Majorana metals

Cargese – June 28th, 2013

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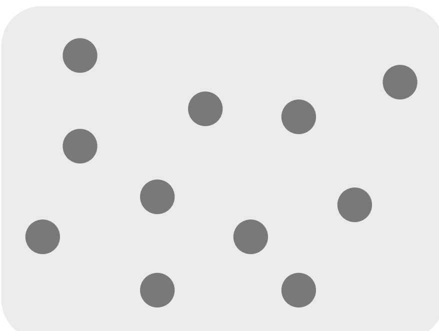
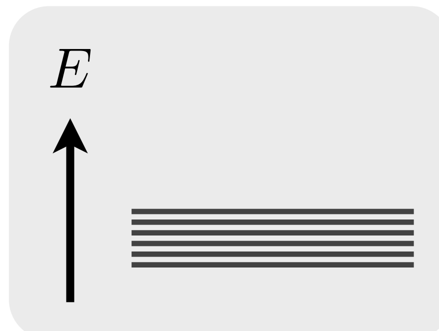
University of Cologne

Motivation

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of ‘accidental’ degeneracies.



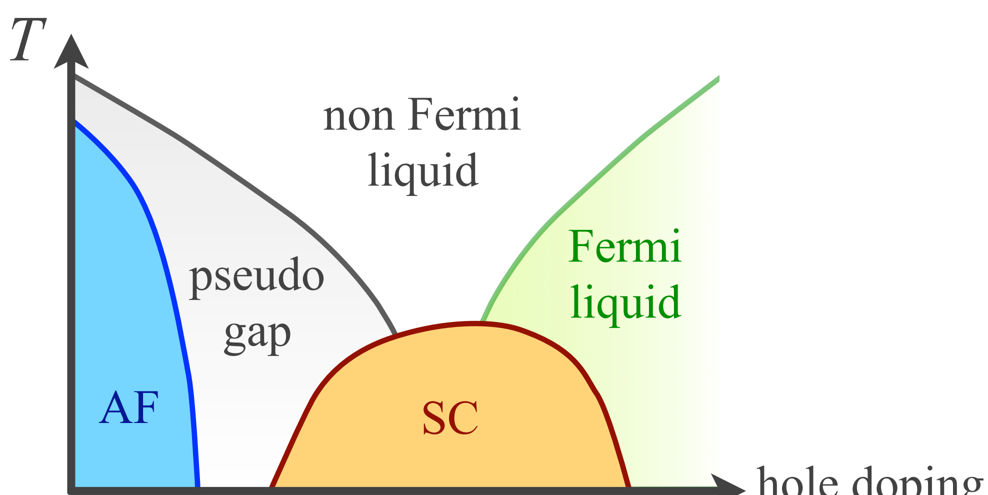
interacting
many-body system



‘accidental’
degeneracy

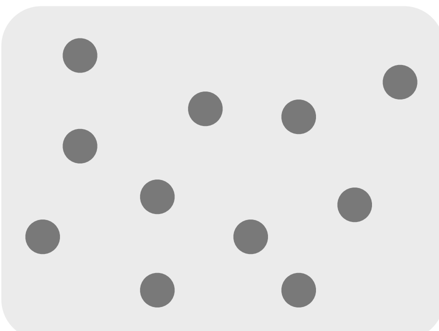
residual effects
select ground state

phase diagram of
cuprate superconductors



Motivation

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of ‘accidental’ degeneracies.



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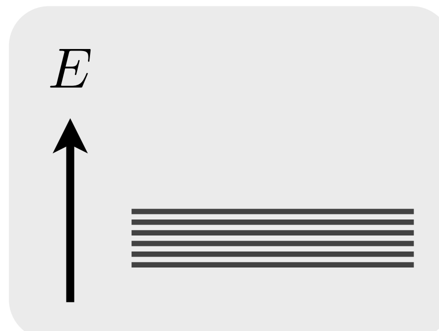


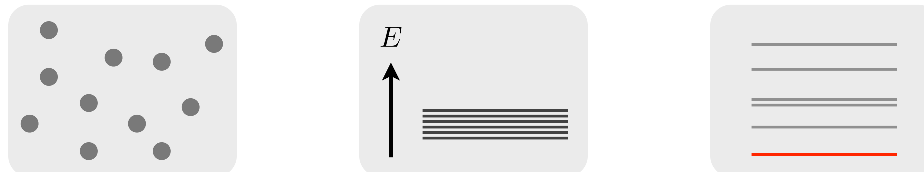
‘accidental’
degeneracy

residual effects
select ground state

But they are also notoriously difficult to handle analytically, due to

- multiple energy scales
- complex energy landscapes / slow equilibration
- strong coupling

Examples – quantum Hall liquids

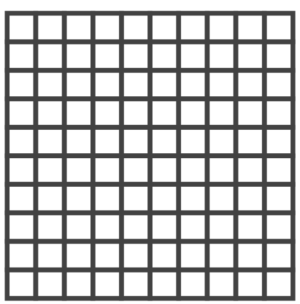


interacting **many-body system**

'accidental' **degeneracy**

residual effects select ground state

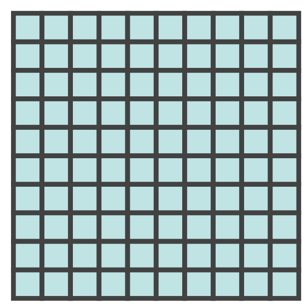
Landau level **degeneracy**



$2\Phi/\Phi_0$

orbital states

integer quantum Hall

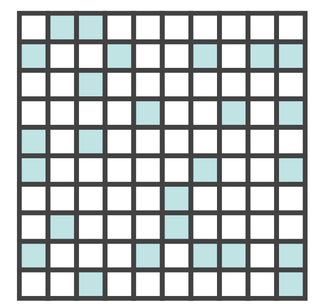


filled level

↓

incompressible liquid

fractional quantum Hall

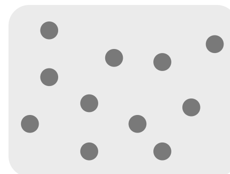


partially filled level

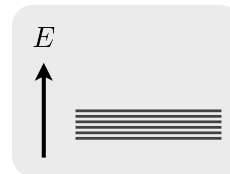
↓ Coulomb repulsion

incompressible liquid

Examples – frustrated magnets

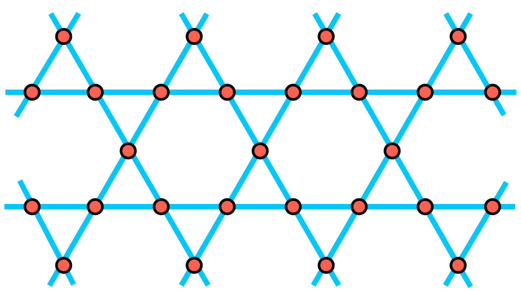


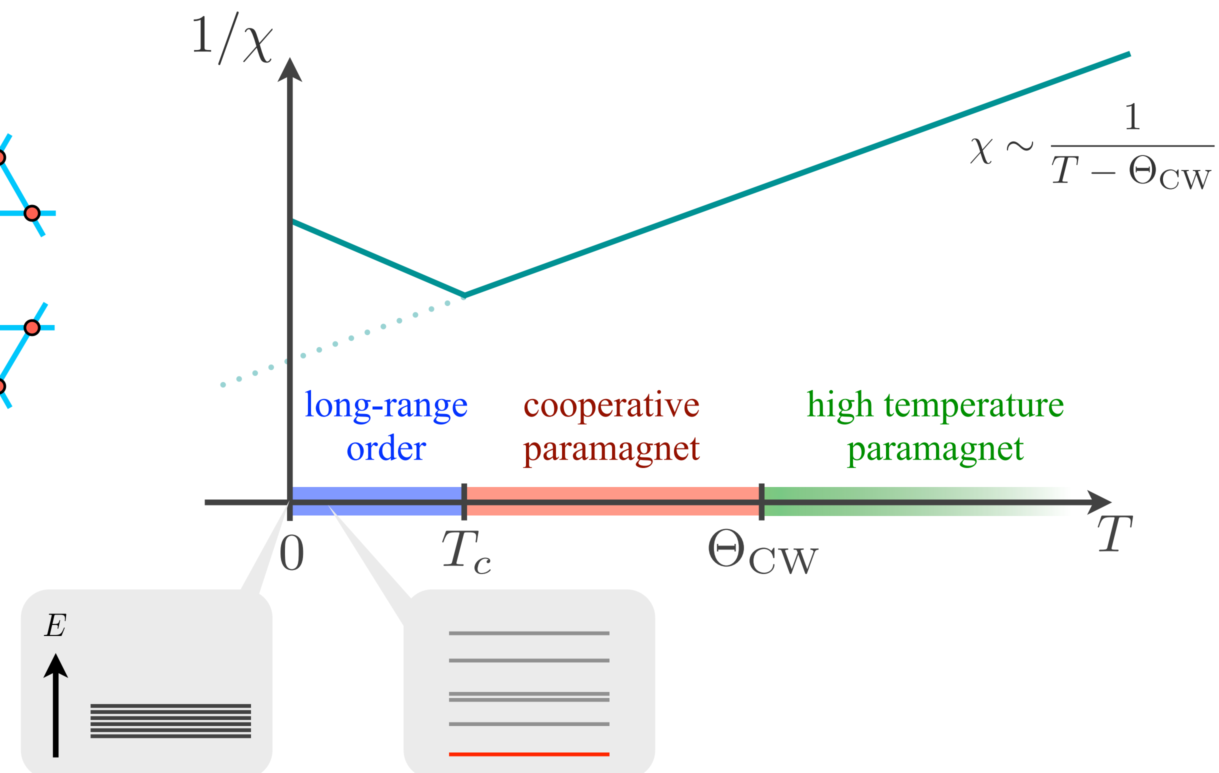
interacting
many-body system



'accidental'
degeneracy

residual effects
select ground state





T=0 residual entropy

long-range order

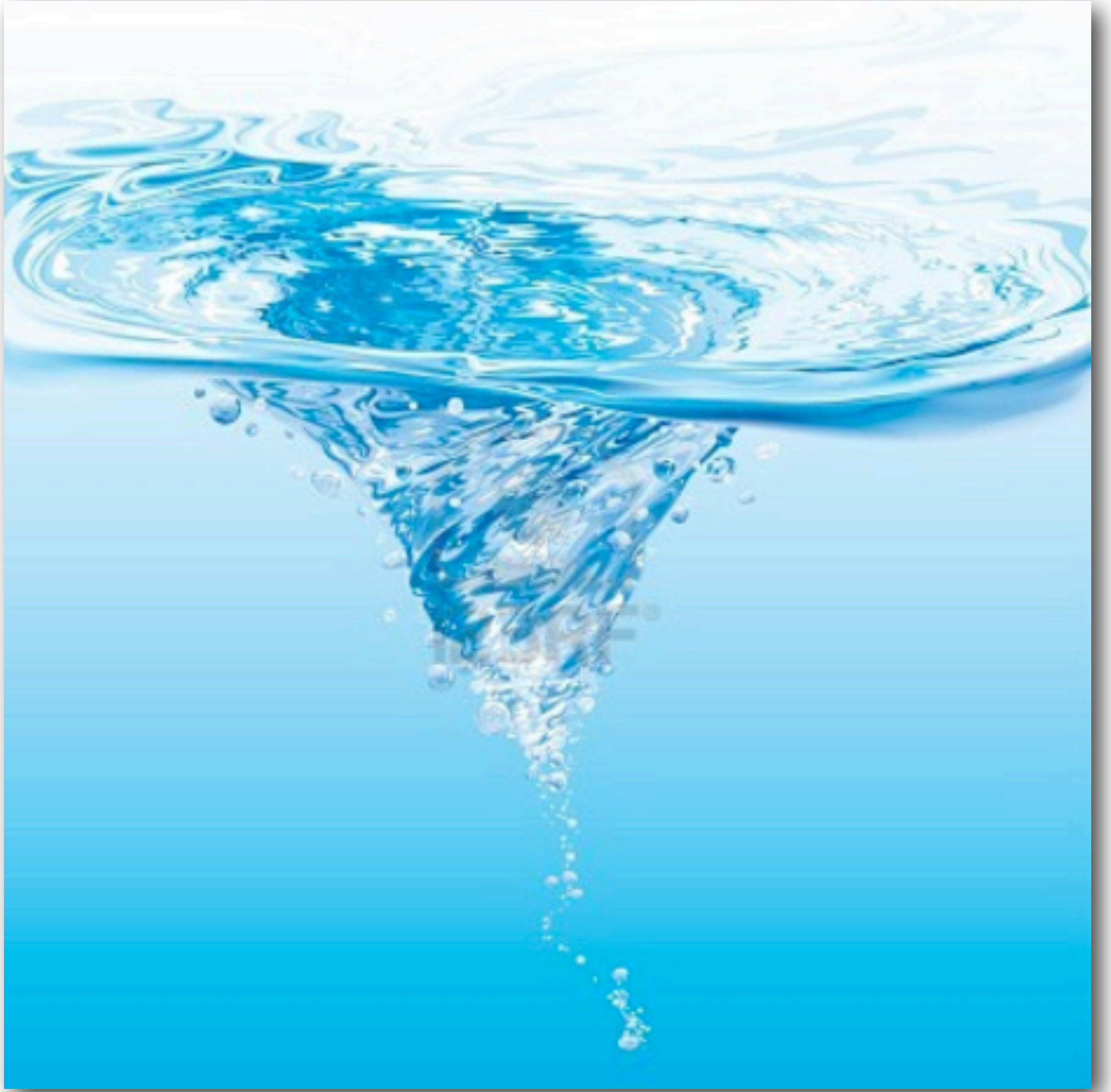
Overview

Vortices, quasiholes, anyons, ...

Interactions

Disorder

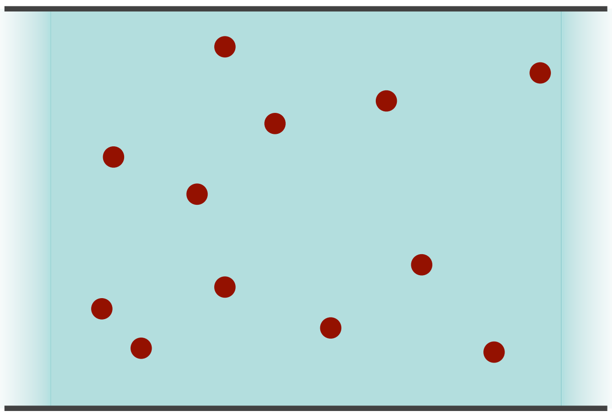
Vortices



Abelian vs. non-Abelian vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

Abelian

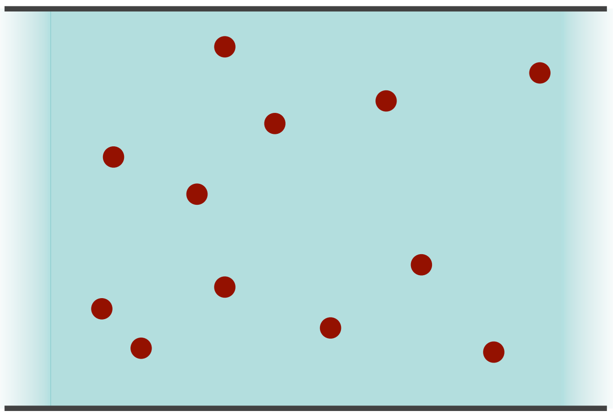


single state

Example:

Laughlin-wavefunction + quasiholes

non-Abelian



(degenerate) **manifold** of states

Manifold of states grows **exponentially** with the number of vortices.

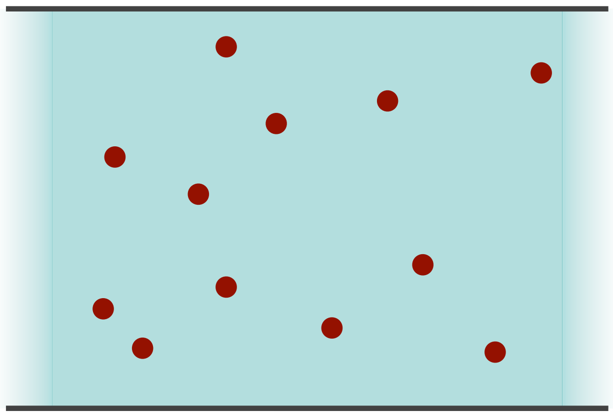
Ising anyons (Majorana fermions) $\sqrt{2}^N$

Fibonacci anyons ϕ^N

Abelian vs. non-Abelian vortices

Consider a set of '**pinned**' vortices at fixed positions.

Abelian

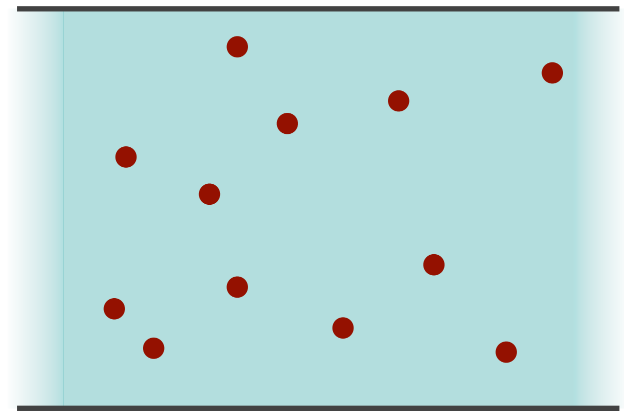


single state

$$\psi(x_2, x_1) = e^{i\pi\theta} \cdot \psi(x_1, x_2)$$

fractional phase

non-Abelian



(degenerate) **manifold** of states

matrix

$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

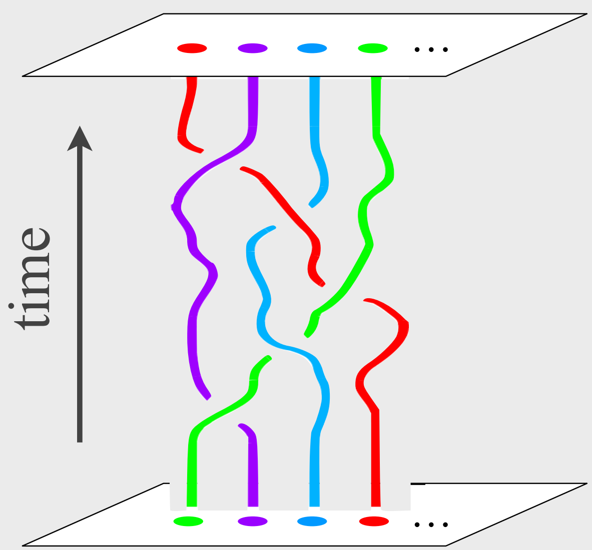
In general *M* and *N* do not commute!

Topological quantum computing

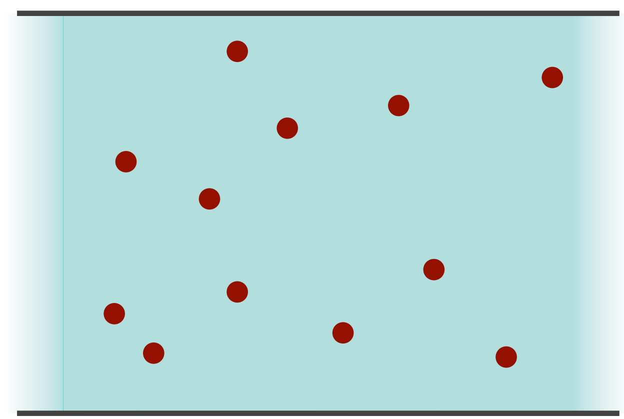
Topological quantum computing

Degenerate manifold = qubit

Employ **braiding** of non-Abelian vortices to perform computing (unitary transformations).



non-Abelian



(degenerate) **manifold** of states

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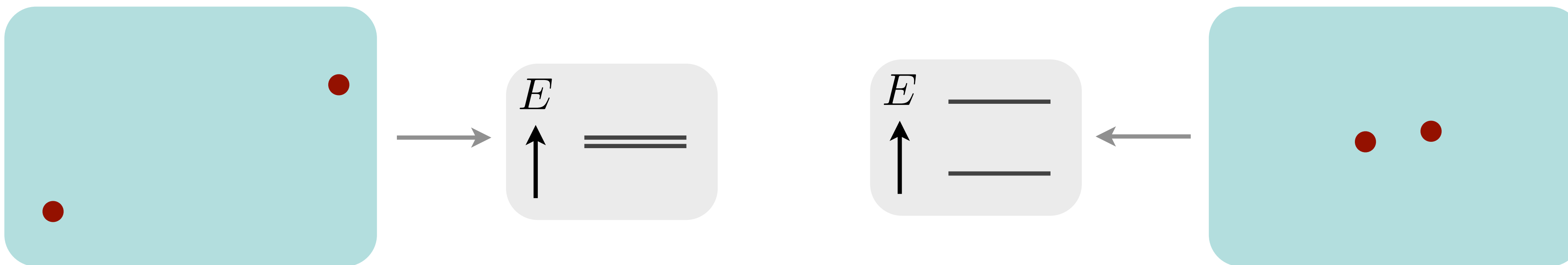
$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

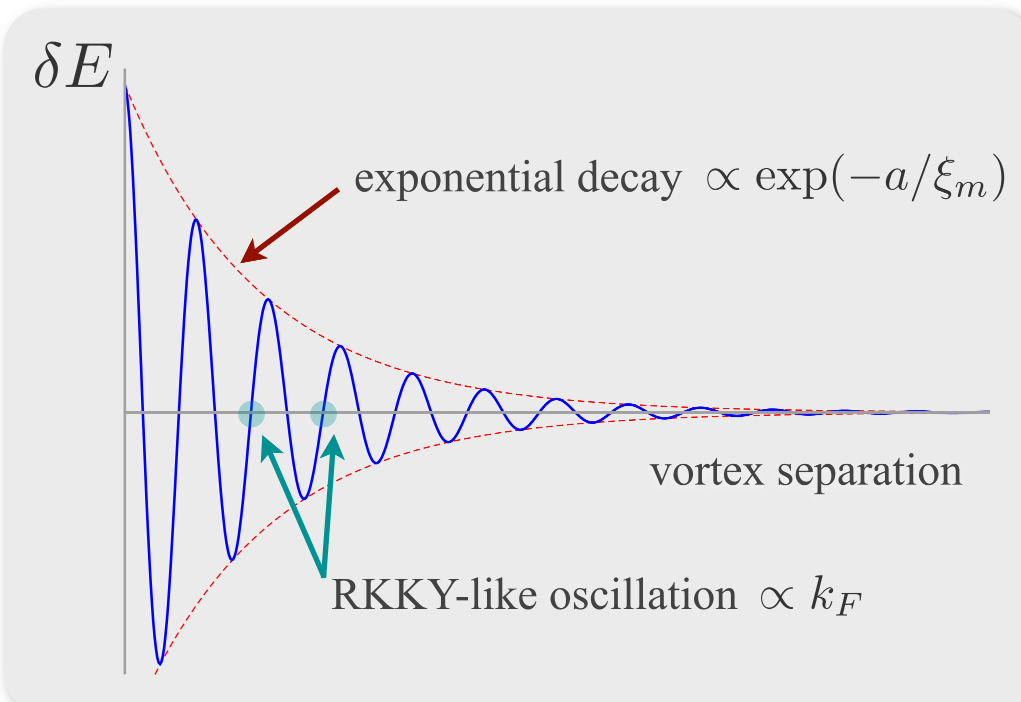
$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

In general *M* and *N* do not commute!

Vortex-vortex interactions

$a \gg \xi_m$ $a \approx \xi_m$





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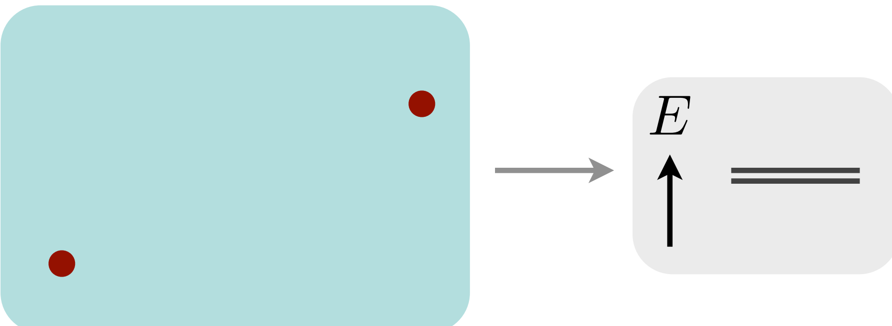

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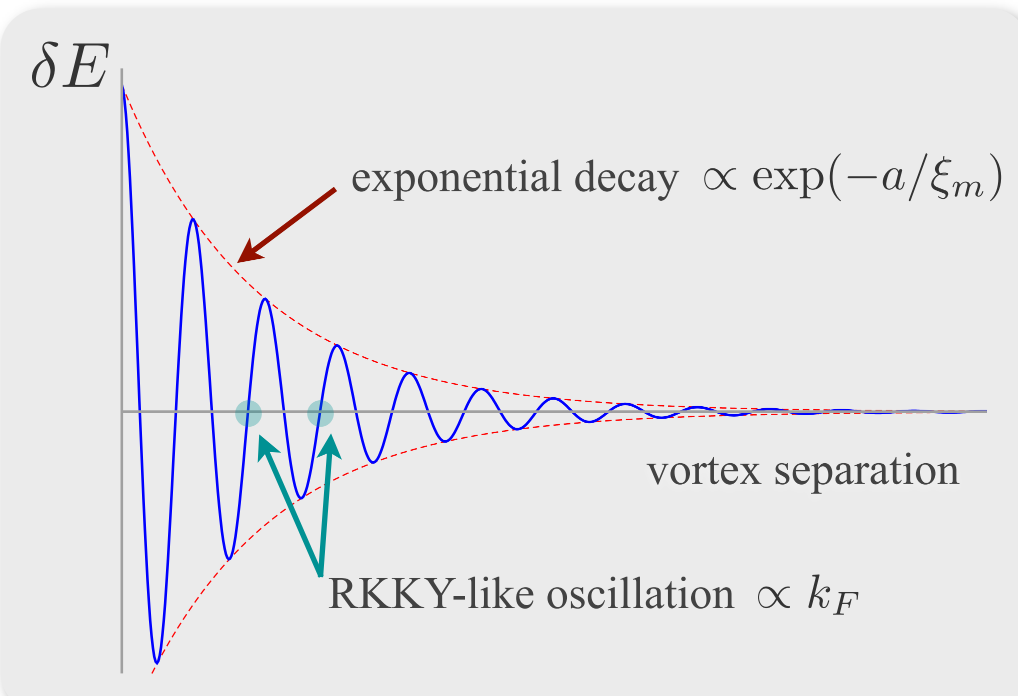
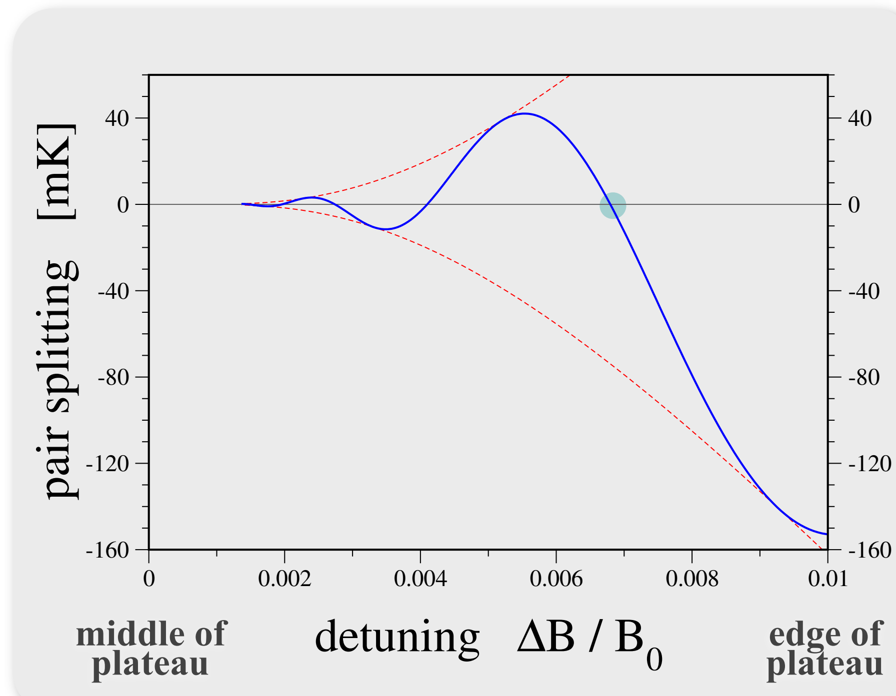




Vortex-vortex interactions

Vortex quantum numbers

$SU(2)_k$ = ‘deformation’ of SU(2)

with finite set of representations

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \frac{\boxed{k}}{2}$$

fusion rules

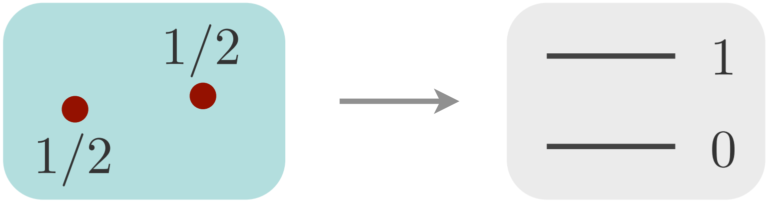
$$j_1 \times j_2 =$$
$$|j_1 - j_2| + (|j_1 - j_2| + 1) + \ldots +$$
$$\min(j_1 + j_2, \boxed{k} - j_1 - j_2)$$

example k = 2

$$1/2 \times 1/2 = 0 + 1$$

Vortex pair

$$1/2 \times 1/2 = 0 + 1$$



Energetics for many vortices

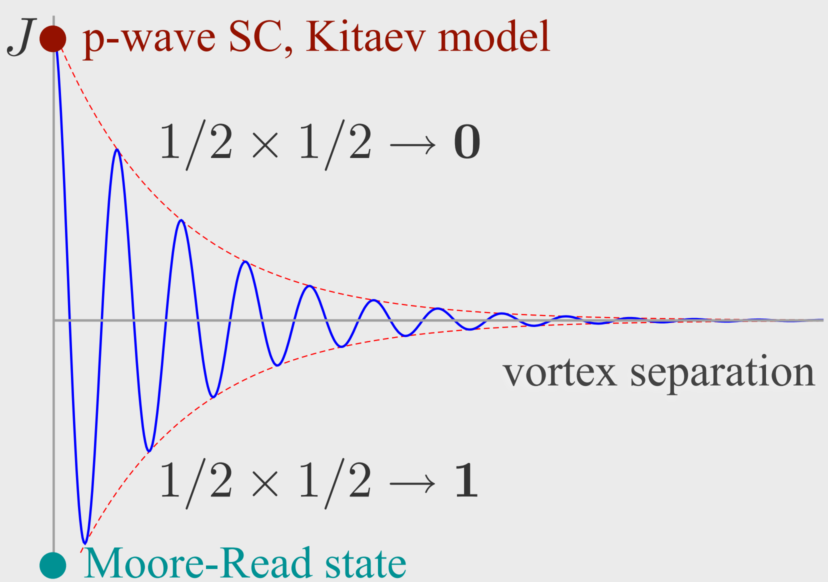
$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

“Heisenberg Hamiltonian”
for vortices

Vortex-vortex interactions

Microscopics

Which channel is favored is
not universal, but microscopic detail.



Vortex pair

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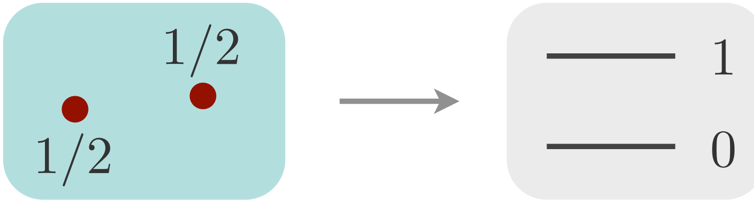

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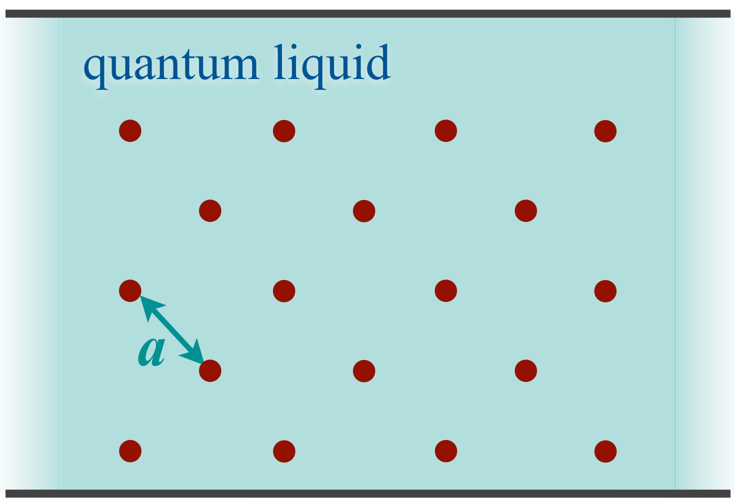
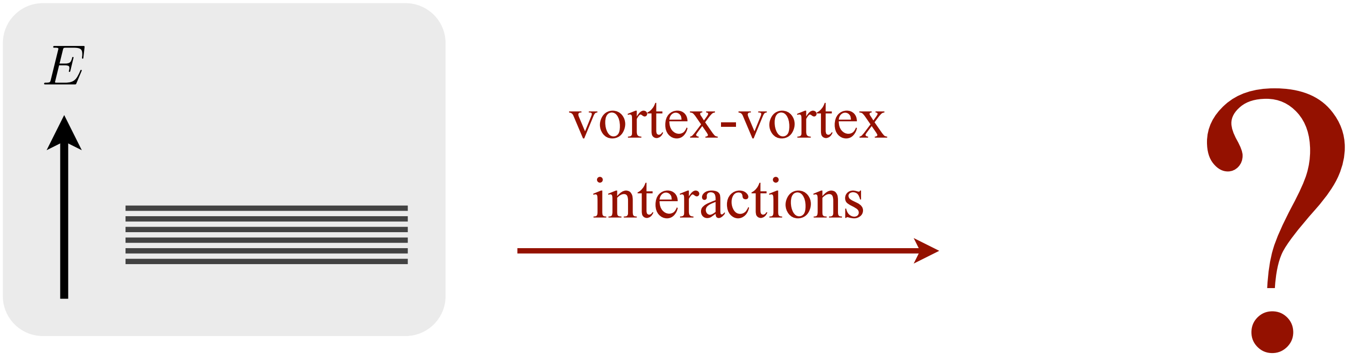
The many-vortex problem

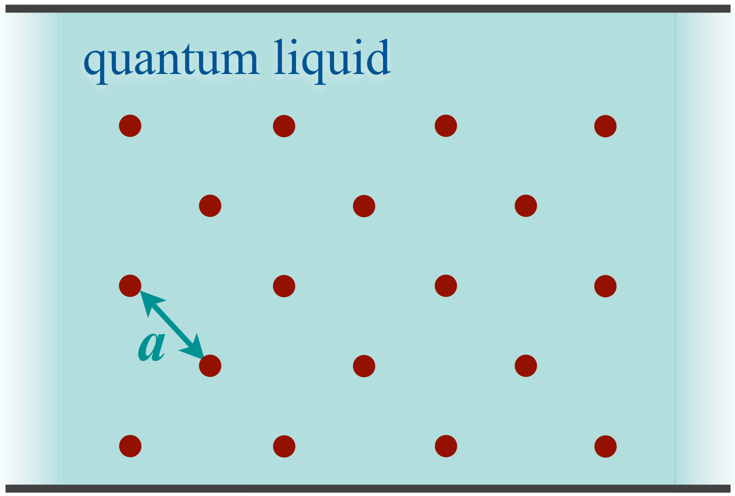


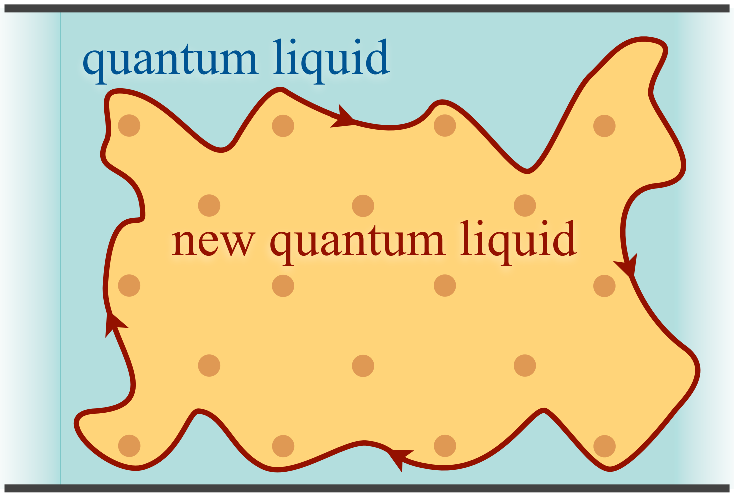
$$a \gg \xi_m$$



macroscopic degeneracy

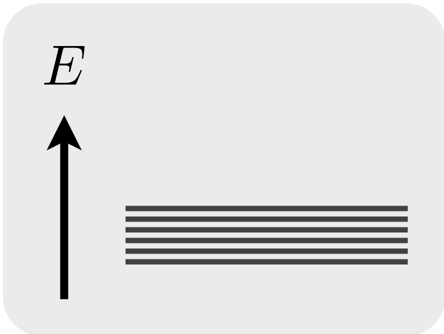
The many-vortex problem



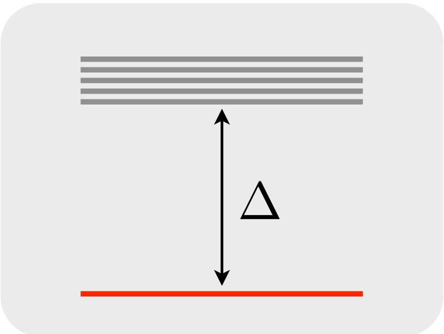


$a \gg \xi_m$

$a \approx \xi_m$



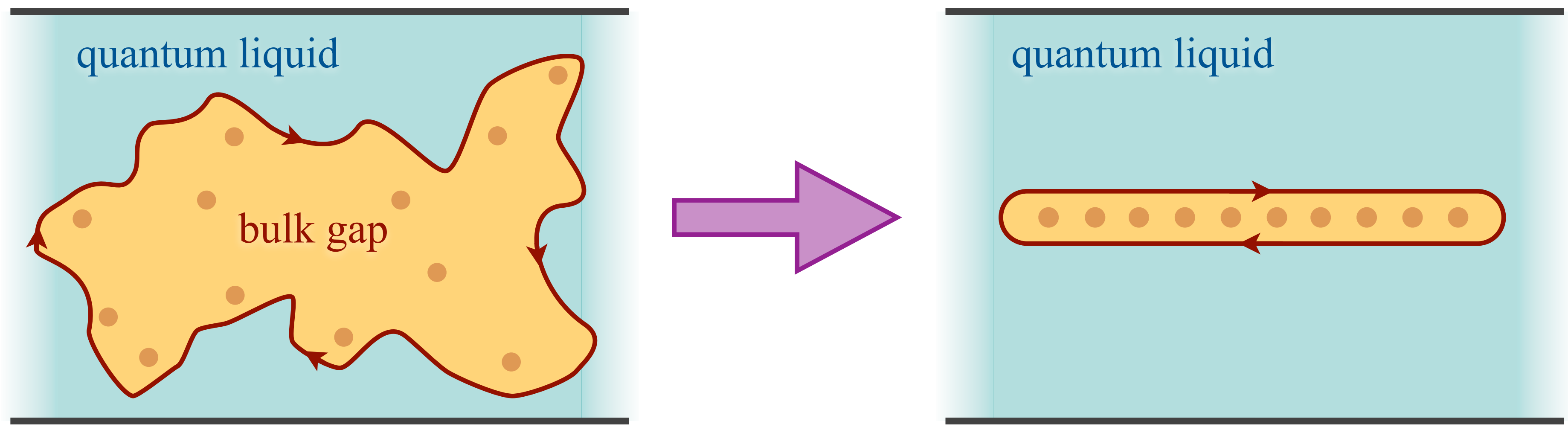
vortex-vortex interactions



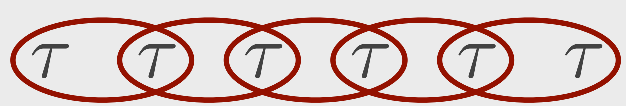
macroscopic degeneracy

unique ground state

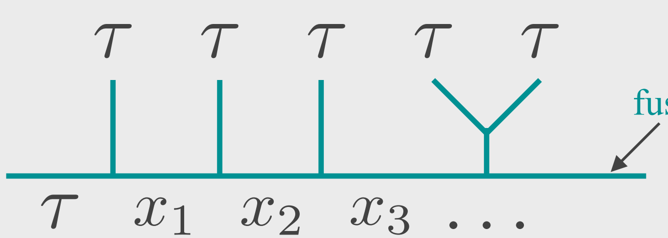
The collective state



Anyonic Heisenberg chains



$(\tau = 1/2)$



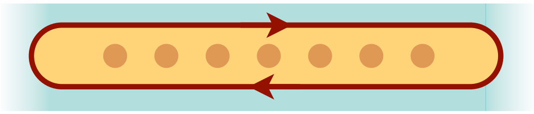
Hilbert space

$$|x_1, x_2, x_3, \ldots\rangle$$

Hamiltonian

$$H = \sum_i F_i \Pi_i^0 F_i$$

F-matrix = 6j-symbol

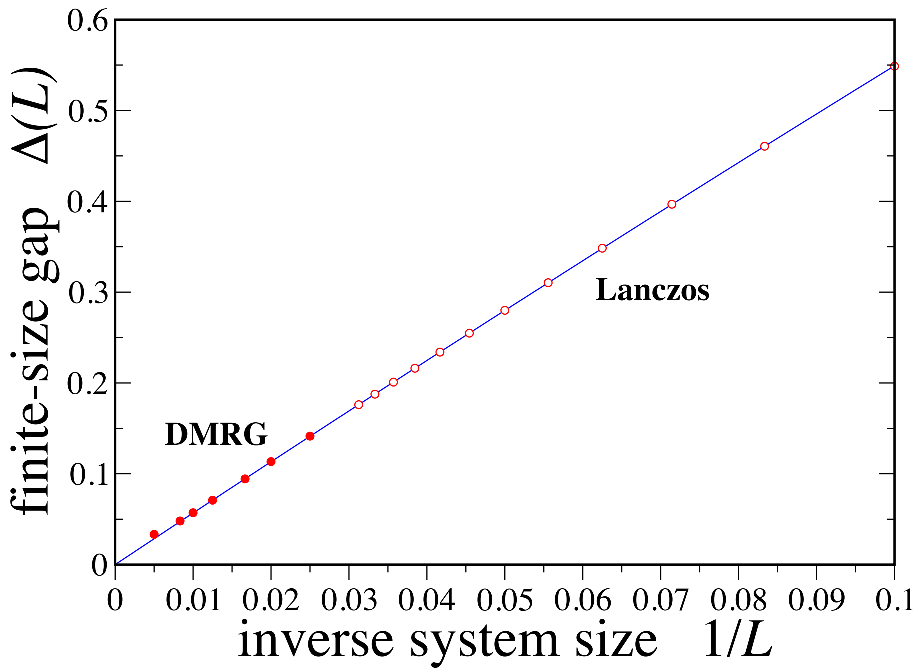


Edge states

Finite-size gap

$$\Delta(L) \propto (1/L)^{\boxed{z=1}}$$

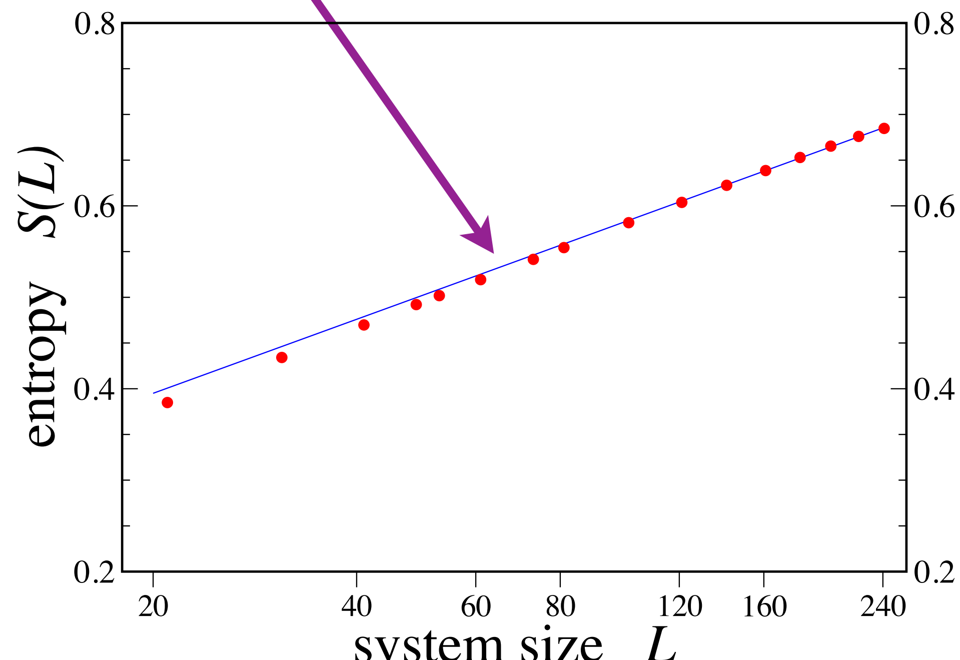
conformal field theory description

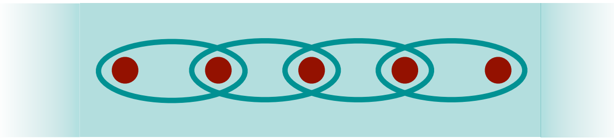


Entanglement entropy

$$S(L) \propto \frac{\boxed{c}}{3} \log L$$

central charge
$c = 7/10$

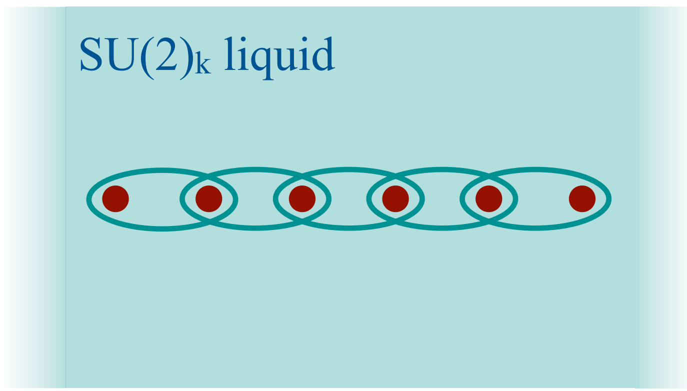




Gapless theories

level k	$1/2 \times 1/2 \rightarrow \mathbf{0}$ 'antiferromagnetic'	$1/2 \times 1/2 \rightarrow \mathbf{1}$ 'ferromagnetic'
2	Ising c = 1/2	Ising c = 1/2
3	tricritical Ising c = 7/10	3-state Potts c = 4/5
4	$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$	$\frac{SU(2)_k}{U(1)}$
5		
k	**k-critical Ising** c = 1-6/(k+1)(k+2)	**Z_k-parafermions** c = 2(k-1)/(k+2)
∞	Heisenberg AFM c = 1	Heisenberg FM c = 2

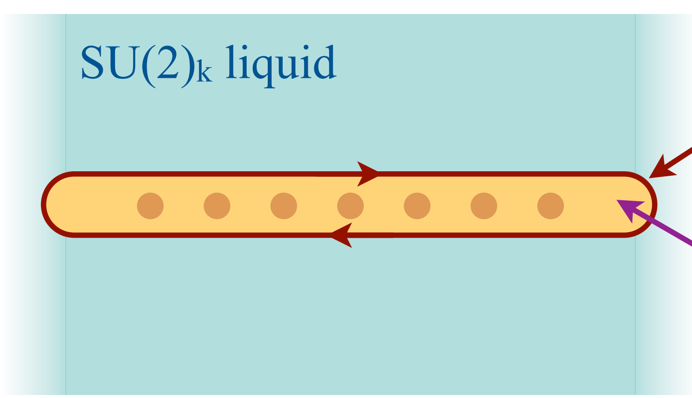
Gapless modes & edge states



critical theory
$1/2 \times 1/2 \rightarrow \mathbf{0}$

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

interactions

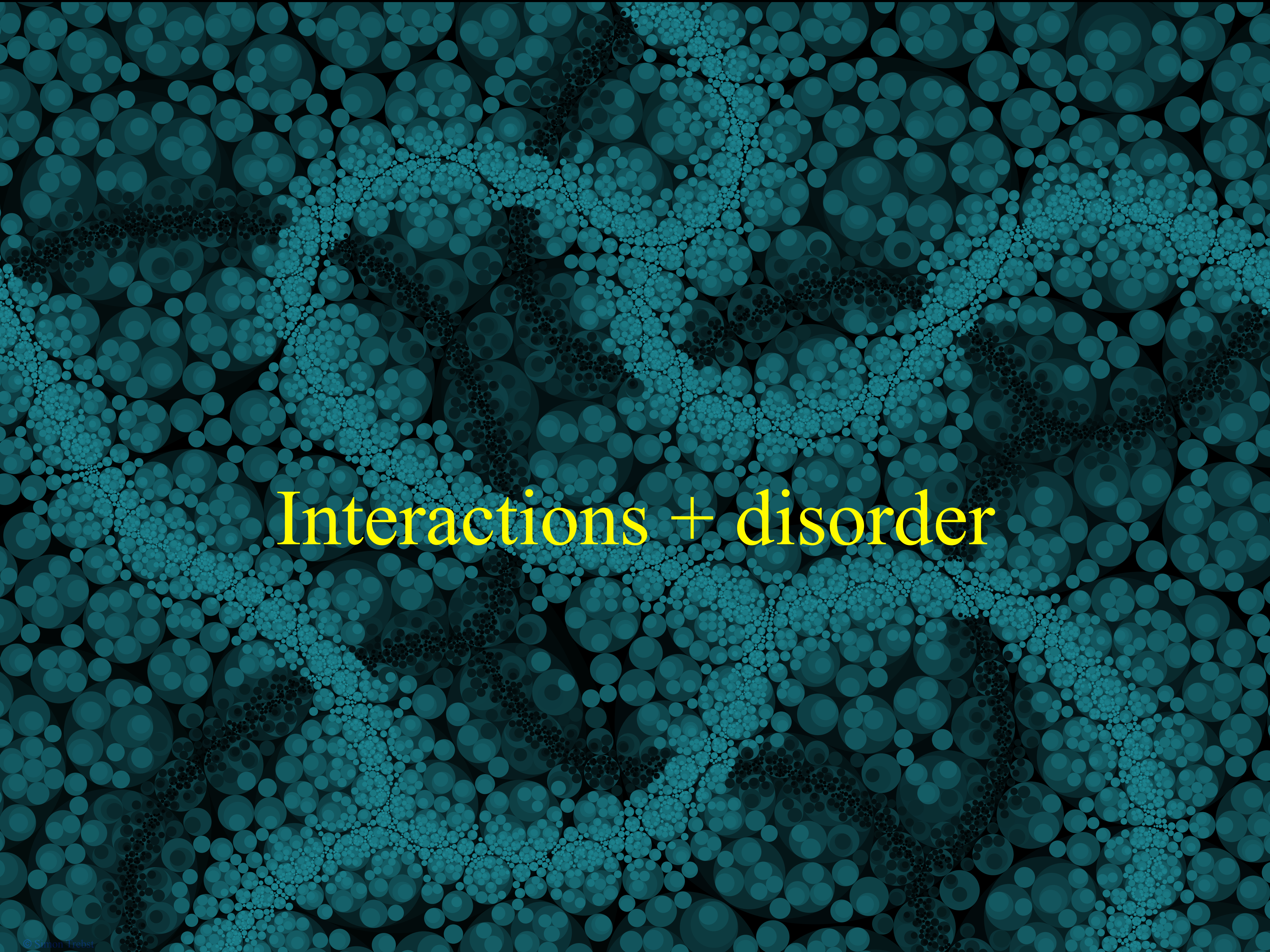


gapless modes = edge states

$$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$$

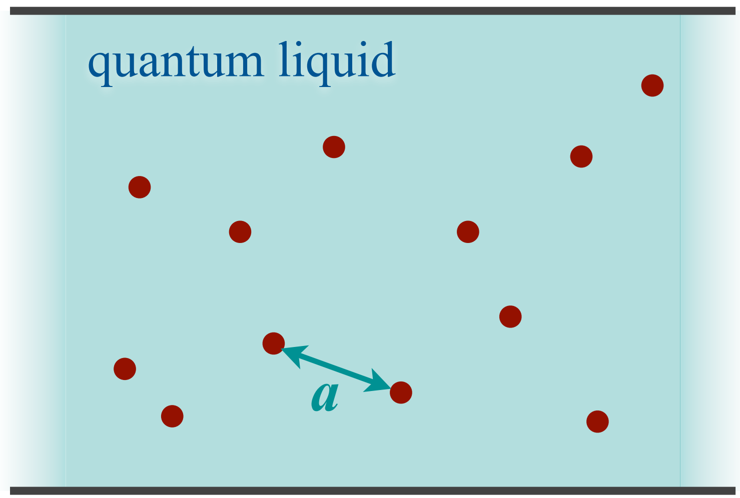
nucleated liquid

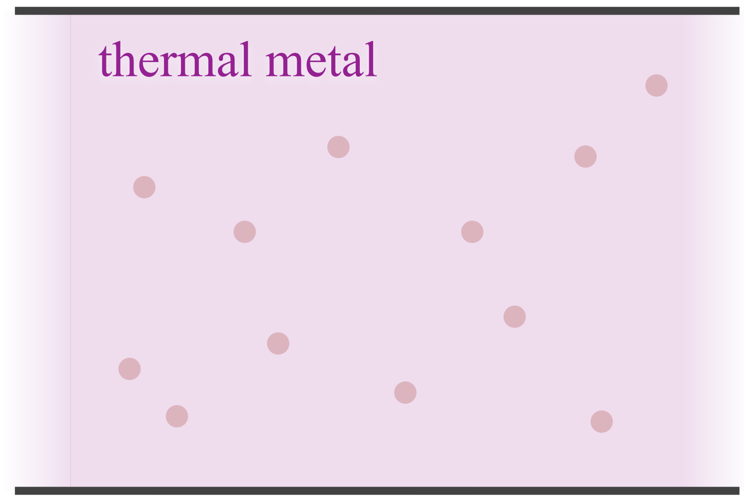
$$SU(2)_{k-1} \times SU(2)_1$$



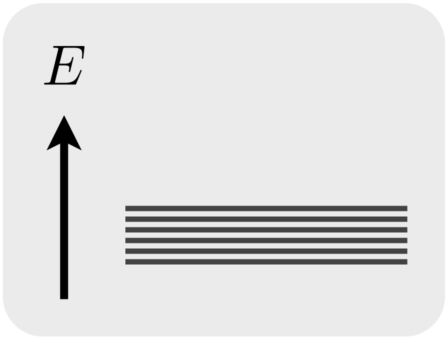
Interactions + disorder

Disorder induced phase transition





$a \gg \xi_m$



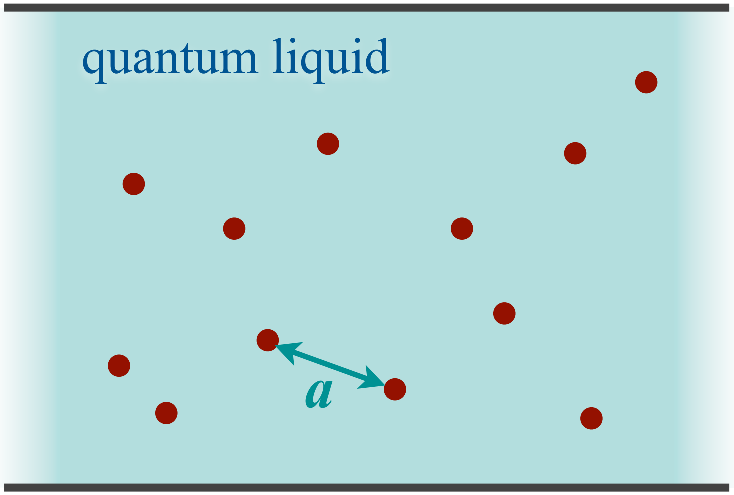
disorder +
vortex-vortex
interactions

$a \approx \xi_m$

macroscopic degeneracy

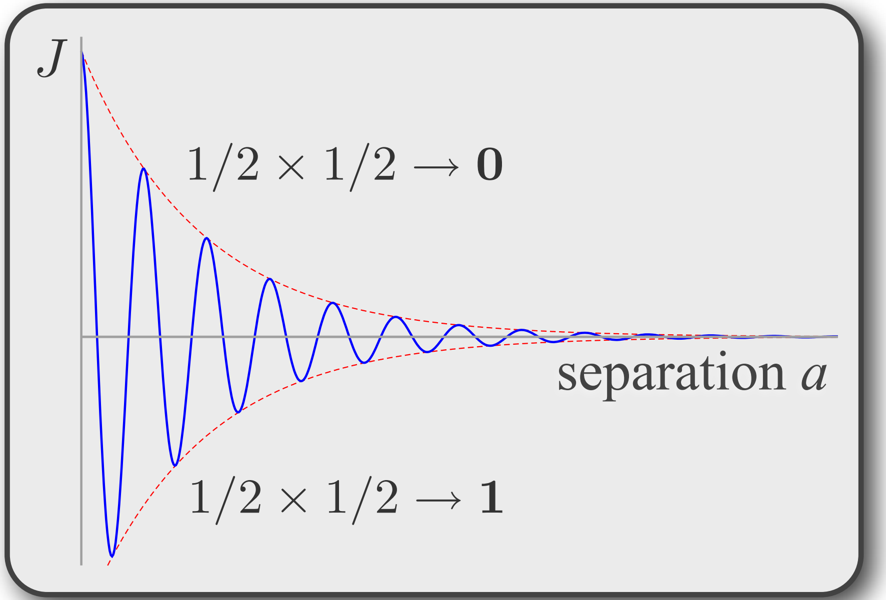
degeneracy is **split**

Interactions and disorder



$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk}$$

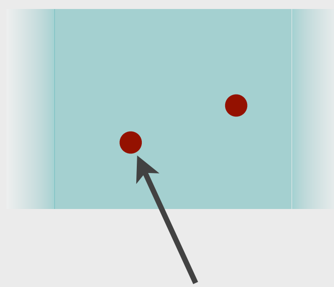
sign disorder
+ strong amplitude modulation



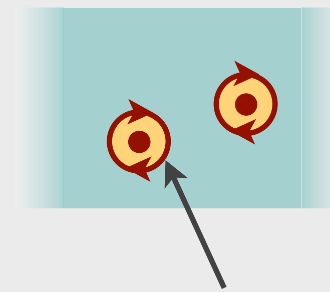
Natural analytical tool:
strong-randomness RG

Unfortunately, this does not work.
The system flows *away* from strong disorder under the RG.
No infinite randomness fixed point.

From Ising anyons to Majorana fermions



SU(2)$_2$
quantum number



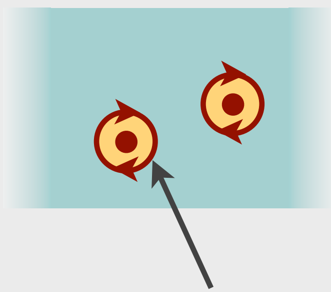
Majorana operator γ_i

$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk} \longrightarrow \mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

interacting Ising anyons
"anyonic Heisenberg model"

free Majorana fermion
hopping model

From Ising anyons to Majorana fermions



Majorana fermion zero mode

Majorana operator γ_i

Majorana operators

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

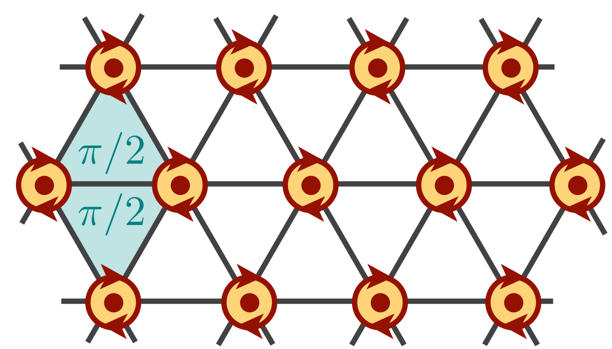
$$\gamma_i^\dagger = \gamma_i$$

$$\gamma_{i1} = (c_i^\dagger + c_i)/2$$

$$\gamma_{i2} = (c_i^\dagger - c_i)/2i$$

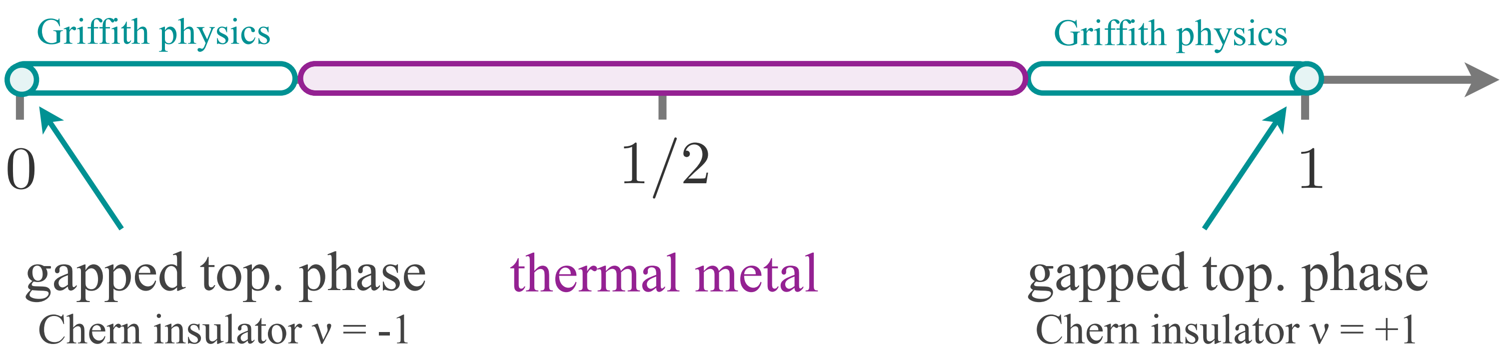
$$\mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

free Majorana fermion hopping model



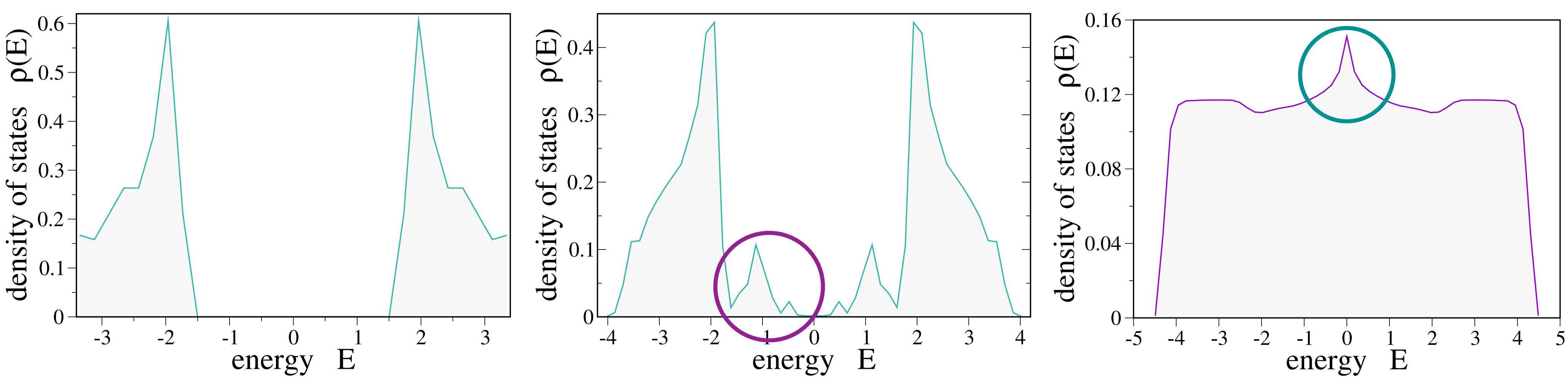
$$\left.\begin{array}{l}\text{particle-hole symmetry } \checkmark \\ \text{time-reversal symmetry } \times\end{array}\right\} \text{symmetry class D}$$

A disorder-driven metal-insulator transition



$$J = \begin{cases} +1, & p \\ -1, & 1\text{-}p \end{cases}$$

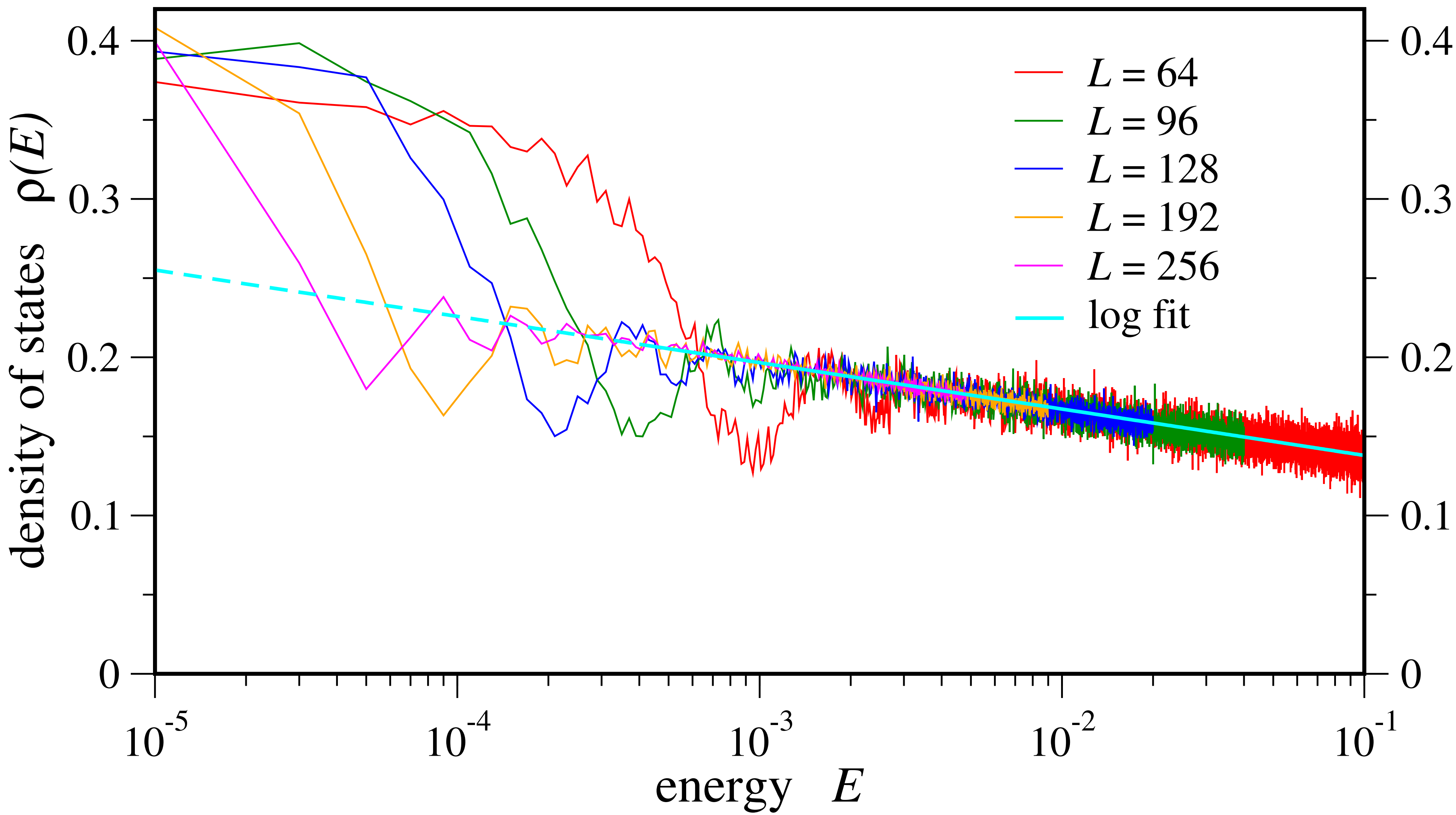
$p = 0$ $\qquad$ $p = 0.02$ $\qquad$ $p = 1/2$



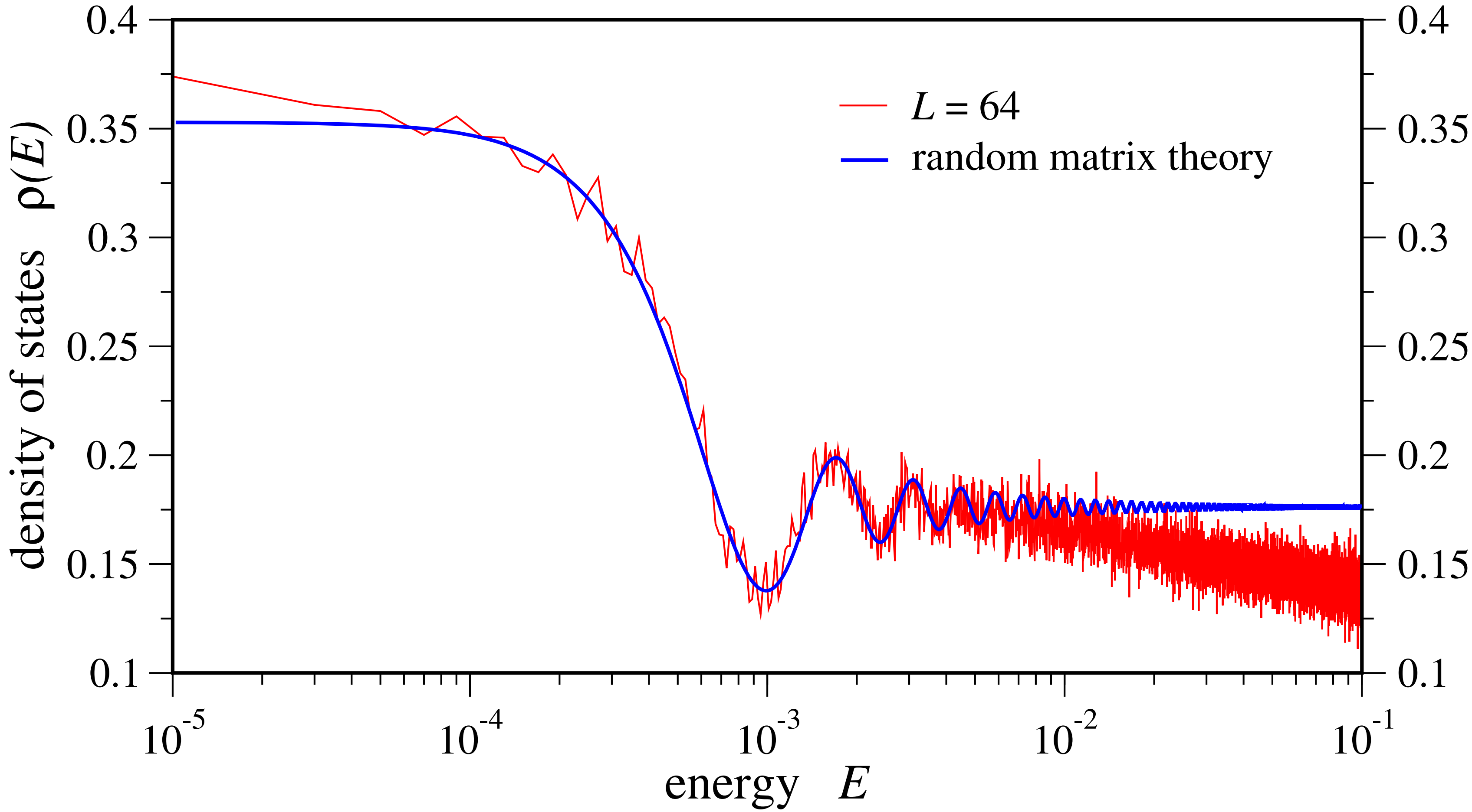
Density of states indicates phase transition.

The thermal metal

Density of states **diverges** logarithmically at zero energy.



Taking a closer look



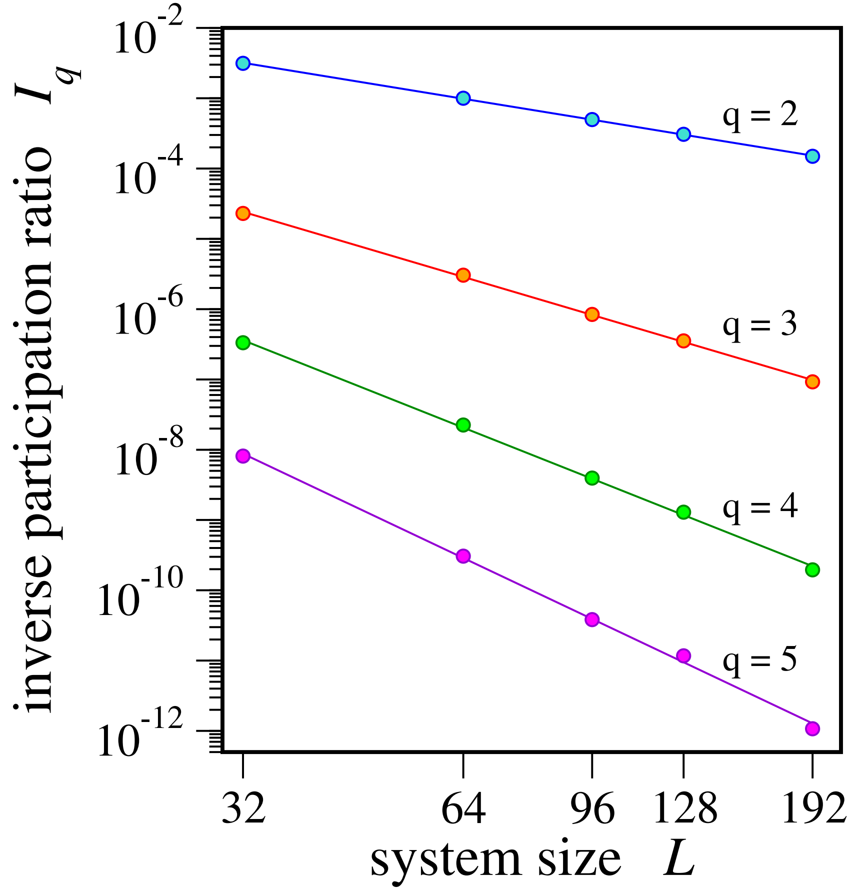
Oscillations in the DOS fit the prediction from **random matrix theory** for symmetry class D

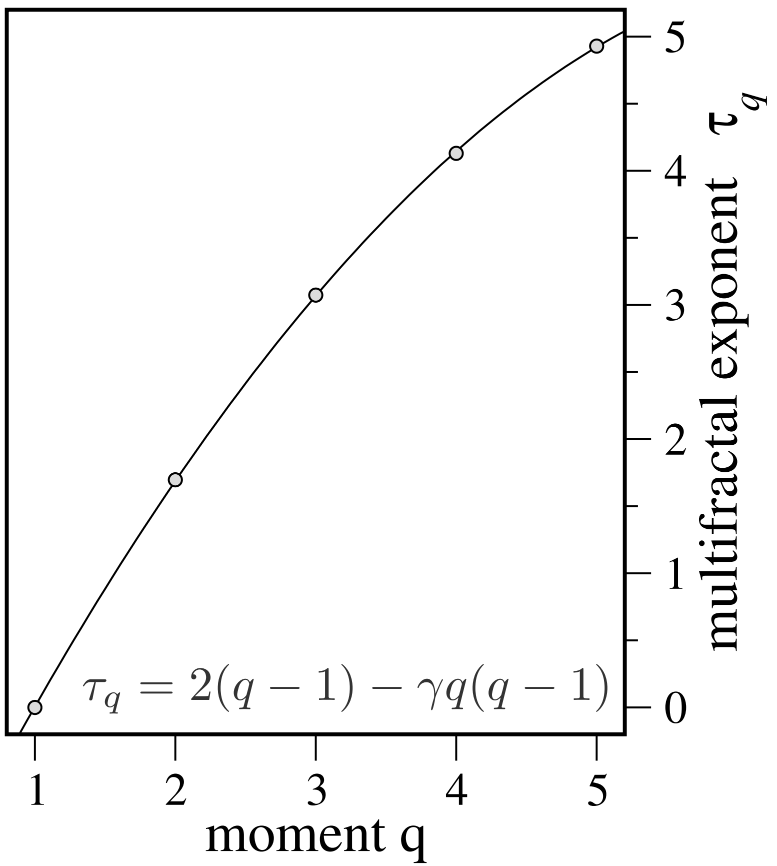
$$\rho(E) = \alpha + \frac{\sin(2\pi\alpha E L^2)}{2\pi E L^2}$$

The thermal metal

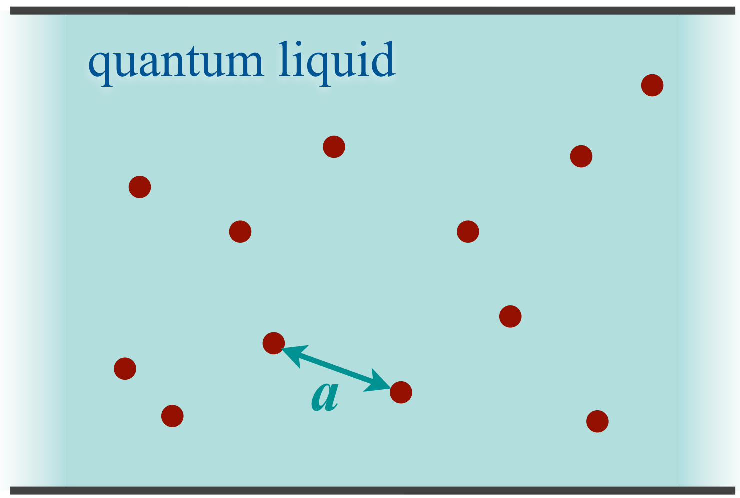
Inverse participation ratios (moments of the GS wavefunction) indicate **multifractal** structure characteristic of a metallic state.

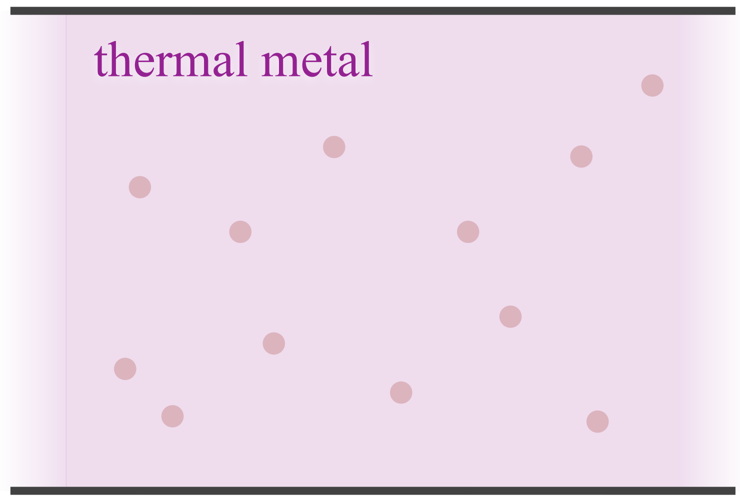
$$I_q = \int d^2r \, |\psi(r)|^{2q} \sim \frac{1}{L^{\tau_q}}$$



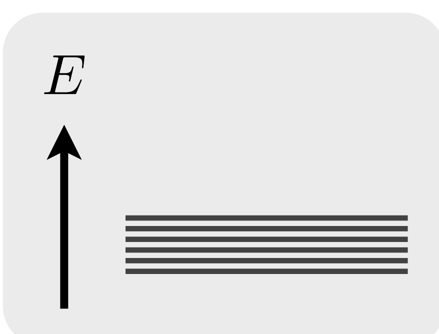


Disorder induced phase transition





$a \gg \xi_m$



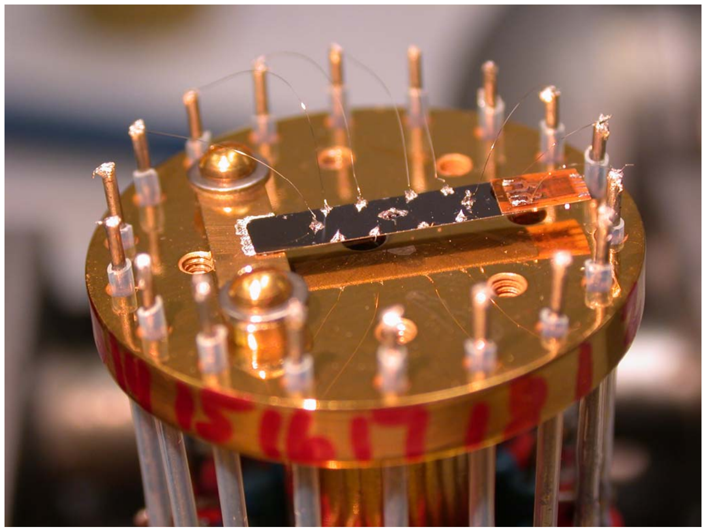
macroscopic degeneracy

disorder +
vortex-vortex
interactions

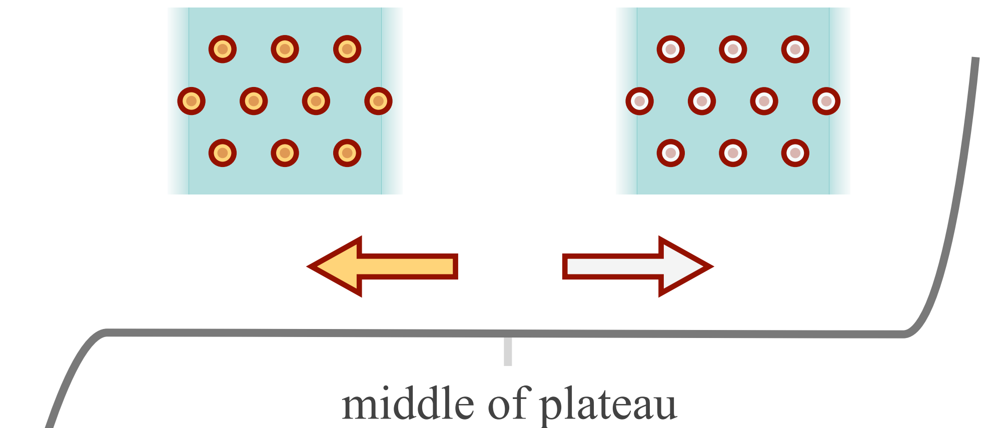
$a \approx \xi_m$

degeneracy is **split**

Heat transport



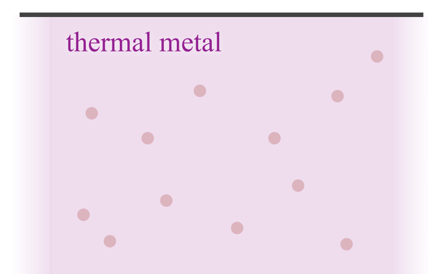
Caltech thermopower experiment

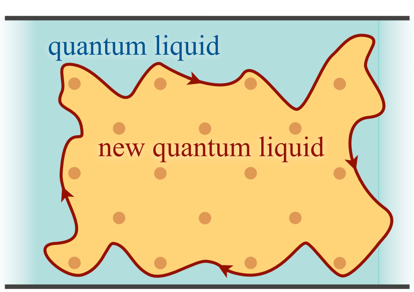


electrical transport remains unchanged

Bulk heat transport
diverges logarithmically
as $T \rightarrow 0$.

$$\kappa_{xx}/T \propto \log T$$



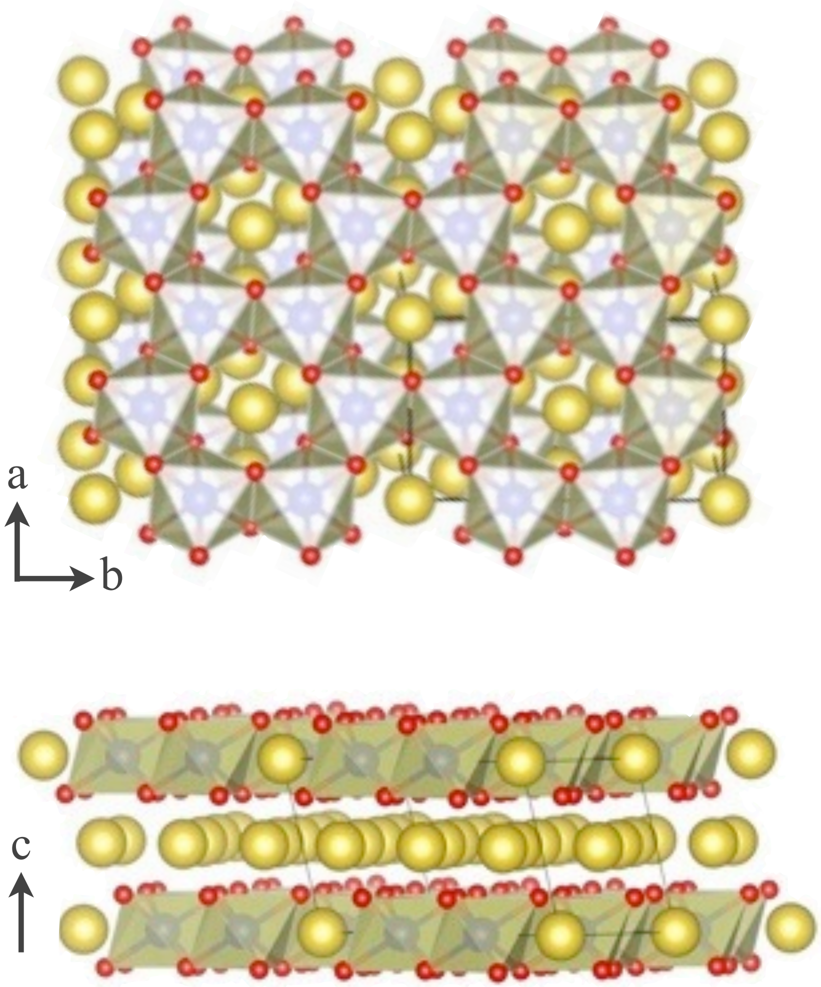


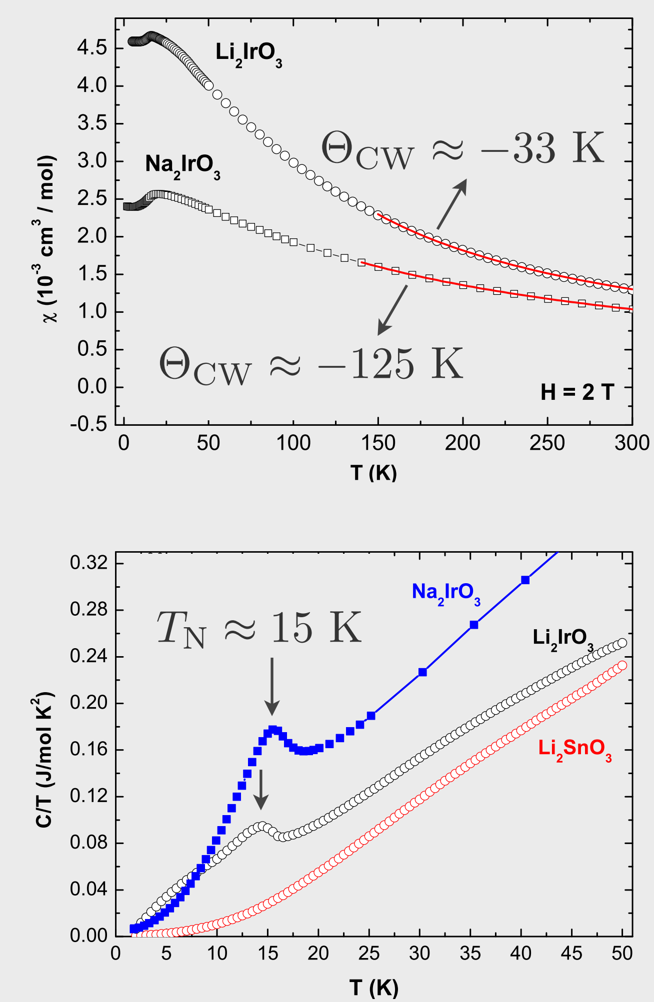
Heat transport
along the sample **edges**
changes quantitatively

Other places to look for Majorana metals …

Spin-orbit assisted Mott physics in Iridates

$(Na,Li)_2IrO_3$

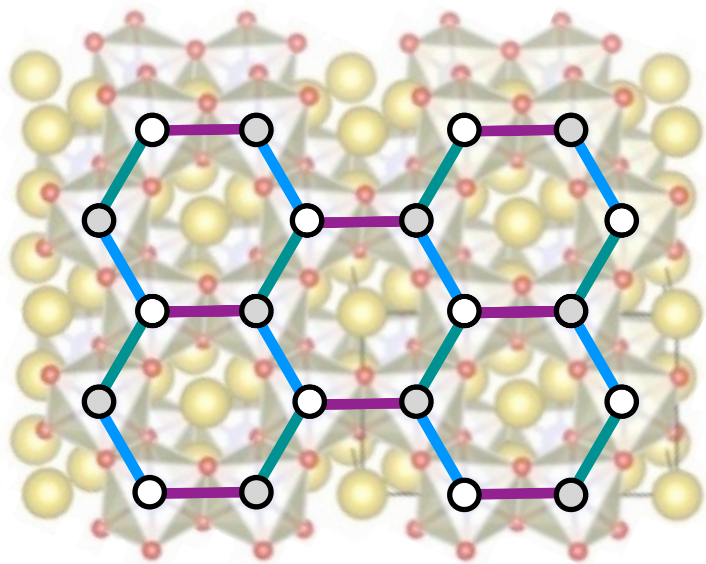


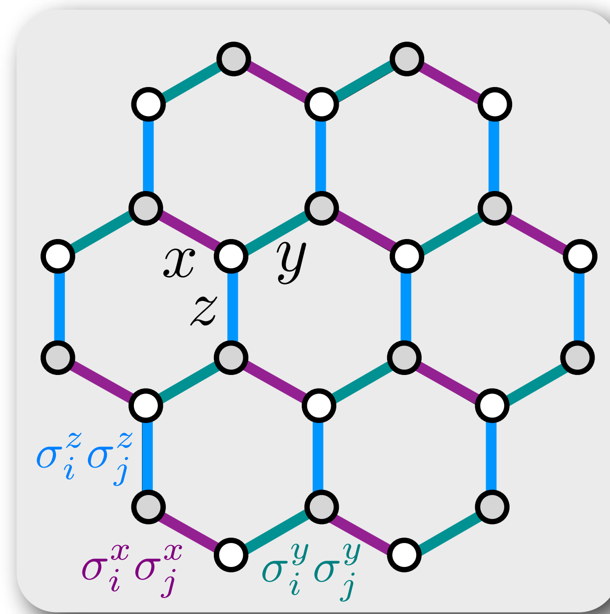


Y. Singh, P. Gegenwart, ST, *et al.*, PRL **108**, 127203 (2012)

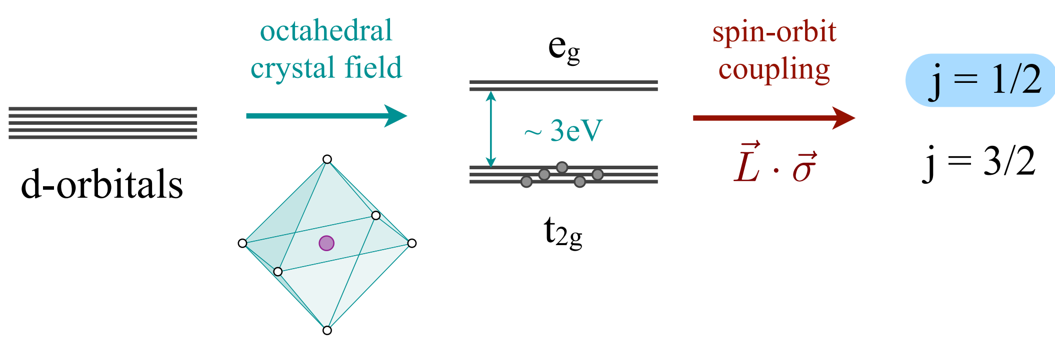
Spin-orbit assisted Mott physics in Iridates

$(Na,Li)_2IrO_3$





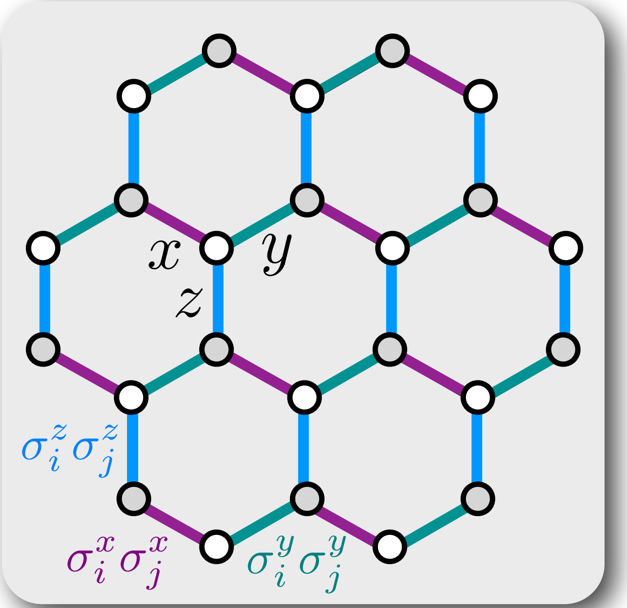
Ir^{4+} $(5d^5)$



$$H_{\text{Kitaev}} = \sum_{\gamma-\text{links}} J_\gamma \sigma_i^\gamma \sigma_j^\gamma$$

Rare combination of a model of **fundamental conceptual importance** (harboring topological phases) and an **exact analytical solution**.

The Kitaev model



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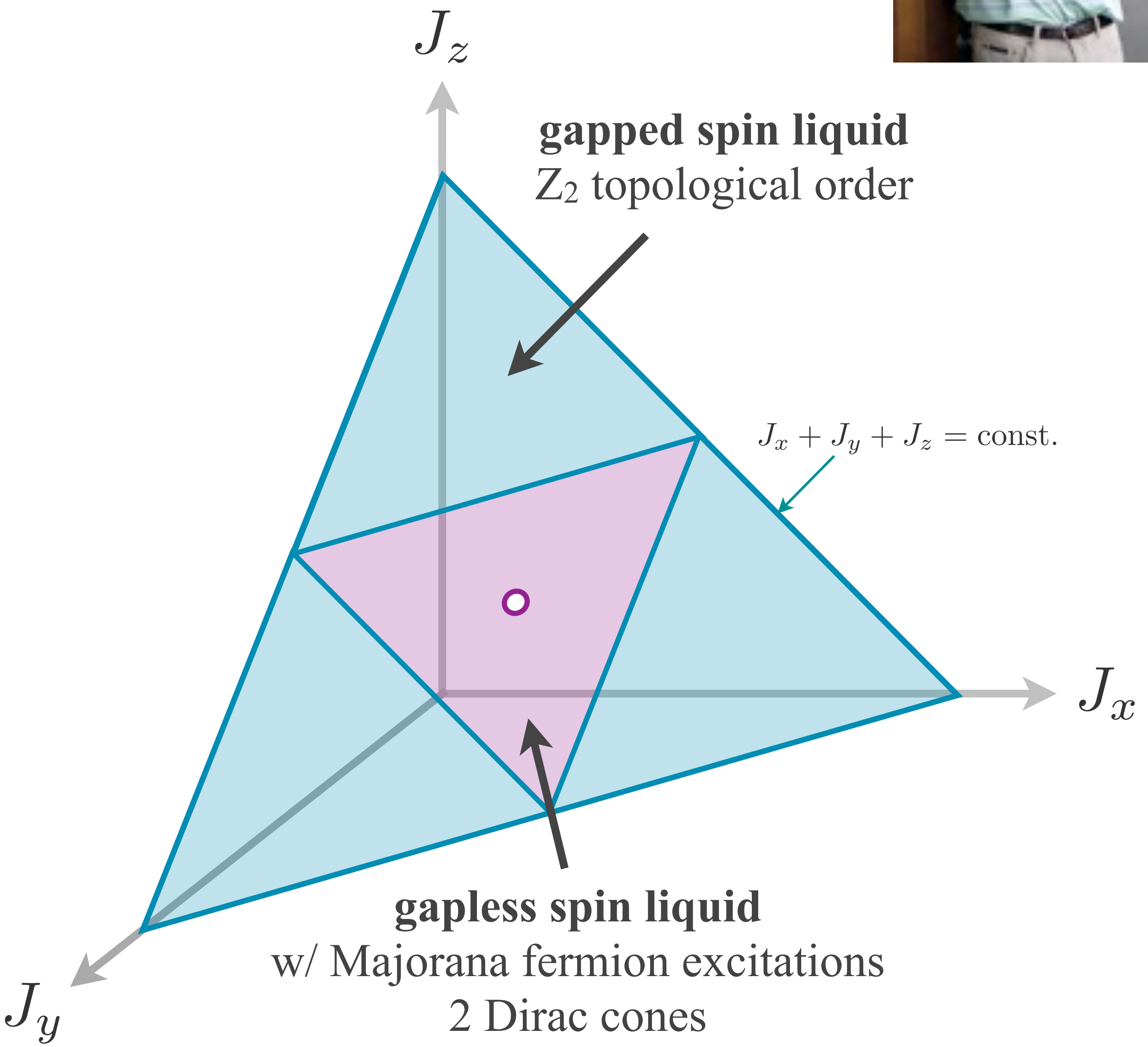
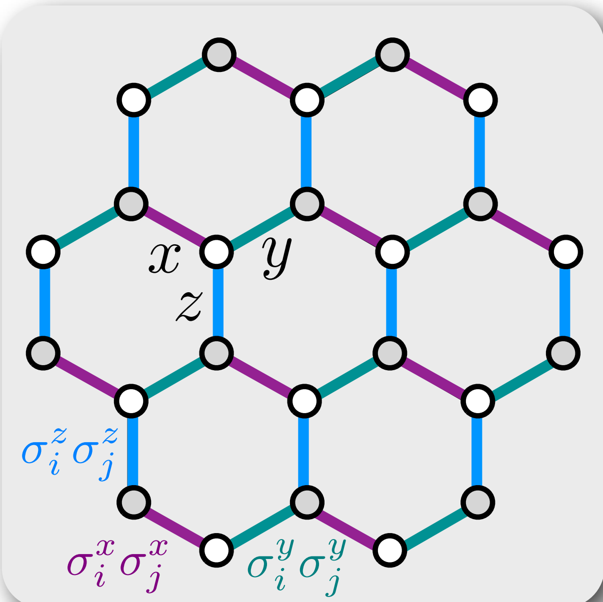
Rare combination of a model of **fundamental conceptual importance** (harboring topological phases) and an **exact analytical solution**.



A. Kitaev, Ann. Phys. **321**, 2 (2006)

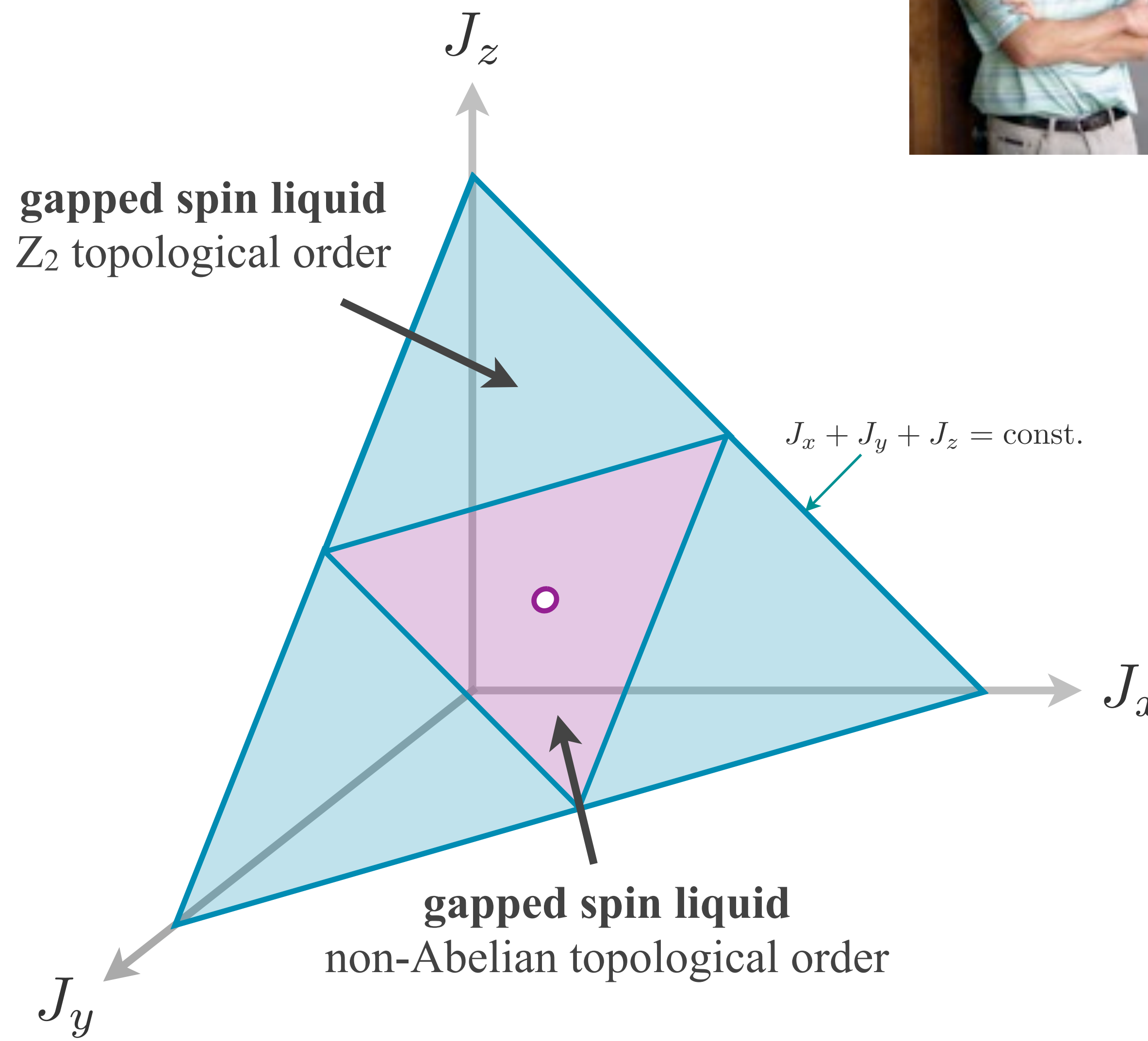
The Kitaev model + field



$$H_{\text{Kitaev+h}} = \sum_{\gamma-\text{links}} J_\gamma \sigma_i^\gamma \sigma_j^\gamma$$

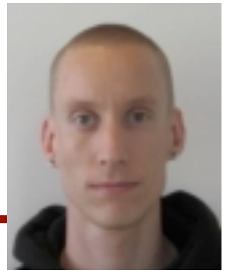
$$\boxed{-\sum_i \vec{h} \cdot \vec{\sigma_i}}$$

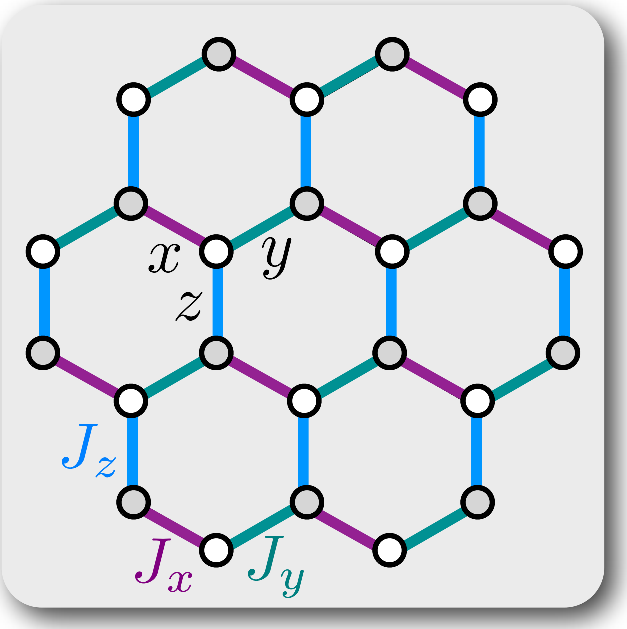
$$\vec{h} = h\,(1,1,1)$$

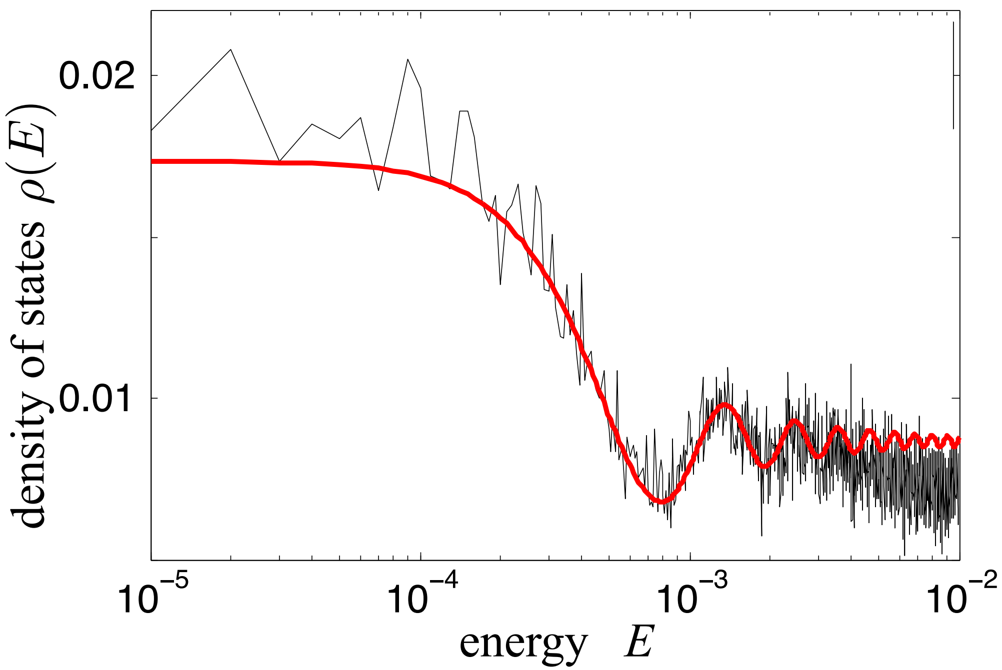


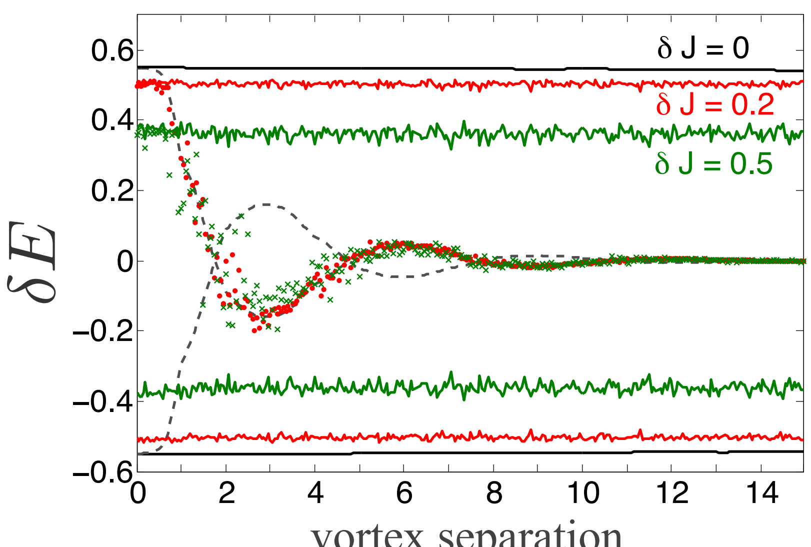
vortices are **Ising anyons** with **Majorana fermion** zero modes

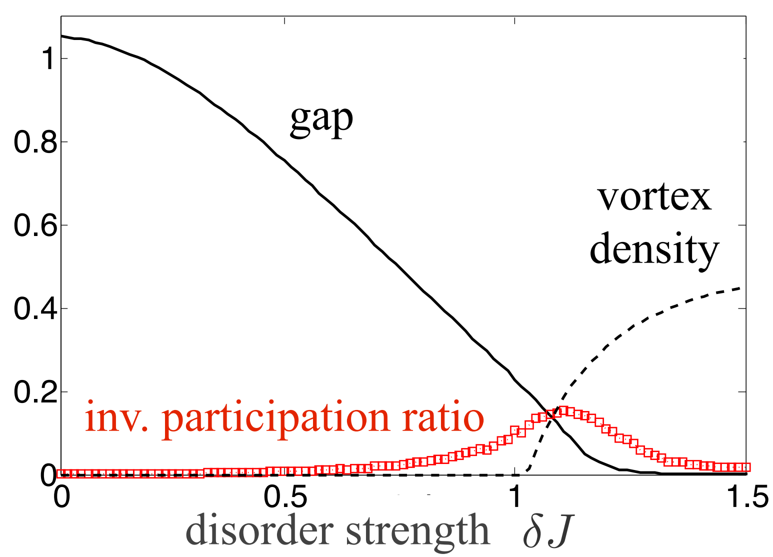
Kitaev model + field + disorder



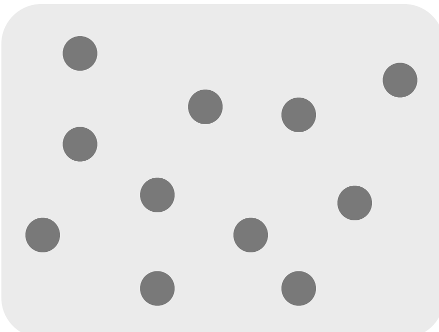




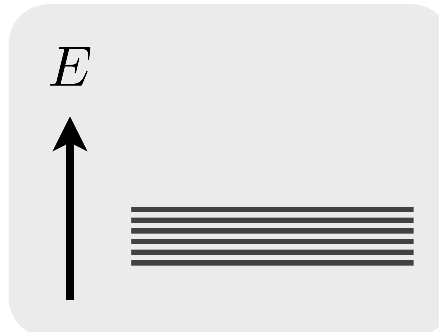




Summary



interacting
many-body system



'accidental'
degeneracy

residual effects
select ground state

Topological excitations + interactions + disorder can give rise to a plethora of collective phenomena.

- Topological **liquid nucleation**
- **Thermal metal**
- Distinct experimental **bulk observable** (heat transport) in search for Majorana fermions.