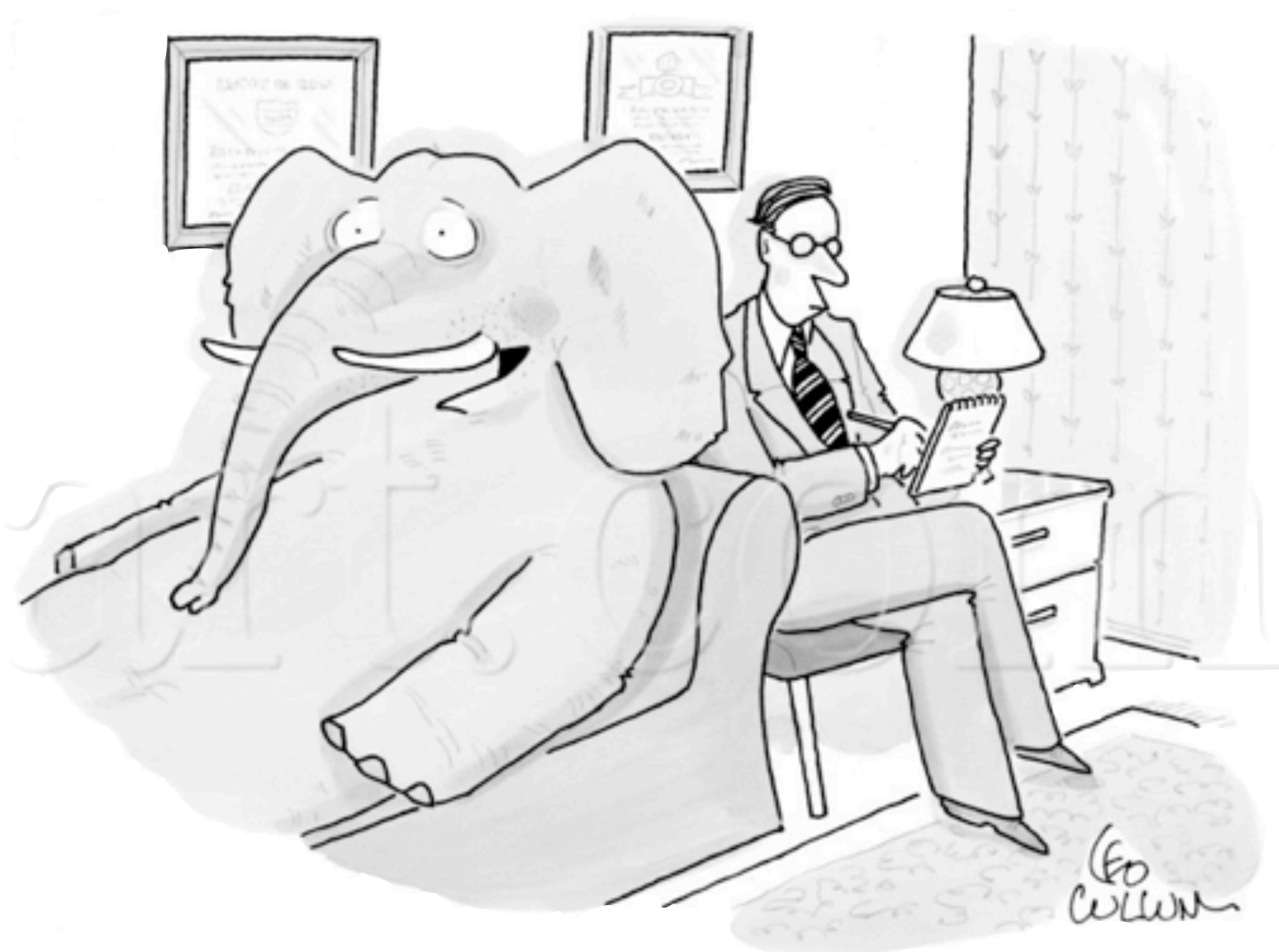
Welche

# Quanten-Sprünge

müssen noch gemacht werden?

# The elephant in the room

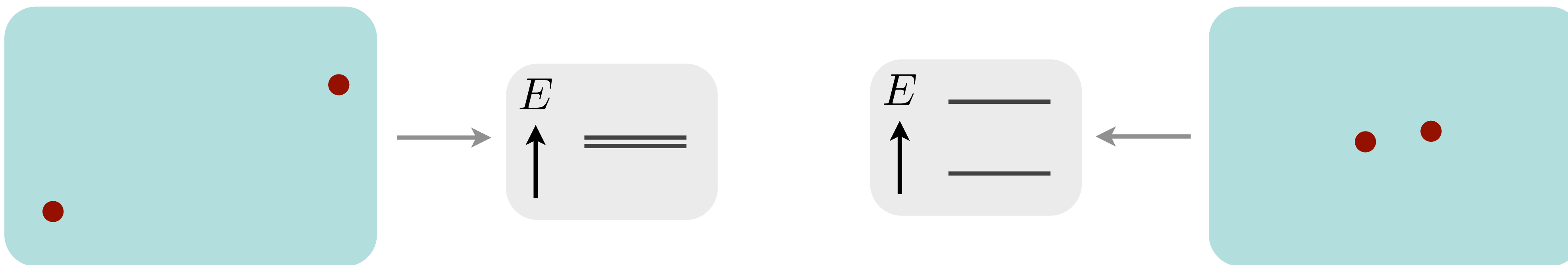
Experimental proof of non-Abelian vortex statistics still open...

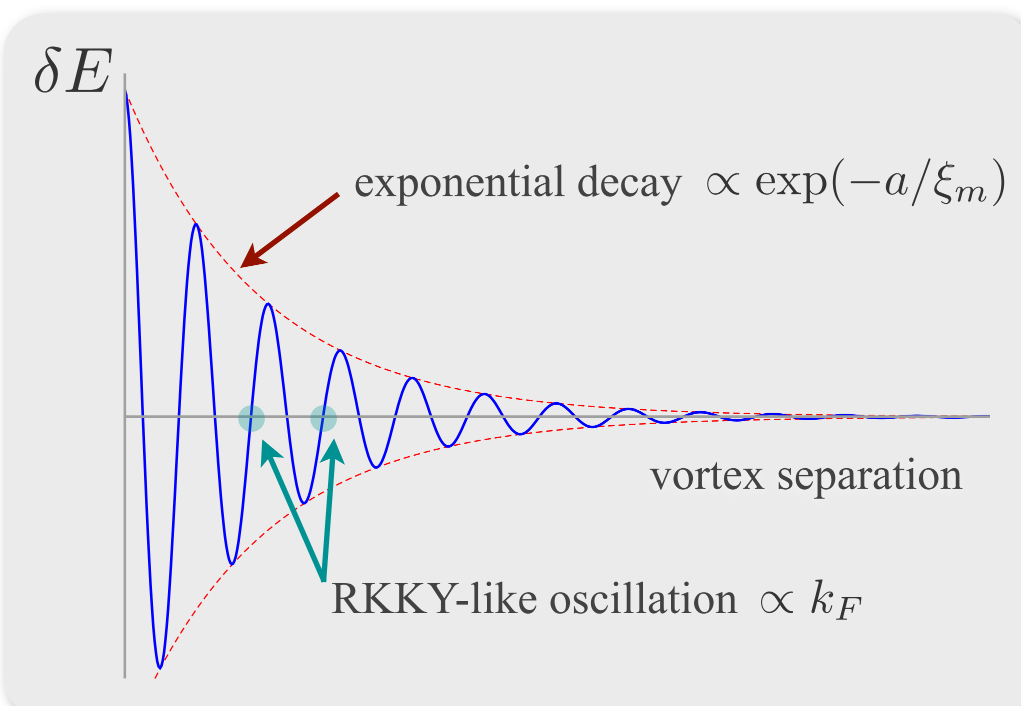


*"I'm right there in the room, and no one even acknowledges me."*

# Vortex-vortex interactions

$a \gg \xi_m$ $a \approx \xi_m$



# Vortex-vortex interactions

**Vortex quantum numbers**

SU(2)$_k$ = ‘deformation’ of SU(2)

with finite set of representations

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \frac{k}{2}$$

fusion rules

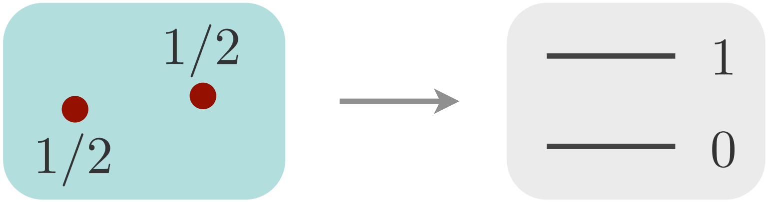
$$j_1 \times j_2 = |j_1 - j_2| + (|j_1 - j_2| + 1) + \ldots + \min(j_1 + j_2, k - j_1 - j_2)$$

example k = 2

$$1/2 \times 1/2 = 0 + 1$$

**Vortex pair**

$$1/2 \times 1/2 = 0 + 1$$

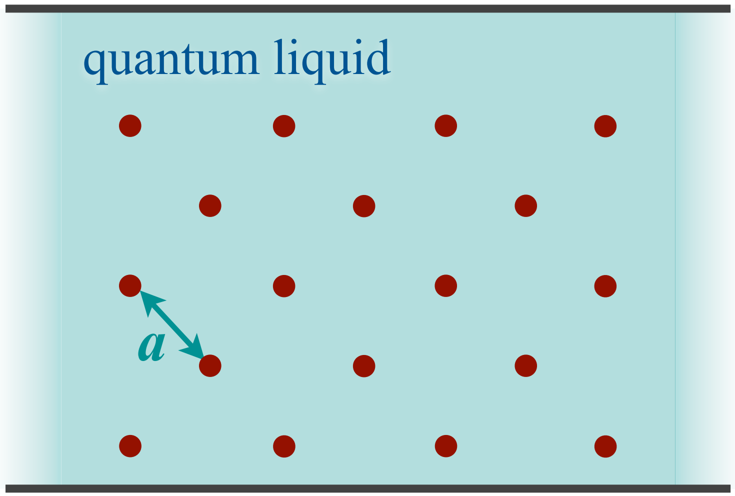



**Energetics for many vortices**

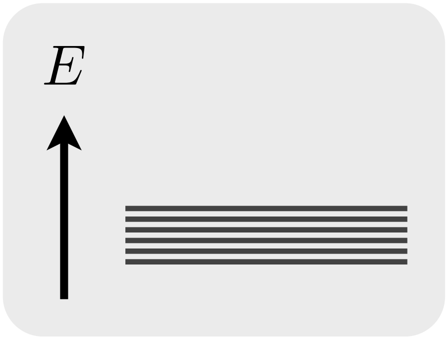
$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

“Heisenberg Hamiltonian”
for vortices

# The many-vortex problem




$$a \gg \xi_m$$

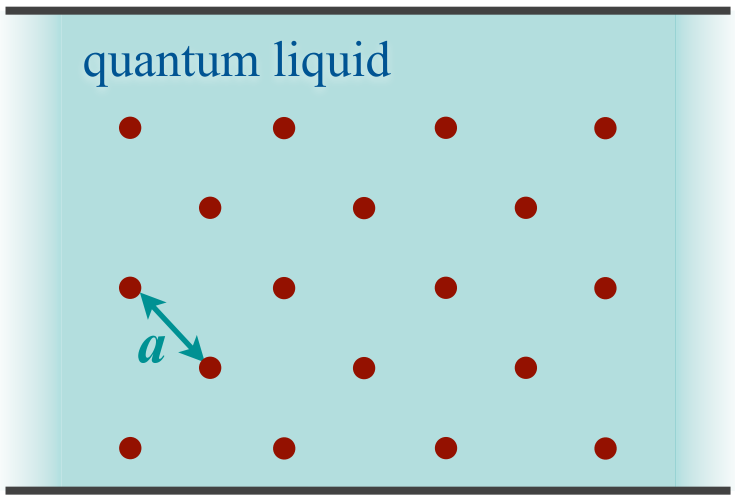


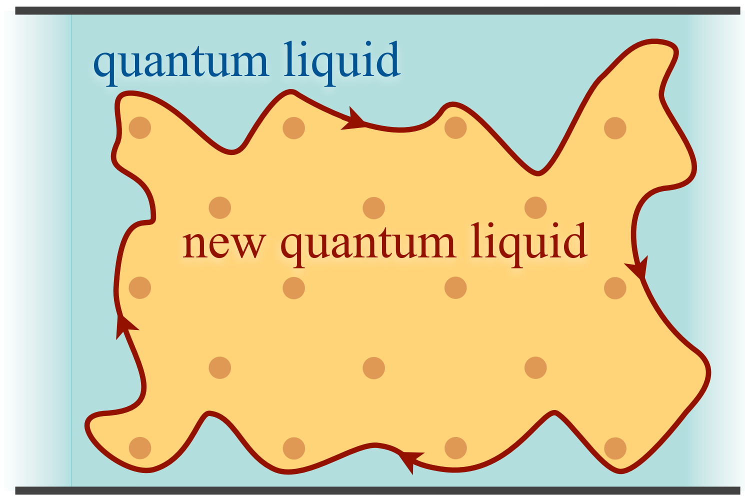

**macroscopic** degeneracy

vortex-vortex interactions

?

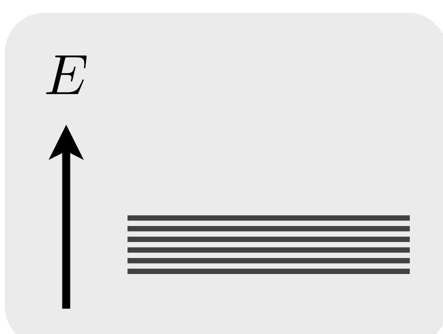
# The many-vortex problem



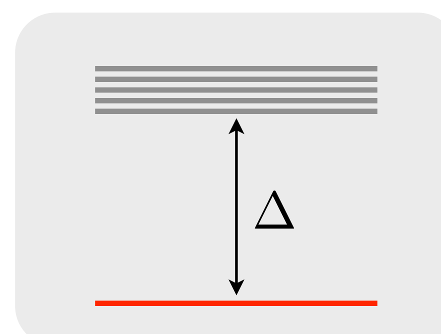




$a \gg \xi_m$

$a \approx \xi_m$



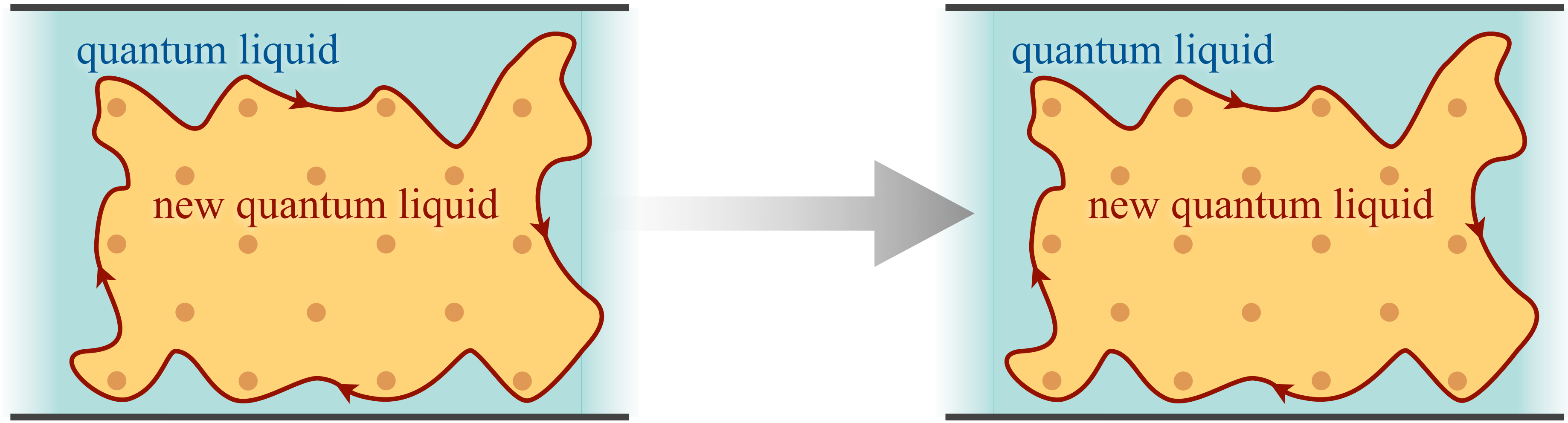

vortex-vortex interactions →

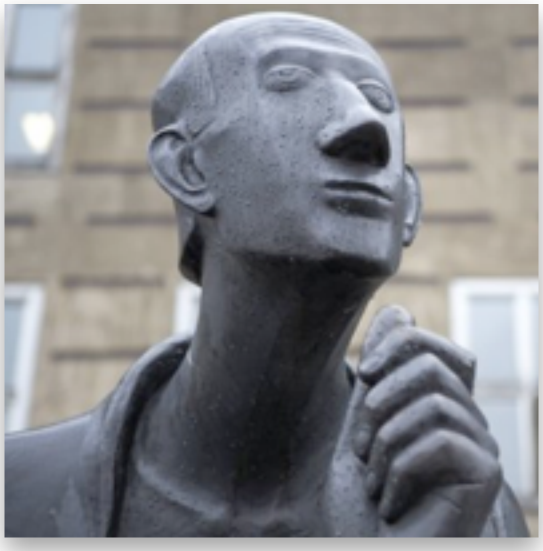



**macroscopic** degeneracy

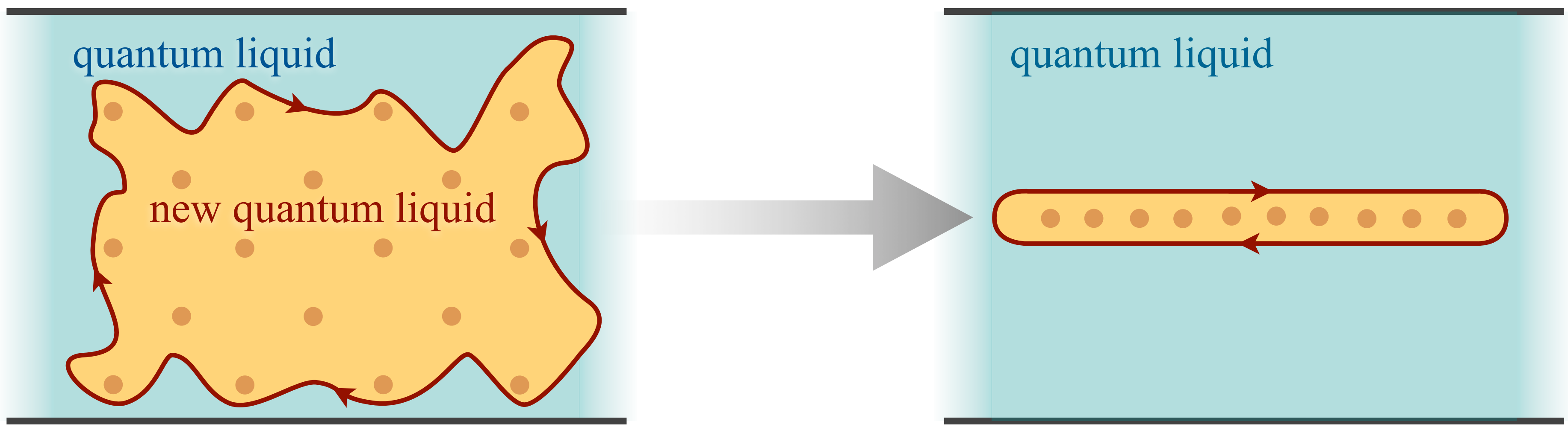
**unique** ground state

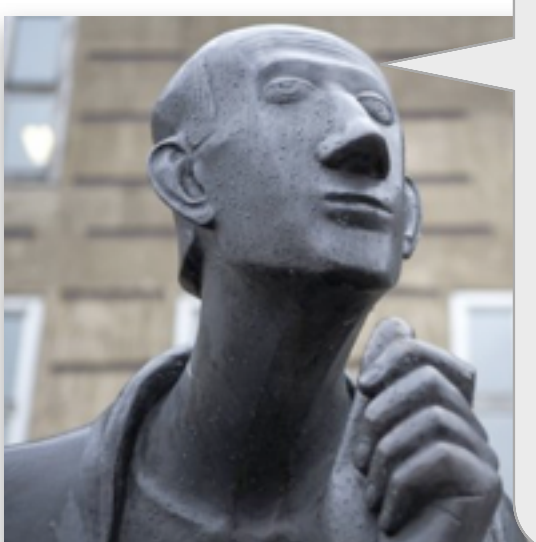
# How do we know this?



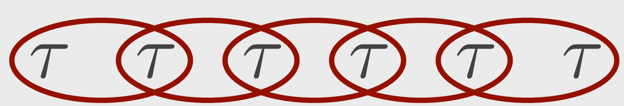



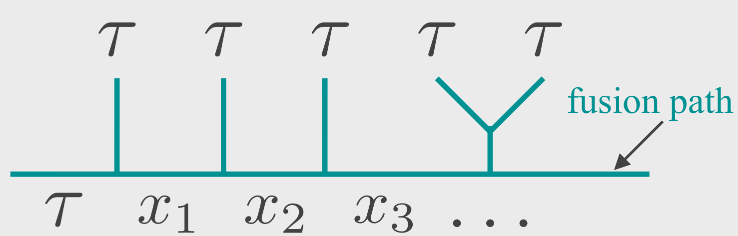
# How do we know this?



## Anyonic Heisenberg chains




## Hilbert space

$$|x_1, x_2, x_3, \ldots\rangle$$

## Hamiltonian

$$H = \sum_i F_i \, \Pi_i^0 \, F_i$$

F-matrix

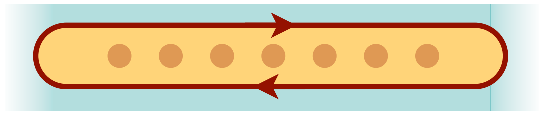
# some number crunching ...

in going from a pair of anyons to the full **many-body** calculation


**CHEOPS**

cluster

9712 Intel Nehalem cores

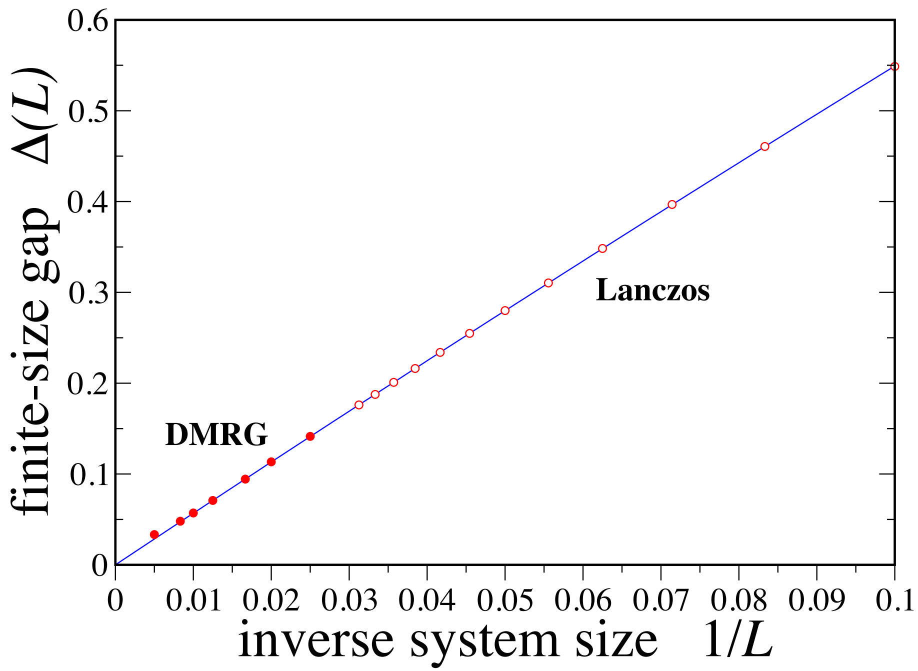


# Edge states

## Finite-size gap

$$\Delta(L) \propto (1/L)^{\boxed{z=1}}$$

conformal field theory description

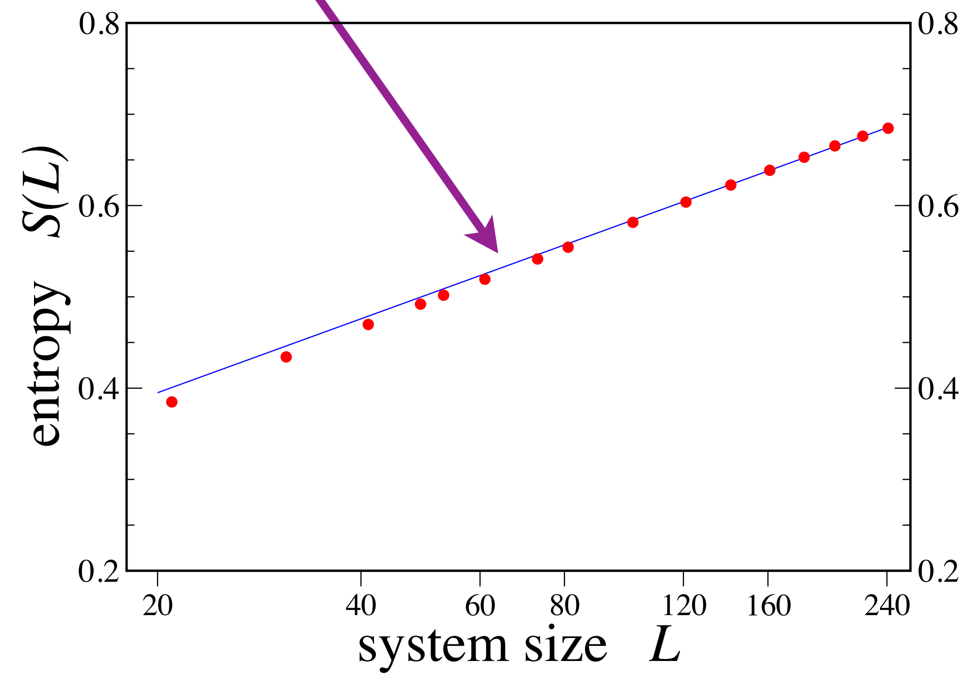



## Entanglement entropy

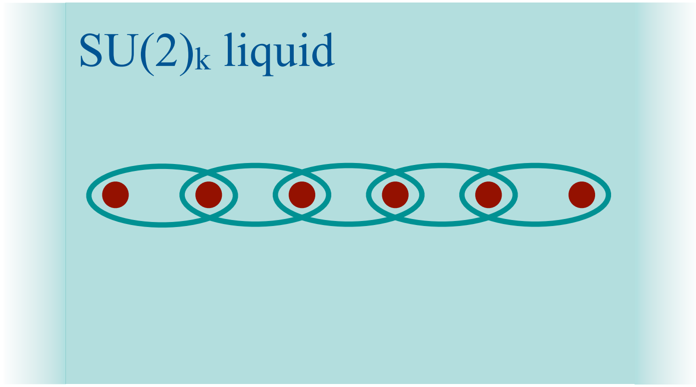
$$S(L) \propto \frac{\boxed{c}}{3} \log L$$

central charge

$c = 7/10$



# Gapless modes & edge states

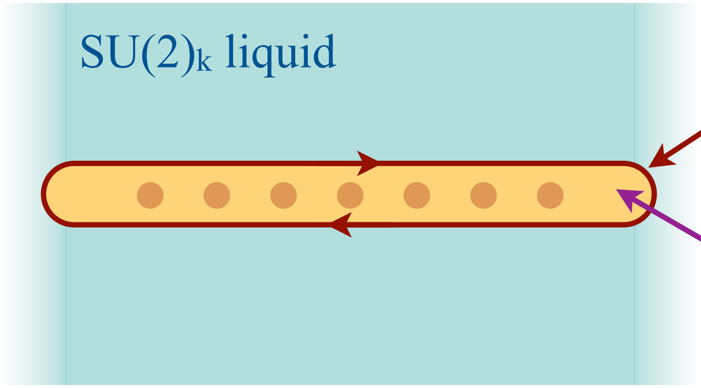



critical theory

$1/2 \times 1/2 \rightarrow \mathbf{0}$

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

interactions

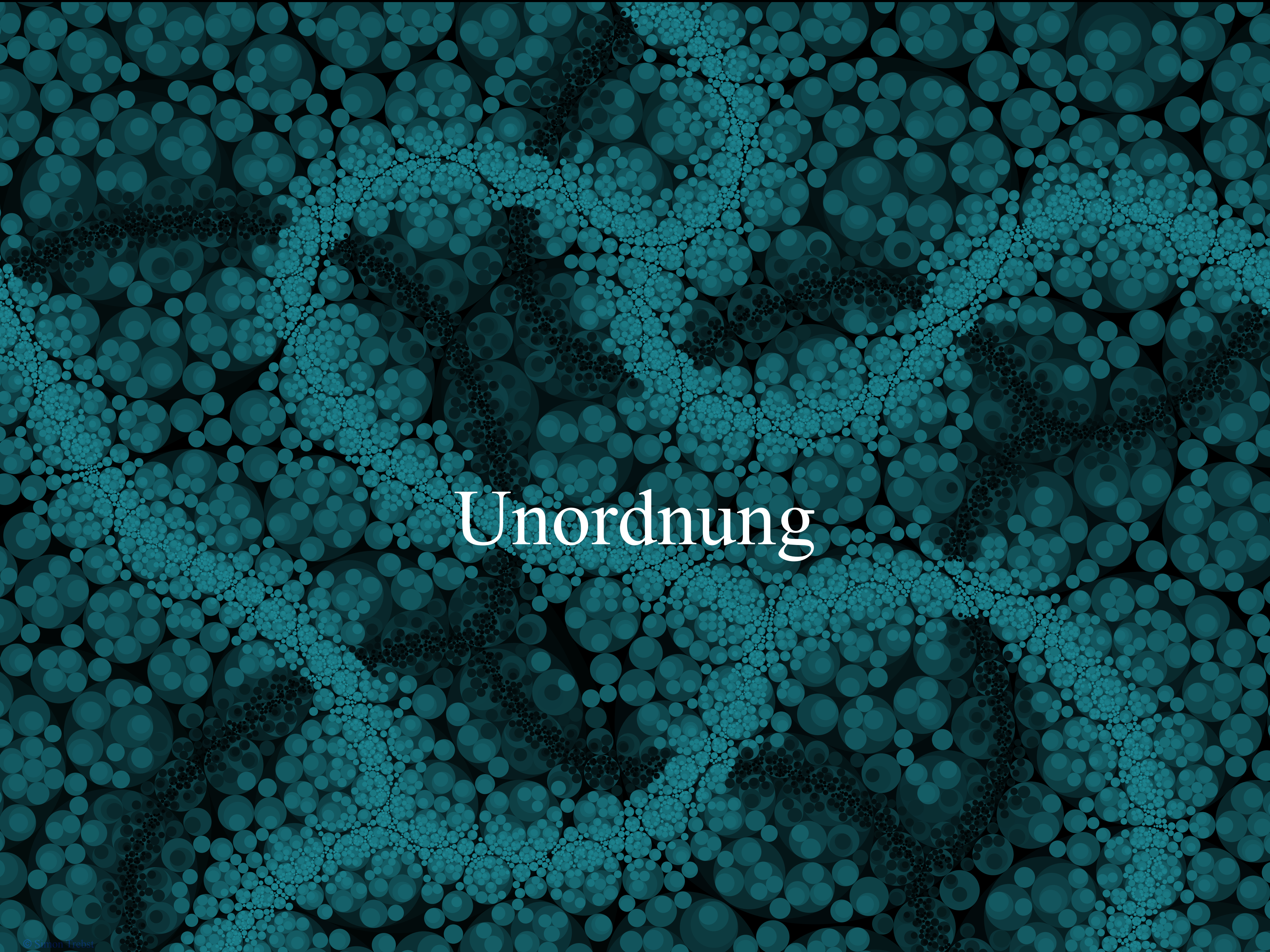



gapless modes = edge states

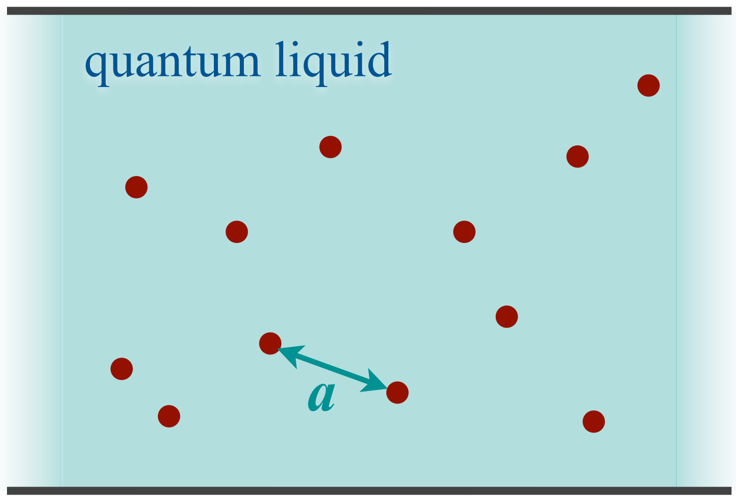
$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

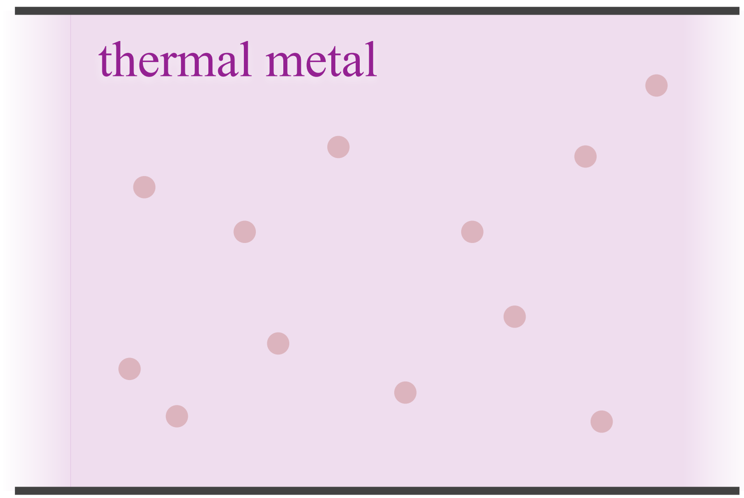
nucleated liquid

$$SU(2)_{k-1} \times SU(2)_1$$

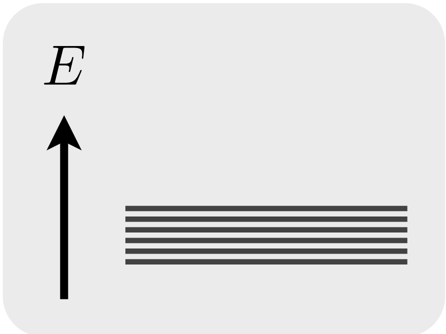

Unordnung

# Disorder induced phase transition




$$a \gg \xi_m$$



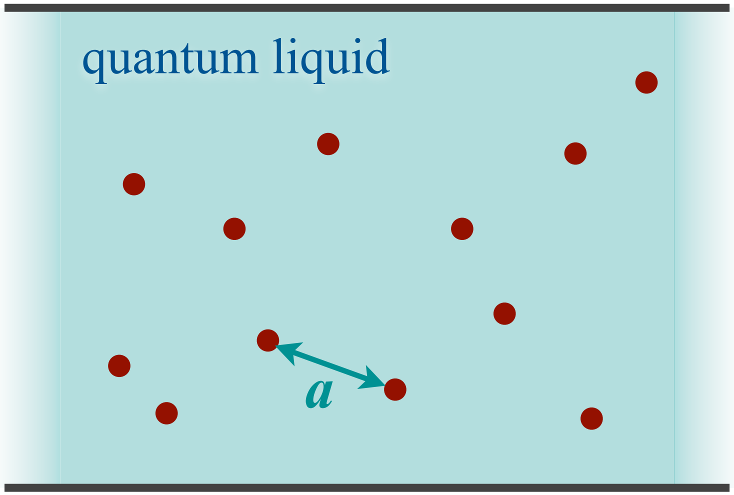

**macroscopic** degeneracy

disorder +
vortex-vortex
interactions

$$a \approx \xi_m$$

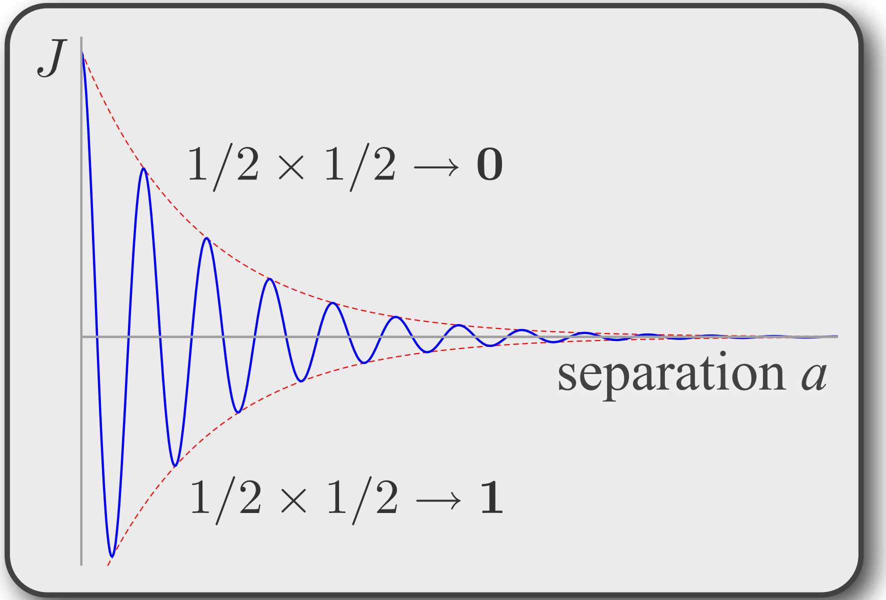
degeneracy is **split**

# Interactions and disorder

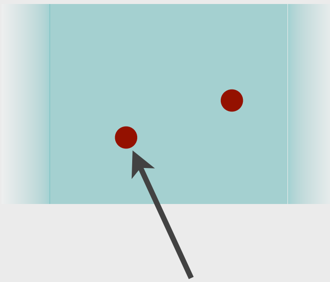



$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk}$$

sign disorder
+ strong amplitude modulation

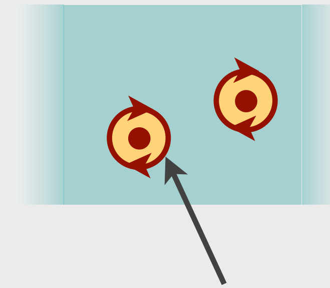


# From Ising anyons to Majorana fermions



Ising anyon

SU(2)$_2$
quantum number



Majorana fermion
zero mode

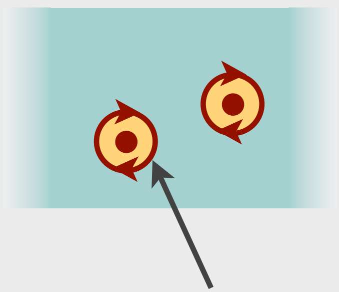
Majorana operator $\gamma_i$

$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk} \longrightarrow \mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

**interacting** Ising anyons
"anyonic Heisenberg model"

**free** Majorana fermion
hopping model

# From Ising anyons to Majorana fermions



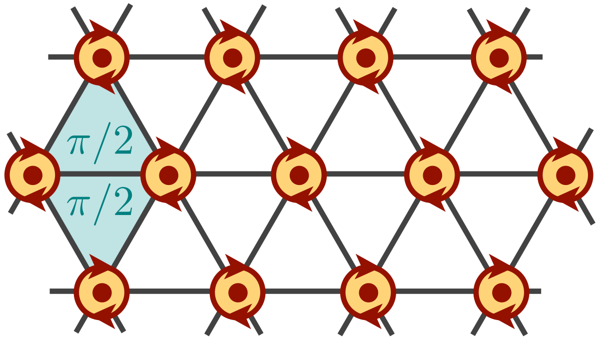
Majorana fermion
zero mode

Majorana operator $\gamma_i$

Ettore Majorana

$$\mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

**free** Majorana fermion
hopping model




particle-hole symmetry ✓ } symmetry class D
time-reversal symmetry ✗

# From Ising anyons to Majorana fermions

PHYSICAL REVIEW B VOLUME 55, NUMBER 2 1 JANUARY 1997-II

**Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures**

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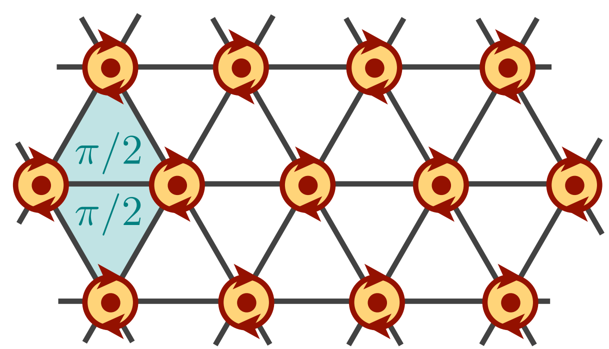
(Received 4 March 1996)

Normal-conducting mesoscopic systems in contact with a superconductor are classified by the symmetry operations of time reversal and rotation of the electron's spin. Four symmetry classes are identified, which correspond to Cartan's symmetric spaces of type $C$, $C$I, $D$, and $D$III. A detailed study is made of the systems

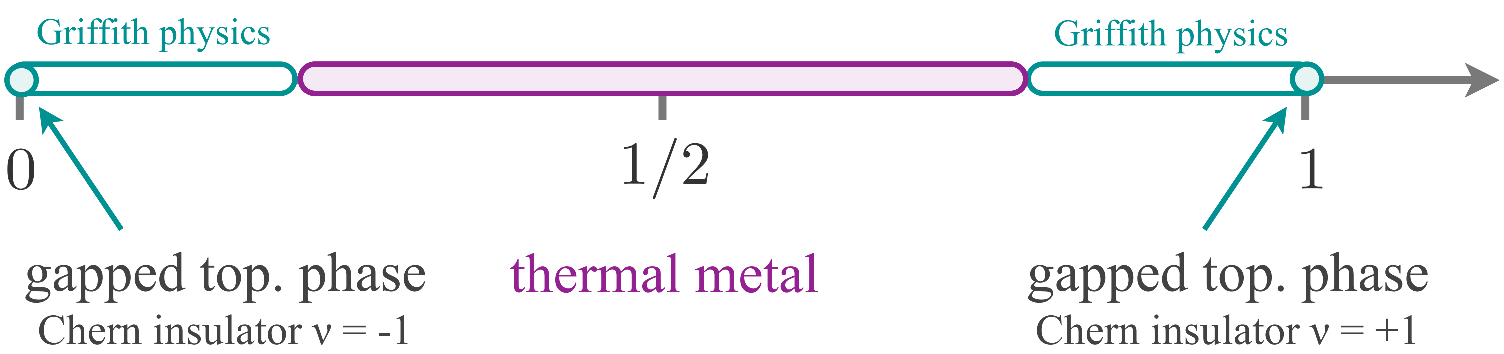
$$\mathcal{H} = -\sum_{\langle jk \rangle} i\mathcal{J}_{jk}\gamma_j\gamma_k$$

**free** Majorana fermion
hopping model



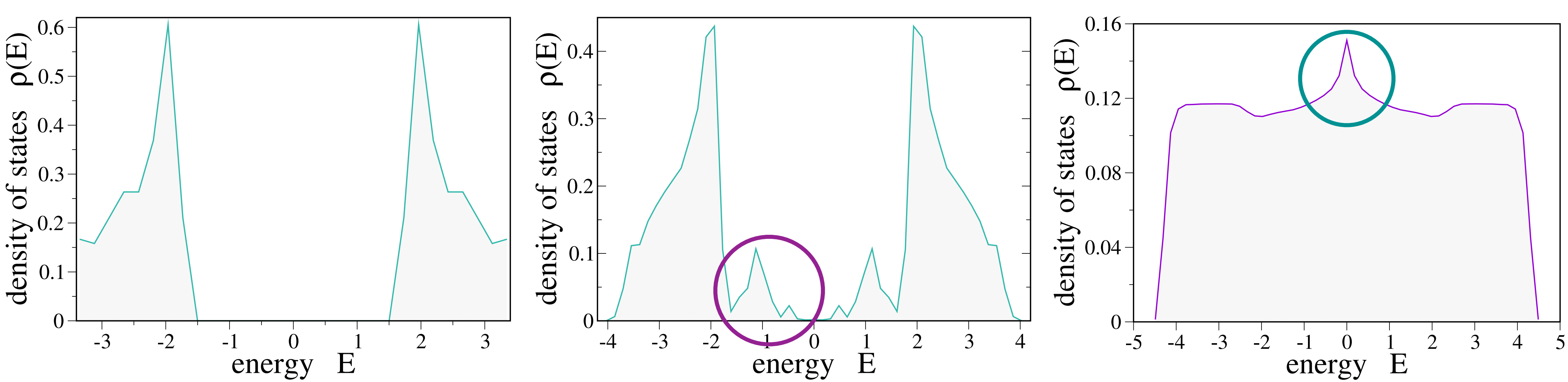

particle-hole symmetry ✓
time-reversal symmetry ✗
} symmetry class D

# A disorder-driven metal-insulator transition



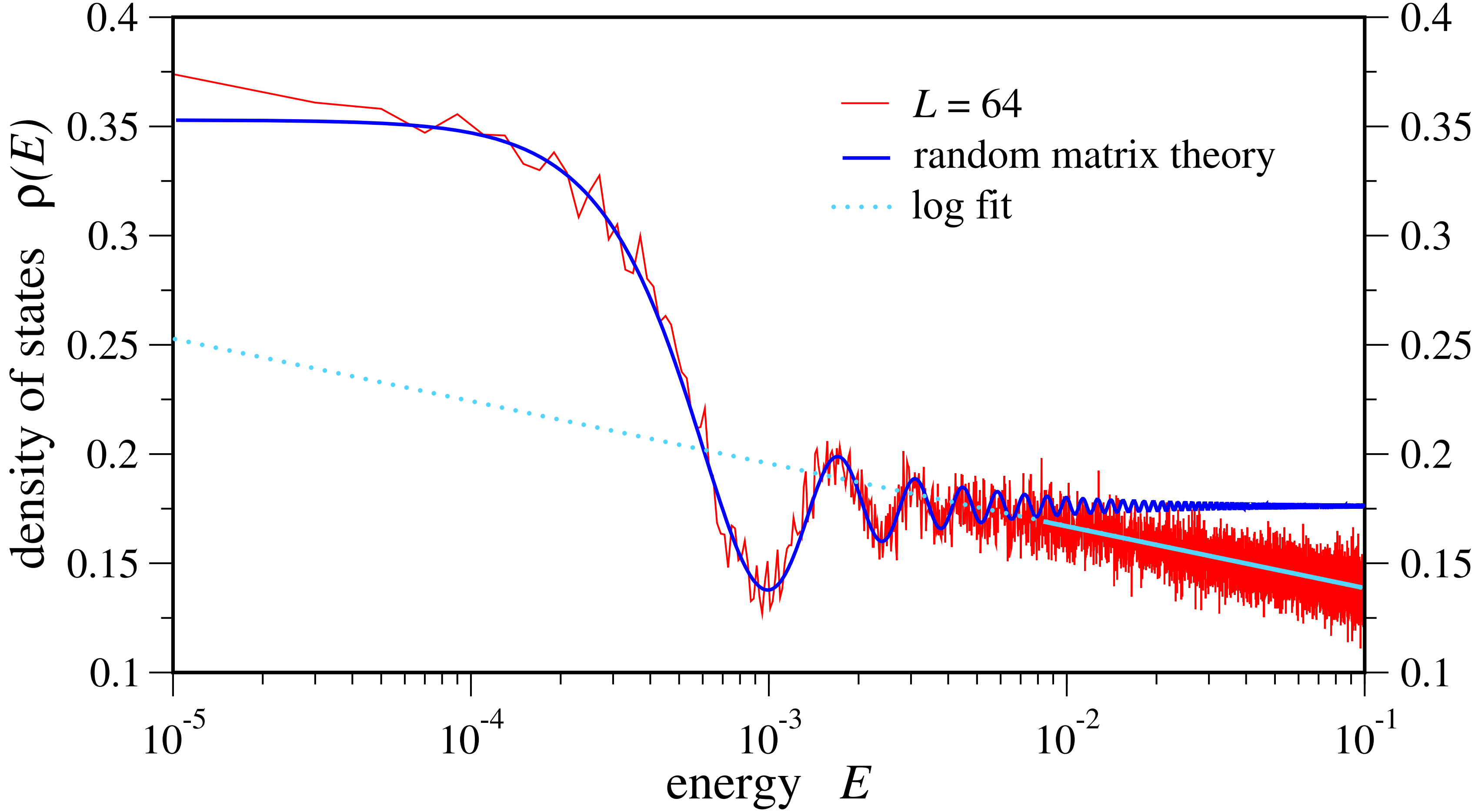

$$J = \begin{cases} +1, & p \\ -1, & 1-p \end{cases}$$

$p = 0$ $\qquad$ $p = 0.02$ $\qquad$ $p = 1/2$




Density of states indicates phase transition.

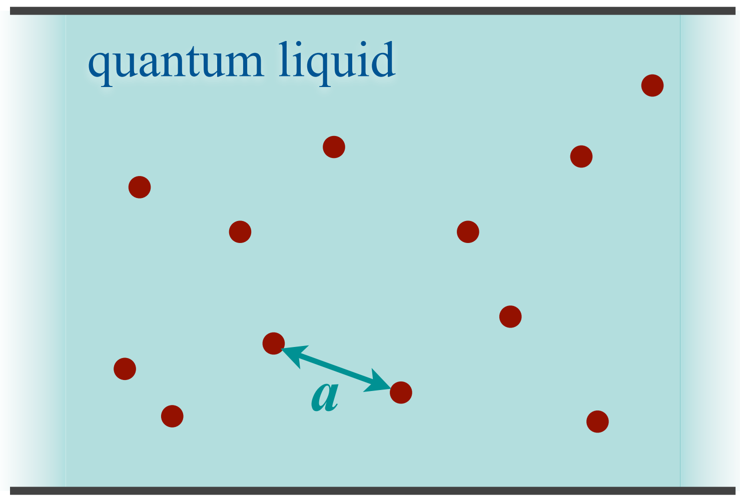
# Taking a closer look

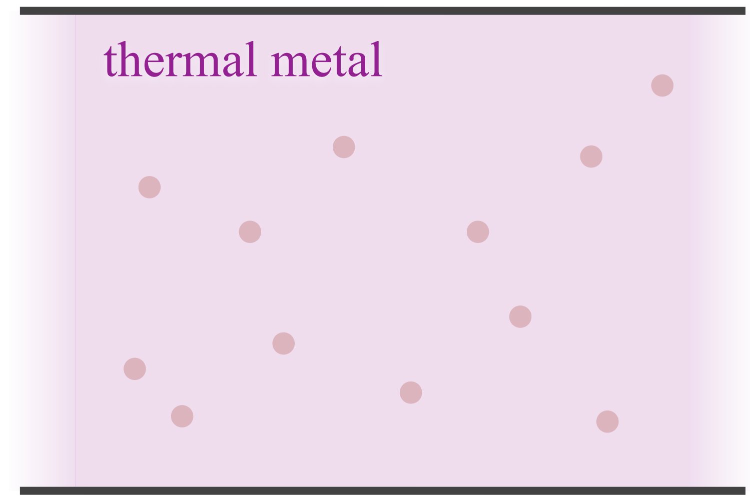



**Oscillations** in the DOS fit the prediction
from **random matrix theory** for symmetry class D

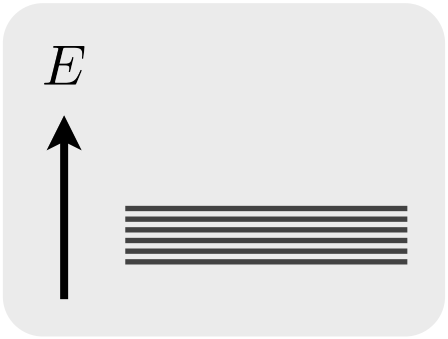
$$\rho(E) = \alpha + \frac{\sin(2\pi\alpha E L^2)}{2\pi E L^2}$$

# Disorder induced phase transition




$$a \gg \xi_m$$



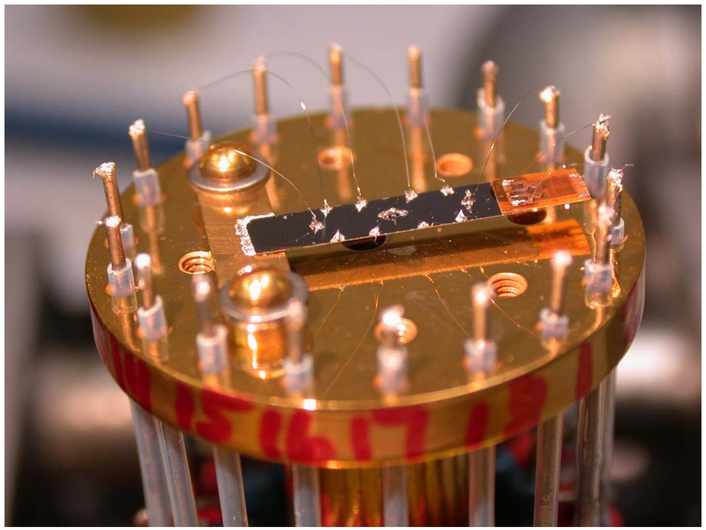

**macroscopic** degeneracy

disorder + vortex-vortex interactions

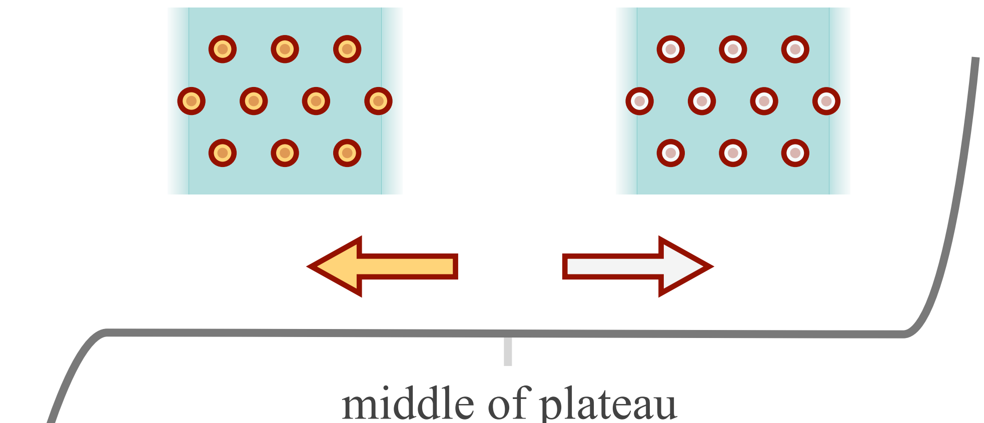
$$a \approx \xi_m$$

degeneracy is **split**

# Heat transport



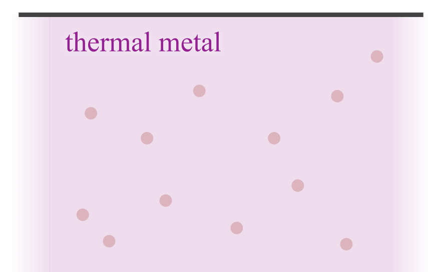
*Caltech thermopower experiment*

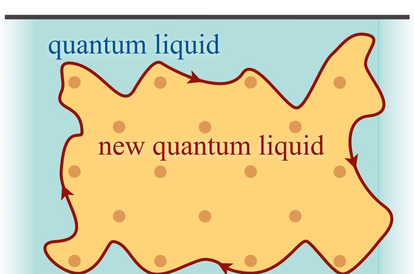



**electrical** transport remains unchanged

**Bulk heat** transport
diverges logarithmically
as T → 0.

$$\kappa_{xx}/T \propto \log T$$

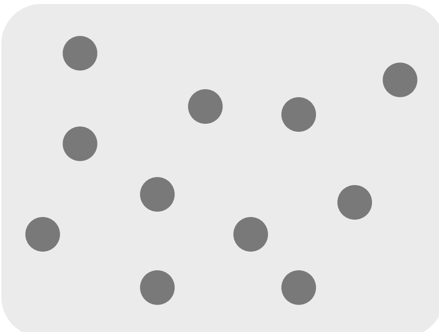


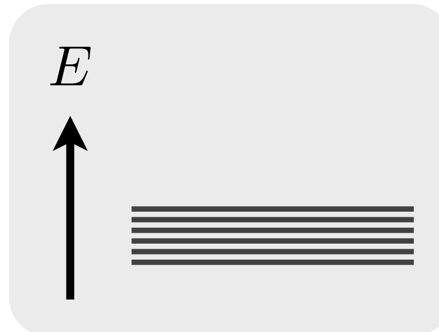

**Heat** transport
along the sample **edges**
changes quantitatively


Das alles und noch viel mehr ...

# Summary



interacting
**many-body system**




‘accidental’
**degeneracy**

**residual effects**
select ground state

Topology + interactions + disorder can give rise to a plethora of collective phenomena.

- Topological **liquid nucleation, thermal metals**
- Distinct experimental **bulk observable** (heat transport) in search for Majorana fermions
- Lord Kelvin was way ahead of his time.