# Decoding quantum many-body teleportation

robust teleportation | weak measurements | wave function deformations

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University of Cologne

CRC183 ENTANGLED STATES OF MATTER

ML4Q MATTER AND LIGHT FOR QUANTUM COMPUTING

ML4Q summer school on "Quantum Error Correction"

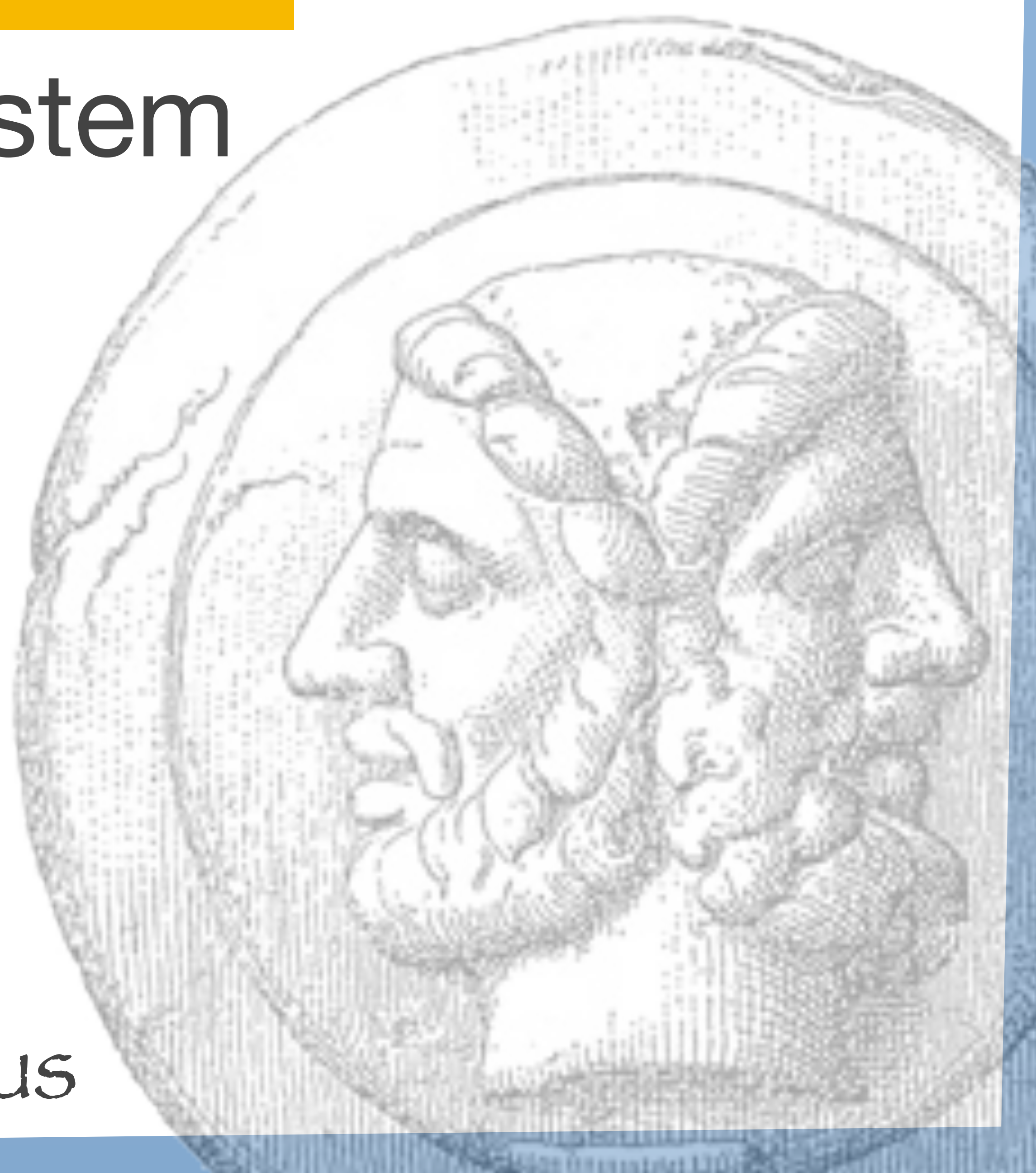
Bonn, September 2024

# quantum measurements


"About your cat, Mr. Schrödinger — I have good news and bad news."

**Quantum** measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system

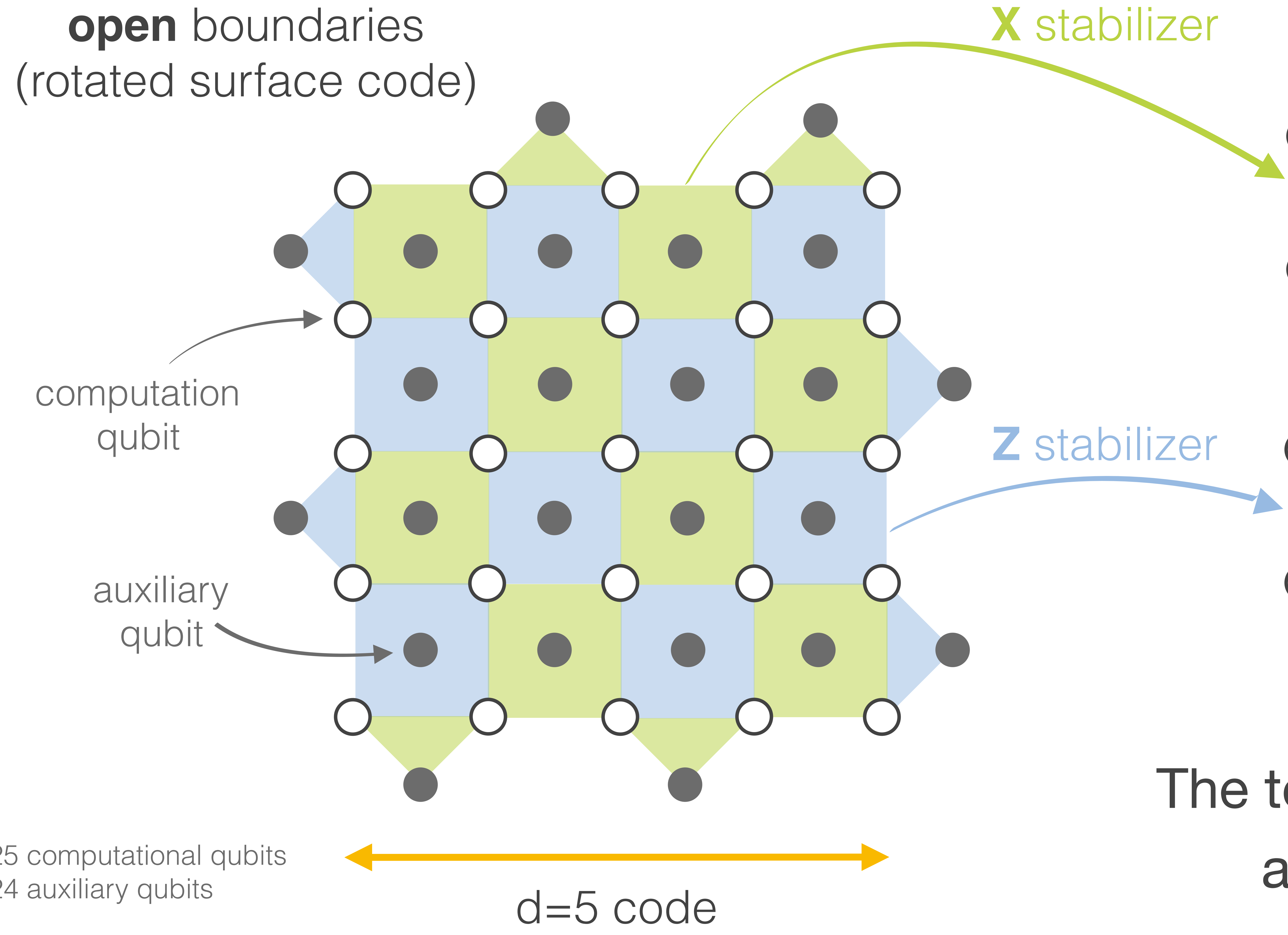


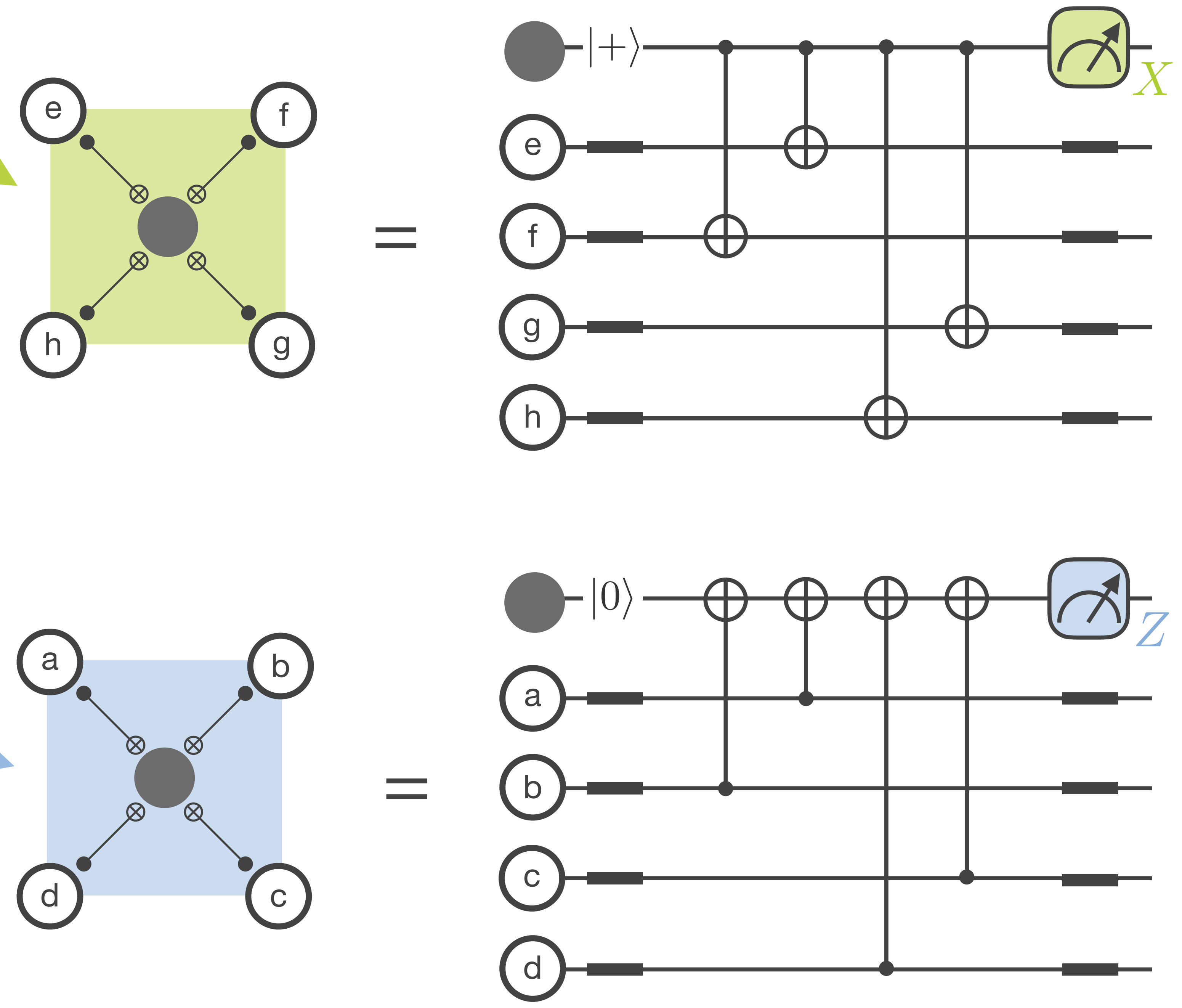
double-faced Janus

# paradigmatic example: **surface code**

Kitaev (1997)

**open** boundaries
(rotated surface code)



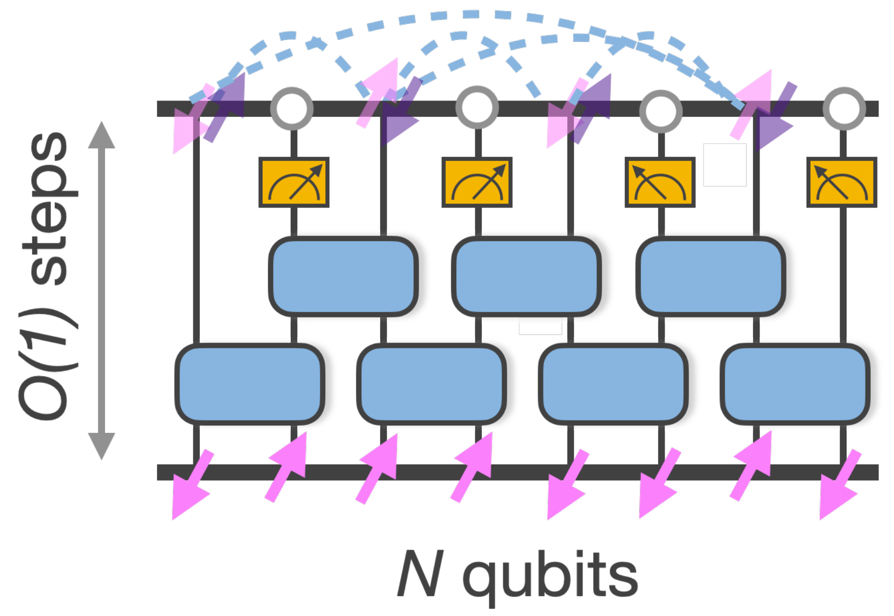

25 computational qubits
24 auxiliary qubits

The toric/surface code was conceived as a **measurement protocol.**

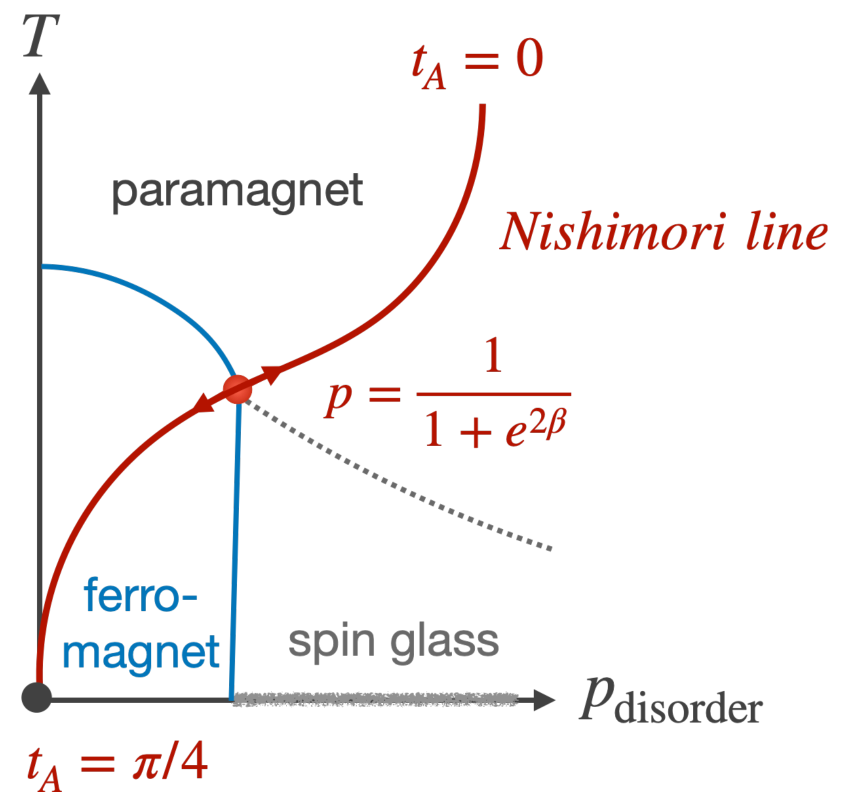
# measurement & entanglement

Guo-Yi Zhu

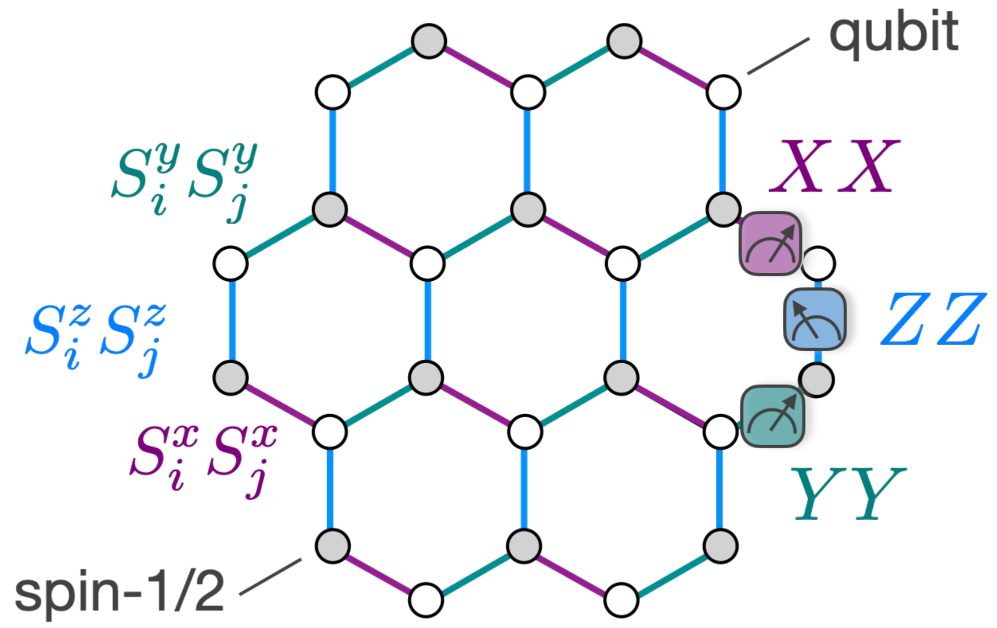
## measurement-assisted **state preparation**



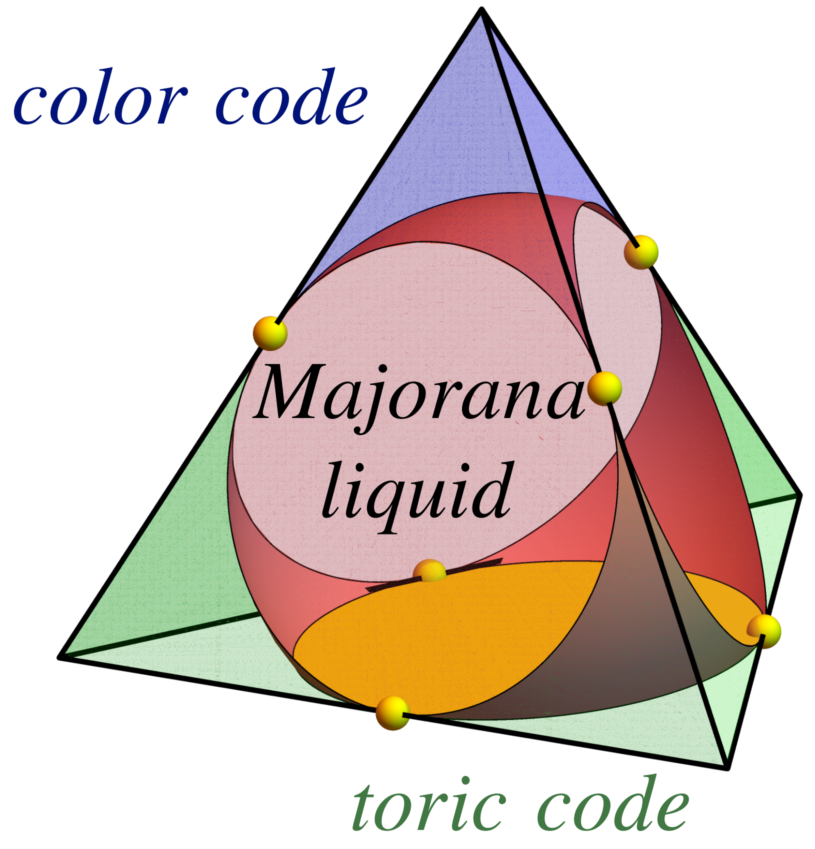

**Nishimori's cat**



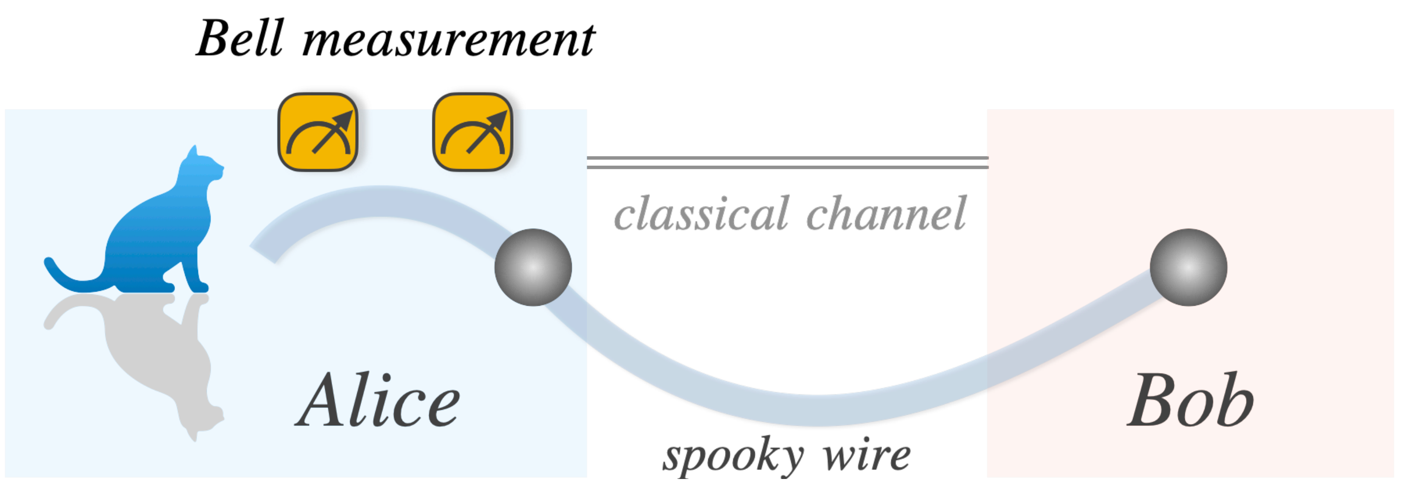

## **measurement-only** quantum dynamics



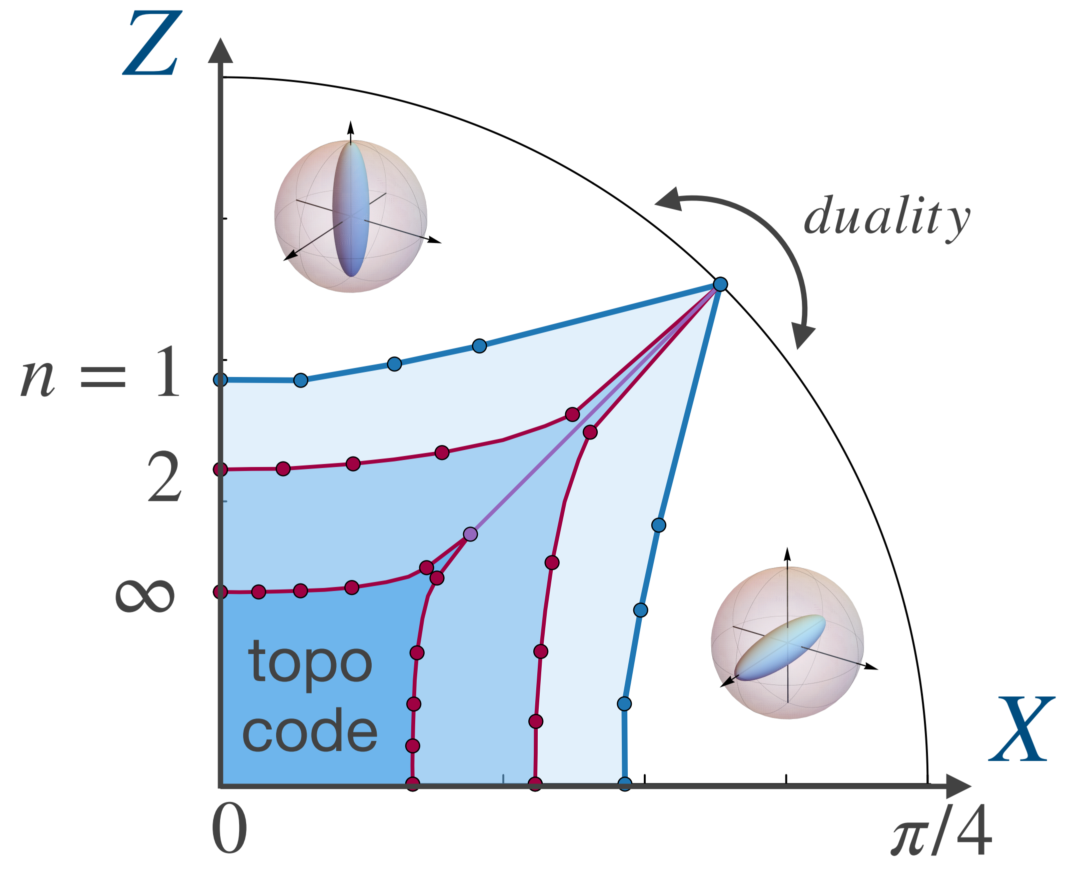

**Kitaev circuits**




## many-qubit **teleportation**




**Bell decoders**



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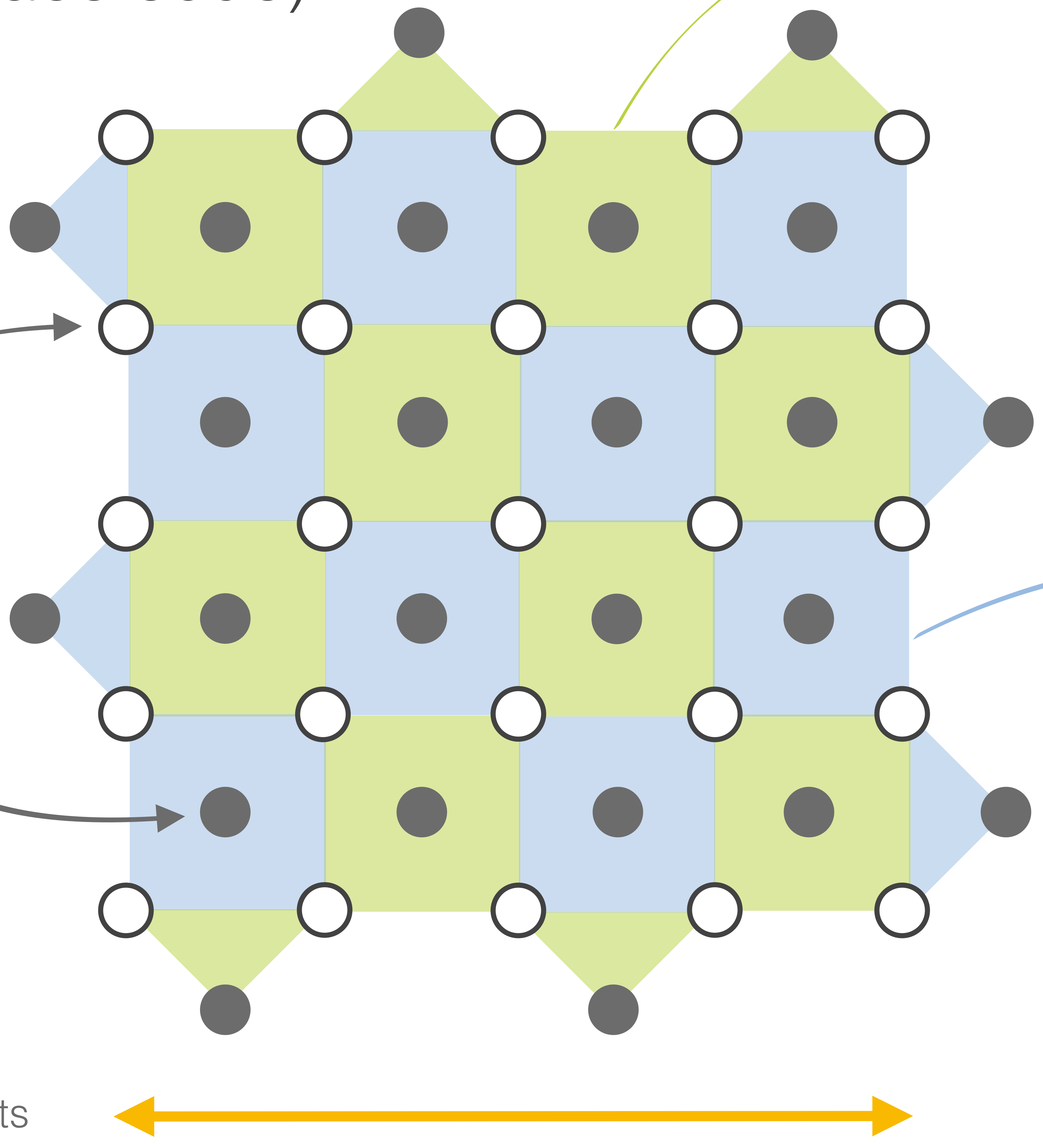
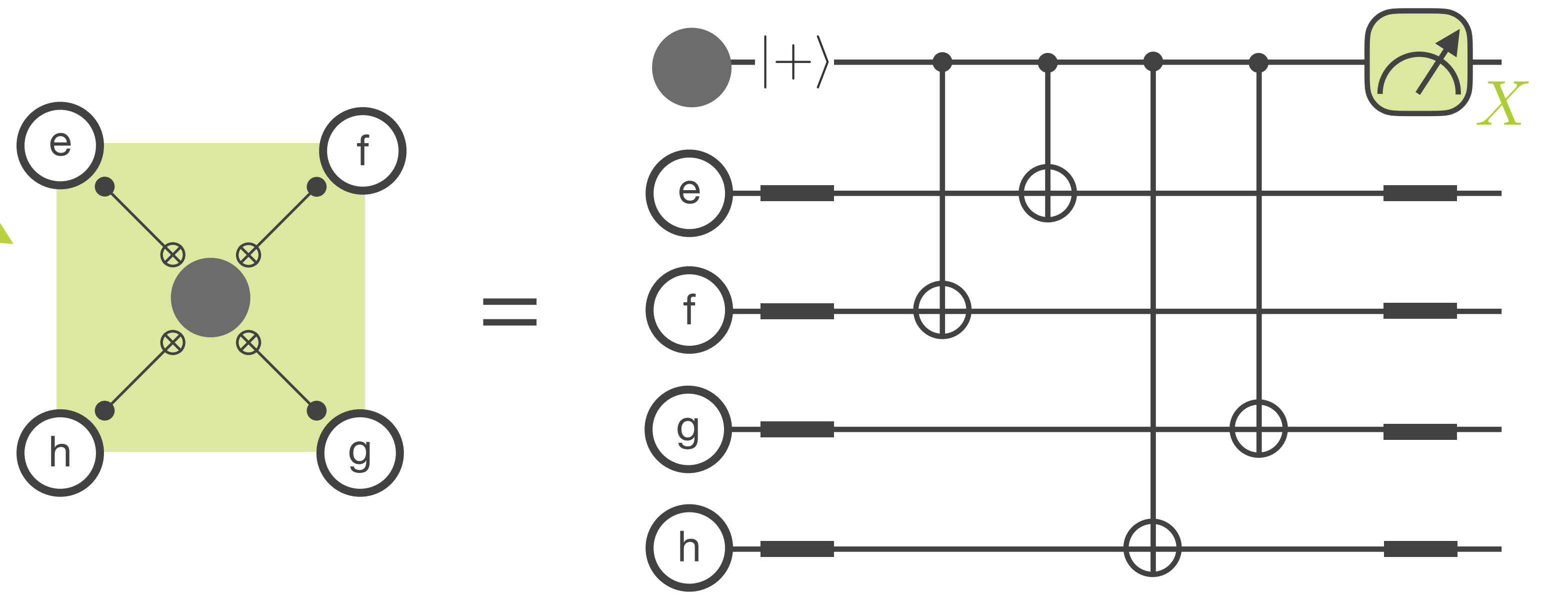


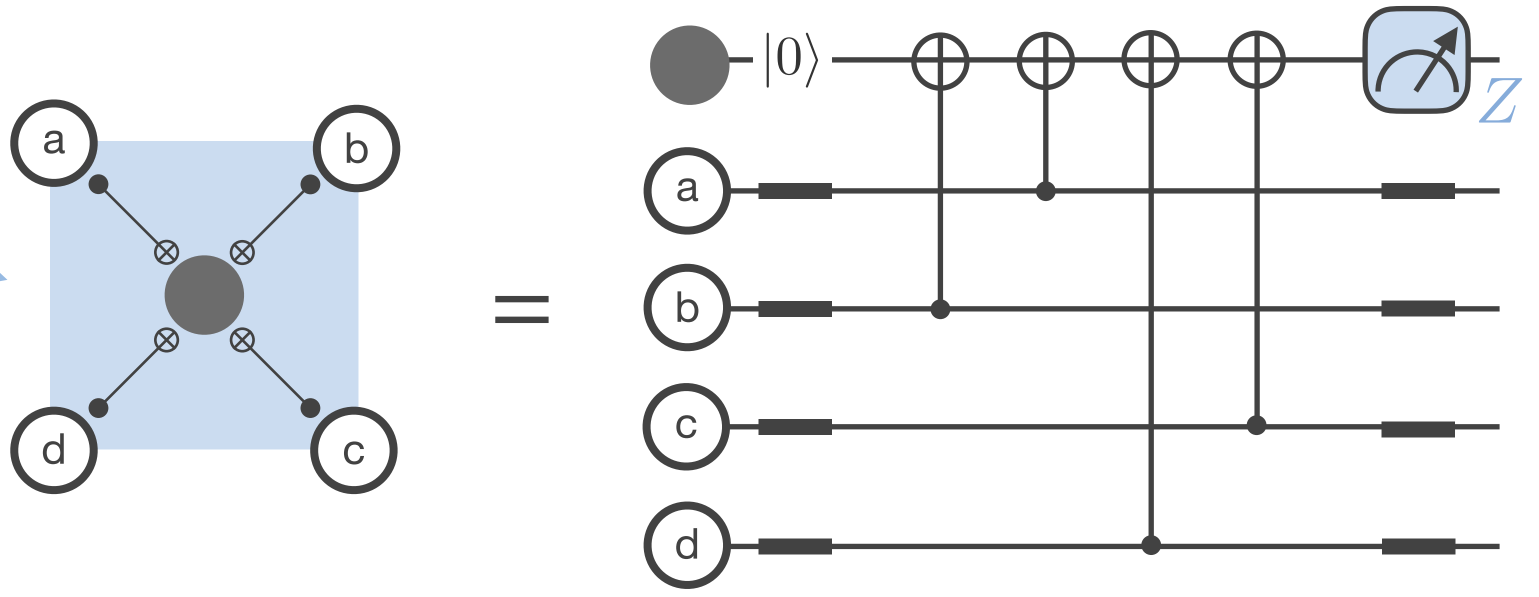

d=5 code

25 computational qubits
24 auxiliary qubits

**X** stabilizer

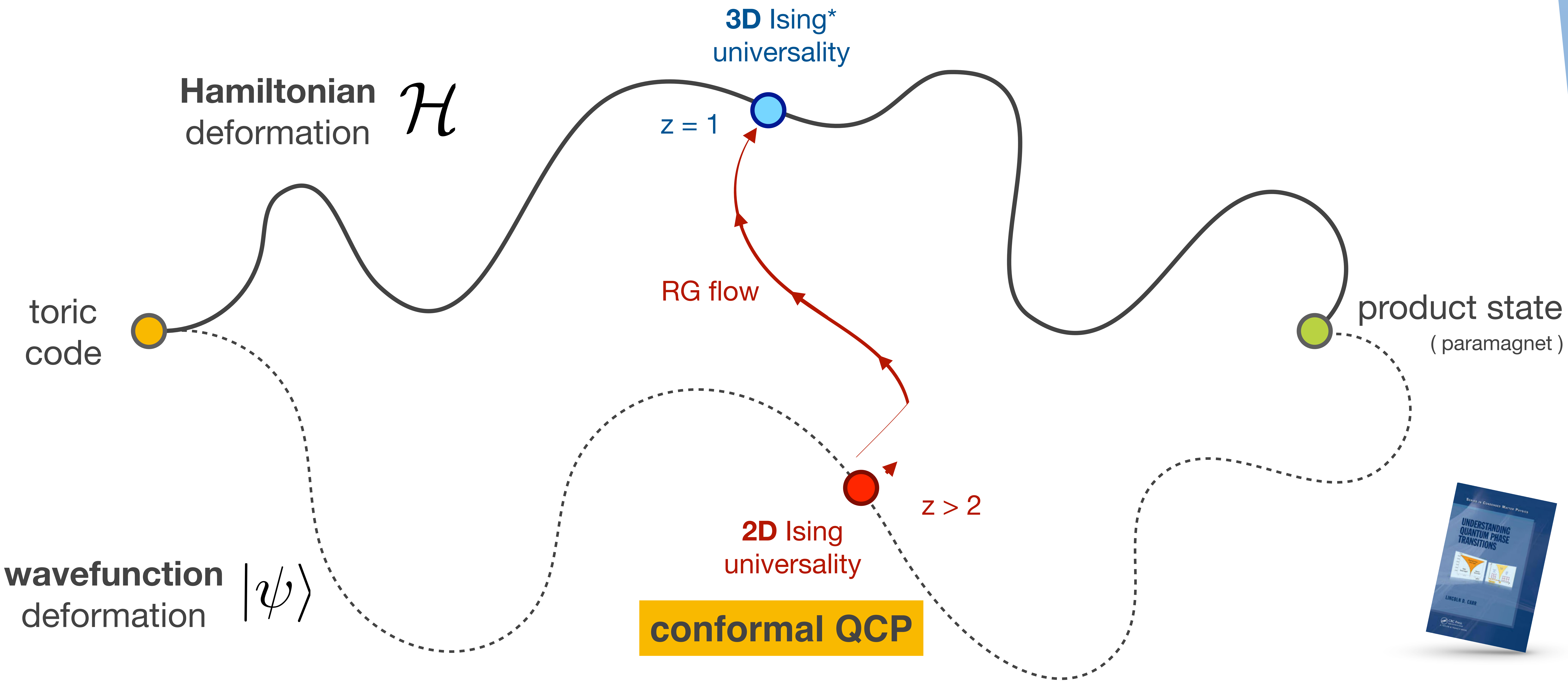



**Z** stabilizer



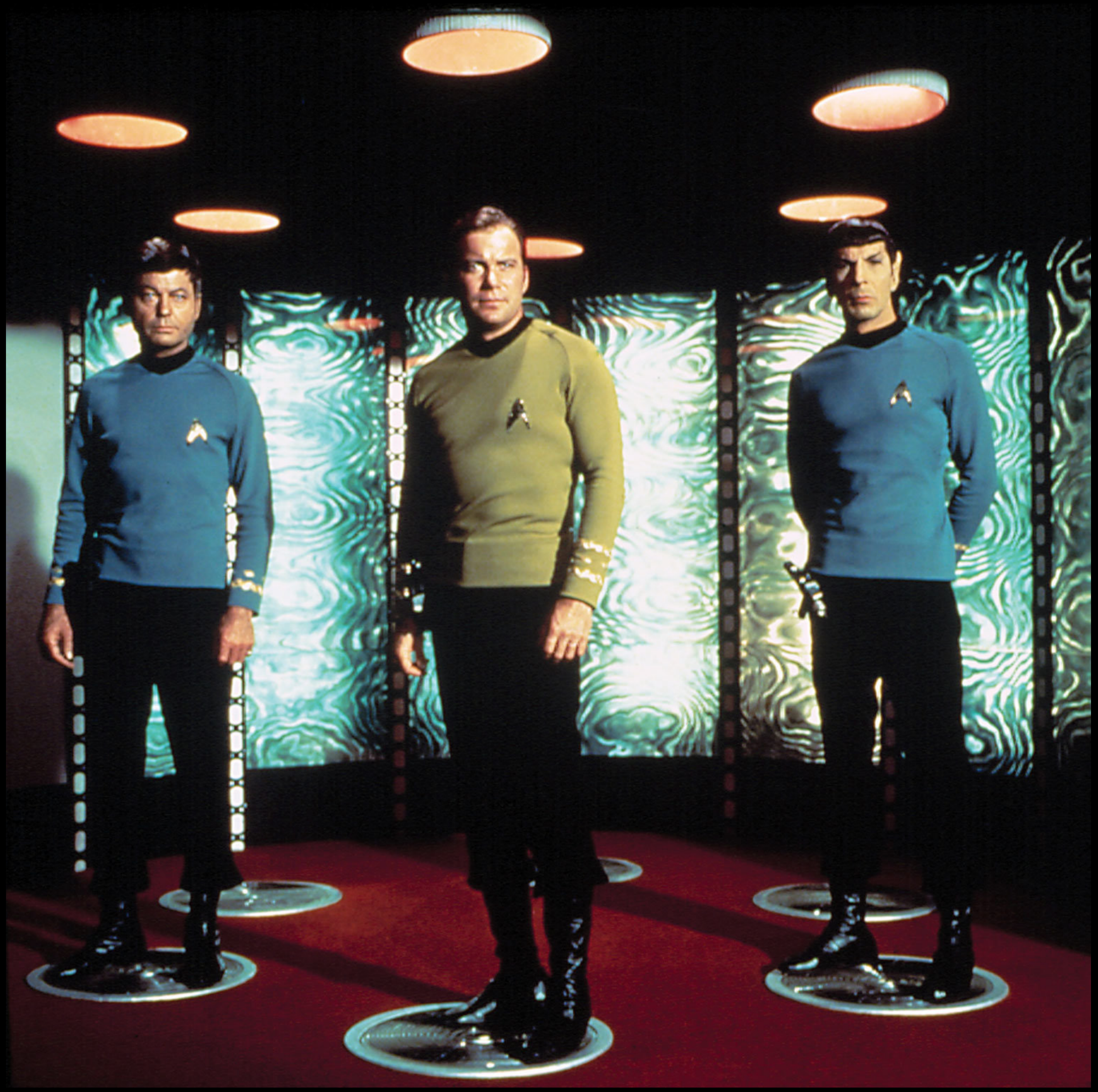

The toric/surface code was conceived as a **measurement protocol.**

# phase transitions & deformations




Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008)

Castelnovo, ST, Troyer (2010),
Isakov, Fendley, Ludwig, ST, Troyer (2011)



**science** meets **fiction**

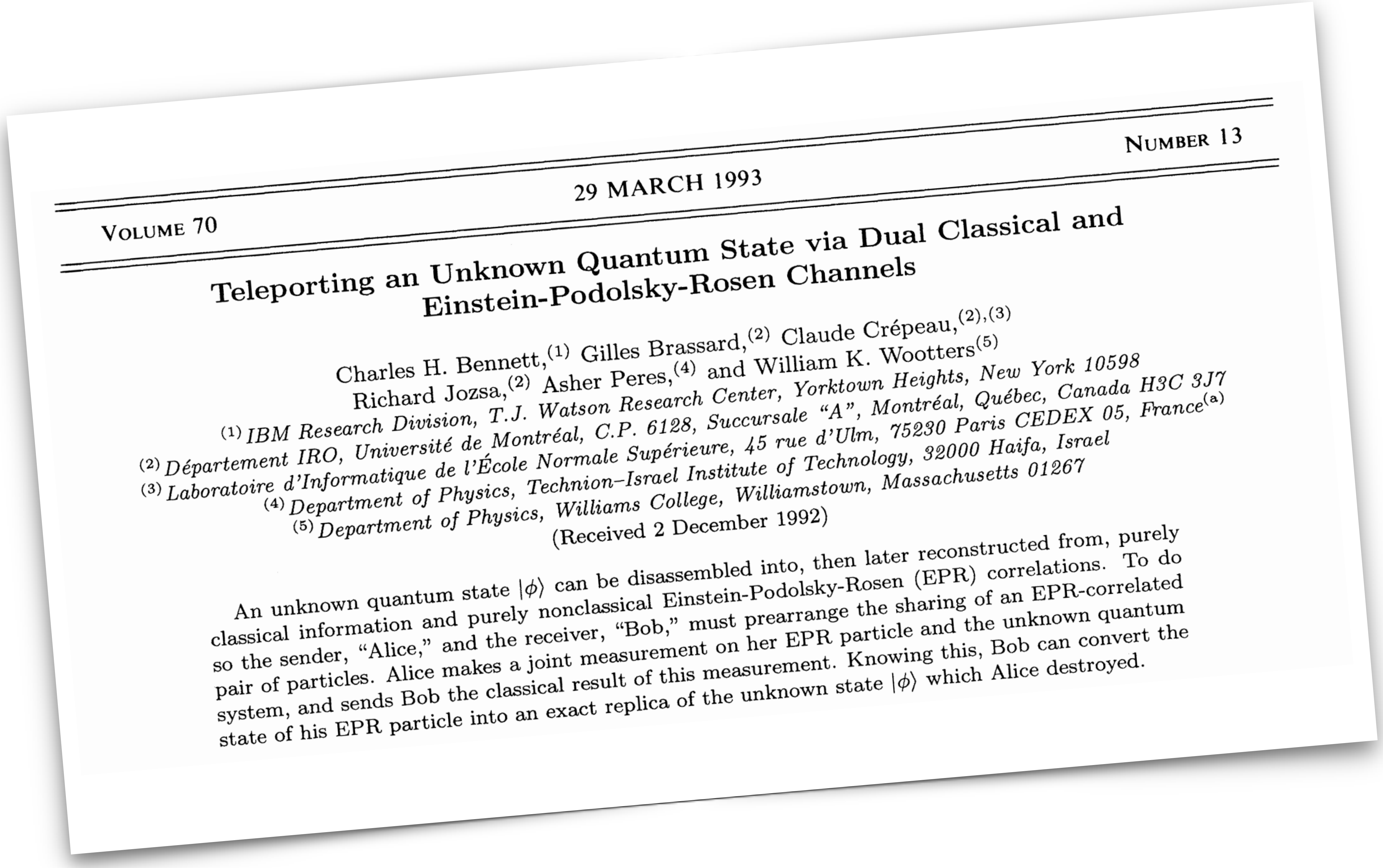
# teleportation

# **quantum** teleportation

teleportation?

**quantum mechanics**

to the rescue …



VOLUME 70 29 MARCH 1993 NUMBER 13

**Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels**

Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2,3] Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]

[1] *IBM Research Division, T.J. Watson Research Center, Yorktown Heights, New York 10598*

[2] *Département IRO, Université de Montréal, C.P. 6128, Succursale "A", Montréal, Québec, Canada H3C 3J7*

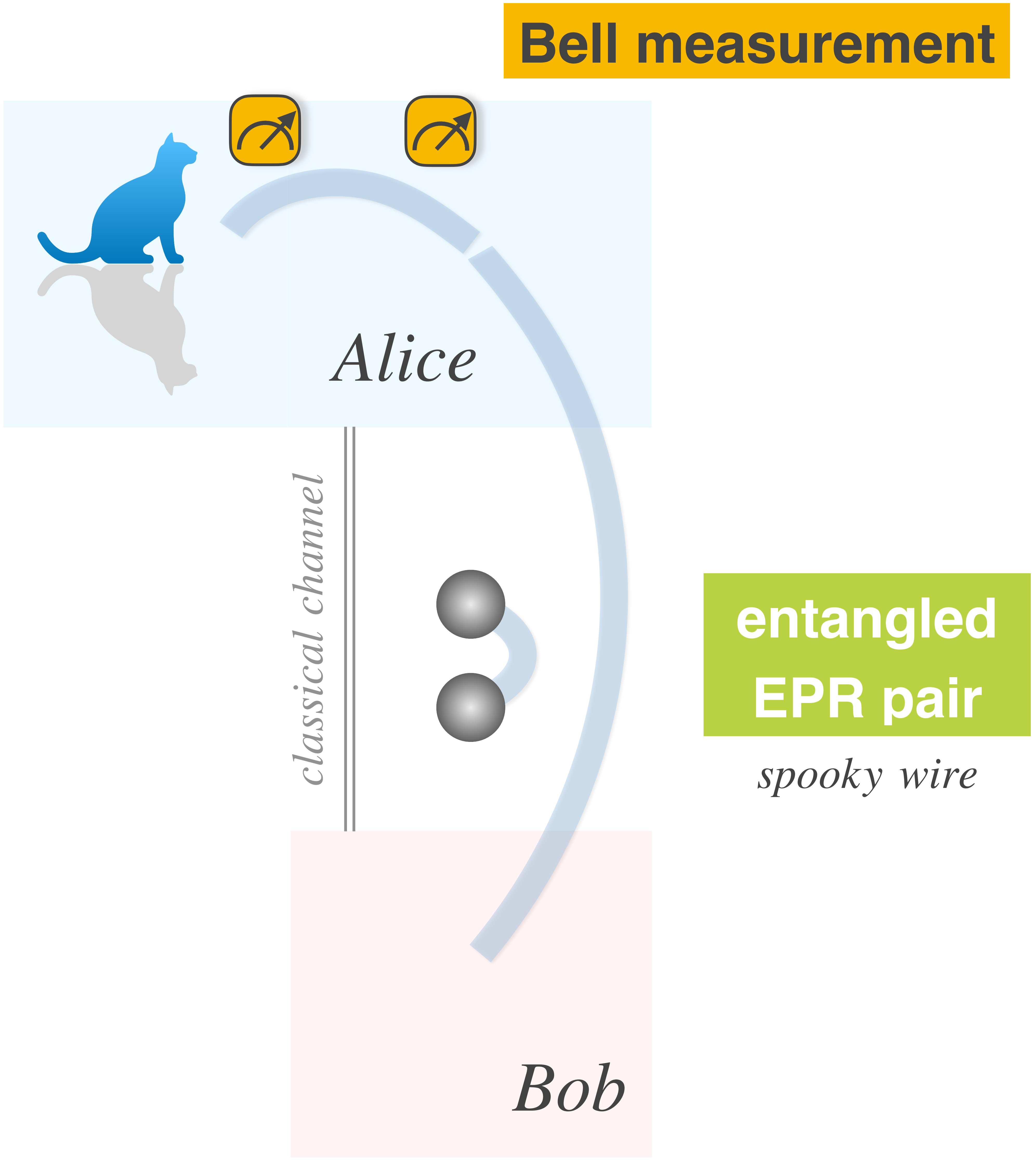
[3] *Laboratoire d'Informatique de l'École Normale Supérieure, 45 rue d'Ulm, 75230 Paris CEDEX 05, France*[a]

[4] *Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel*

[5] *Department of Physics, Williams College, Williamstown, Massachusetts 01267*

(Received 2 December 1992)

An unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations. To do so the sender, "Alice," and the receiver, "Bob," must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state $|\phi\rangle$ which Alice destroyed.



# **quantum** teleportation

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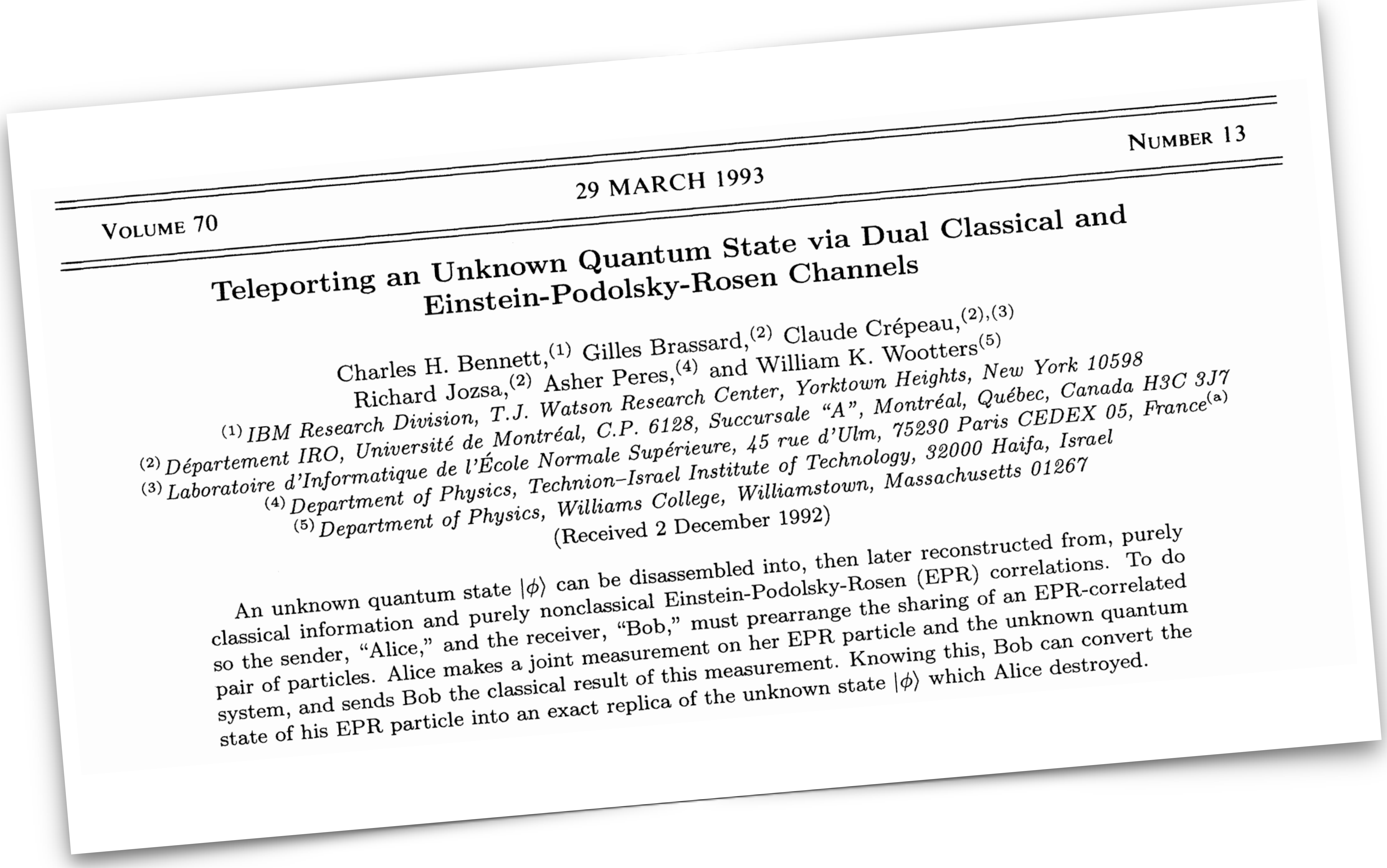
to the rescue …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**experimental** quantum teleportation (1997)


D. Bouwmeester, …., A. Zeilinger, Nature **390**, 575 (1997)

**3 m** teleportation on demand (2014)

The New York Times

W. Pfaff, …, R. Hanson, Science **345**, 532 (2014)

**100 km** optical fibre channel (2016)

Q.-C. Sun *et al.*, Nature Photonics **10**, 671 (2016)

**1000 km** earth-satellite channel (2017)

J.-G. Ren *et al.* Nature **549**, 70 (2017)

# **quantum** teleportation

teleportation?

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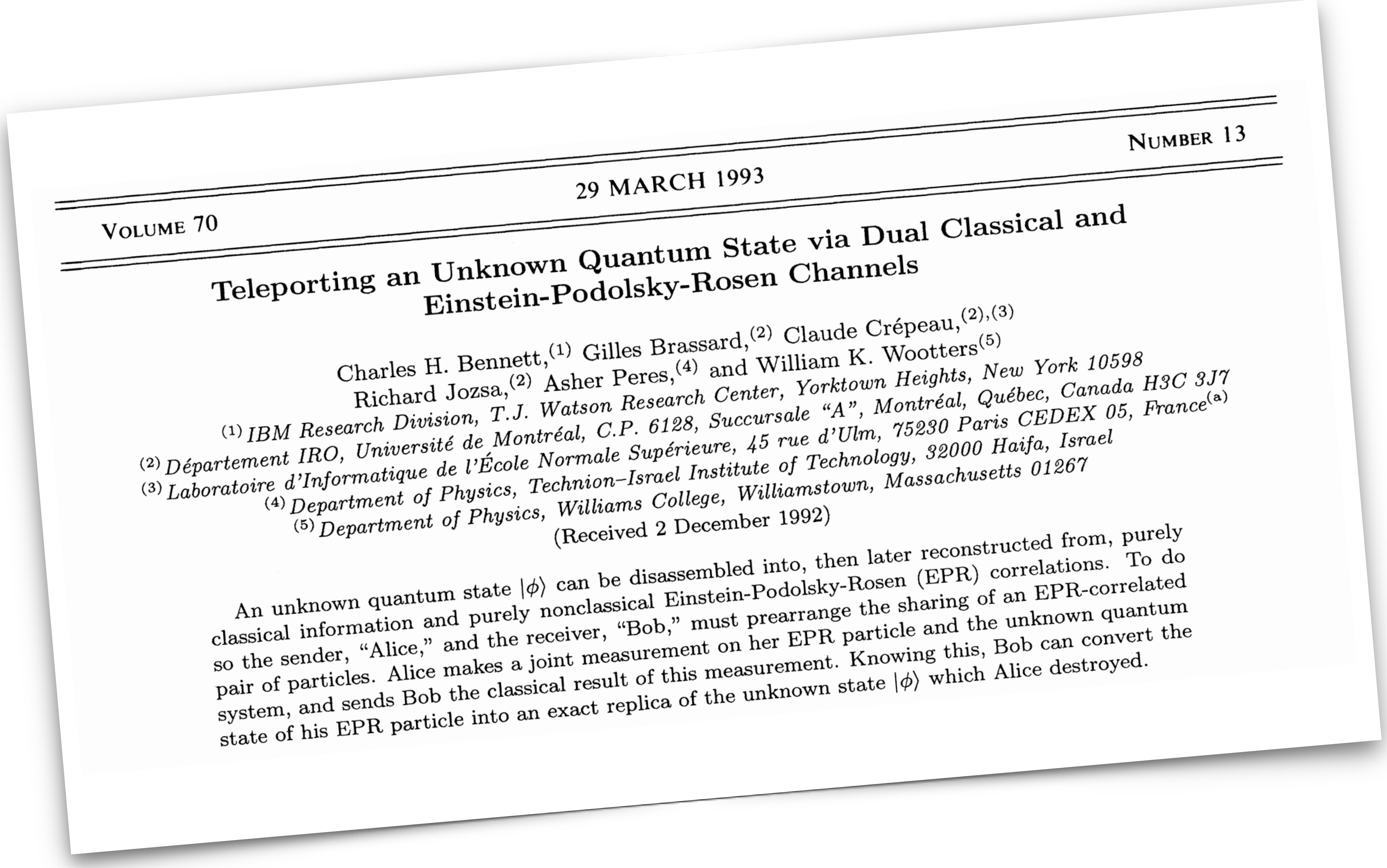
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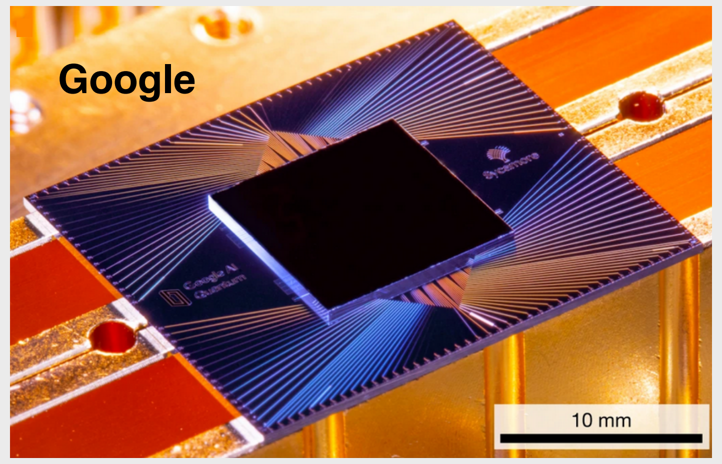
**single qubit** teleportation

**few-qubit** teleportation

noisy intermediate-scale

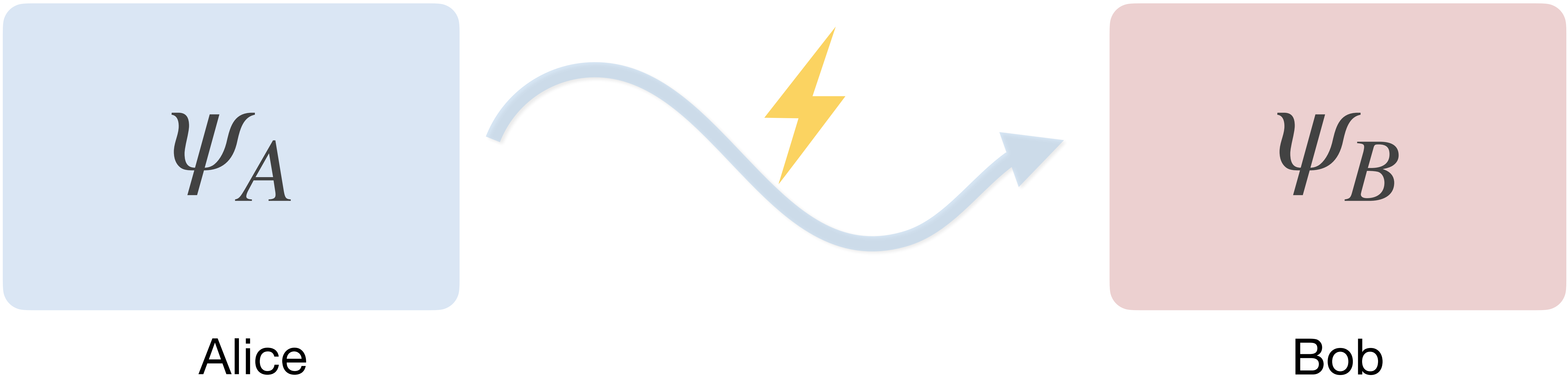
**NISQ quantum devices**




**many-body** teleportation

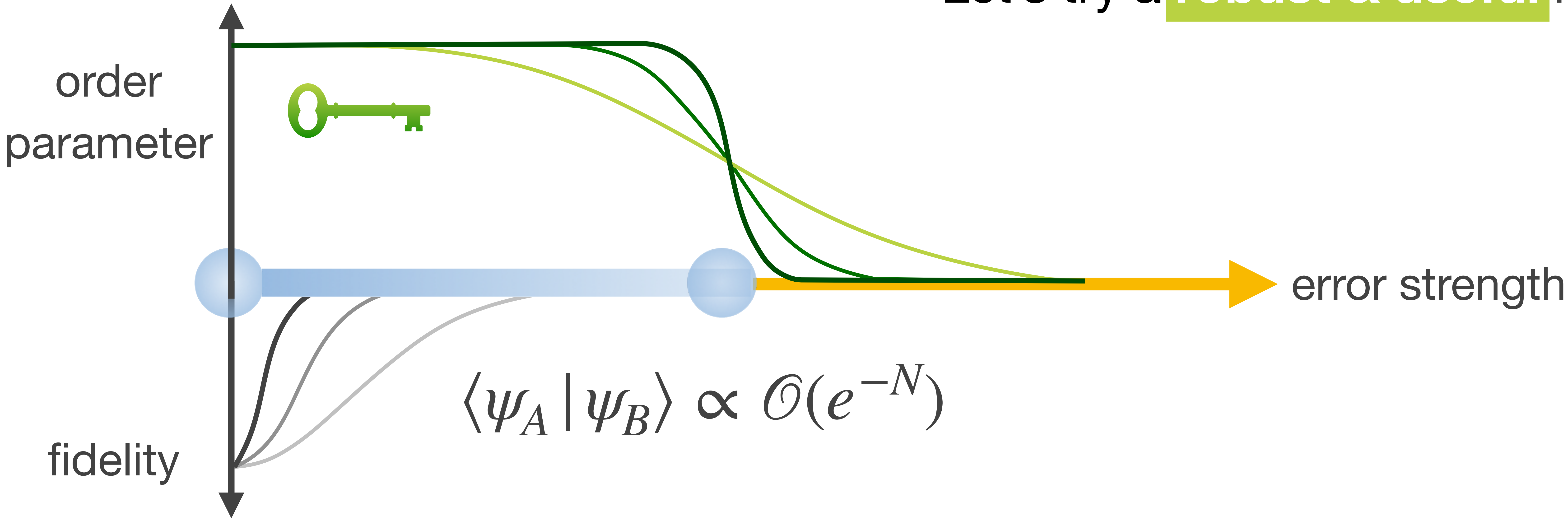
**Star Trek**

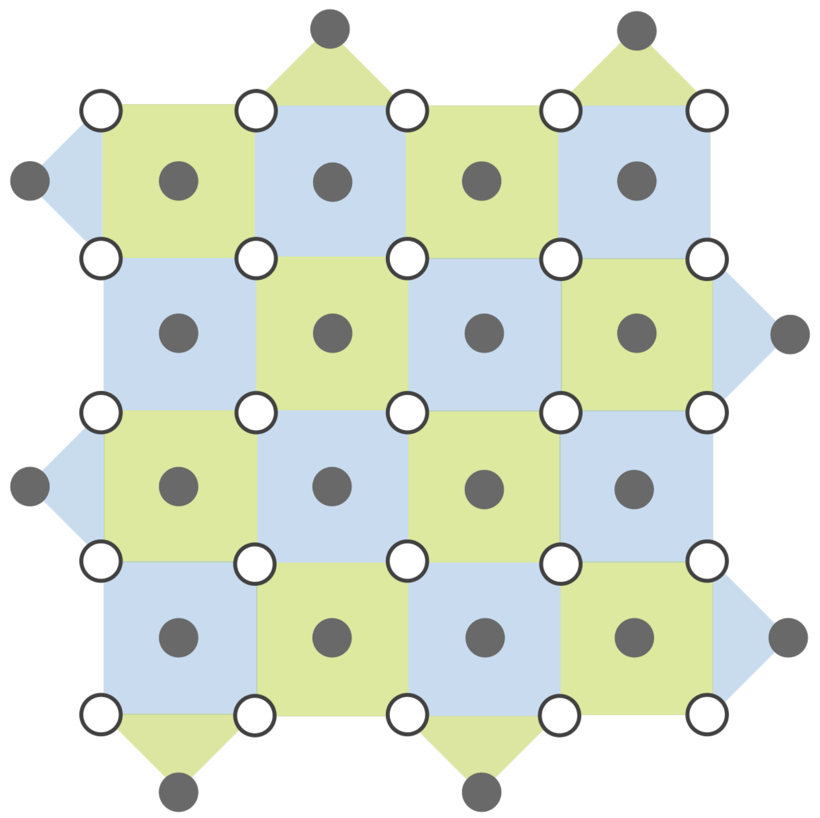
# teleportation of **quantum matter**




Q: Can **quantum matter** be teleported under **coherent error** ?

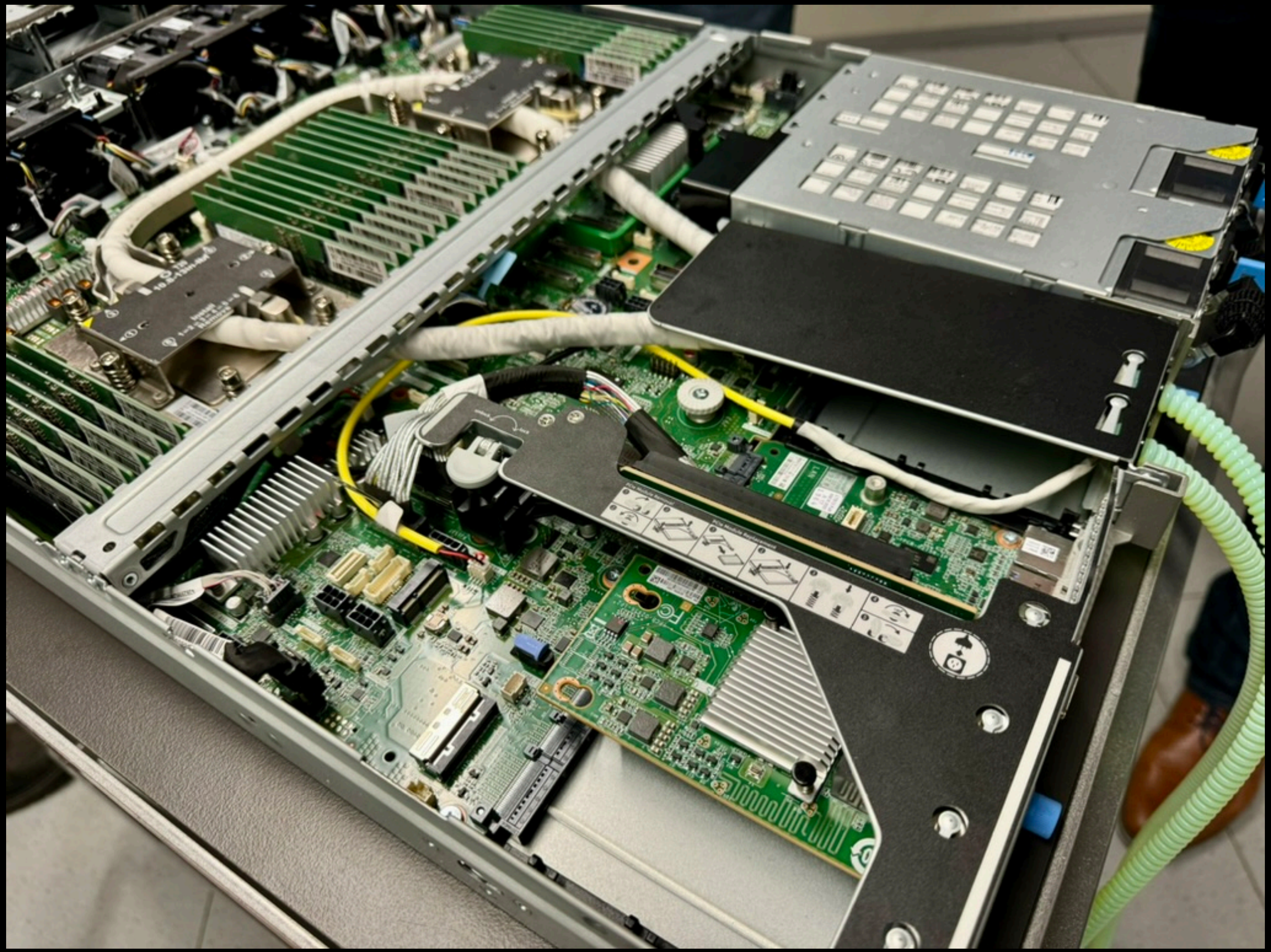
Let's try a **robust & useful** many-body state



surface code / logical qubit

# von Neumann architecture



RAMSES HPC cluster @ University of Cologne,  September 2024

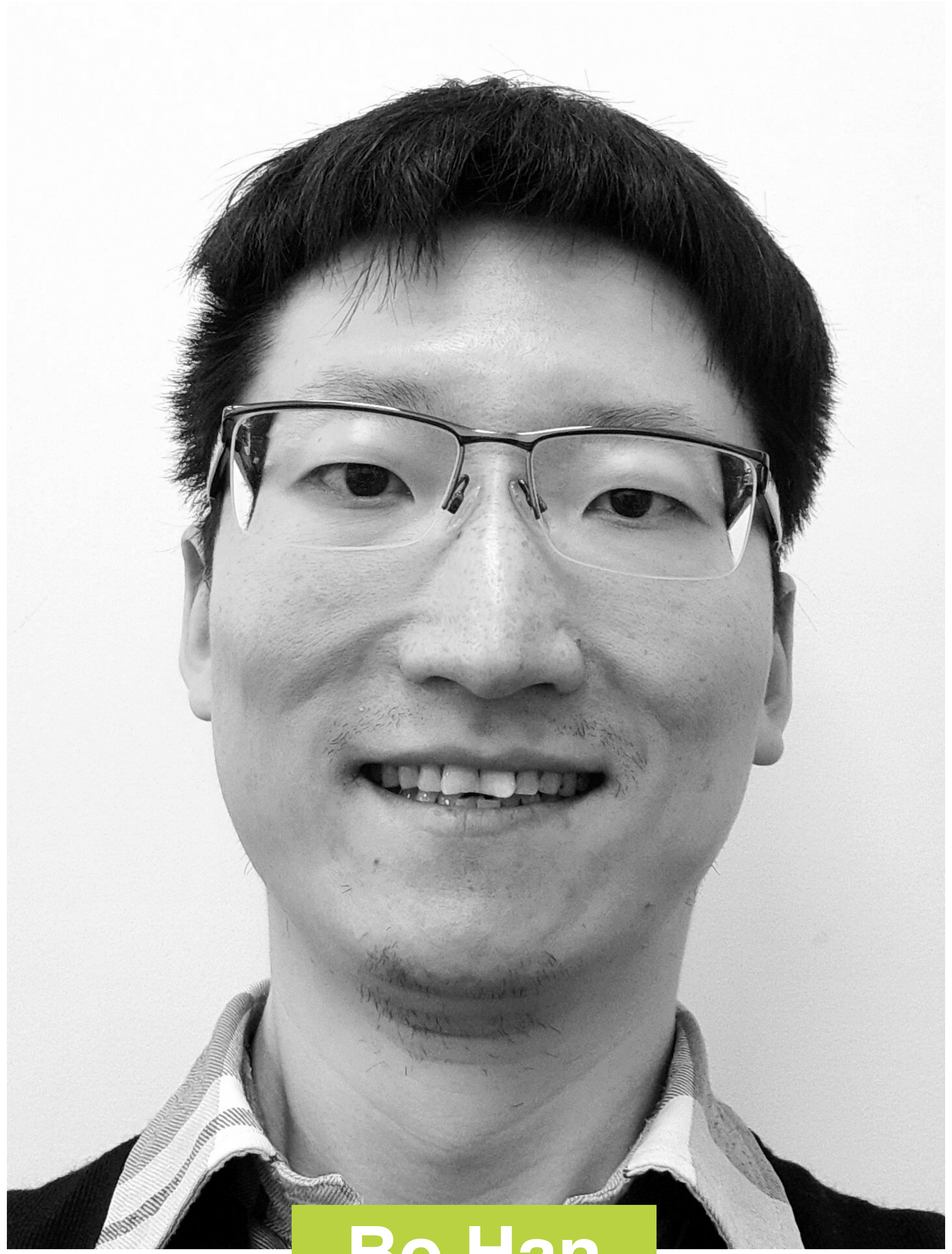
# the **team**

**Finn Eckstein**

University of Cologne

**Guo-Yi Zhu**

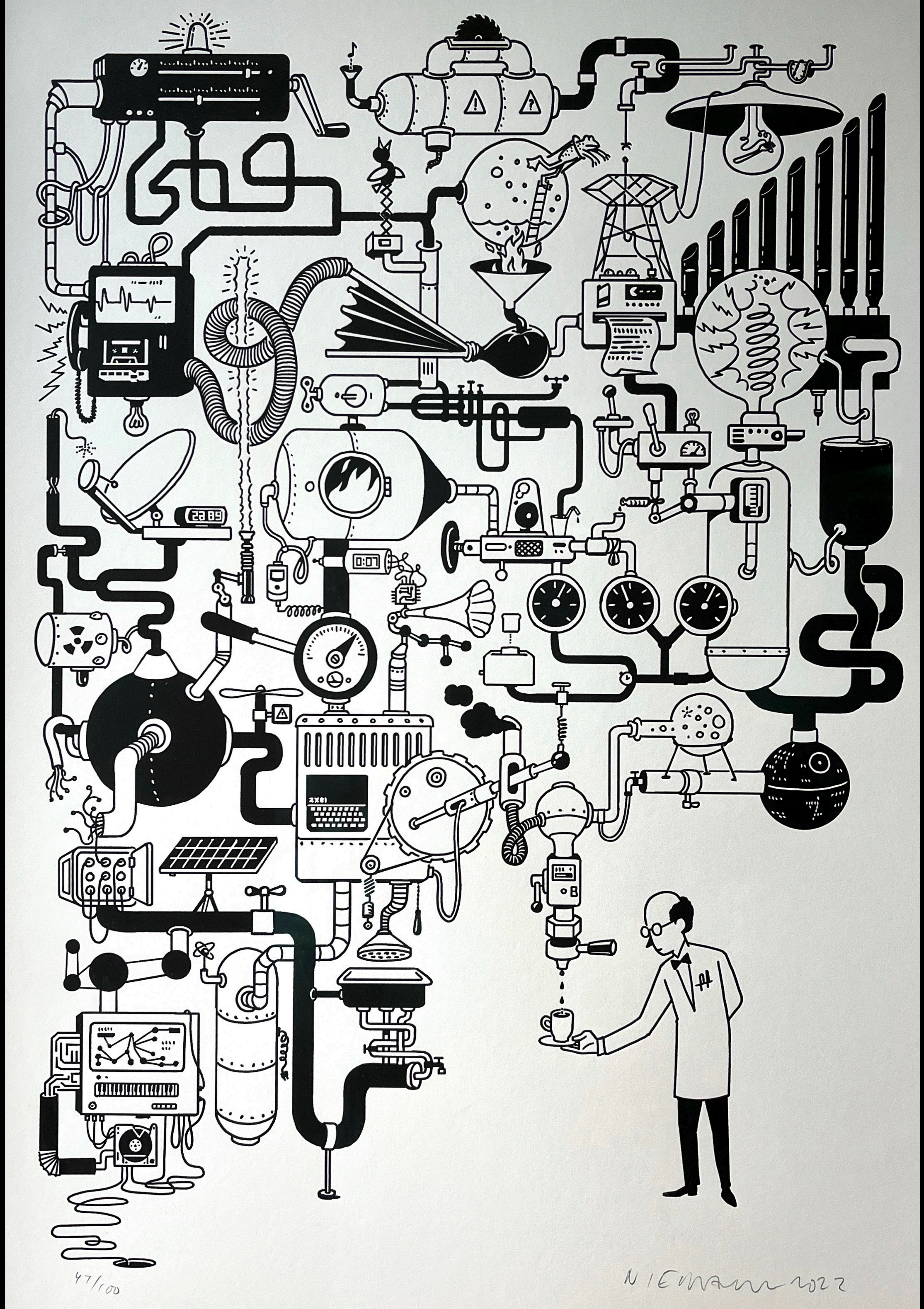
University of Cologne
Hong Kong University of Science and Technology



**Bo Han**

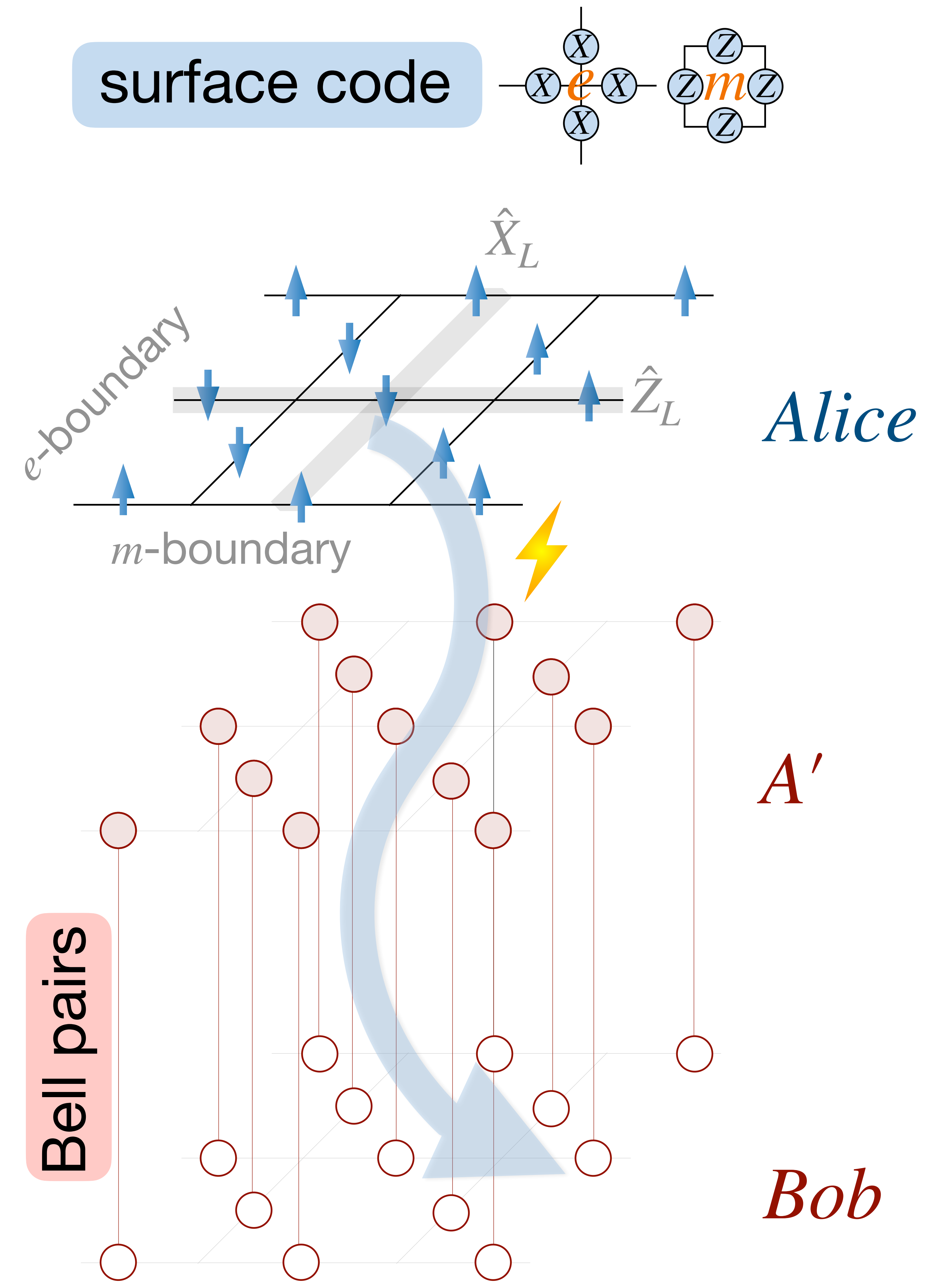
Weizmann Institute of Science

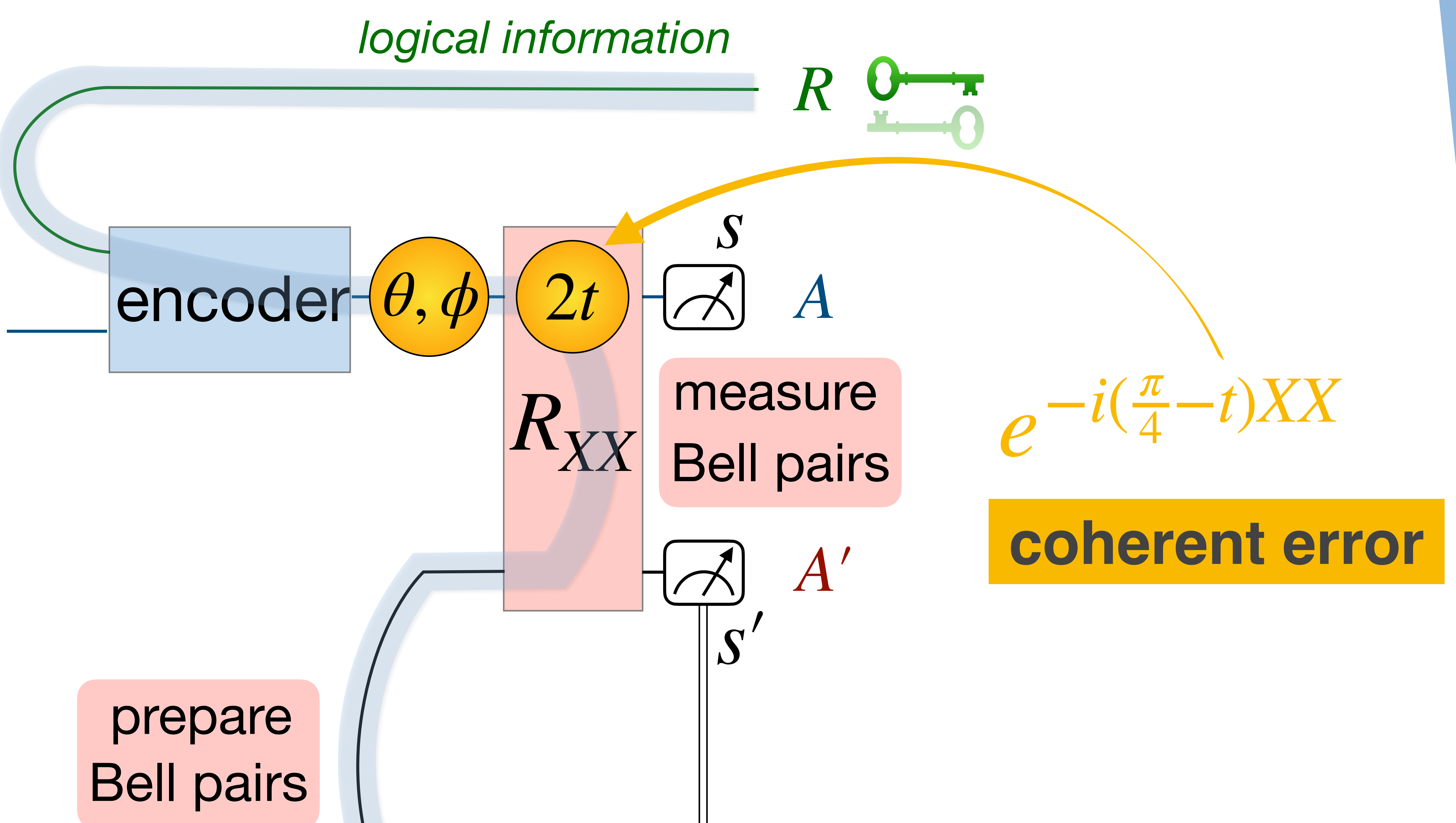
# teleportation protocol

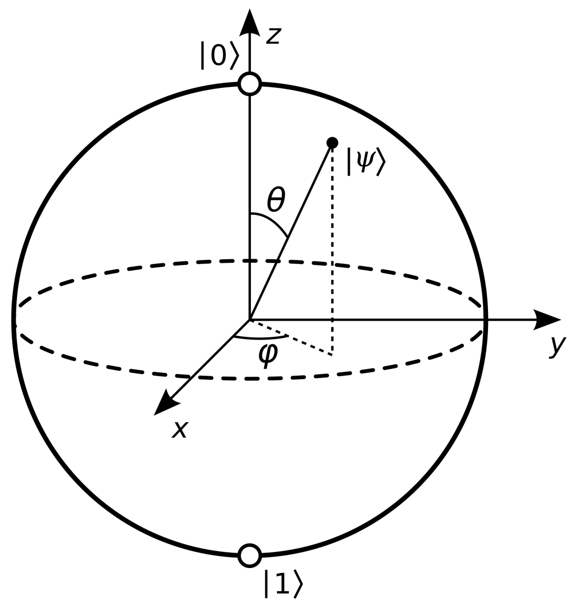


Christoph Niemann, New Yorker Cover

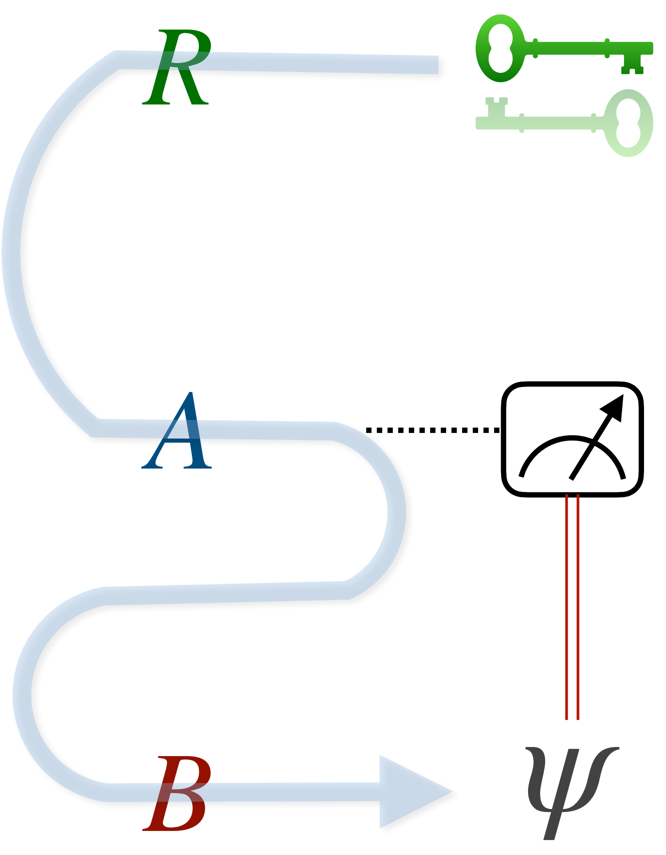
# protocol & **quantum circuit**



# **diagnose** robust teleportation



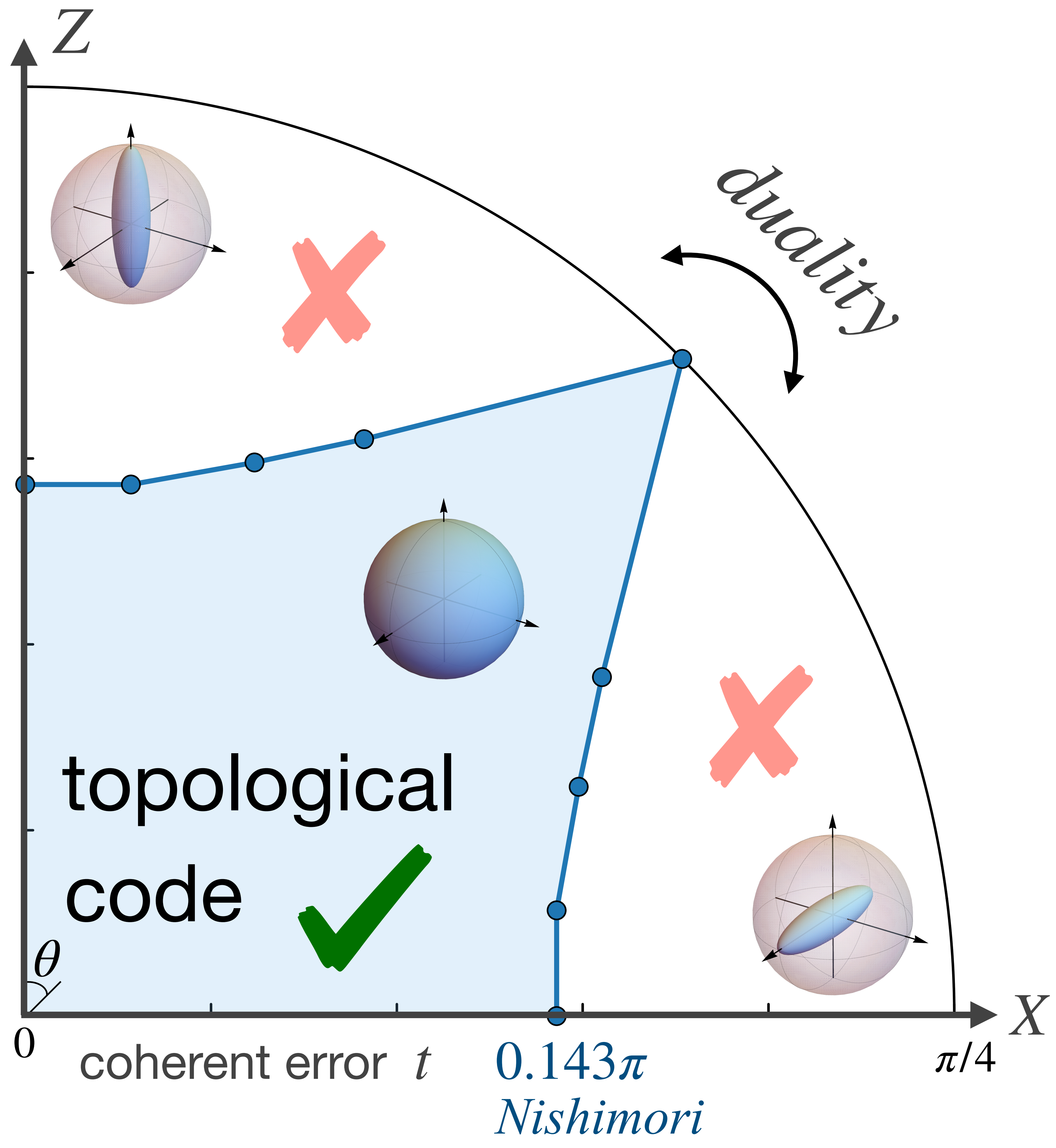

$$I_c = S_{RA} - S_A$$

$$= S_{AB} - S_{RAB}$$

$$= \sum_{\mathbf{s}} P(\mathbf{s}) S_B(\mathbf{s})$$

**coherent information** **channel capacity**

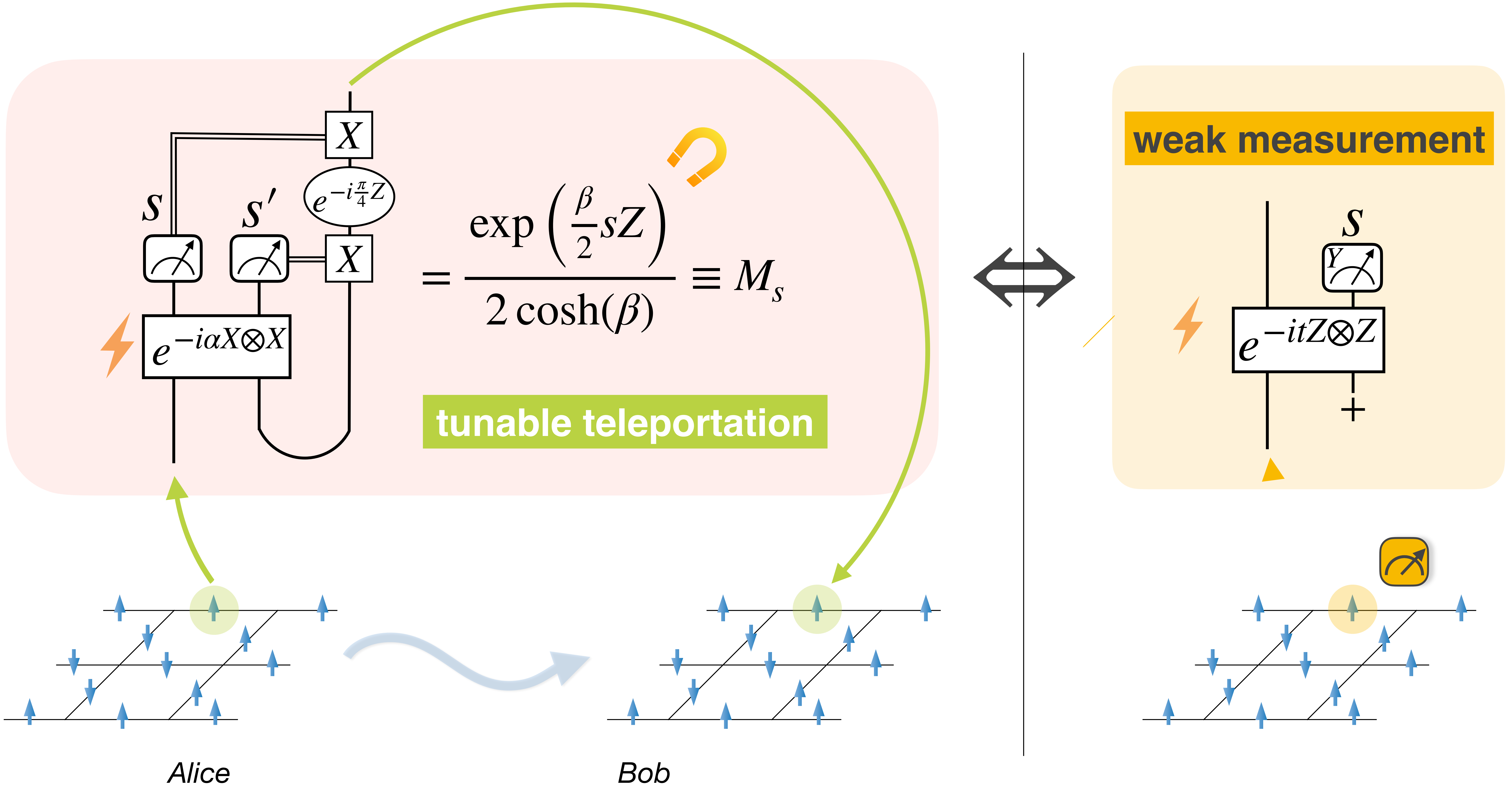
- Can **Alice steal** the key?
- Can **Bob decode** the key with classic info shared by Alice?
- **Ensemble average** of logical entropy (size of code space).




$$\rho_{RAB} = \sum_{\mathbf{s}} P(\mathbf{s}) \, |\Psi(\mathbf{s})\rangle_{RB}\langle\Psi(\mathbf{s})| \otimes |\mathbf{s}\rangle_A\langle\mathbf{s}|$$

Schumacher, Nielsen 1996; Lloyd 1997; Gullans, Huse 2020;
Fan, Bao, Vishwanath, Altman 2023; Colmenarez, Huang, Diehl, Müller 2023

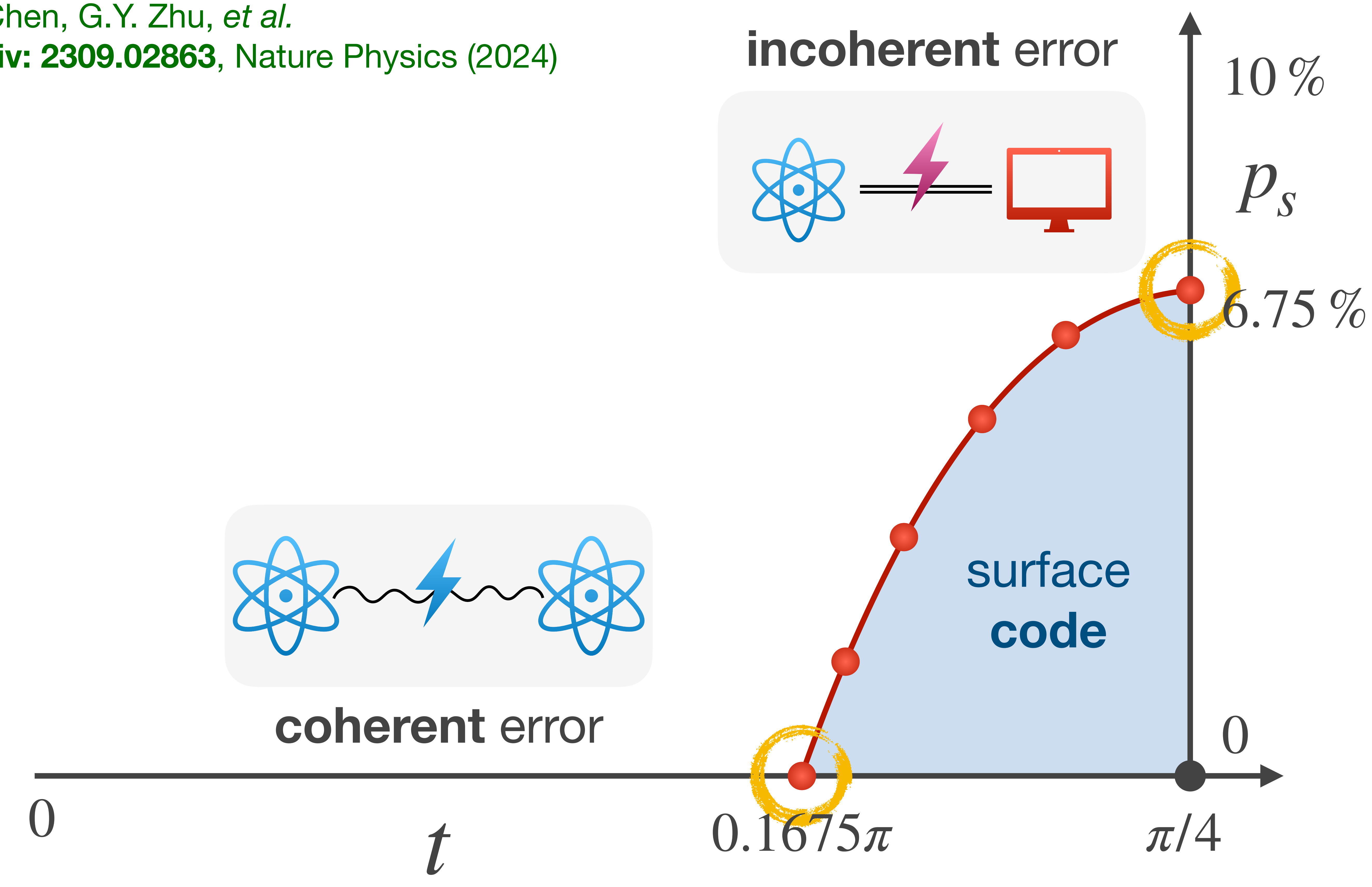
# **physical qubits** / teleportation vs. measurement

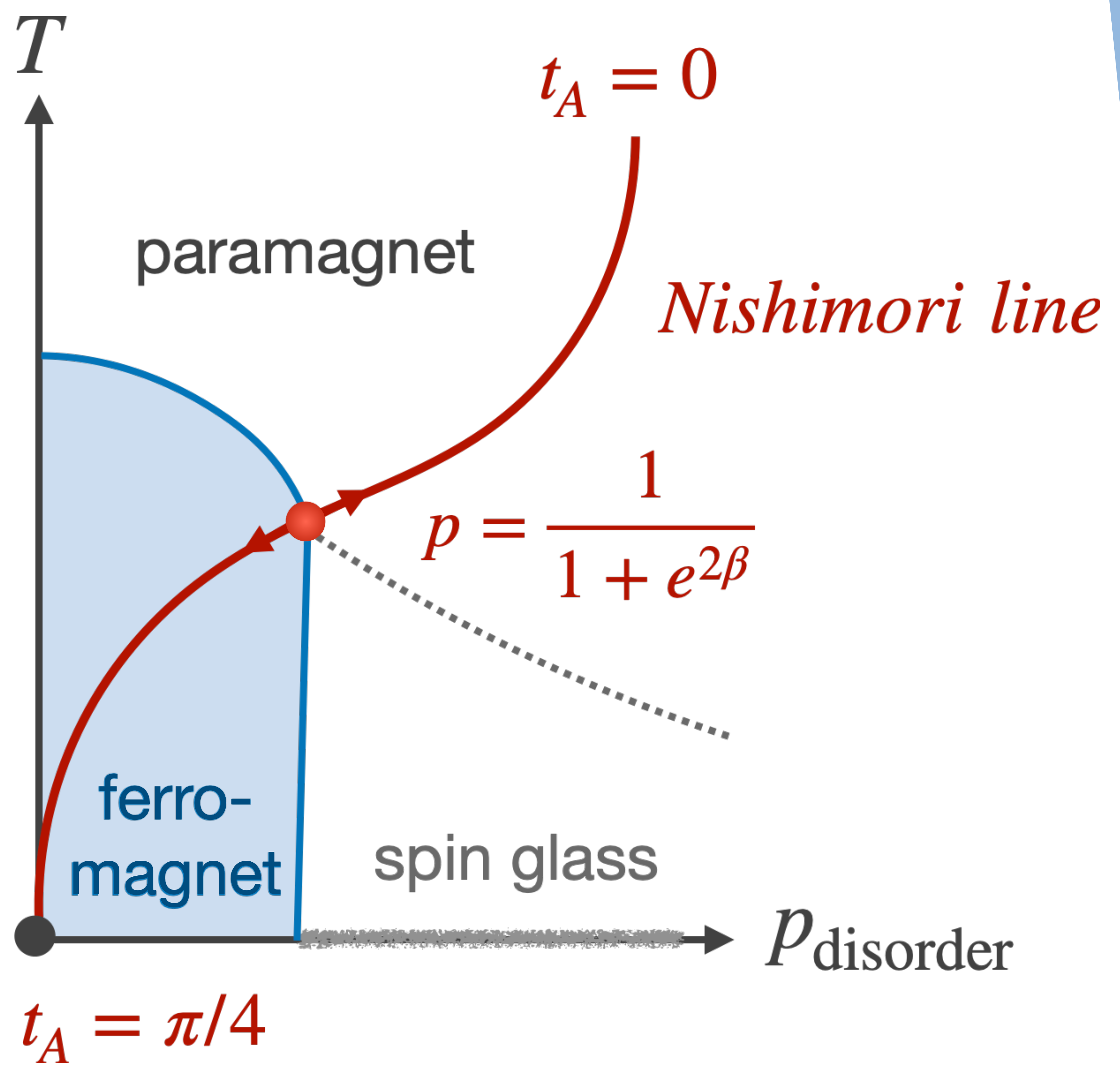


# coherent vs. incoherent errors

Dennis, Kitaev, Landahl, Preskill 2002

E. Chen, G.Y. Zhu, *et al.*
**arXiv: 2309.02863**, Nature Physics (2024)



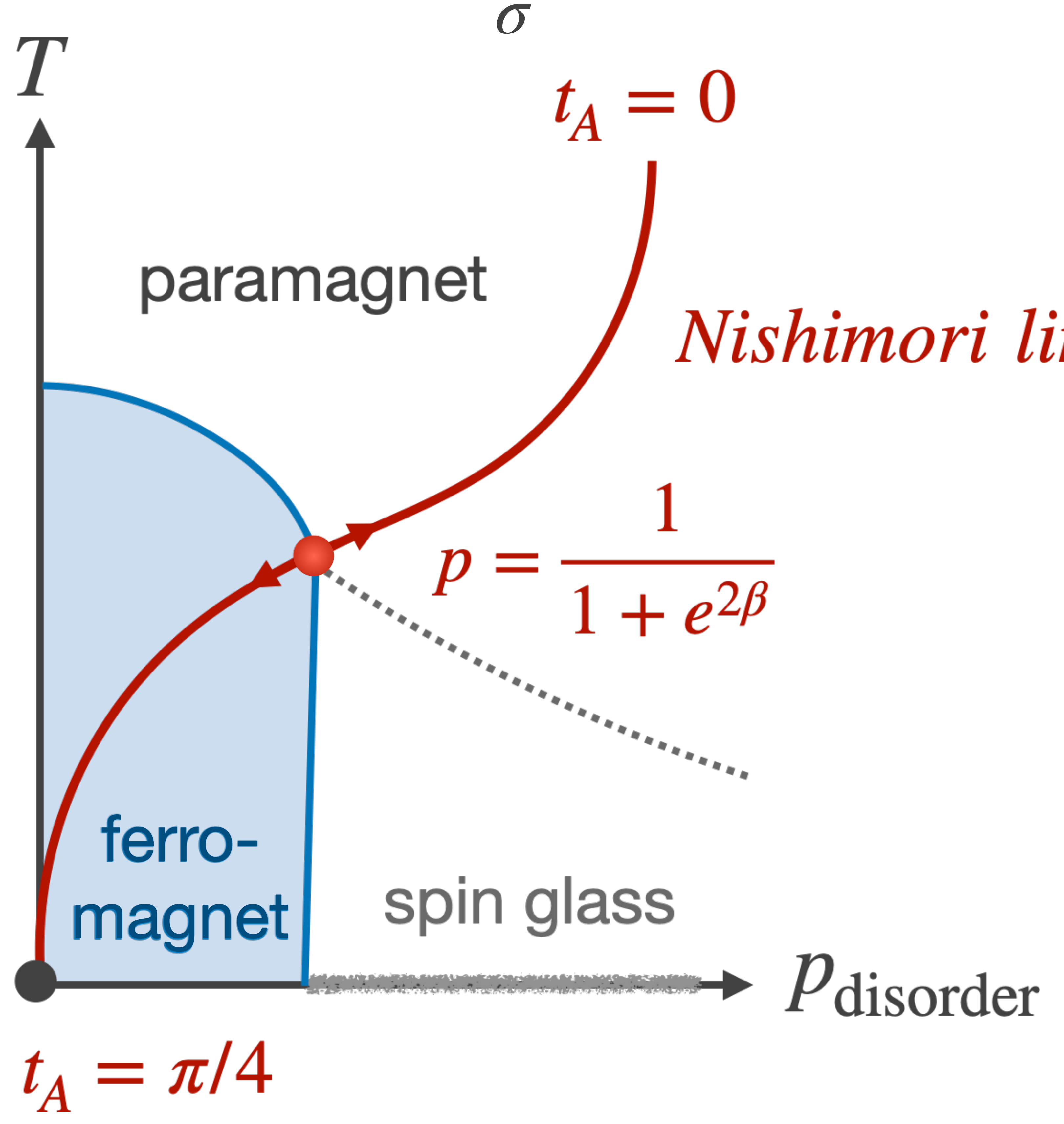

**entire** phase diagram is mapped to **Nishimori line**

$$\tilde{p} = \frac{1-(1-2p_s)\sin(2t_A)}{2}$$

# **Nishimori** criticality

map to **random bond Ising model**

$$P(s) = \sum_{\sigma} e^{-\beta \sum_{ij} s_{ij} \sigma_i \sigma_j}$$




$$\rho = \sum_{\mathbf{s}} \underbrace{|\mathbf{s}\rangle\langle\mathbf{s}|}_{Alice} \otimes \underbrace{M_{\mathbf{s}} |\Psi\rangle\langle\Psi| M_{\mathbf{s}}}_{Bob}$$

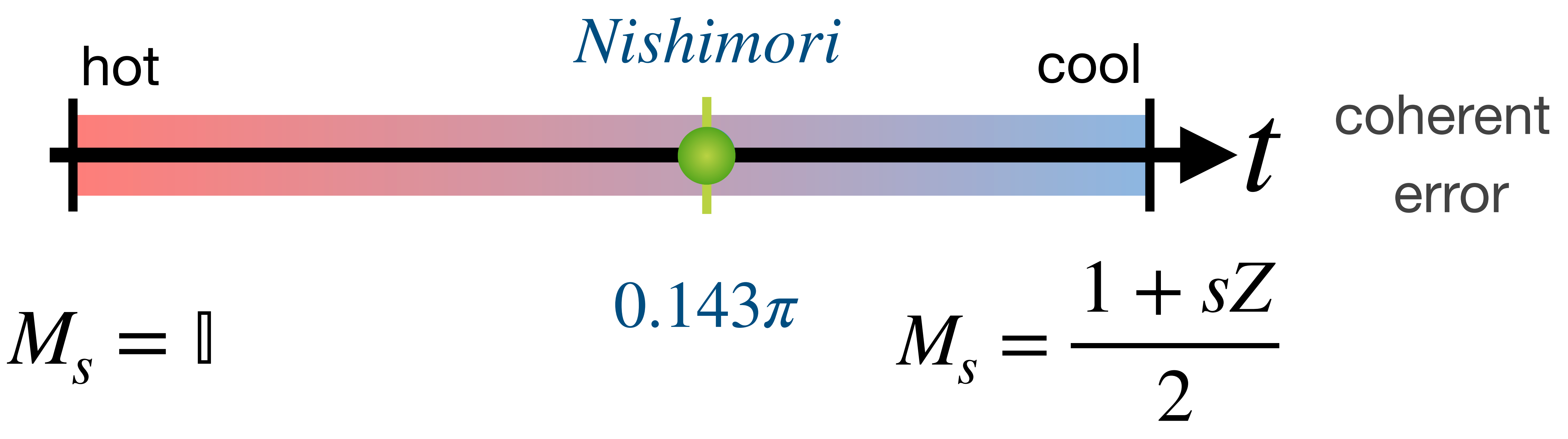
$\theta = 0$

$M_s = e^{\frac{\beta}{2} sZ}$

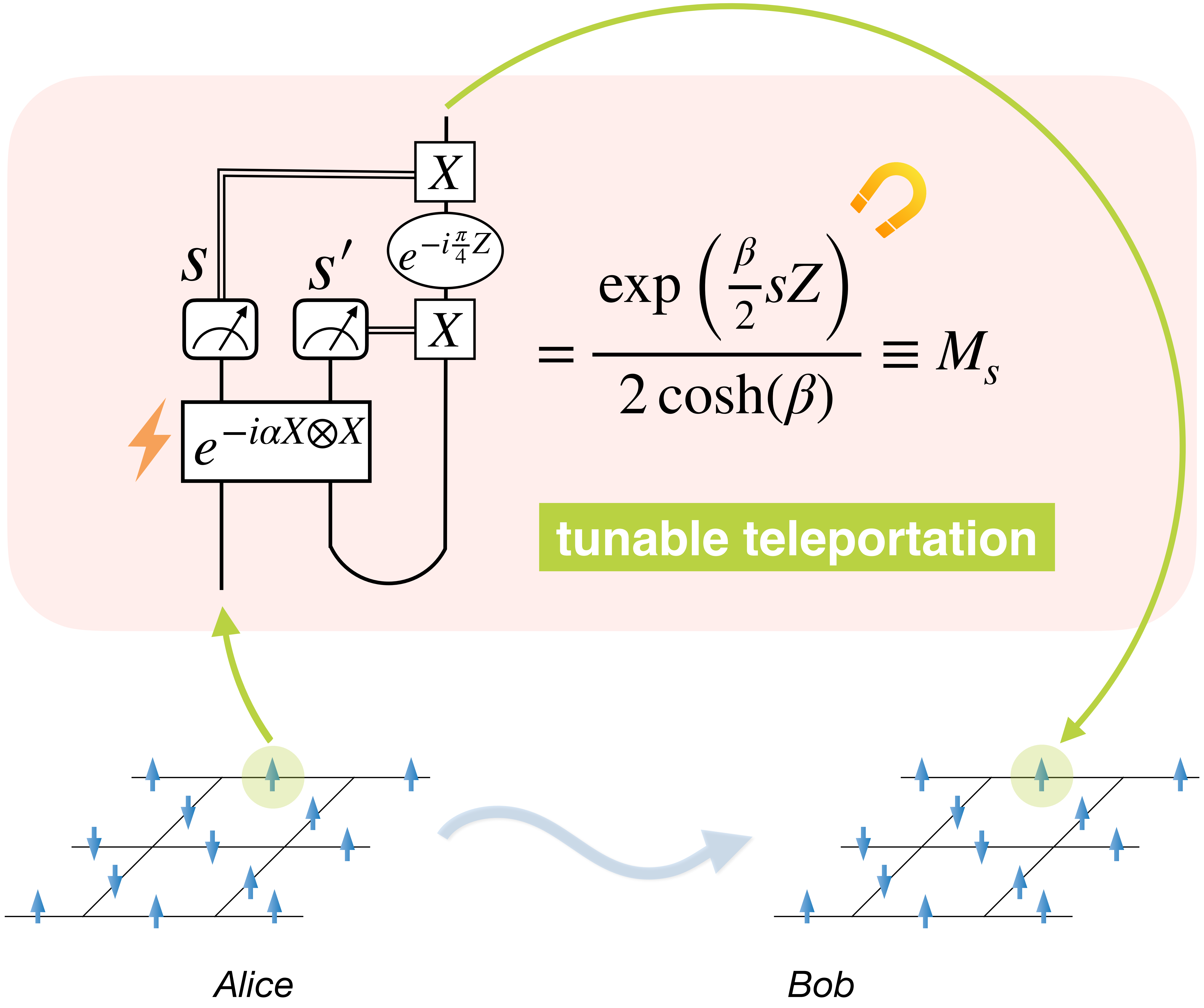
**Born's rule** $\langle\Psi | M_{\mathbf{s}}^2 | \Psi\rangle$

**disorder** $P(s)$

**temperature** $Z(s)$

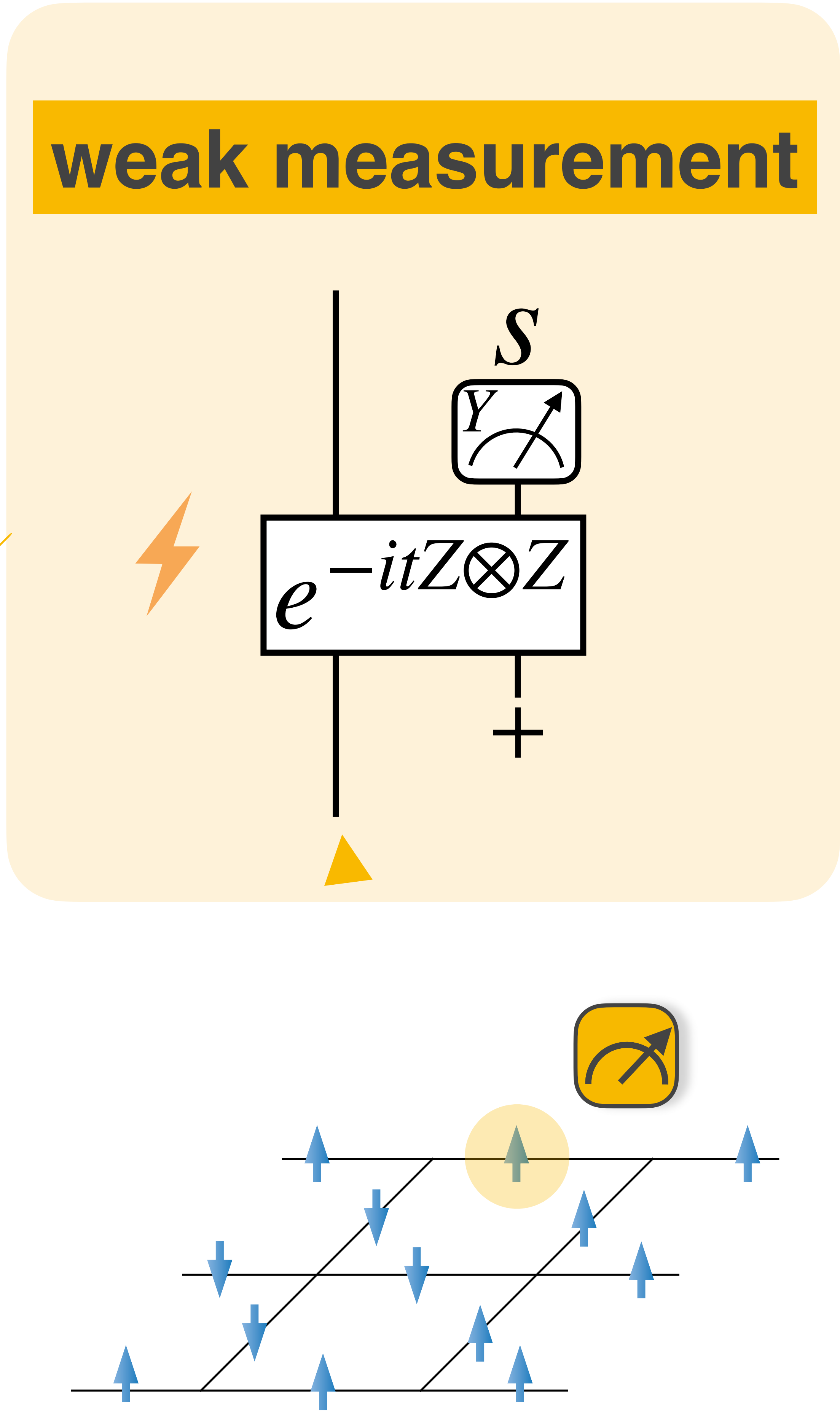


# **physical qubits** / teleportation vs. measurement

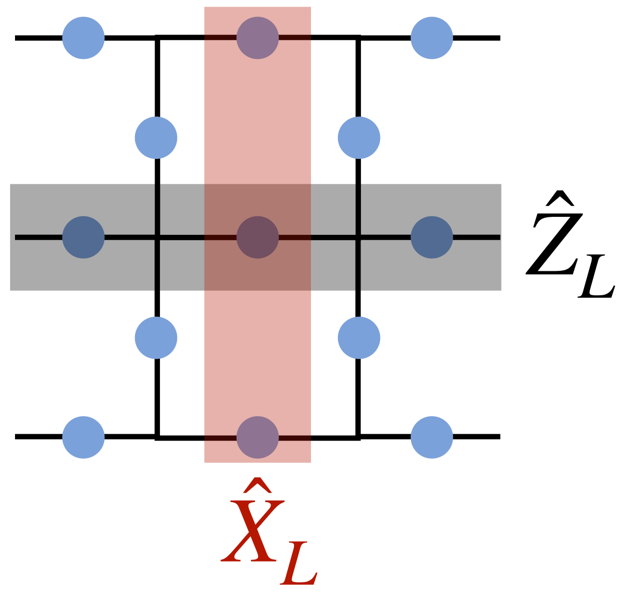



$$= \frac{\exp\left(\frac{\beta}{2}sZ\right)}{2\cosh(\beta)} \equiv M_s$$

$\Leftrightarrow$



# **logical qubits** / anyon condensation



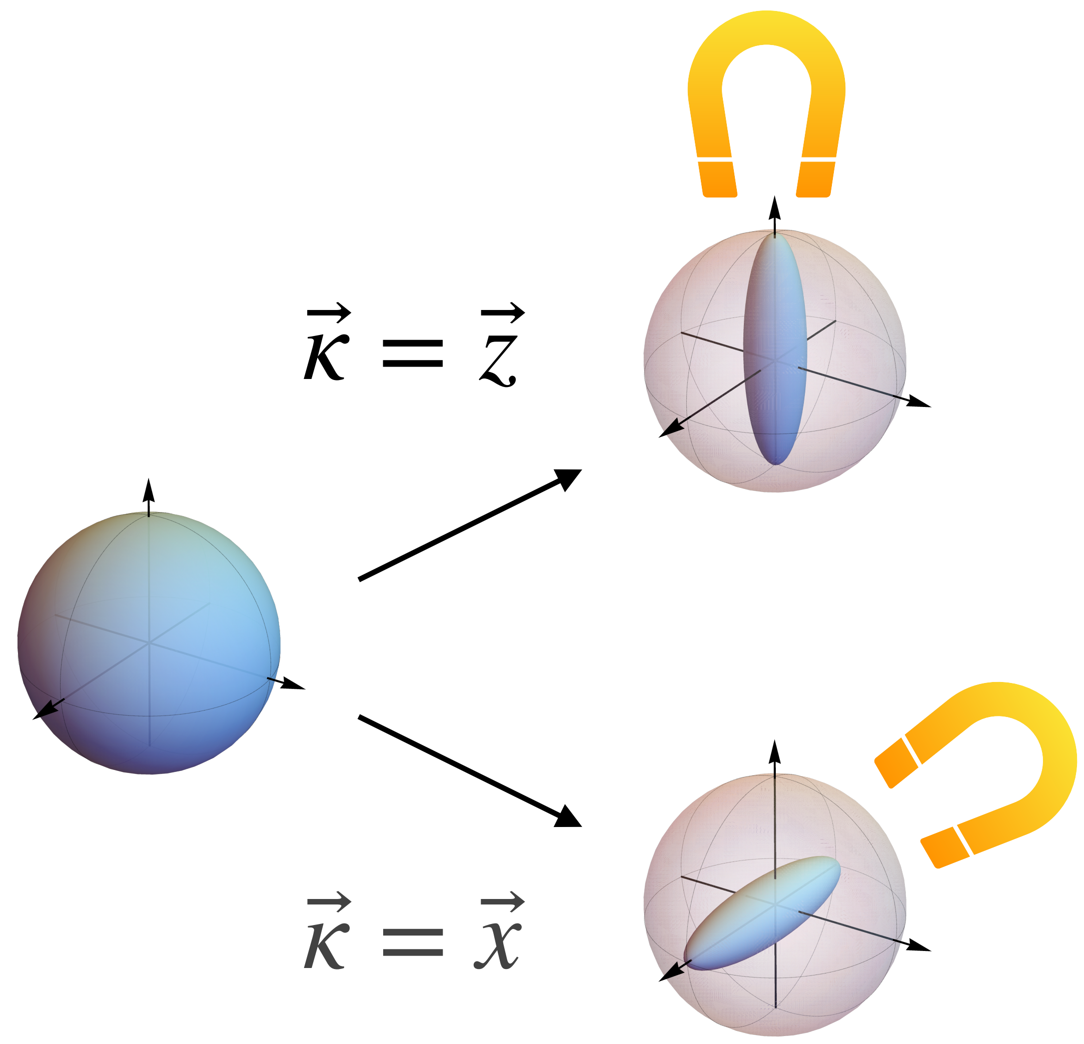

- **deformation** in logical space $\quad P_{\mu\nu}(\mathbf{s}) := \langle \psi_\mu \,|\, M_{\mathbf{s}}^\dagger M_{\mathbf{s}} \,|\, \psi_\nu \rangle$

- logical **density matrix**

$$\rho_R(\mathbf{s}) = \frac{1}{2P(\mathbf{s})} \begin{pmatrix} P_{++}(\mathbf{s}) & P_{+-}(\mathbf{s}) \\ P_{+-}^*(\mathbf{s}) & P_{--}(\mathbf{s}) \end{pmatrix} = \frac{1 + \vec{\kappa} \cdot \vec{\sigma}}{2}$$

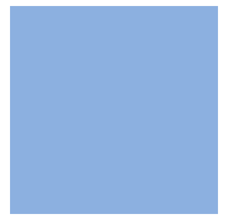
- **polarization / purification** of logical qubit

$$\vec{\kappa} = \left( \frac{P_{++} - P_{--}}{P_{++} + P_{--}} \,,\; \frac{2\,|P_{+-}|}{P_{++} + P_{--}} \right)$$



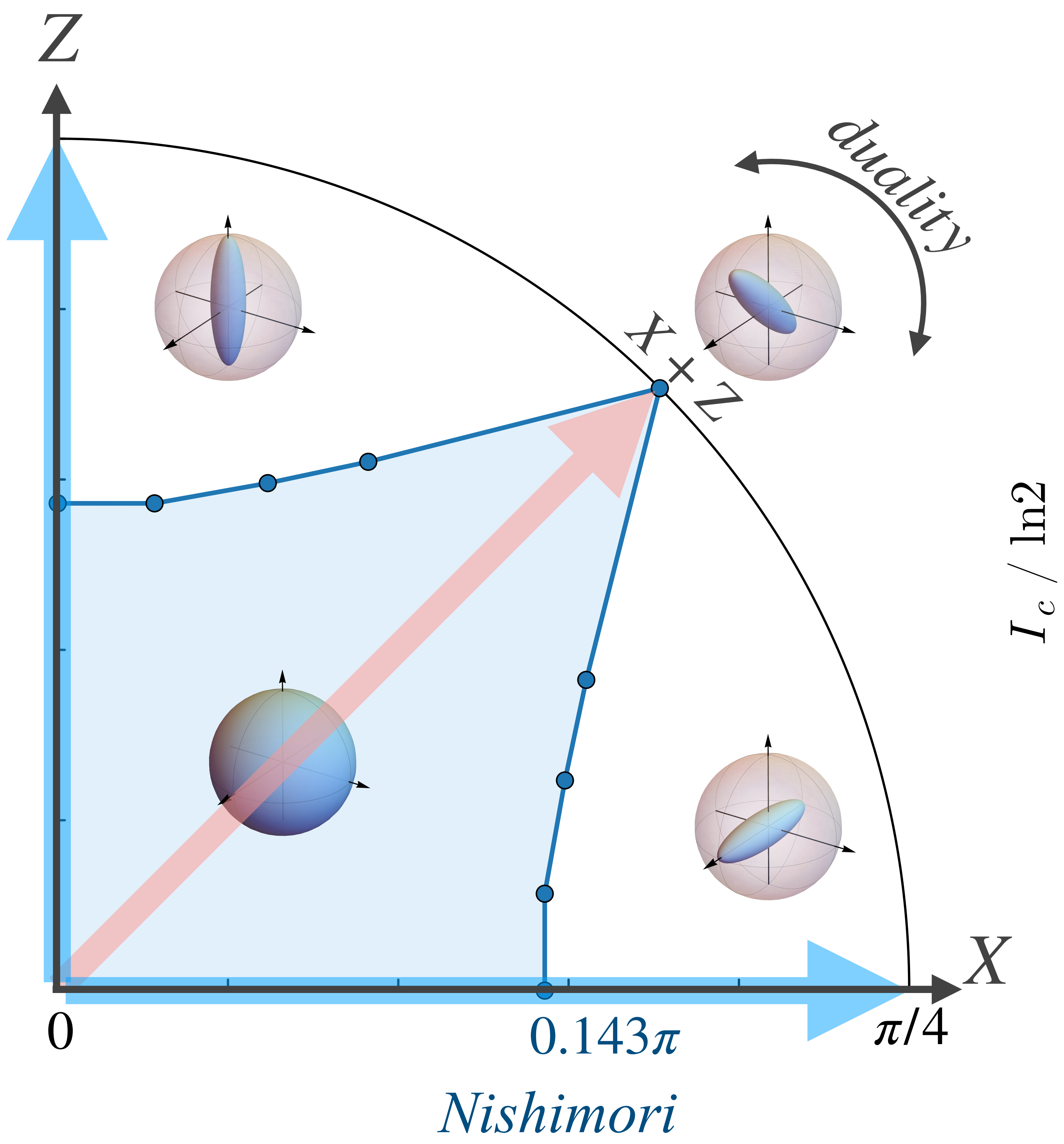

- **anyon mechanism** $\quad$ *confinement* $\quad$ *condensation*

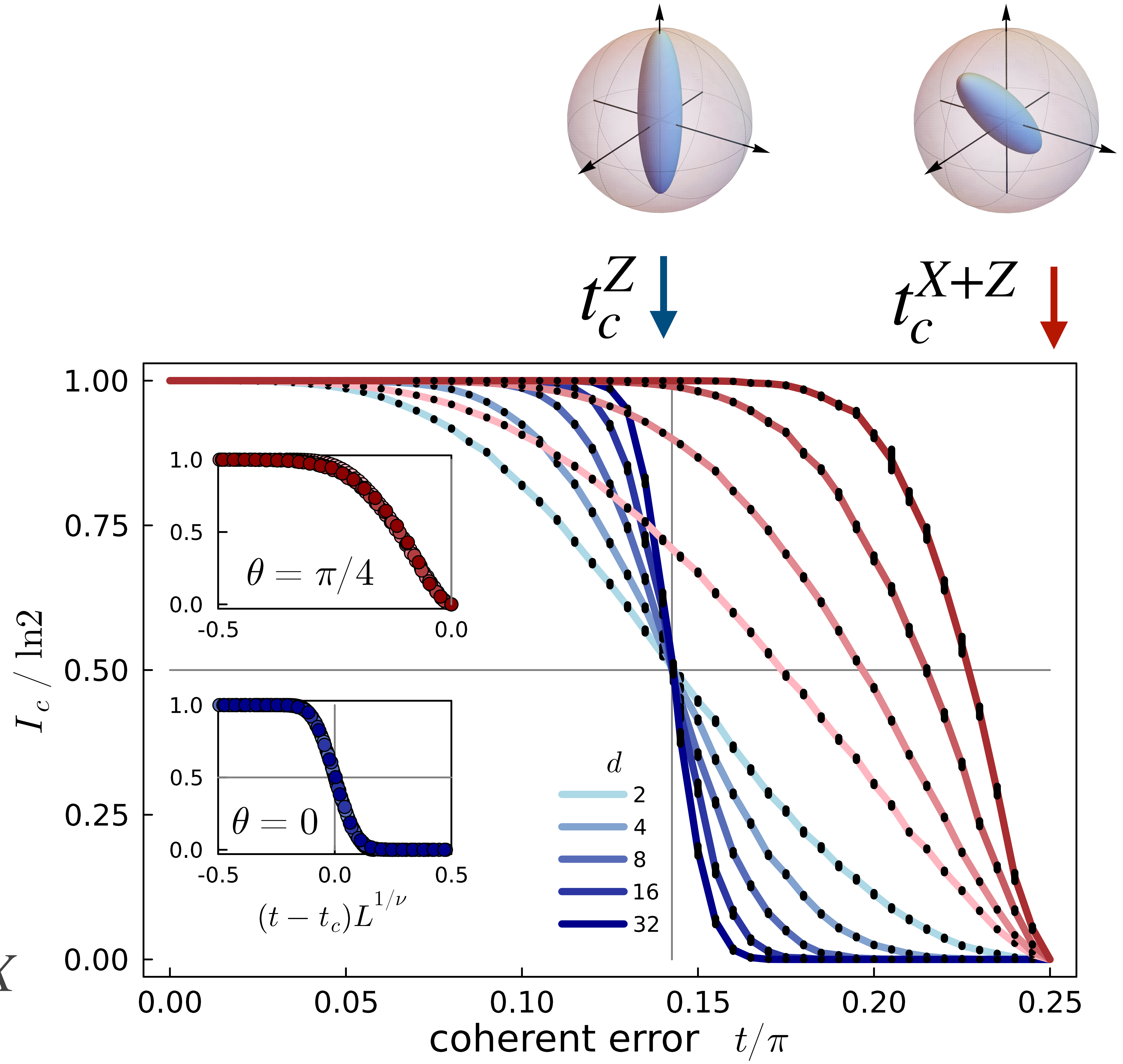
$\cancel{e}$ $\qquad \psi \sim vac + e + ee + \ldots$

$|\psi_+\rangle =$ 

$|\psi_-\rangle = e$  $e$

# **thresholds** / phase transitions



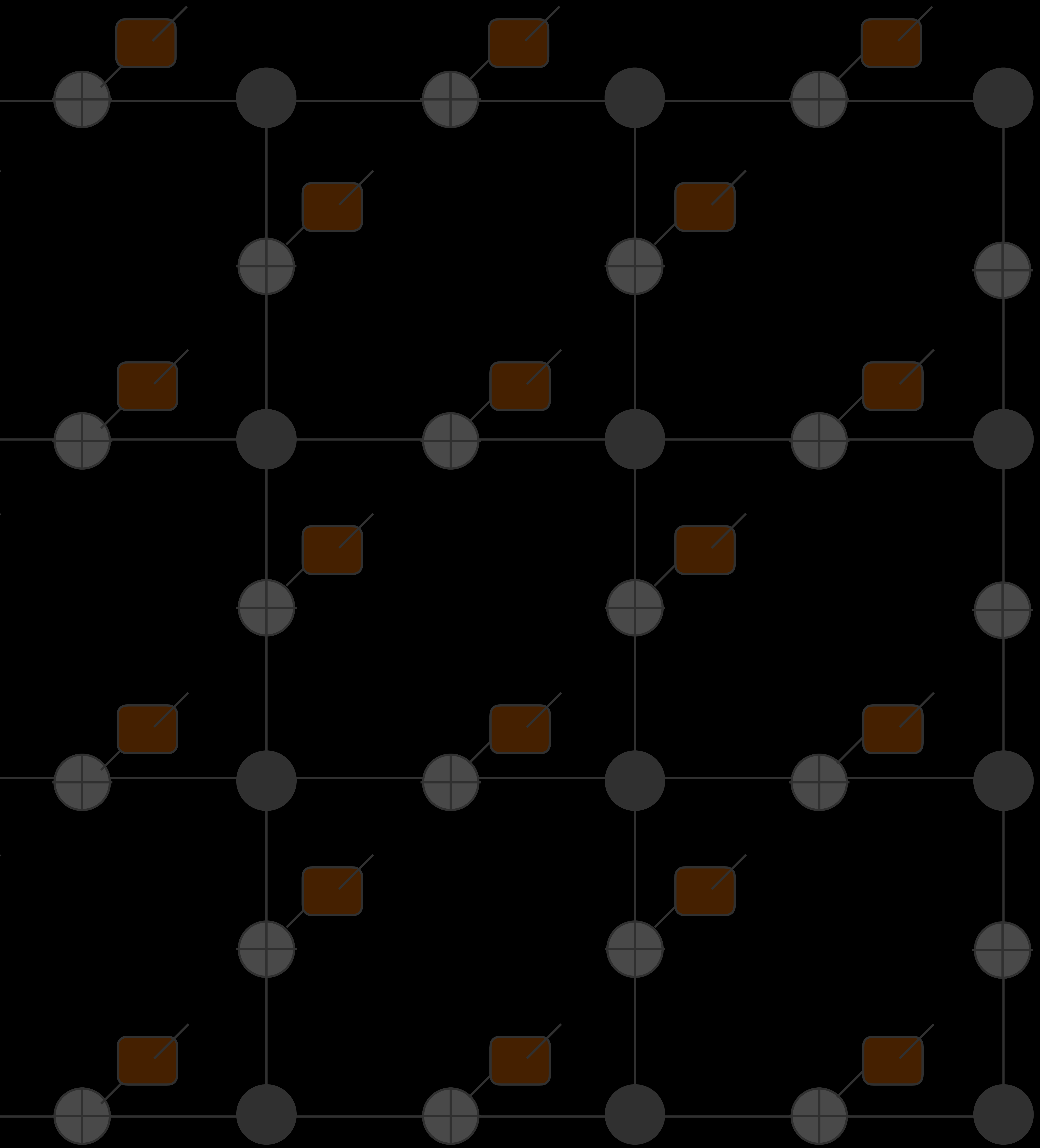




$\nu = 1.6(1)$ $\nu = 1.8(1)$

angle dependence due to **competition** of **anyon condensation**

**self-dual** direction
**∞ threshold**

teleportation succeeds even for **infinitesimal coupling**



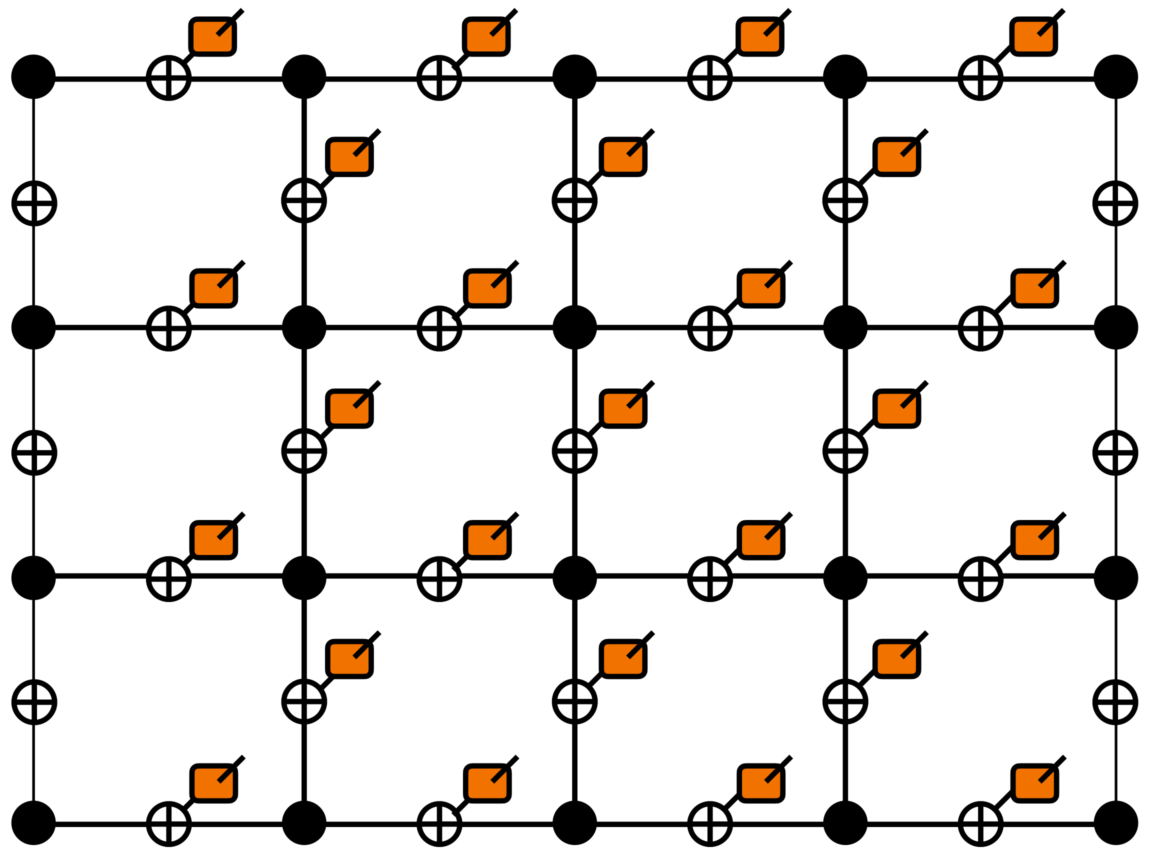
**stat mech perspective**

# tensor network & statistical model

## (2+0) dimensional

**deformed wave function**

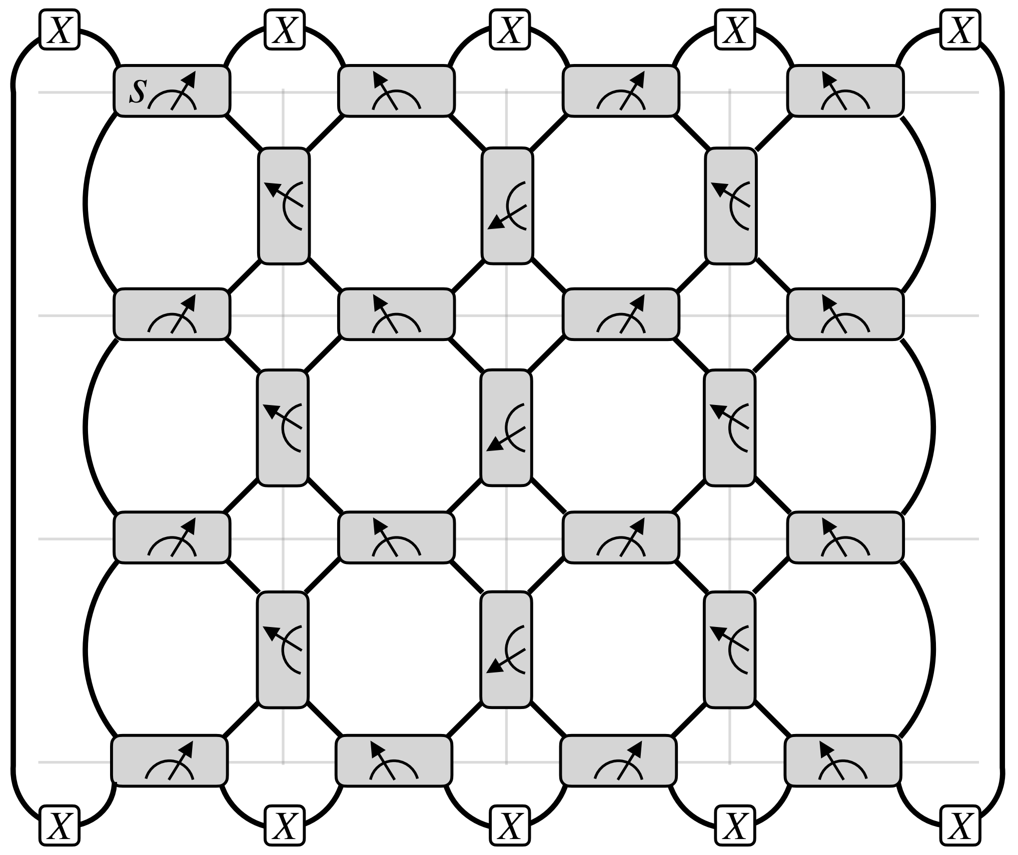
$|\psi(\mathbf{s})\rangle =$



## (1+1) dimensional

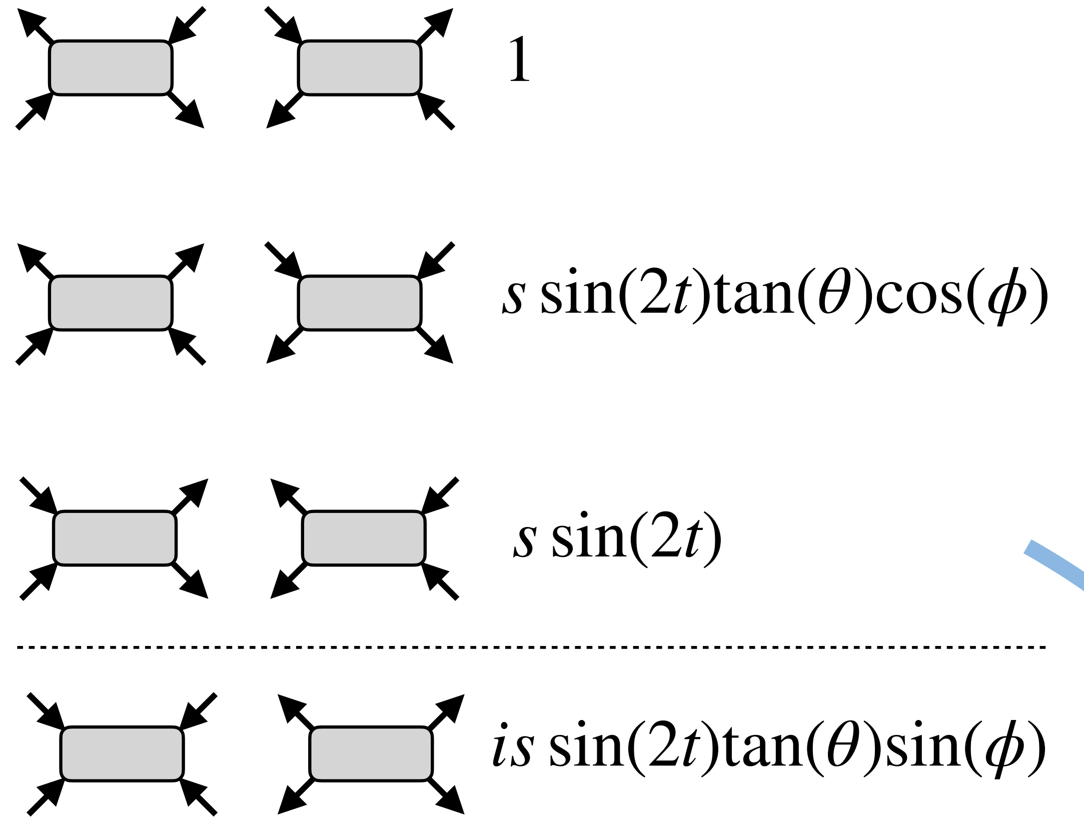
**random circuit**

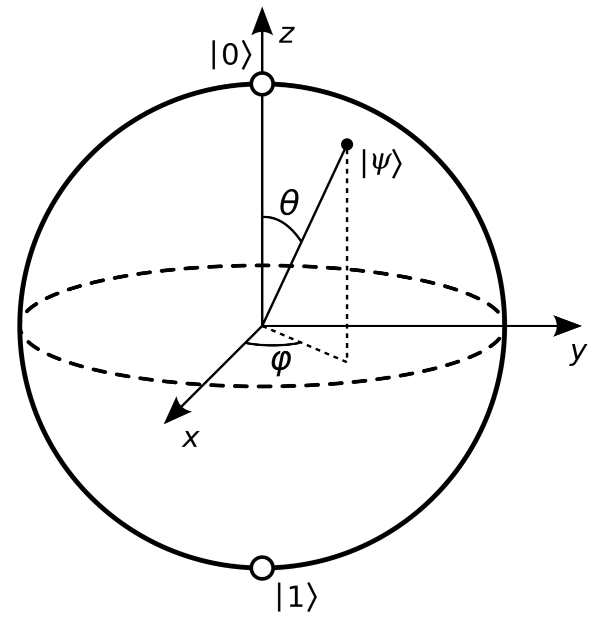
$\langle\psi(\mathbf{s})|\psi(\mathbf{s})\rangle =$

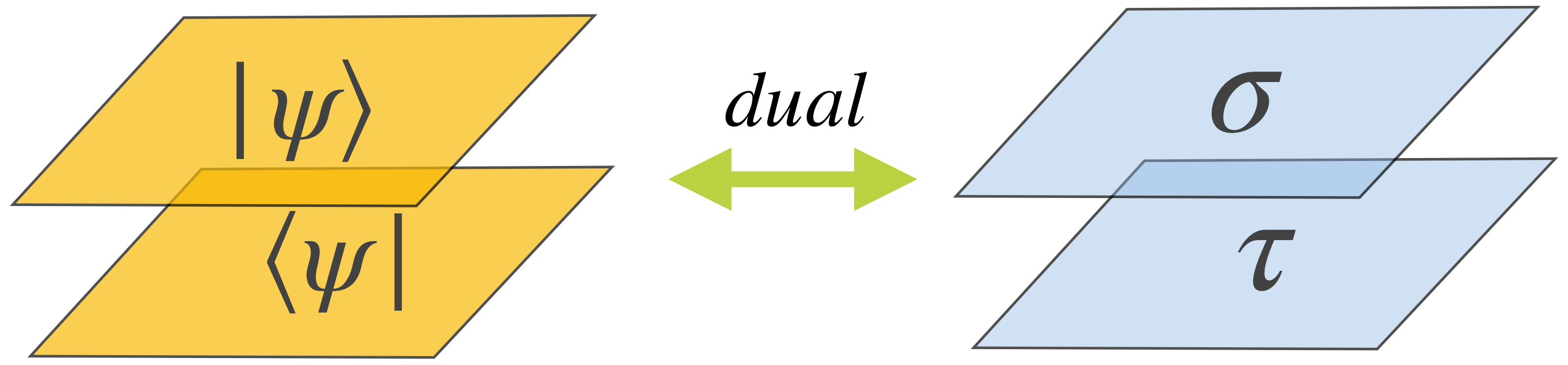



## 2-dimensional

**classical 8-vertex model**




$\tanh J = \sin(2t)\cos(\theta)$

$e^{-2K} = \sinh(J)\tanh(\theta)$

**Ashkin-Teller model**

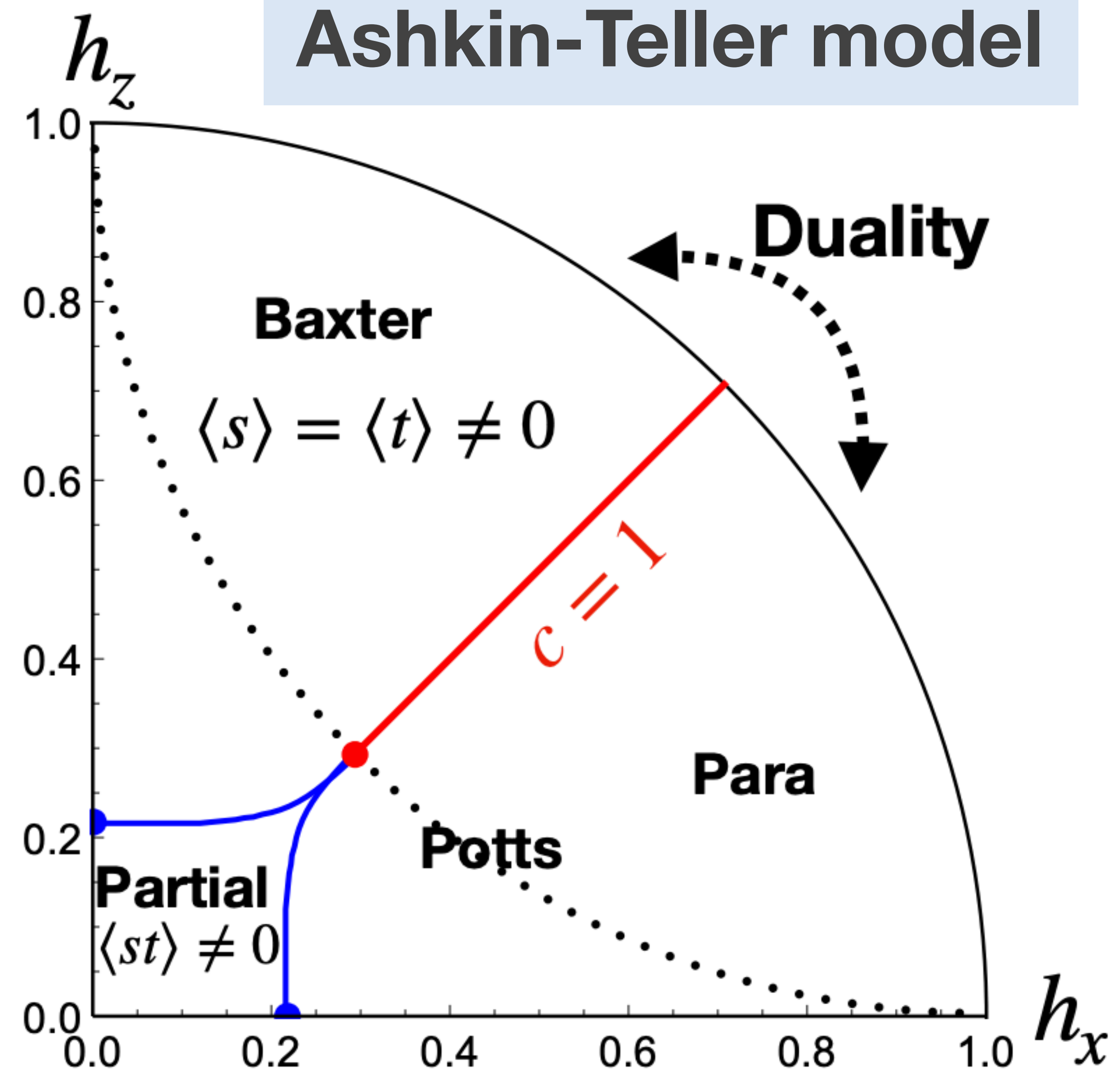
$$-E_{ij} = Js_{ij}\frac{\sigma_i\sigma_j + \tau_i\tau_j}{2} + i\phi\frac{\sigma_i\sigma_j - \tau_i\tau_j}{2} + \left(2K + i\pi\frac{1-s_{ij}}{2}\right)\frac{\sigma_i\sigma_j\tau_i\tau_j - 1}{2}$$

# ∞-**replica** model / post-selection

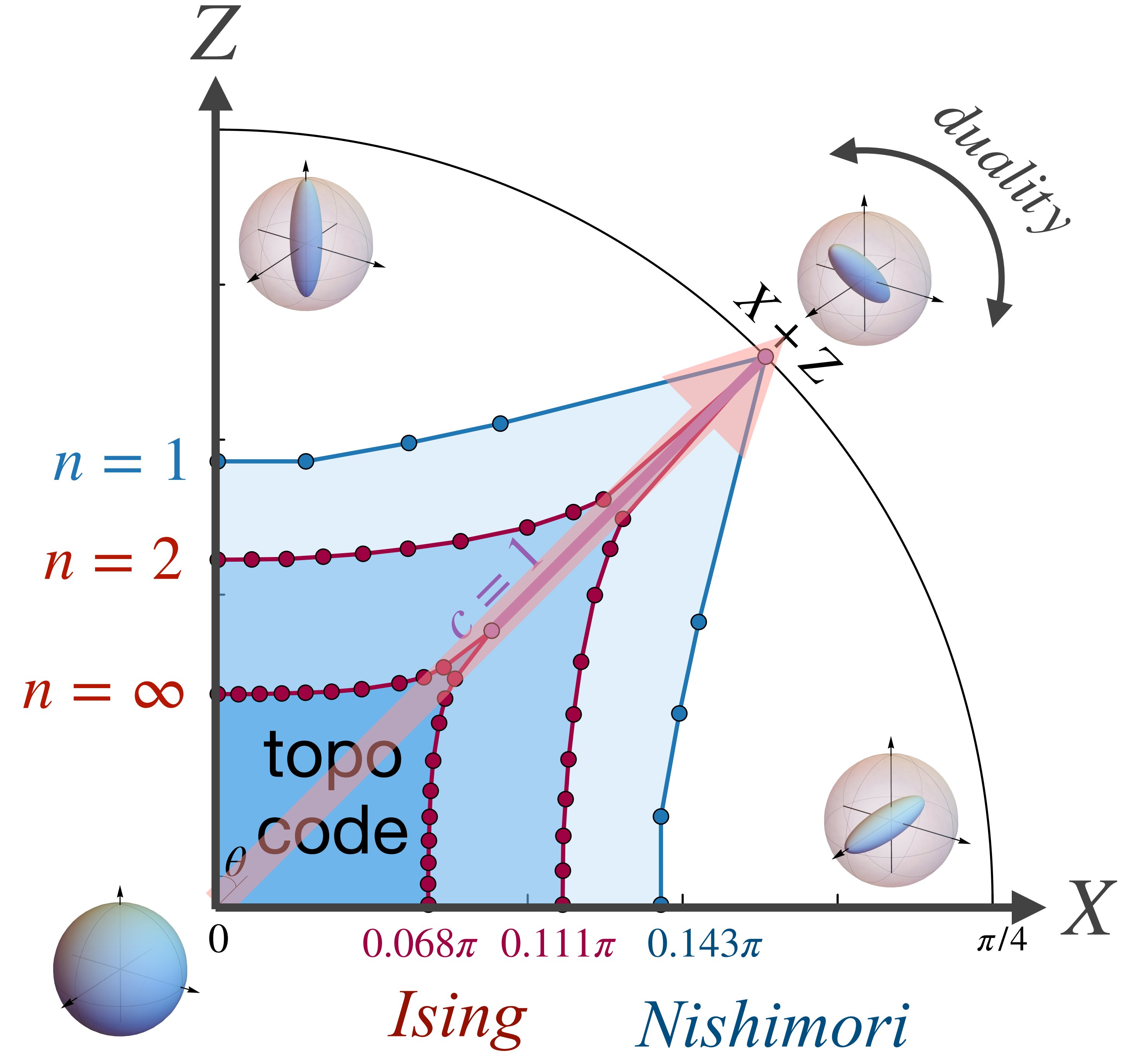
$\sum_{\mathbf{s}} P(\mathbf{s})^{\infty}$ distills out **most probable** configuration $s = +1$ **no randomness**

exactly solvable

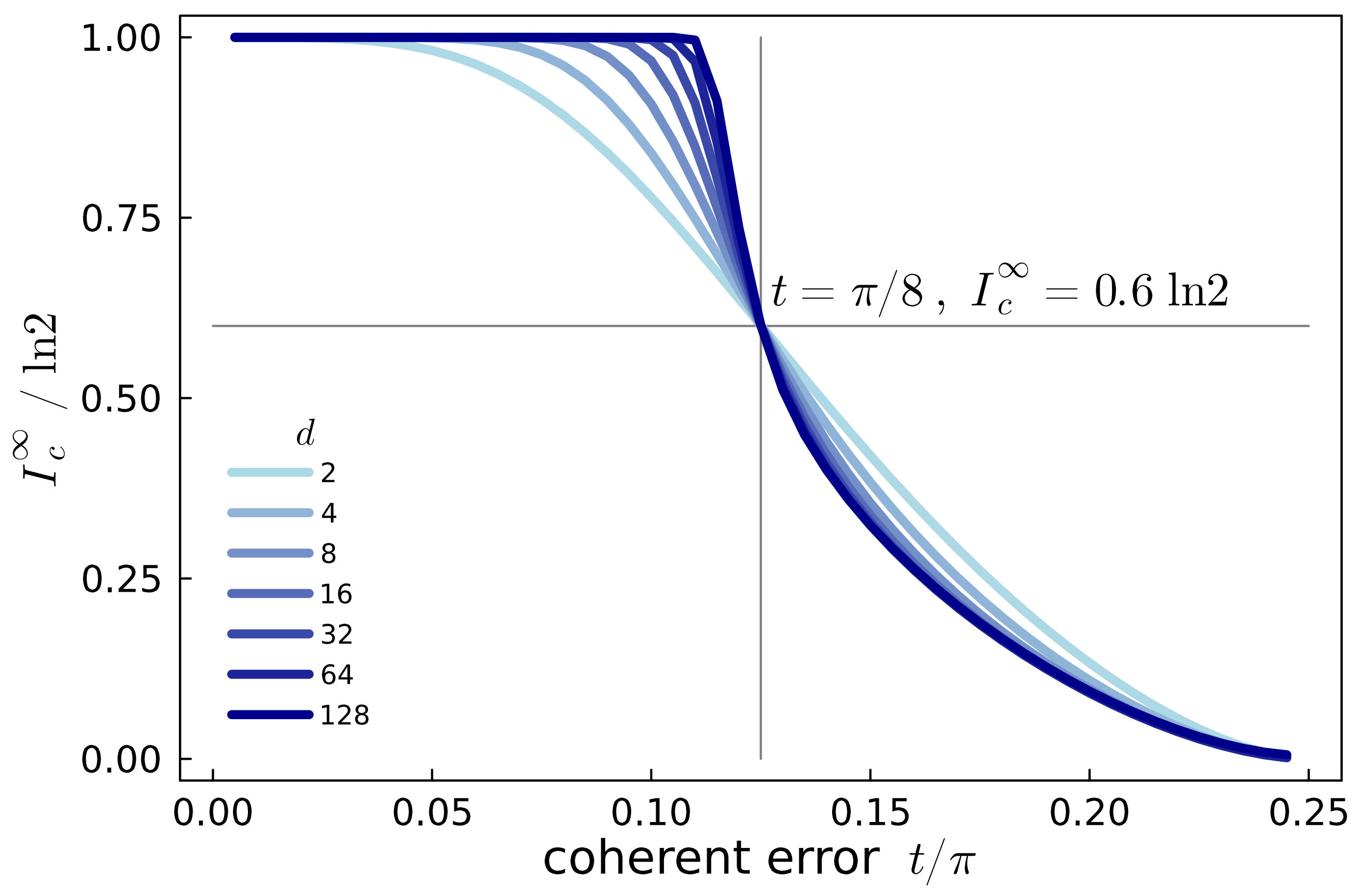
**Ashkin-Teller model**




Baxter 1982




**Kosterlitz-Thouless transition**





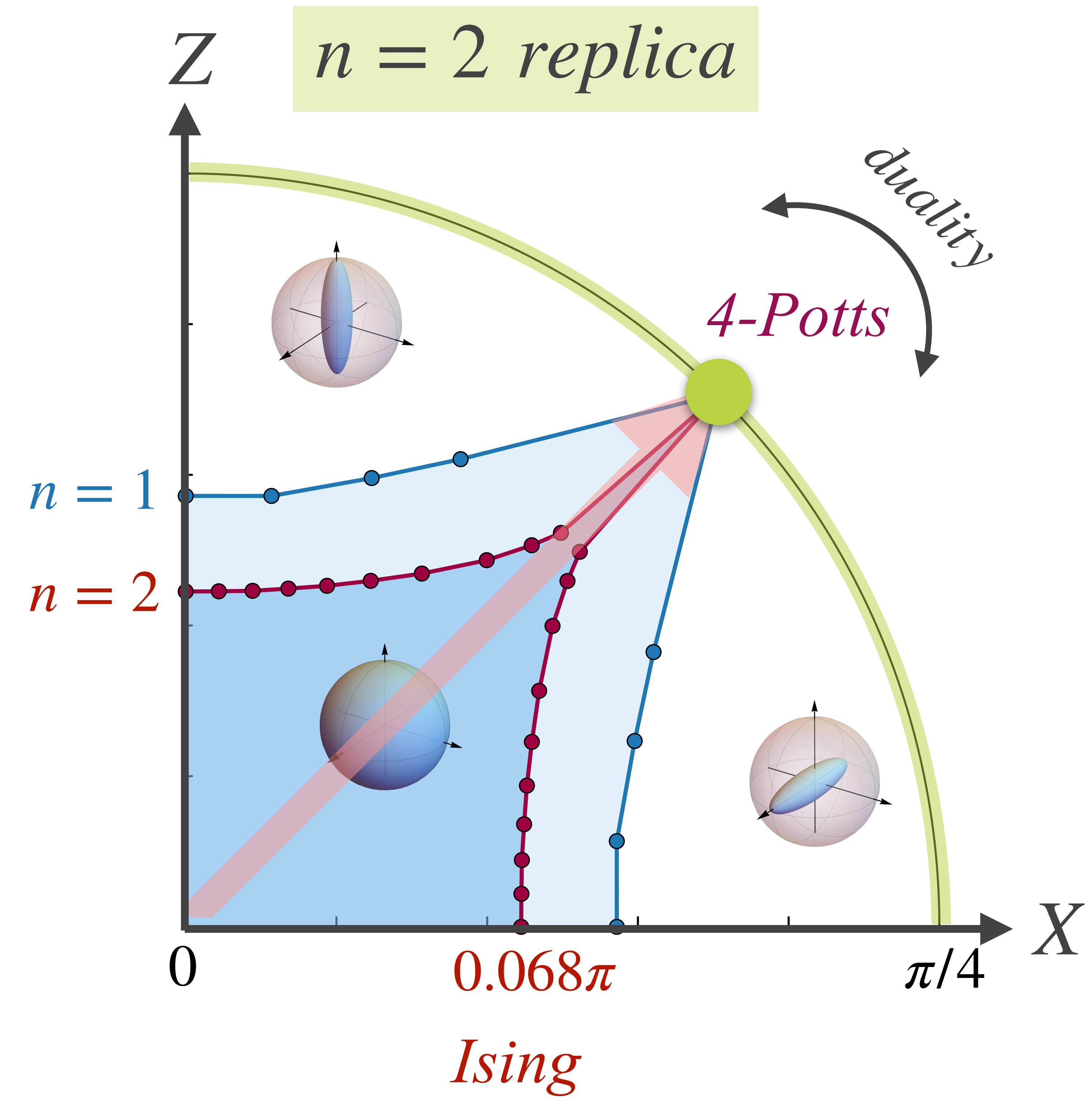
**wave function deformation**

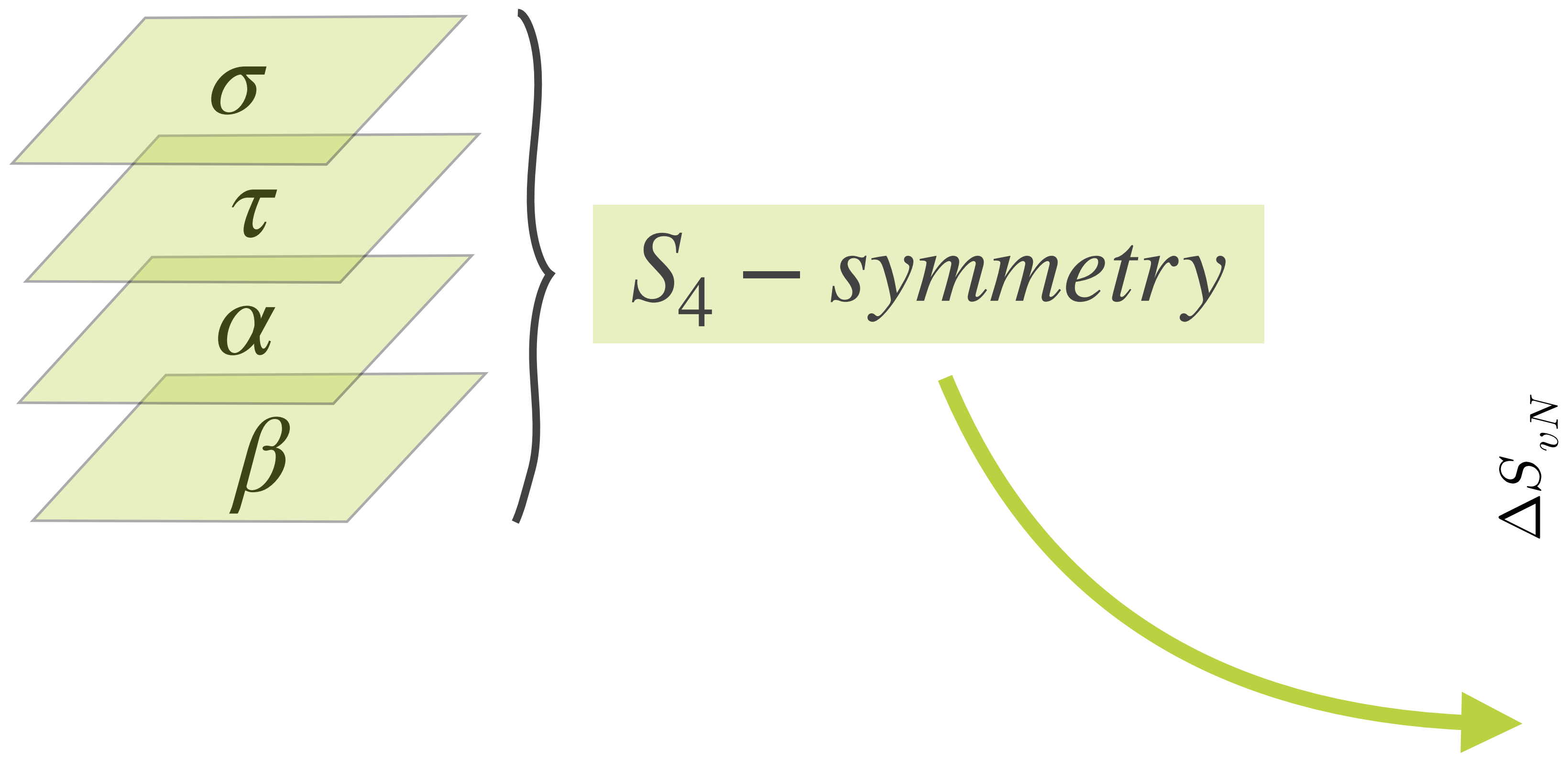
Ardonne, Fendley, Fradkin 2004; Zhu & Zhang, PRL 2019

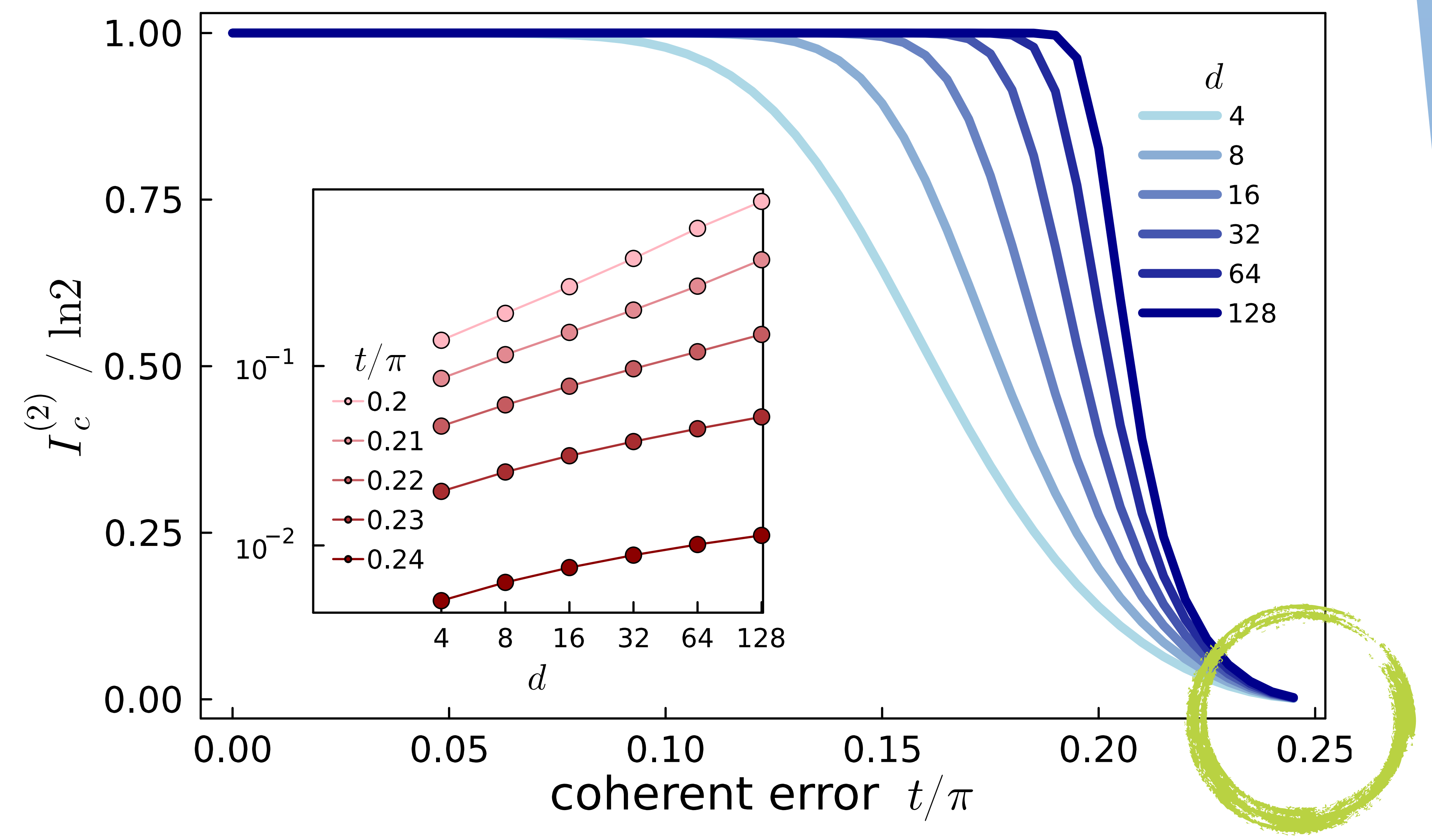
# **2-replica** model

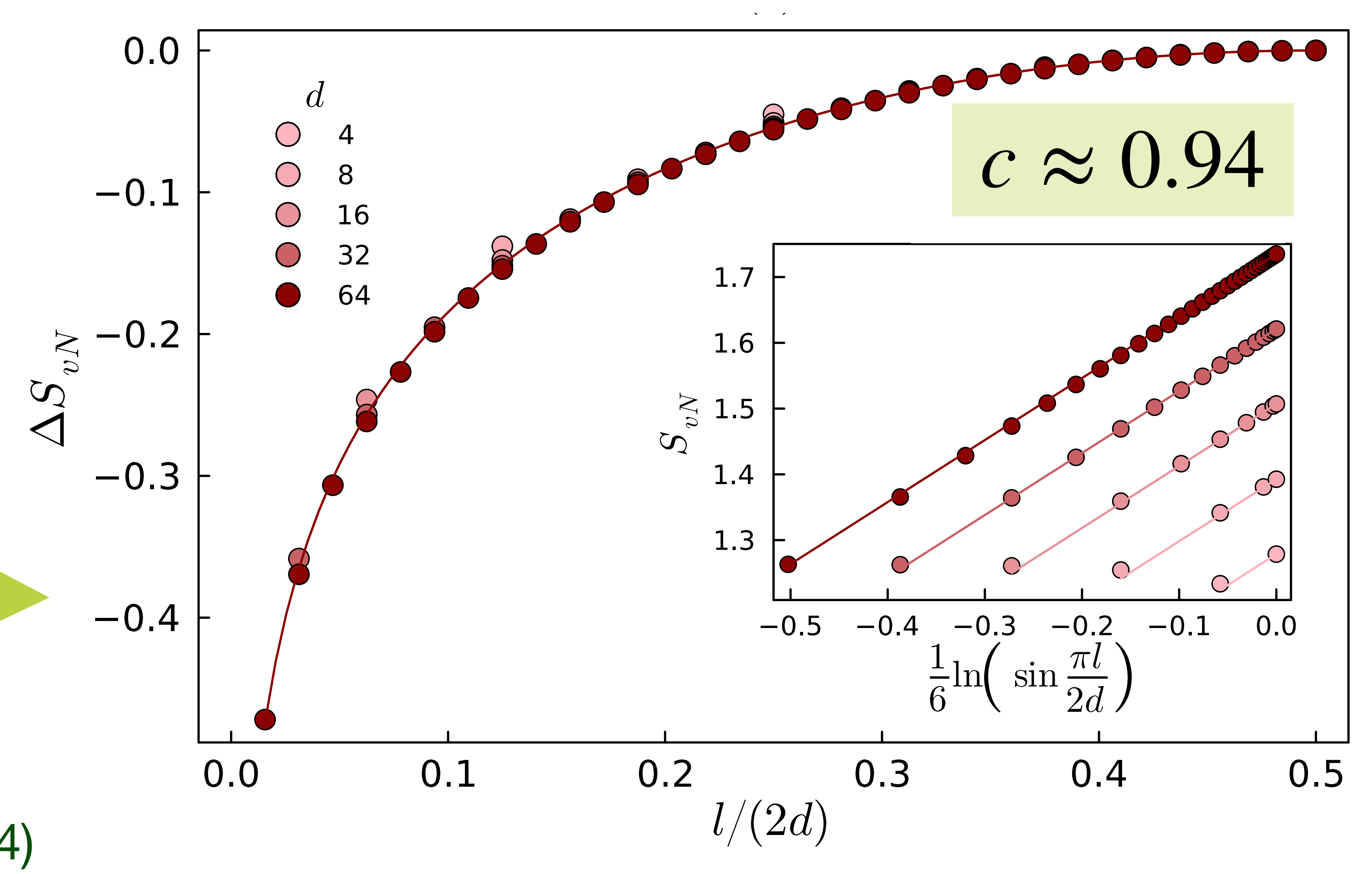
**n-th order coherent information**

$$I_c^{(n)} = \frac{1}{1-n} \ln \frac{\mathrm{tr}(\rho_{RA}^n)}{\mathrm{tr}(\rho_A^n)} = \frac{1}{1-n} \ln[\mathrm{tr}\rho_R(\mathbf{s})^n]_n$$

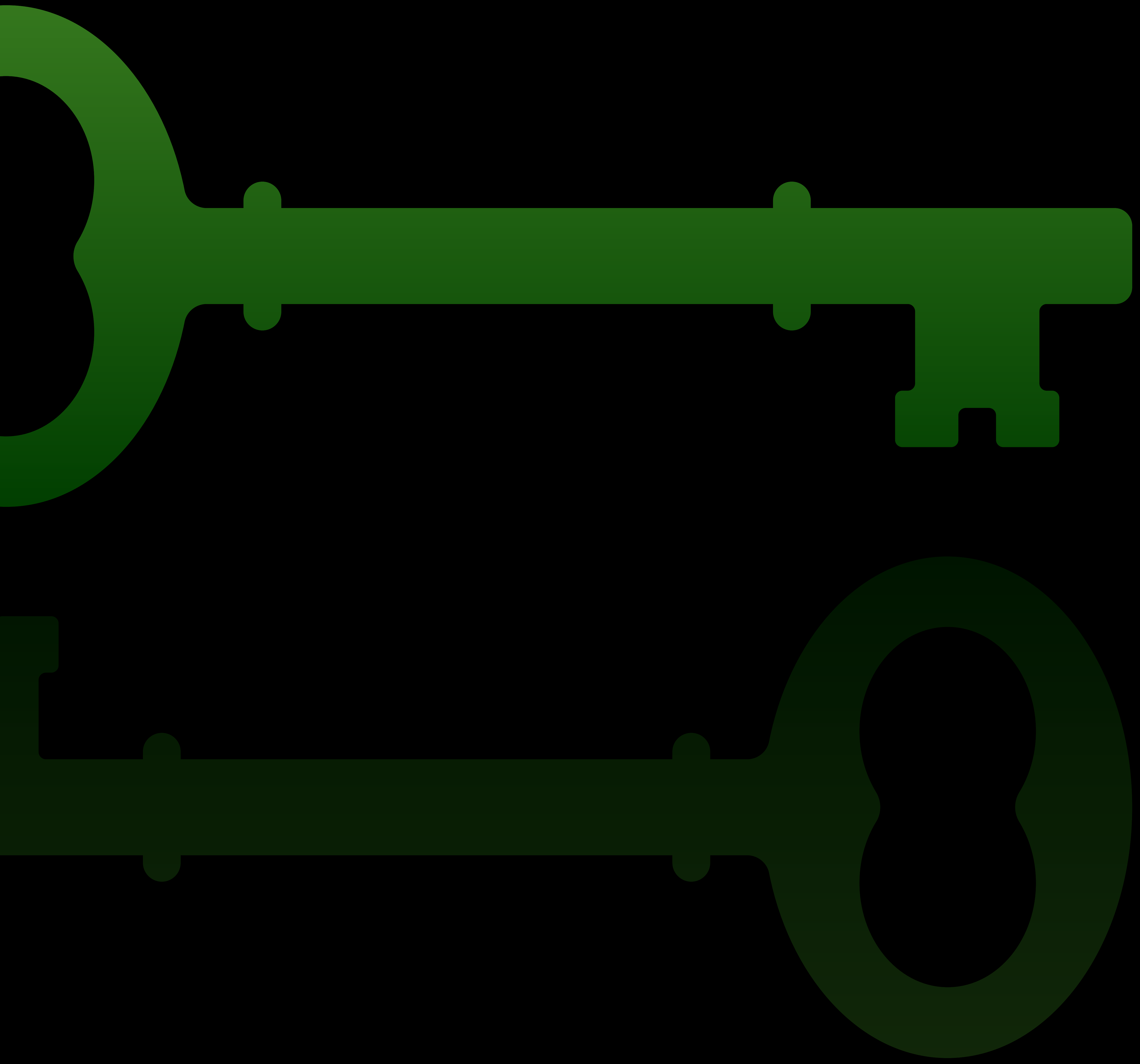



(2-replica study, see also Chen & Grover '24)

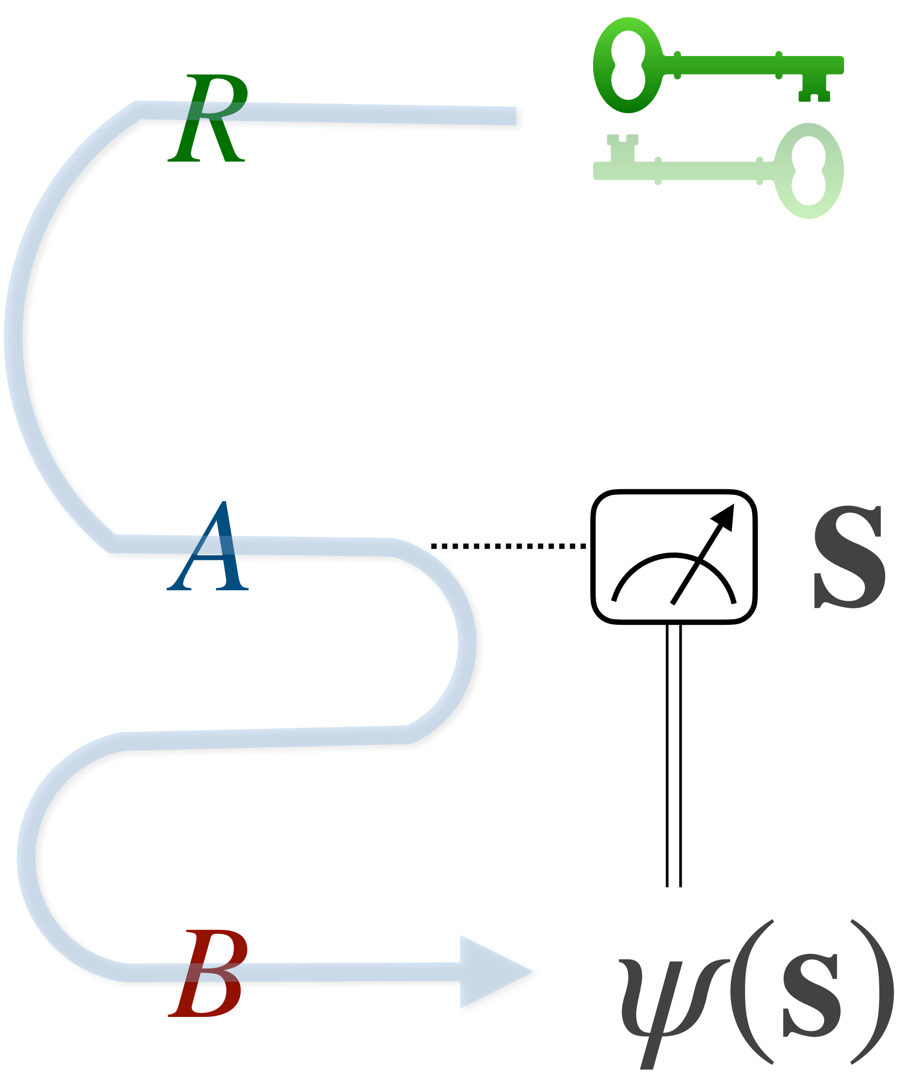


# decoding teleportation

# **decoding** many-qubit teleportation

Who has the key / logical qubit?

- A quantum key **cannot be cloned.**
- Does the logical information **leak to Alice** or **flow to Bob** ?




*Alice?* $I_c = S_{RA} - S_A$

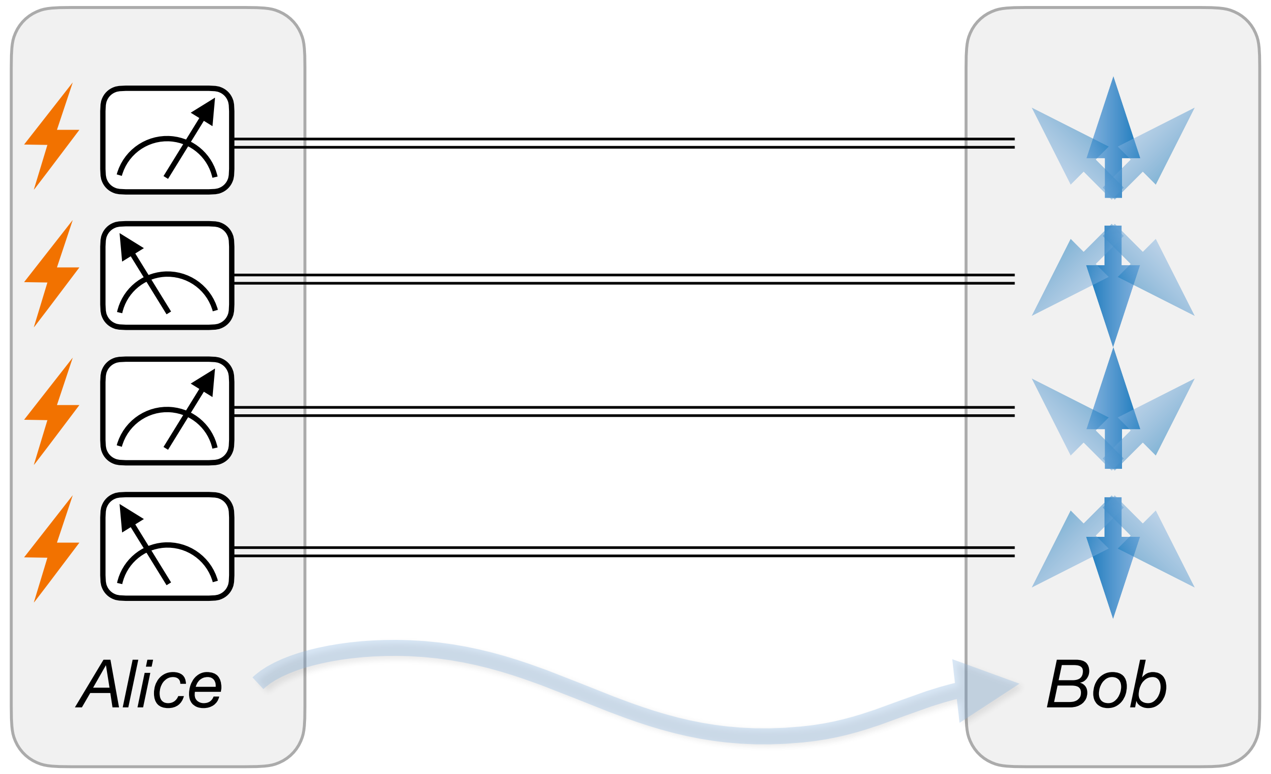
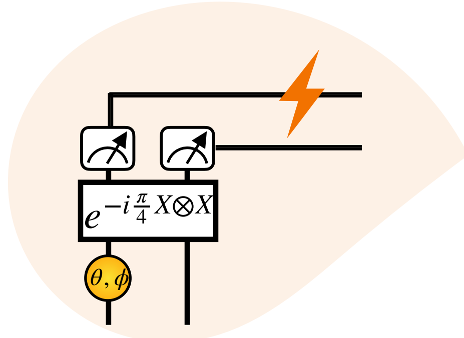
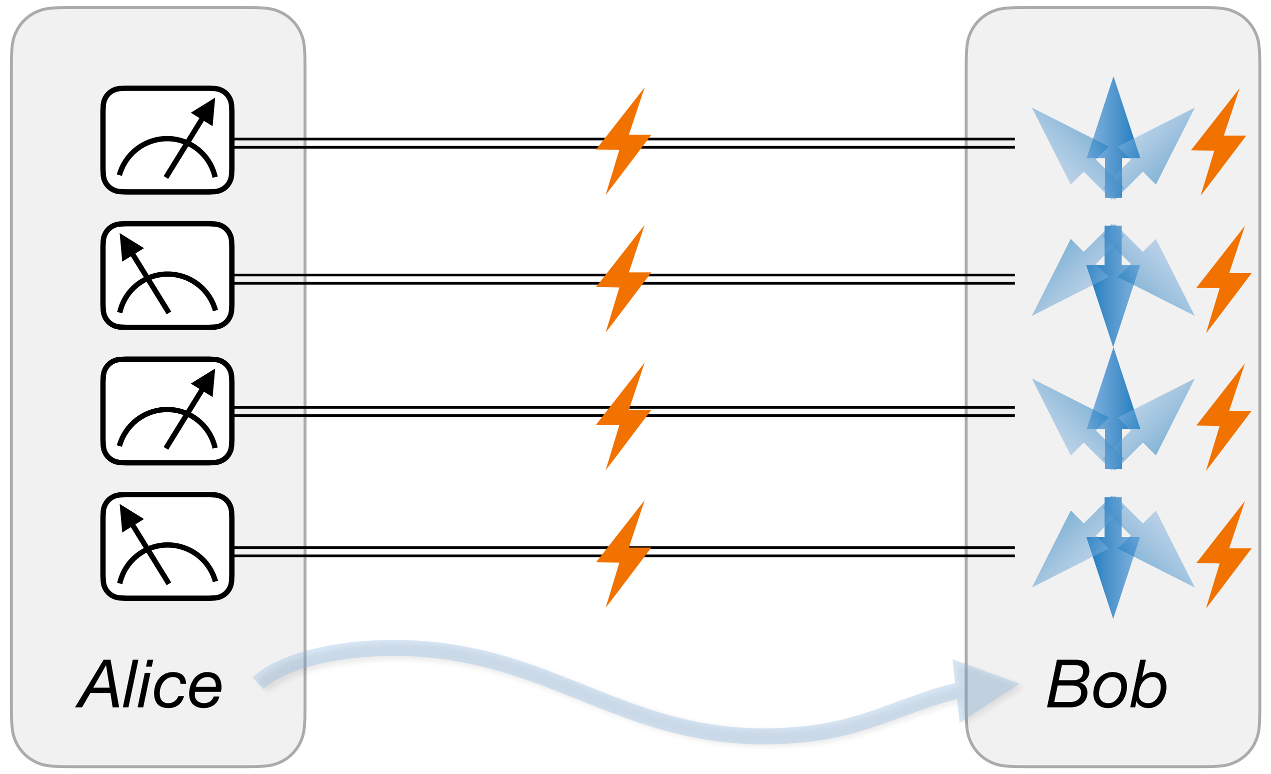
*Scalable* decoder
for entanglement transition
Gullans & Huse, PRL 2020

*Bob?* $I_c = S_{AB} - S_{RAB}$

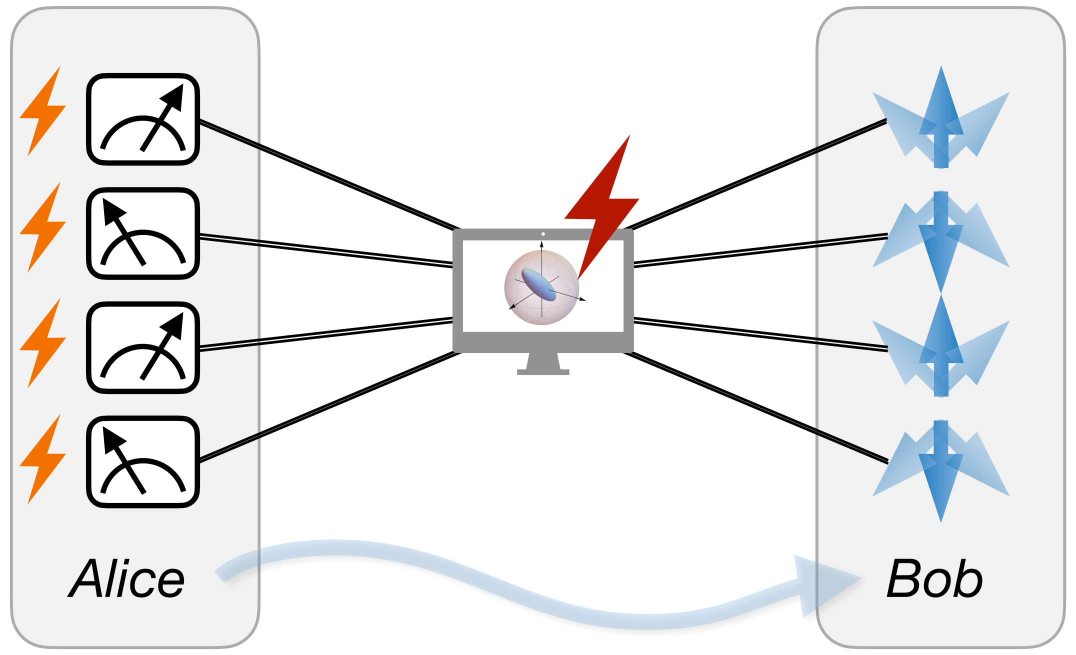
**error correction ?**

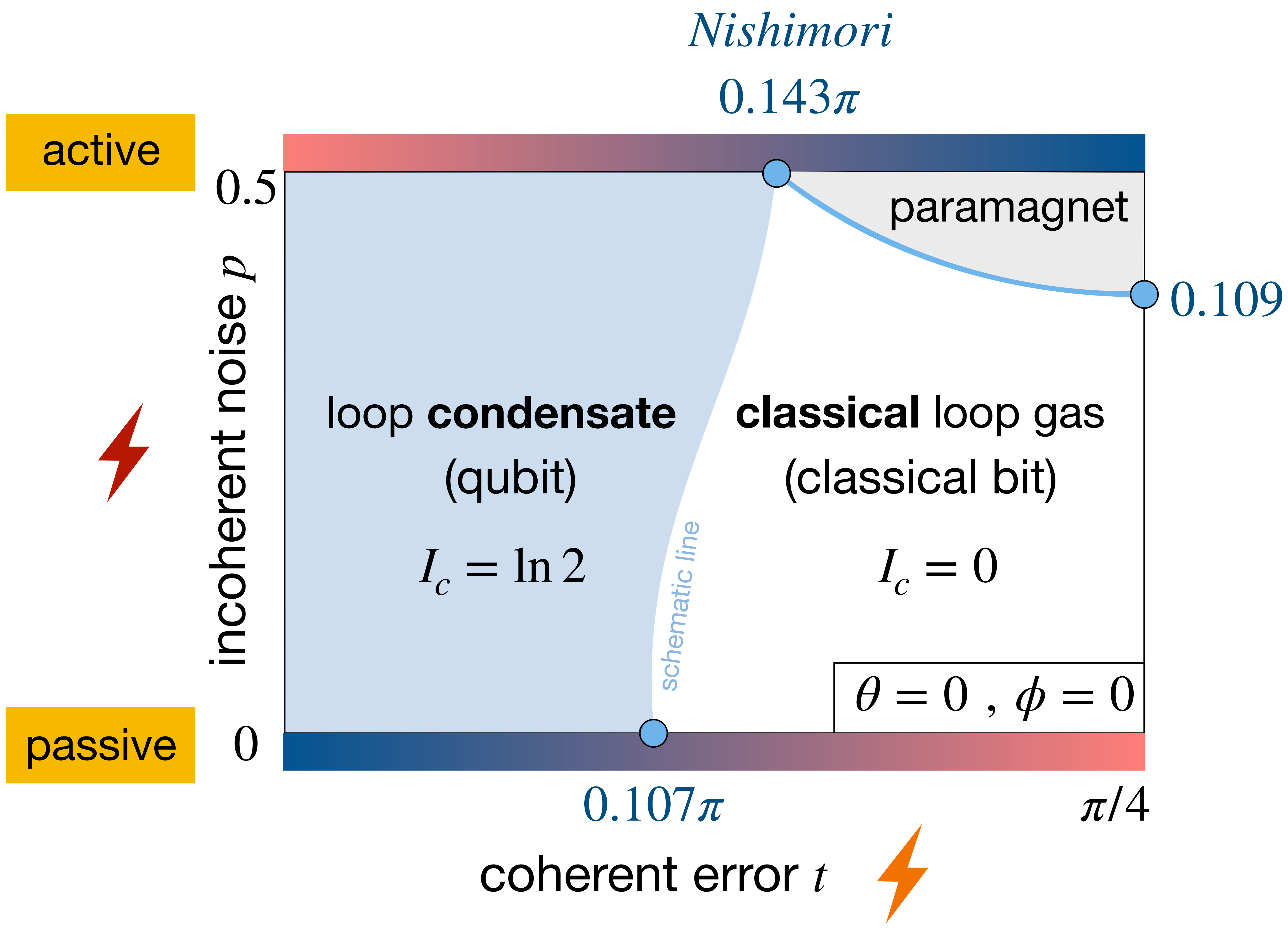
# **decoding** many-qubit teleportation

## **passive** teleportation | **active** teleportation

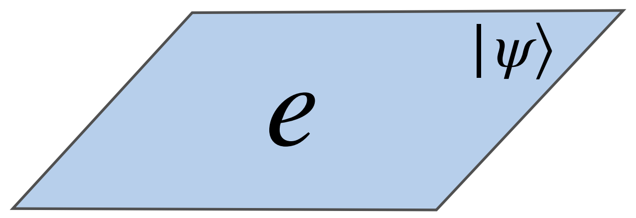
| | **coherent** error w/o decoder | **incoherent** error | **coherent** error w/ decoder |
|---|---|---|---|
| |  |  |  |
| $Z$ error<br>$\theta = 0$ | $t_c = 0.107\pi$ | $p = \sin^2(t)$<br>$p_c = 10.9\,\%$ | $t_c = 0.143\pi$ |
| $\theta = \pi/4$<br>$X + Z$ error | $t_c = \pi/4$ | $p_c = 50\,\%$<br>(2-replica result, see also Chen & Grover ’24) | $t_c = \pi/4$<br>(our finding) |

# interpolating active & passive teleportation

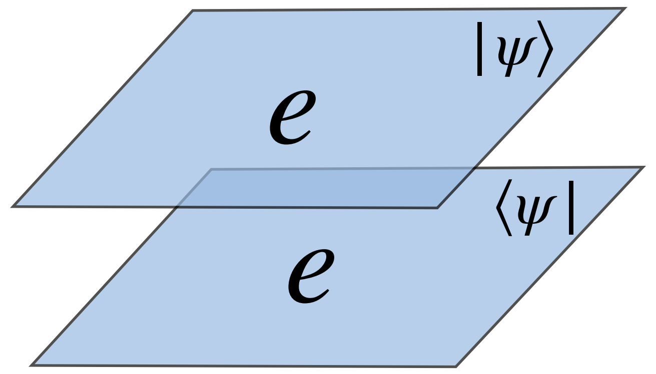



**anyon condensation**




in **single** Hilbert space



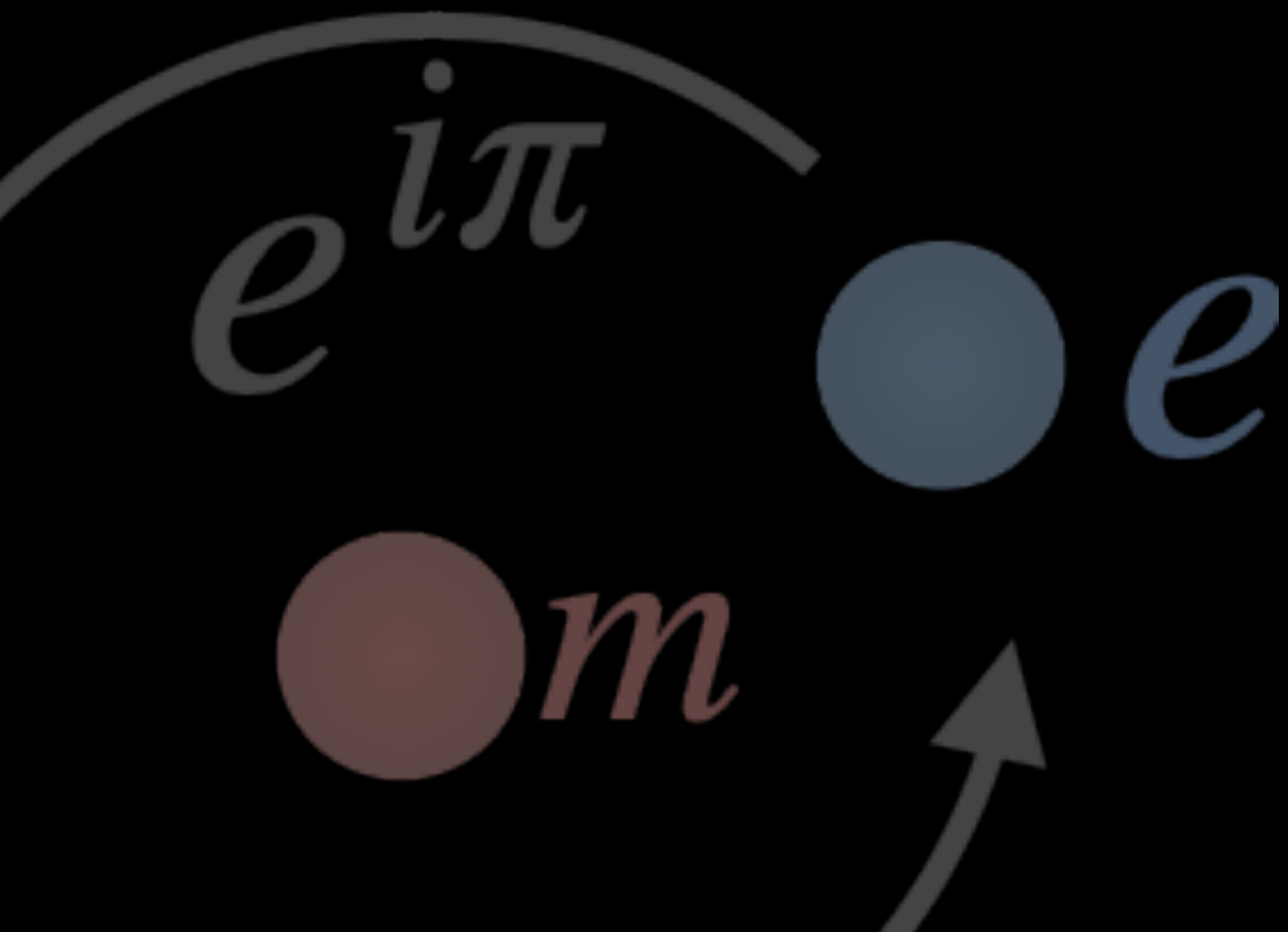

in **double** Hilbert space

dual to Nishimori's cat realized in **IBM experiment**

Chen, GYZ, Verresen, Seif, Bäumer, Layden, Tantivasadakarn, Zhu, Sheldon, Vishwanath, Trebst, Kandala 2023

Dennis, Kitaev, Landahl, Preskill 2002; Fan, Bao, Vishwanath, Altman 2023; Lee, Jian, Xu, 2023; Li, Mong, 2023; …

# summary



# conclusions

Guo-Yi Zhu

- **teleportation of many-body state / logical qubit**

  self-duality ⇒ **optimal threshold**

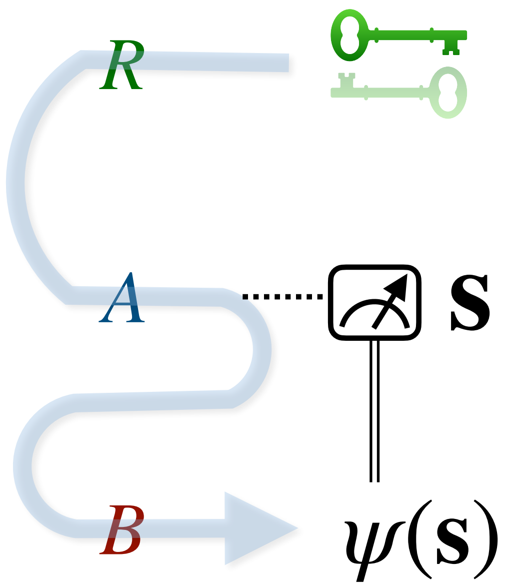
- **topological order**

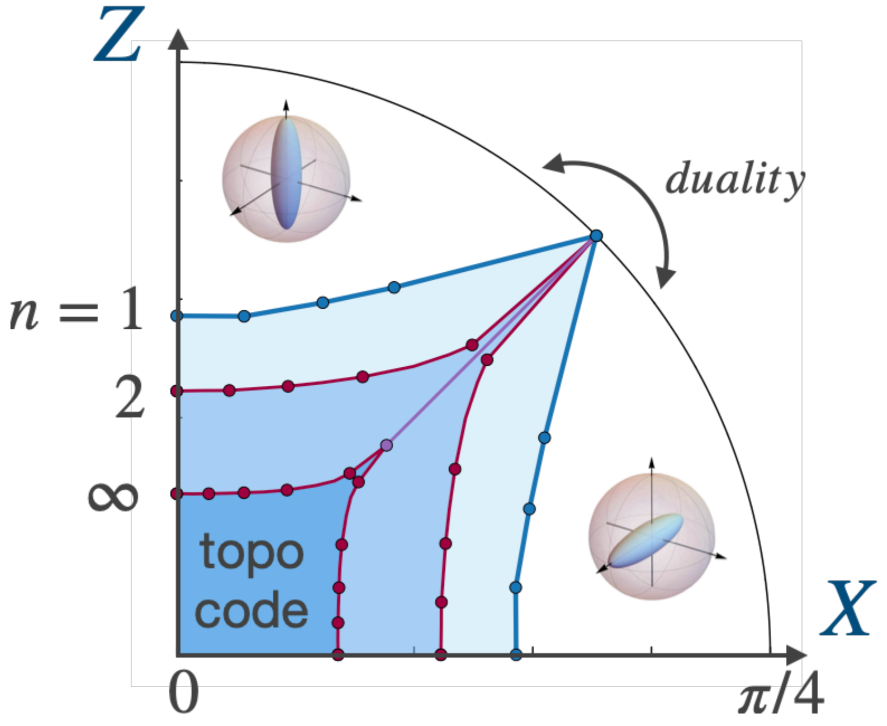
  competing **anyon condensation** phase transitions

- **spin glass model**

  self-dual 4-state Potts | complex Ashkin-Teller model

- **experimentally feasible** in multiple NISQ platforms




tunable teleportation

↕

weak measurement

↕

wave function deformation

*decoherence*

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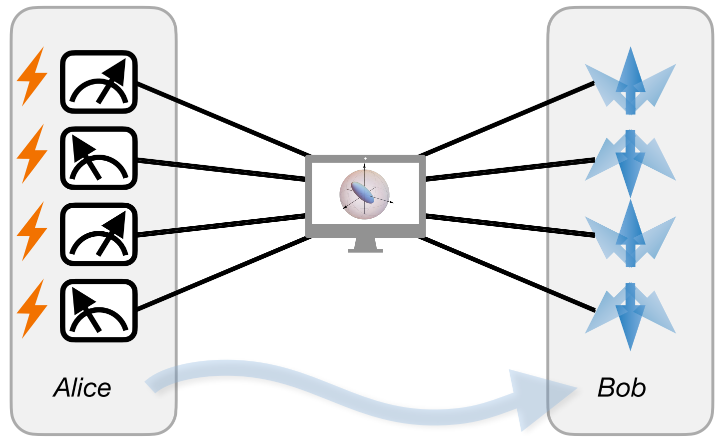
**Outlook**

- non-unitary CFTs & non-Hermitian topology?
- coherent error + incoherent noise?




F. Eckstein, B. Han, ST, G.Y. Zhu
**arXiv: 2403.04767**, PRX Quantum (2024)