Nishimori's Cat

Stable long-range entanglement
from finite-depth unitaries & weak measurements

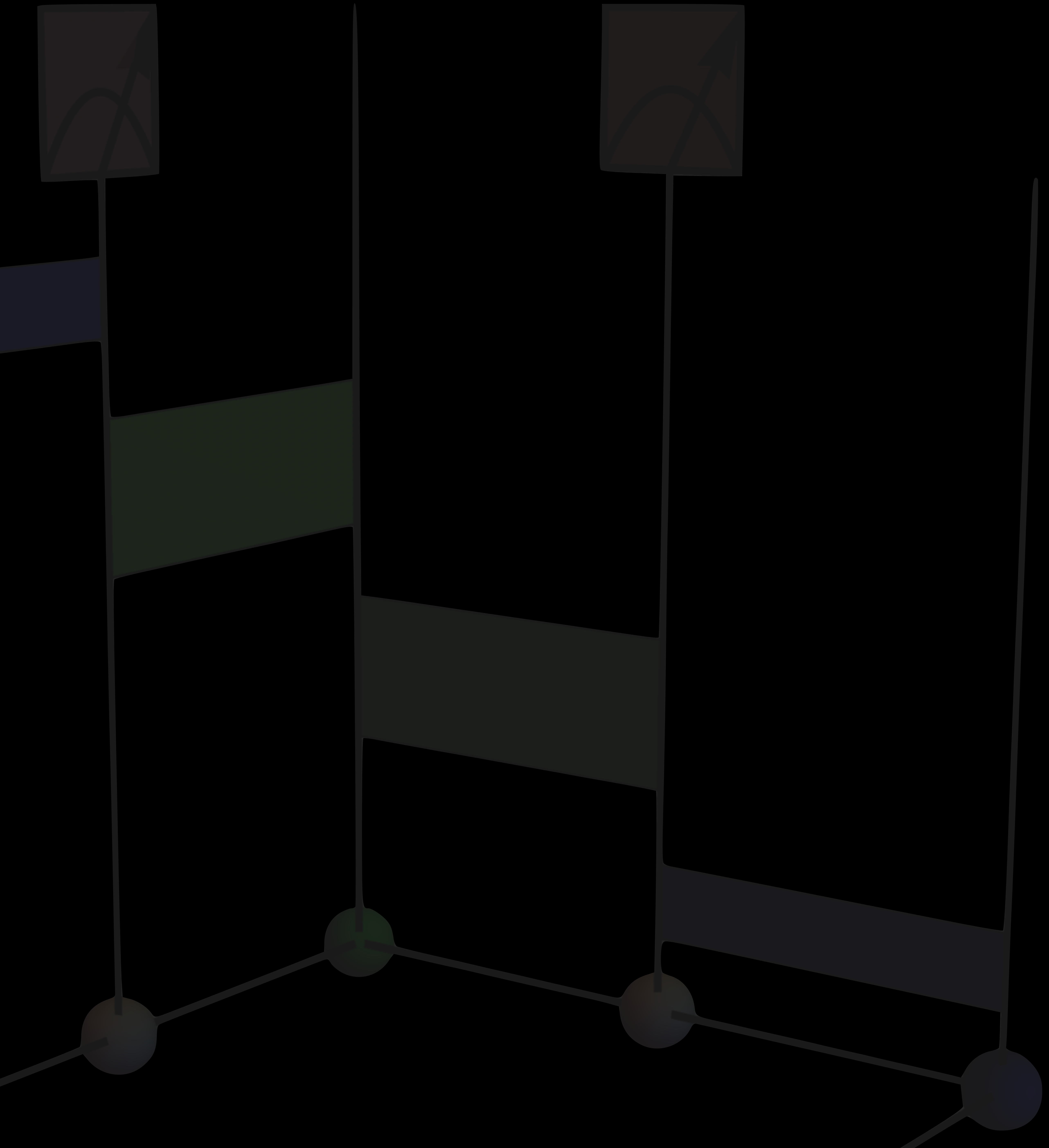
Simon Trebst
University of Cologne

CRC183
ENTANGLED STATES OF MATTER

A Quantum Many-Body Handshake:
Theory and Simulation meet Experiment
Weizmann Institute of Science, December 2022

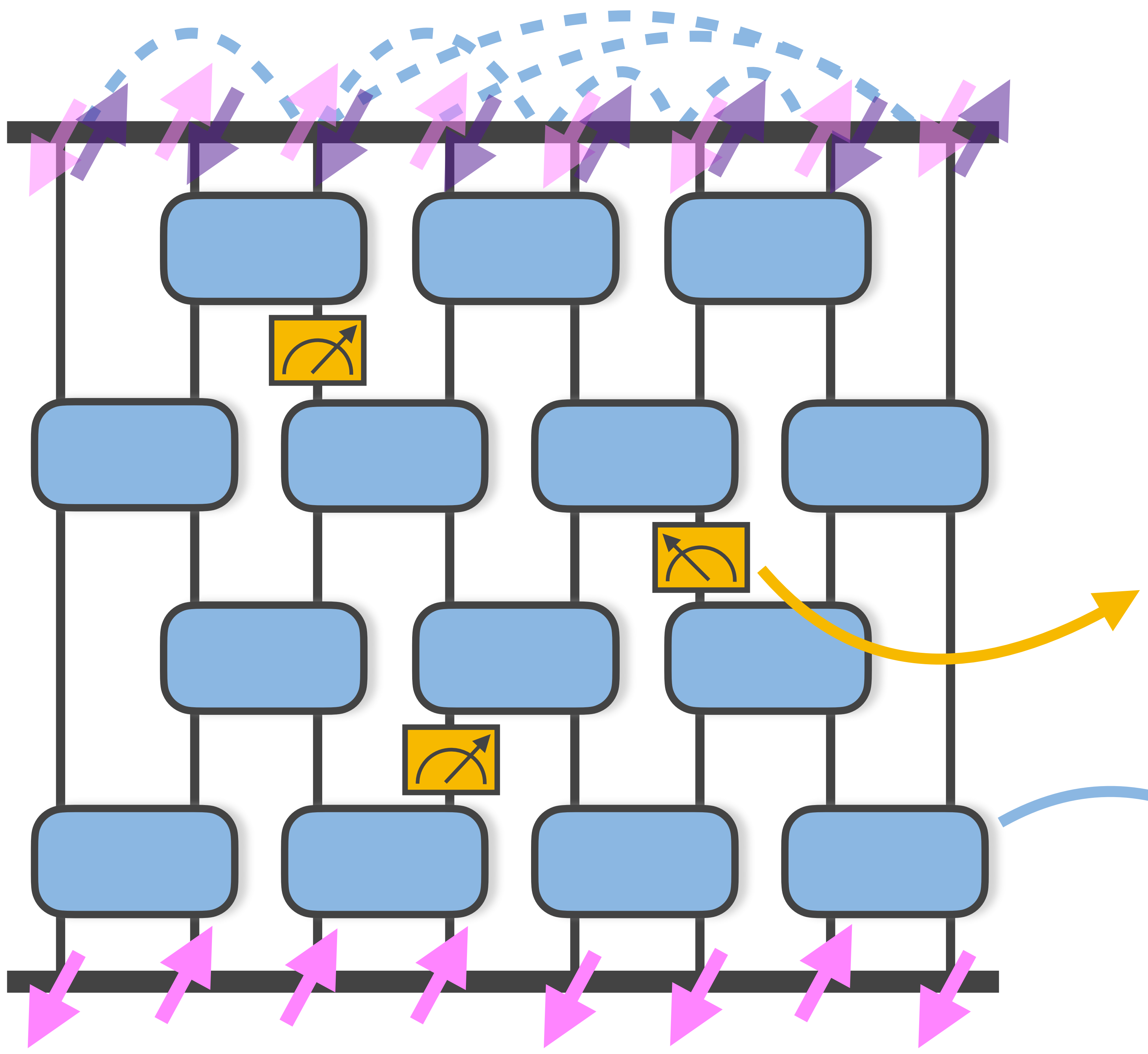
A Quantum Many-Body Handshake:
Theory and **Simulation** meet **Experiment**

A Quantum Many-Body Handshake:
Theory and **Simulation** meet **Experiment**



monitored quantum circuits

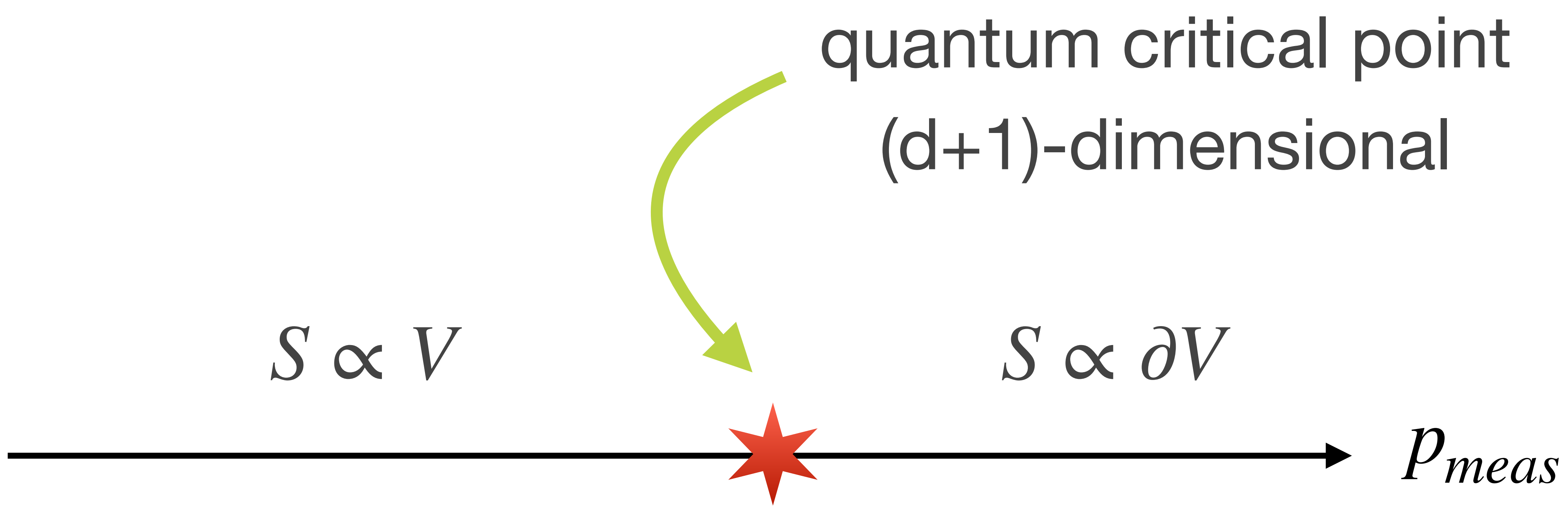
measurement-induced **phase transitions**



measurements (non-unitary)

gates (unitary)

monitored quantum circuit



volume law entangled — **area law** entangled

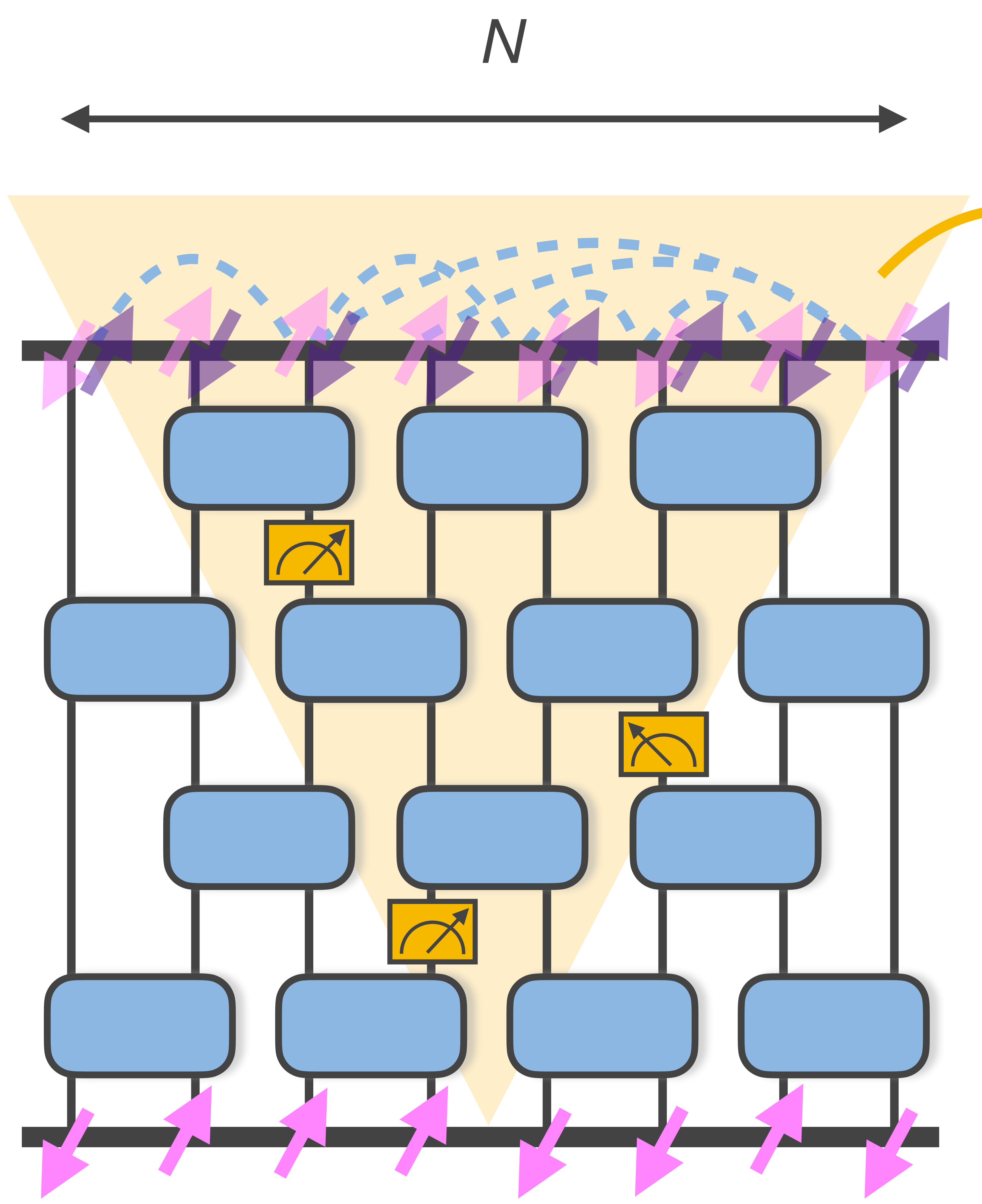
- spontaneous symmetry breaking orders
- symmetry protected topological orders
- intrinsic topological orders

Non-deterministic, deep circuits.

Fisher, Khemani, Nahum, Vijay, review 2022

measurement-based **state preparation**

Lieb Robinson bounds, Bravyi, Hastings, Verstraete 2006; Raussendorf 2001; Cirac 2008; Cirac 2021; Vishwanath 2022



information **light-cone**

monitored quantum circuit

local, unitary gates

circuit depth $\propto O(N)$

local, unitary gates **+ measurements**

circuit depth $\propto O(1)$

Greenberger–Horne–Zeilinger

$$|\psi\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

cat state

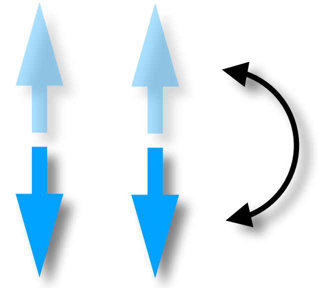
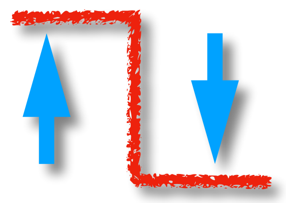
deterministic, shallow circuits?

preparing **cat states**

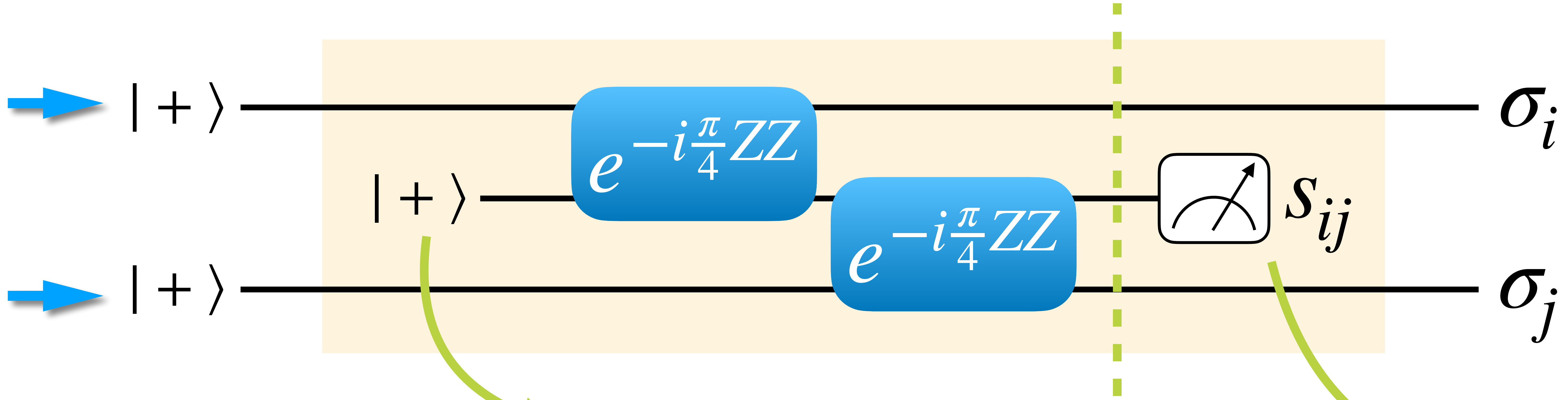
Greenberger–Horne–Zeilinger

$$|\psi\rangle = \begin{matrix} |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \\ + \\ |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle \end{matrix}$$

cat state

- Ising symmetry  ✓
- domain wall  ✗

$$\lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle_c = 1$$



$$\left\{ \begin{matrix} |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle & s = -1 & ✓ \\ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle & s = +1 & ✗ \end{matrix} \right.$$

ancilla qubit

$$|++\rangle = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

cluster state

$$s_{ij}^x = -\sigma_i^z \sigma_j^z$$

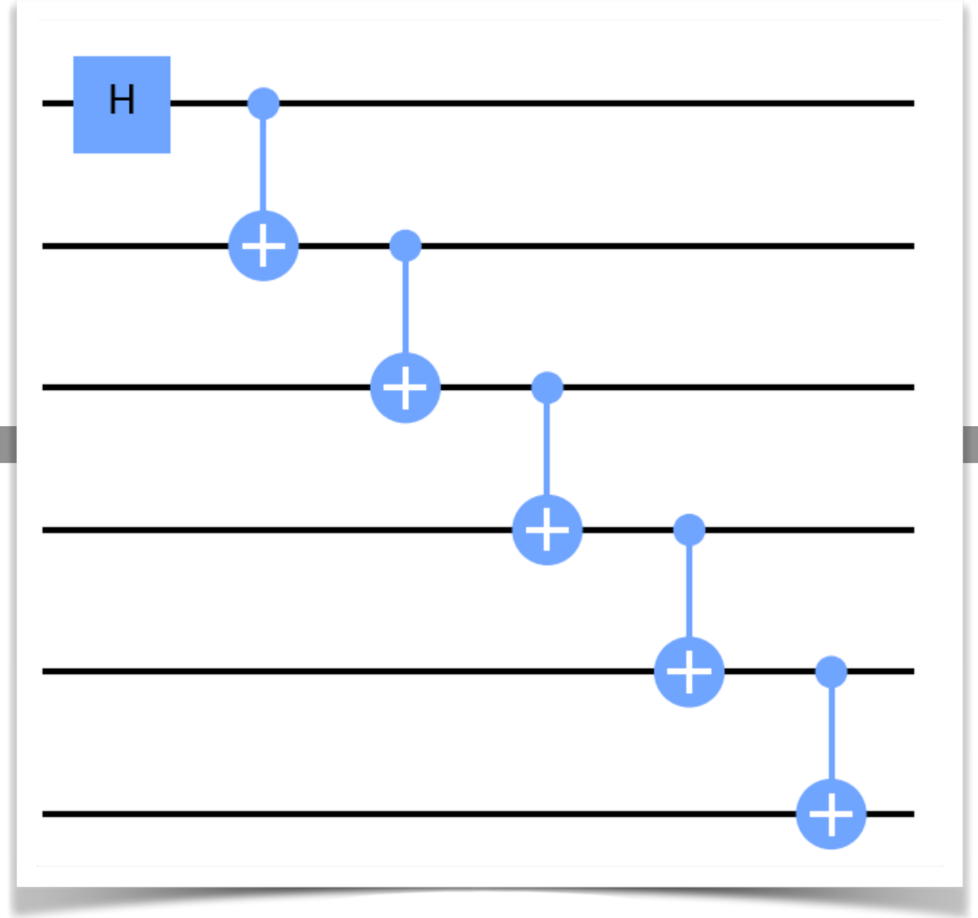
measure ancilla
= collapse domain-walls

Raussendorf 2001; Cirac 2021

preparing **cat states**

unitaries

$|\uparrow\rangle^{\otimes N}$



Greenberger–Horne–Zeilinger

$$|\psi\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

circuit depth $\propto O(N)$

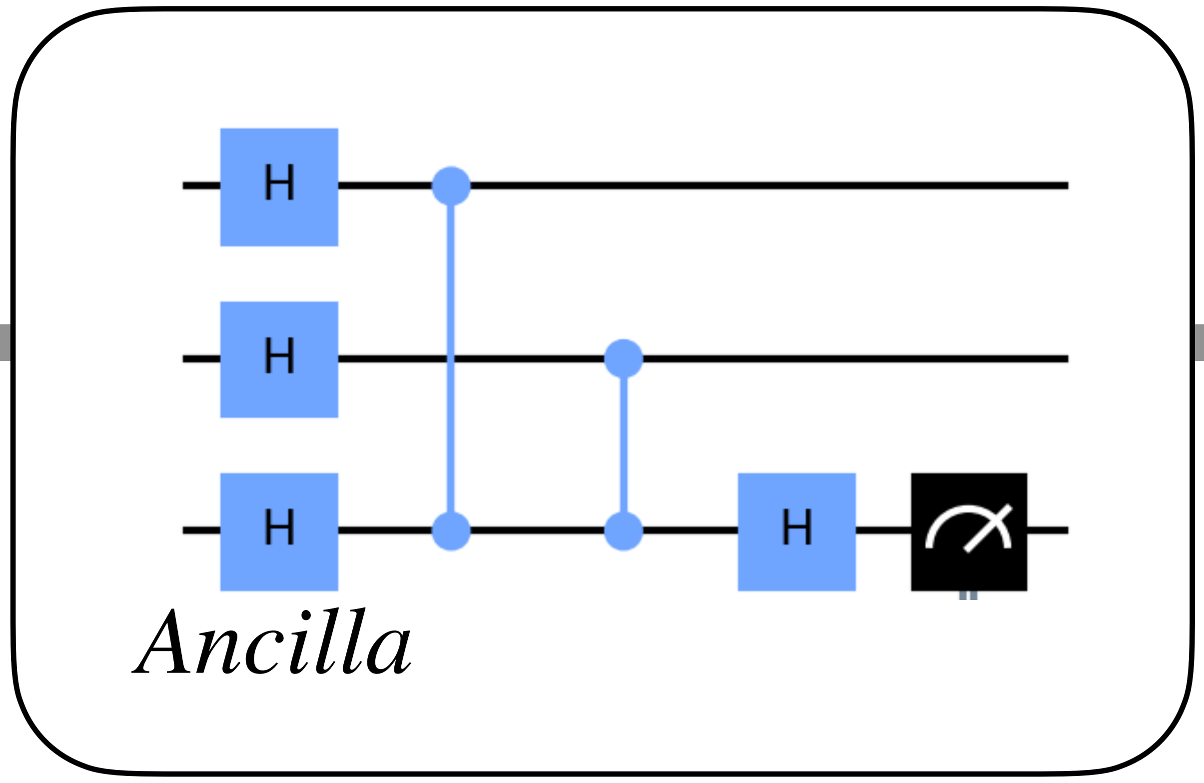
decoder

domain wall correction

measurements

ZZ parity check for *every bond*

$|\uparrow\rangle^{\otimes N}$



$$CZ \sim e^{-i\frac{\pi}{4}ZZ}$$

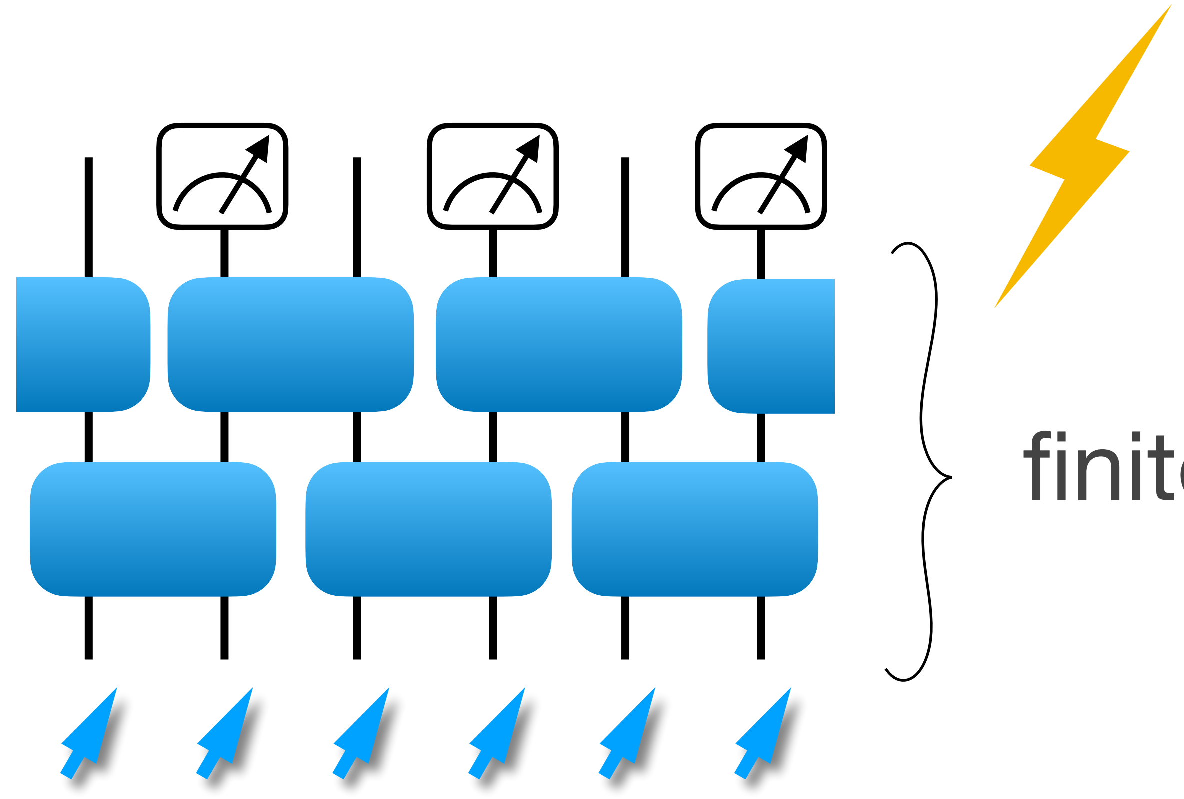
$$|\psi\rangle = |\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle$$

glassy GHZ state

circuit depth $\propto O(1)$

circuit **imperfections**

coherent noise



finite depth

gate imperfections

$$CZ \sim e^{-i\frac{\pi}{4}ZZ}$$

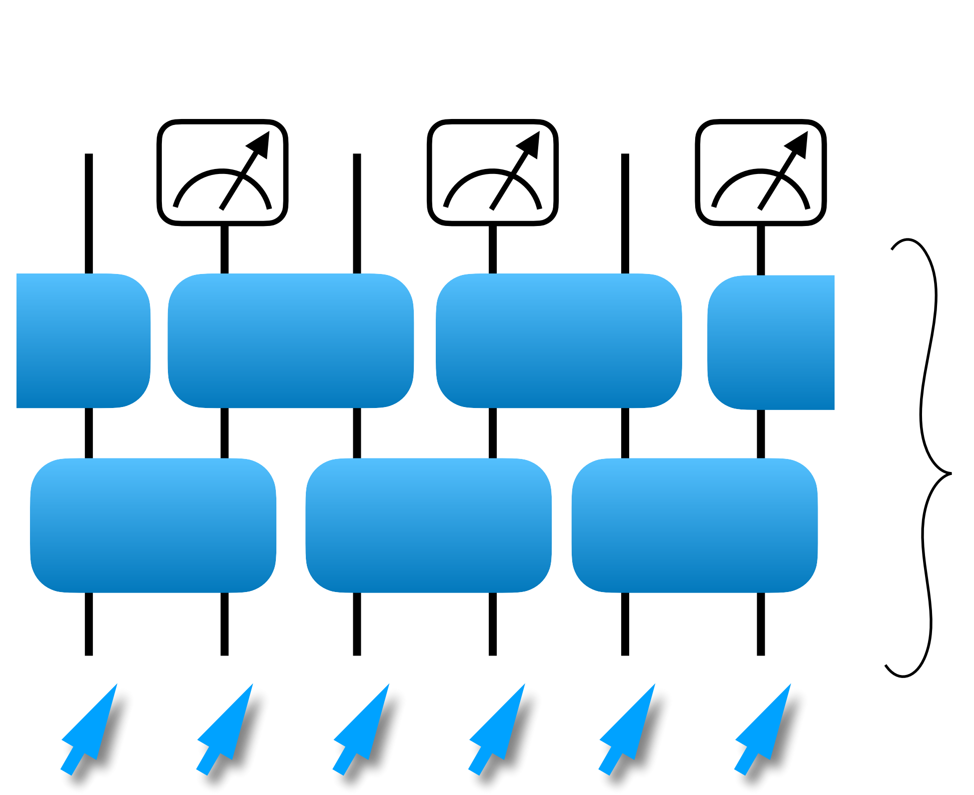
incomplete rotation → **weaken** measurement

conceptual question

Does the formation of **long-range entanglement** in these engineered states entail a similar **notion of stability** as known fromquantum ground states?

circuit **imperfections**

coherent noise



finite depth

gate imperfections

$$CZ \sim e^{-i\frac{\pi}{4}ZZ}$$

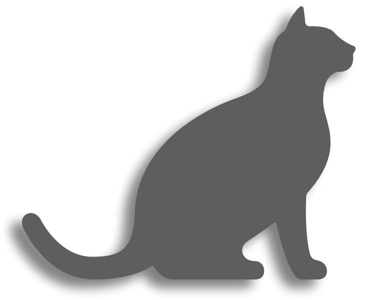
incomplete rotation → **weaken** measurement

conceptual answer

weak measurements — SRE | strong measurements — LRE → $\beta_{measurement}$

finite stability threshold

Nishimori's cat



meet the team

arXiv:2208.11136

Guo-Yi Zhu

Nat Tantivasadakarn

Ashvin Vishwanath

Ruben Verresen

University of Cologne

Harvard

shallow quantum circuit

weak measurements

LRE ?

SRE

$t_A = \pi/4$

$t_B < \pi/4$

gate imperfection

product

strong measurements

GHZ

measurement (ancillas)

cluster

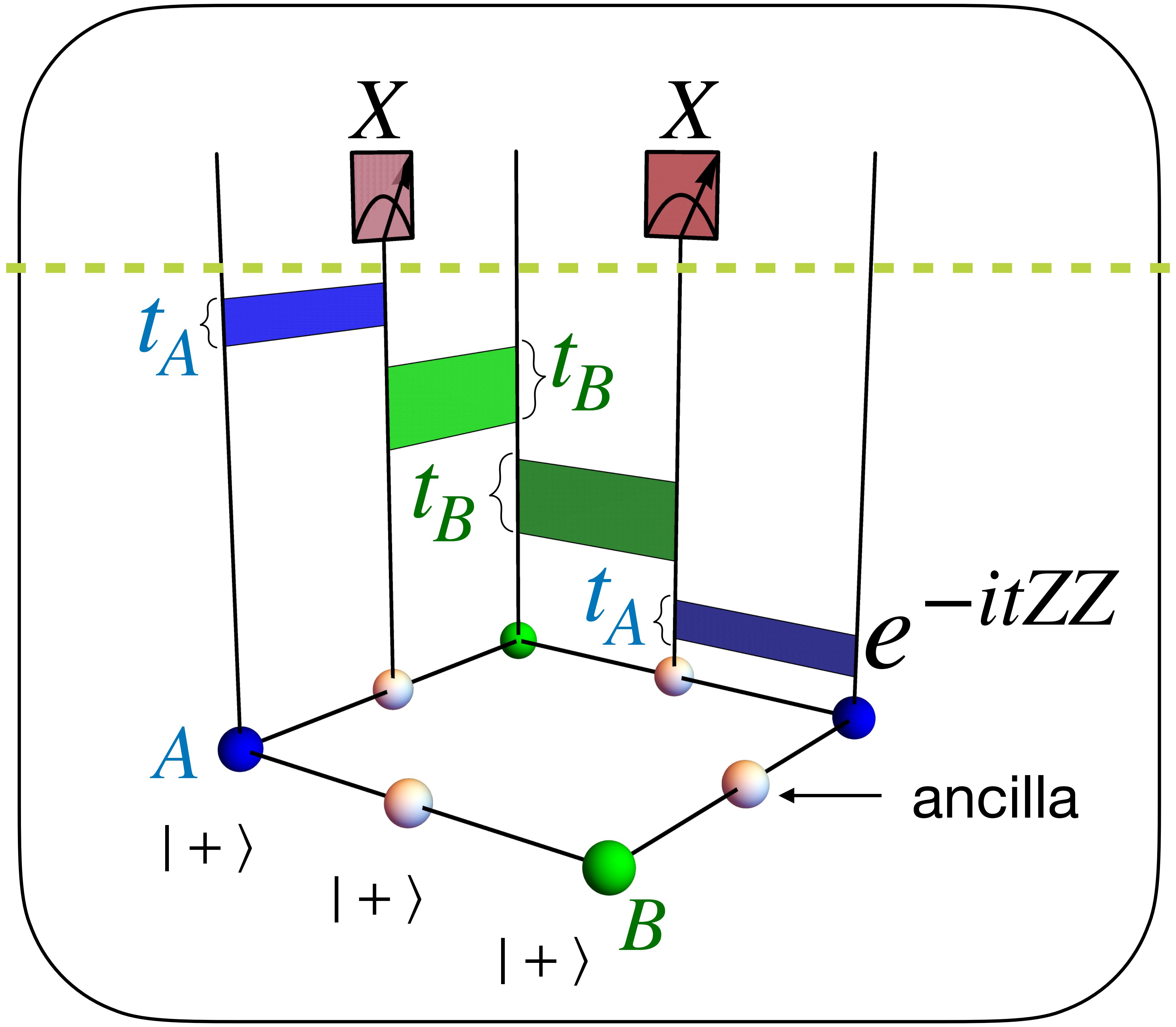
unitary evolution

$t_A = \pi/4$

$t_B = \pi/4$

product

$\{s_l^x \rightarrow s = \pm 1\}$ **disordered** state $\langle \{s\} | \psi \rangle$



post-measurement state

unitary evolution times
lock temperature & coupling

$$\langle \{s\} | \psi \rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} | + \rangle^{\otimes N}$$

$$\tanh \frac{\beta}{2} J_+ = \tan t_A \tan t_B$$

$$\tanh \frac{\beta}{2} J_- = -\tan t_A \cot t_B$$

interprete as
classical
stat mech model

$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} (J_{s_{ij}} \sigma_i \sigma_j + h s_{ij})}$$

random bond Ising model

thermal fluctuations and
disorder are **locked**

post-measurement state

$\langle \{s\} | \psi \rangle$

interprete as **classical stat mech model**

$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} (J_{s_{ij}} \sigma_i \sigma_j + h s_{ij})}$$

random bond Ising model

thermal fluctuations and disorder are **locked**

$$\tanh \frac{\beta}{2} J_+ = \tan t_A \tan t_B$$

$$\tanh \frac{\beta}{2} J_- = -\tan t_A \cot t_B$$

	weak measurement	**strong** measurement	
real time t_A, t_B	0	$\pi/4$	
imag time β	0	$+\infty$	
	"high temperature"	"low temperature"	

Nishimori physics

random bond Ising model

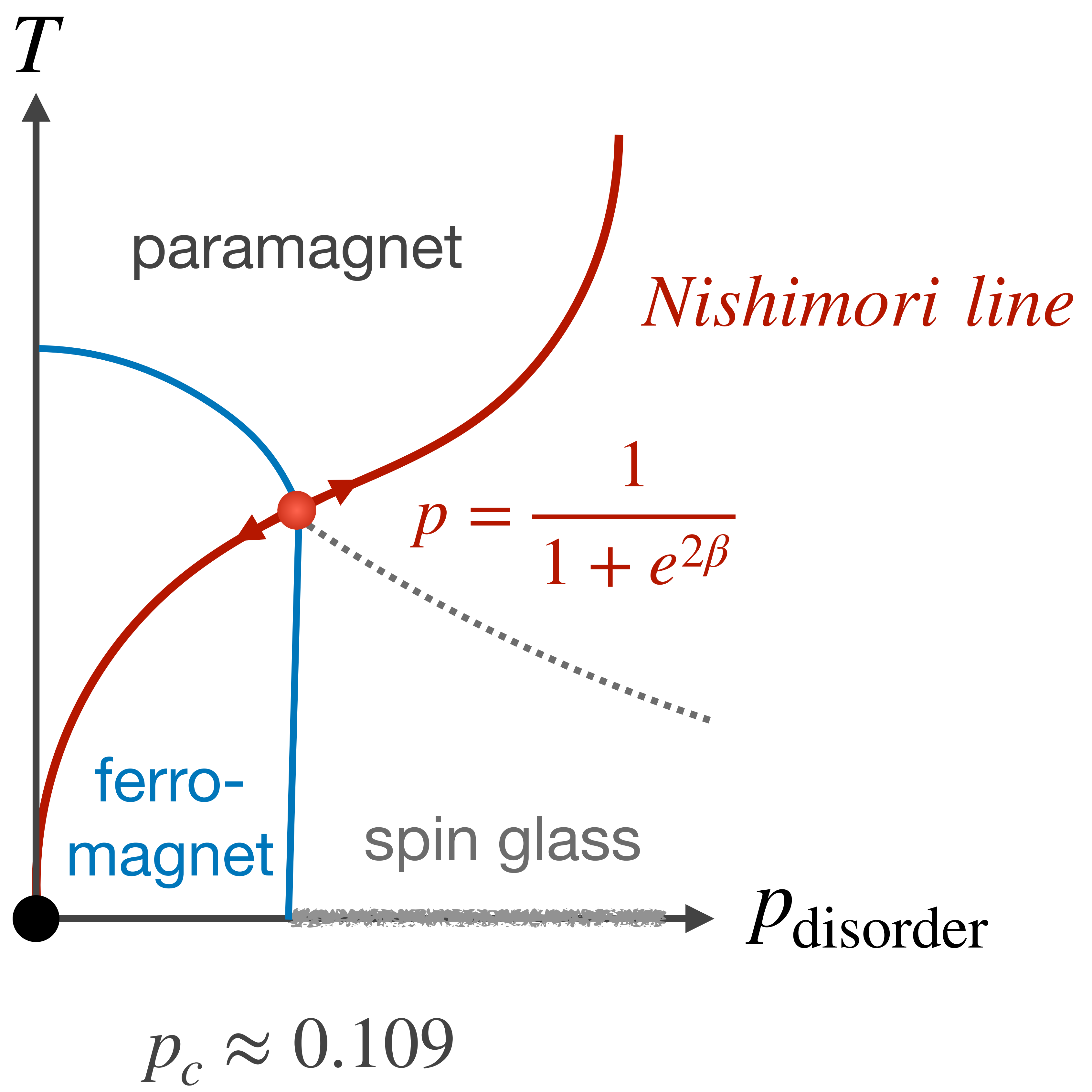
thermal fluctuations and disorder are **locked**

Nishimori (1981)

disorder "temperature" = thermal "temperature"

uncorrelated disorder
gauge symmetry

- internal energy is analytic
- correlation (in-)equalities
- free energy = frustration entropy
- RG scaling axis
- unstable multi-critical point
- separate FM / PM / SG phases
- reentrant phase boundary



Nishimori physics

random bond Ising model

thermal fluctuations and disorder are **locked**

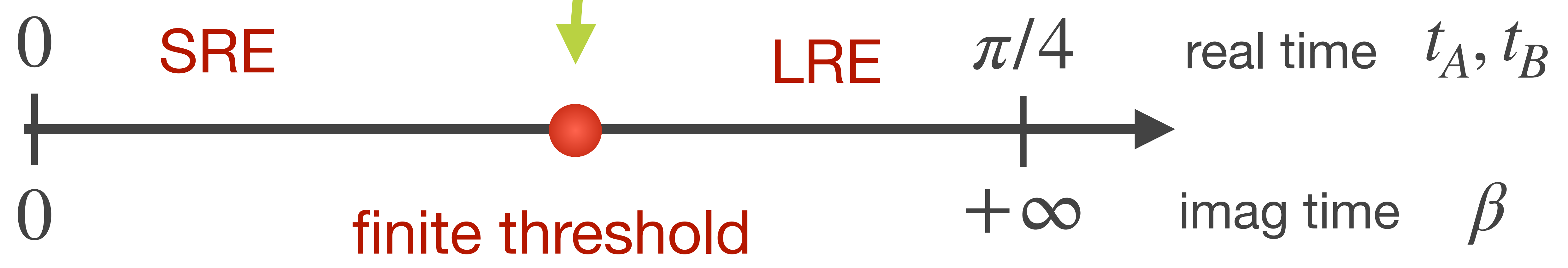
Nishimori (1981)

disorder “temperature” = thermal “temperature”

uncorrelated disorder
gauge symmetry

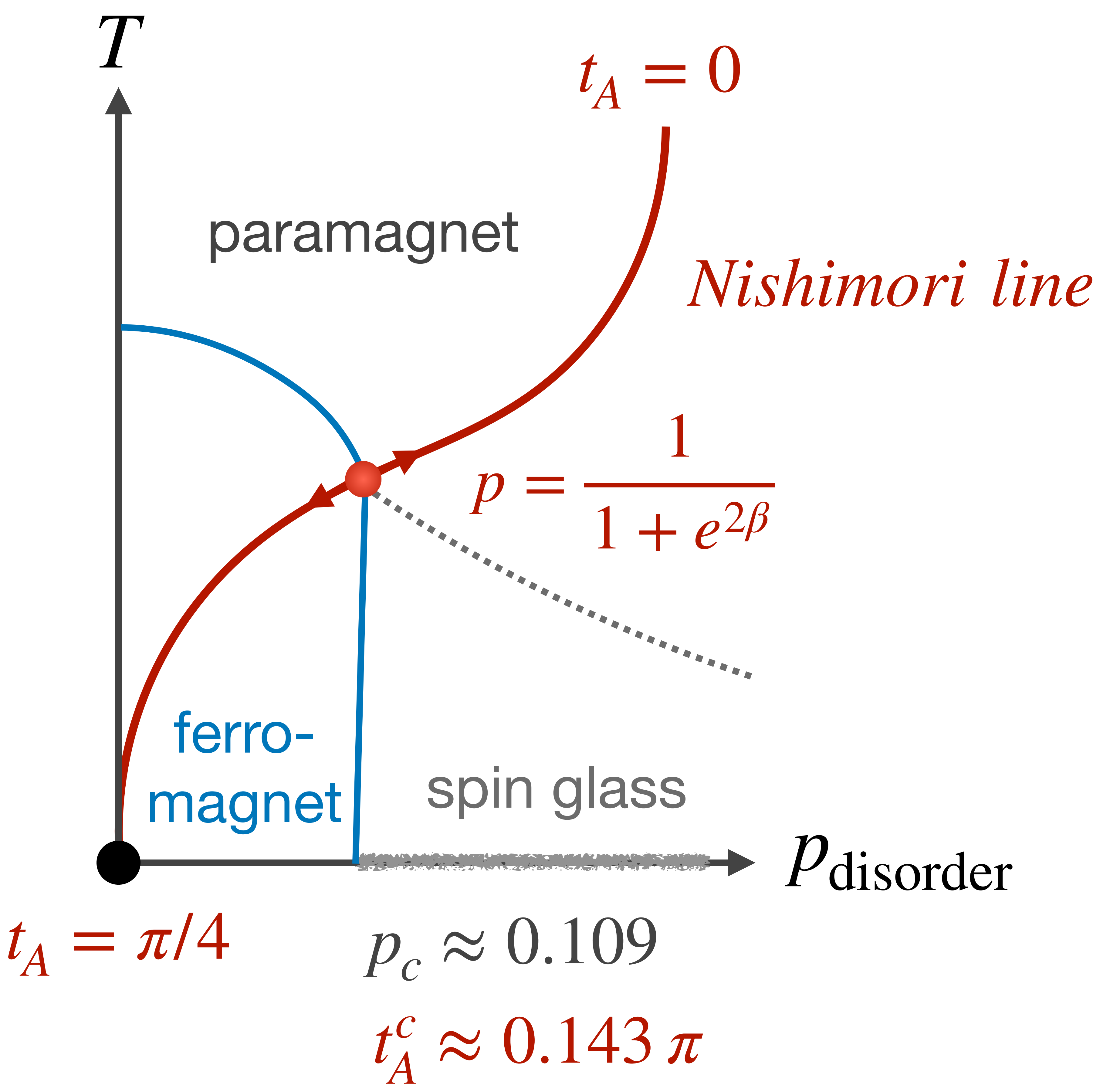
weak measurement

strong measurement

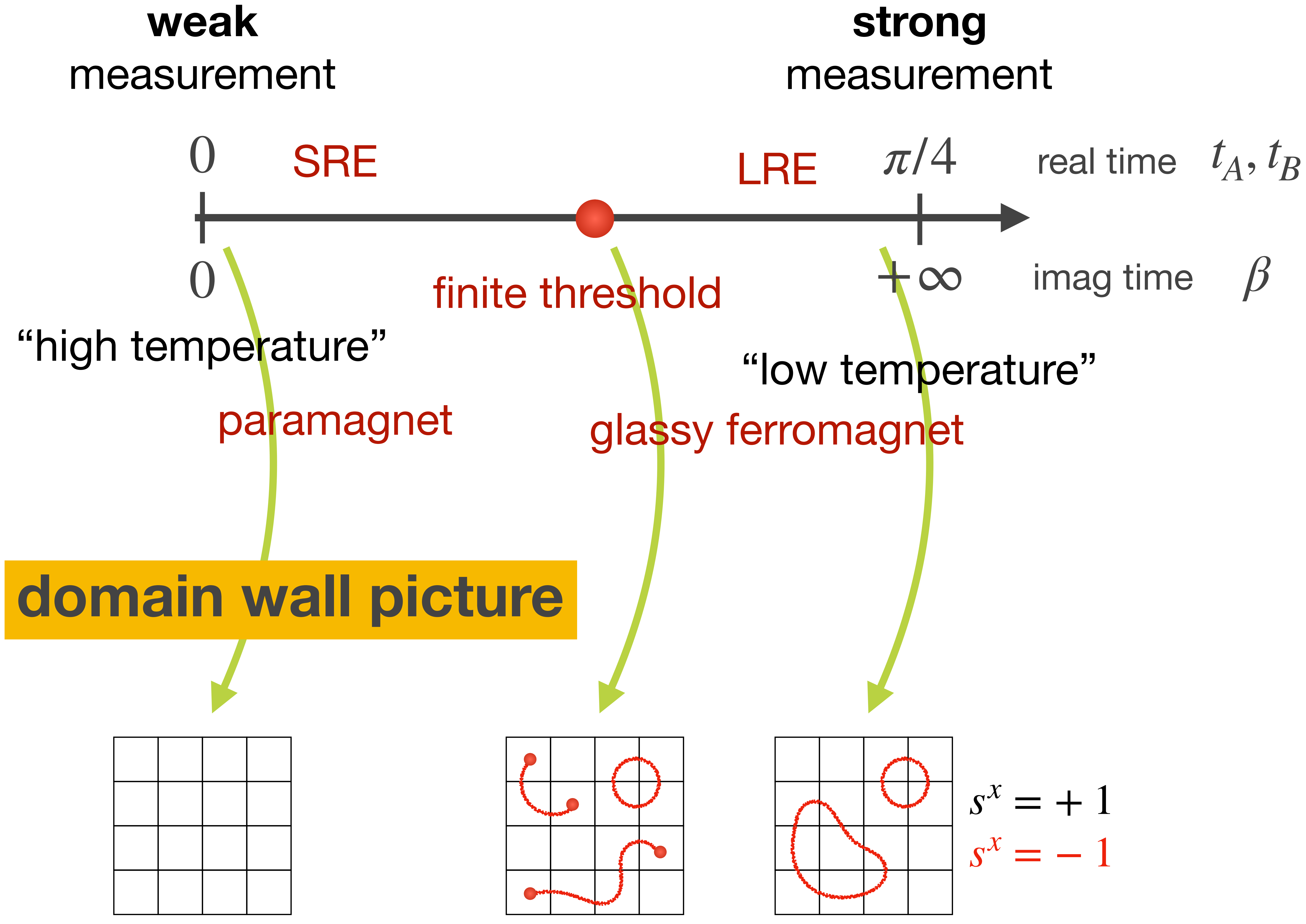


“high temperature”
paramagnet

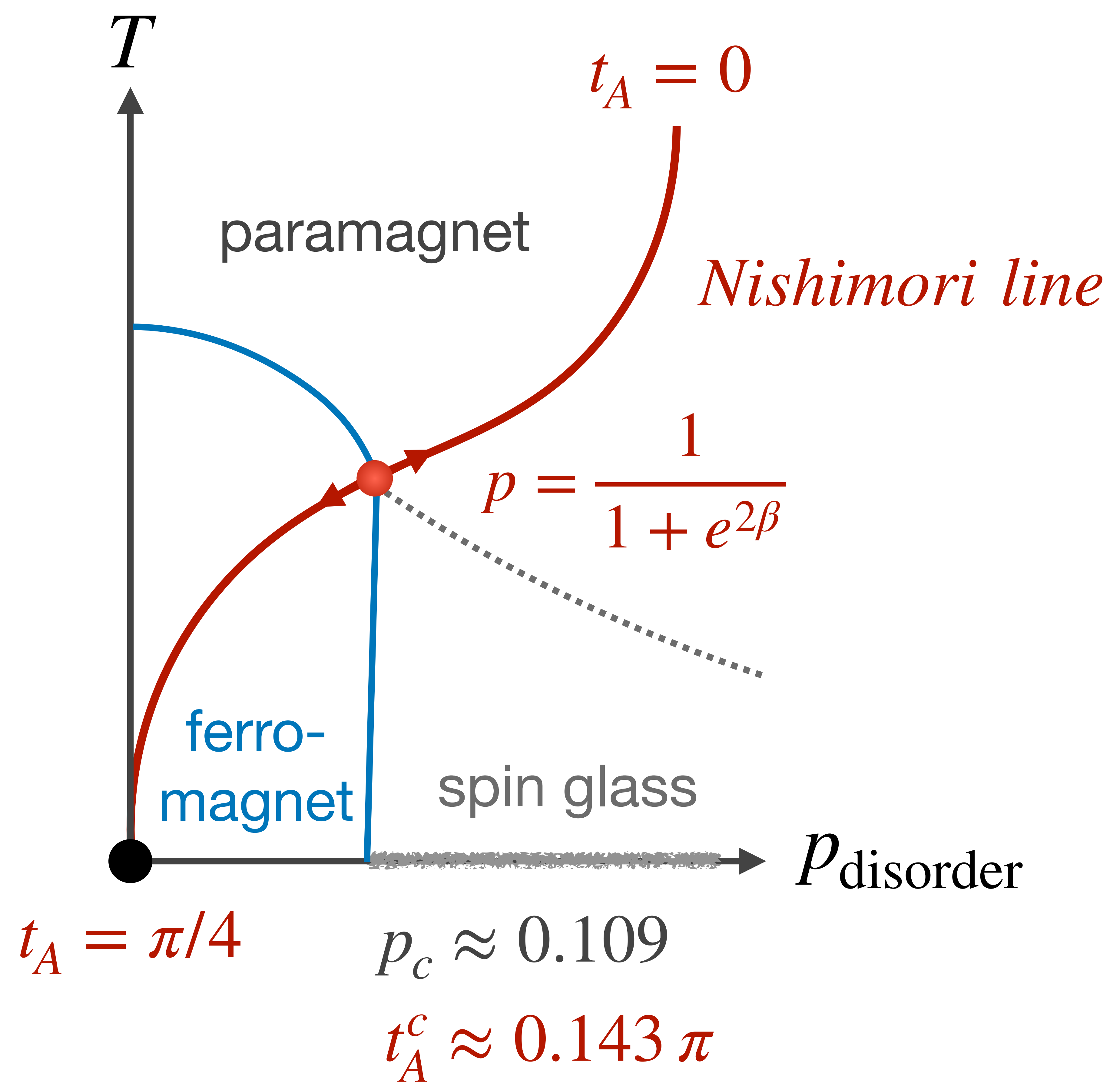
“low temperature”
glassy ferromagnet



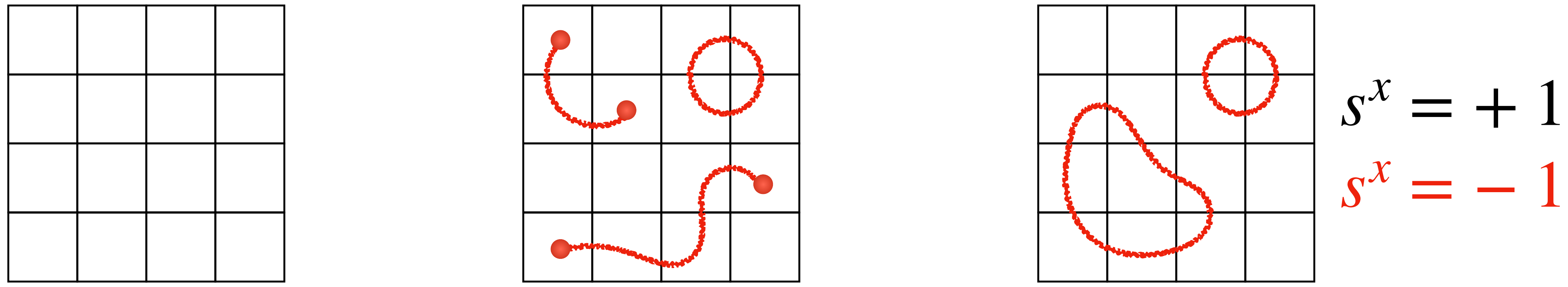
decoding **Nishimori** physics



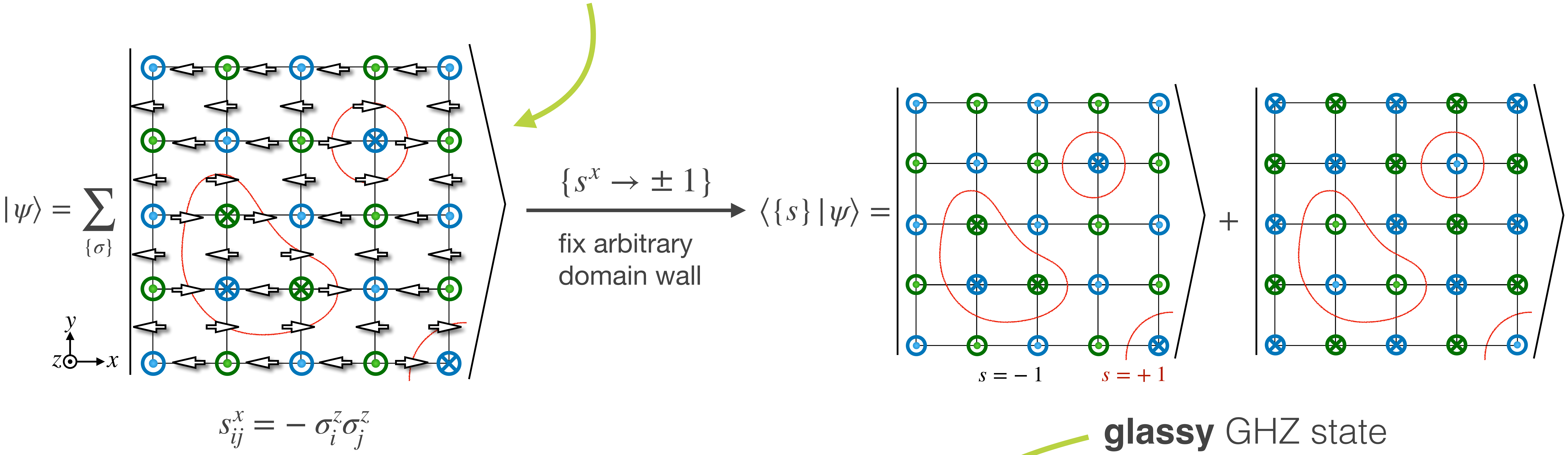
Ising vortex disorder



decoding **Nishimori** physics



domain wall picture

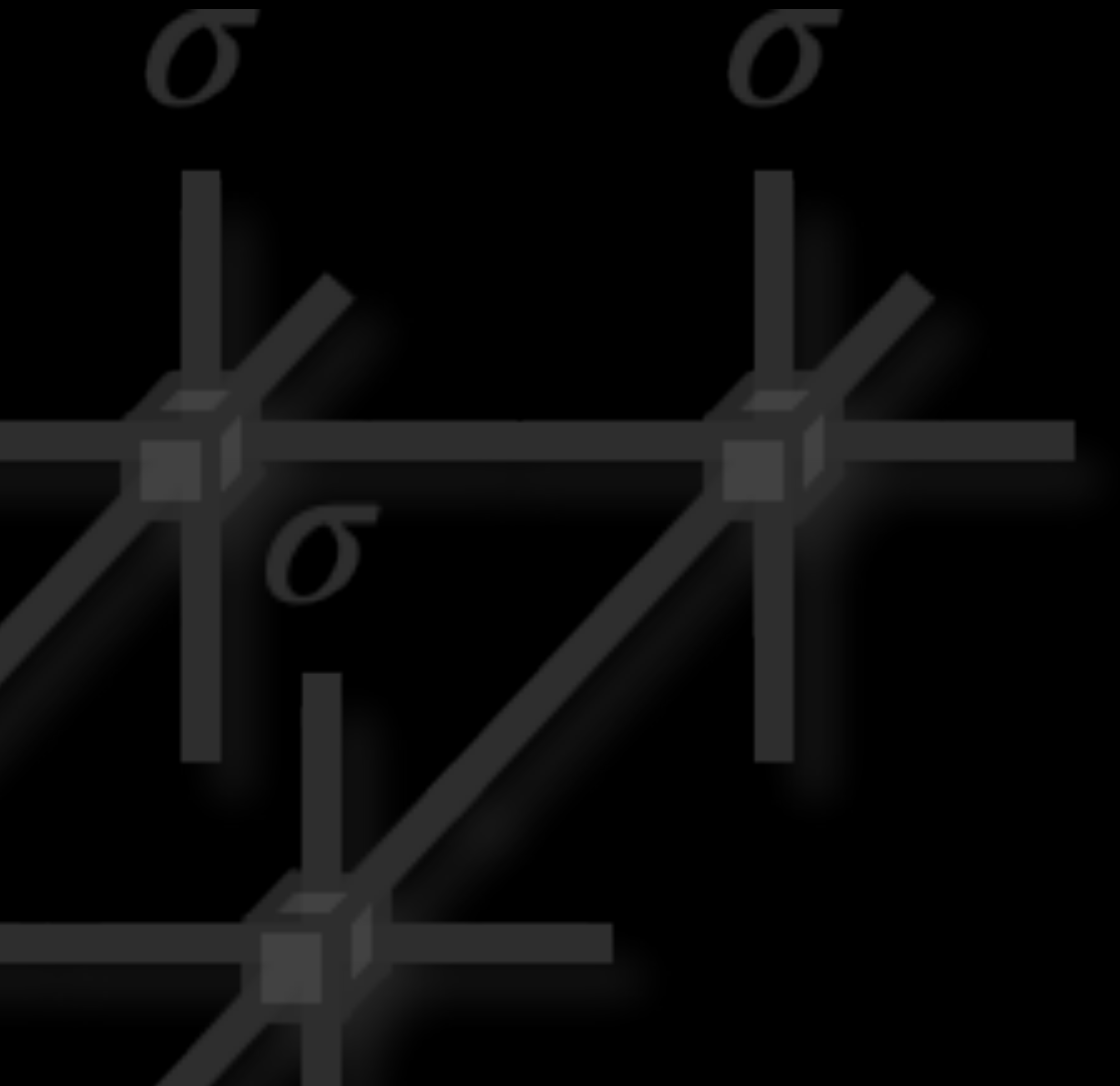


glassy GHZ state

decoder

flip "wrong" domains

clean GHZ state



tensor network calculations

hybrid tensor network & Monte Carlo $p_{\{s\}} \propto Z_{\{s\}}$

two **degrees of freedom**

{s} traced by Monte Carlo

{sigma} traced by tensor network

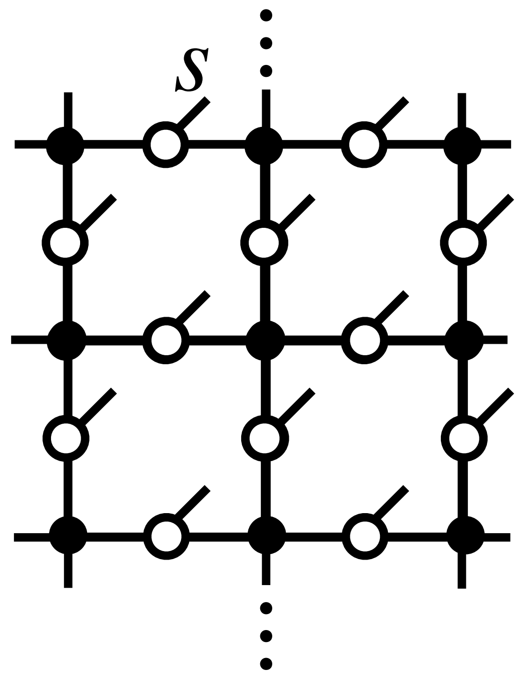
$$q = \sum_{\{s\}} p_{\{s\}} \, \langle \psi_\sigma | \sigma_j | \psi_\sigma \rangle^2_{\{s\}}$$

quantum average

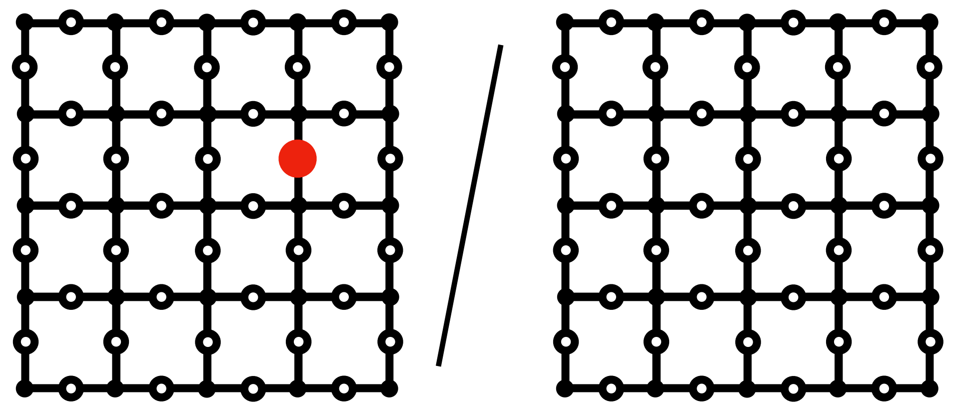
tensor network

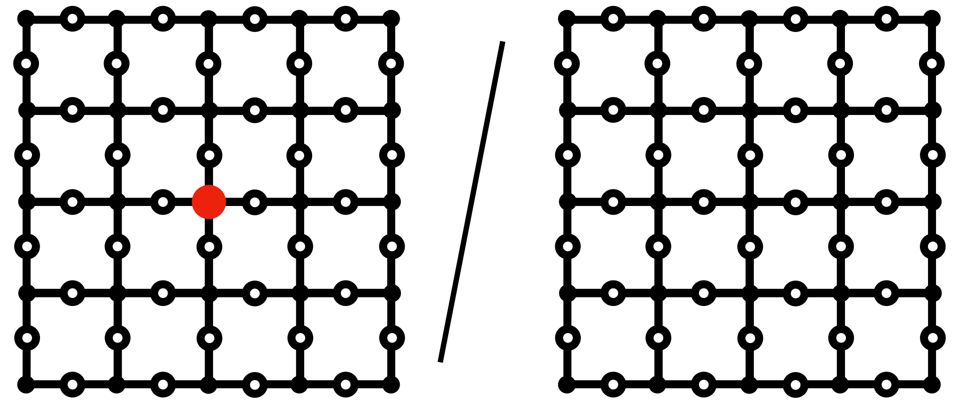
measurement (disorder) average

Monte Carlo

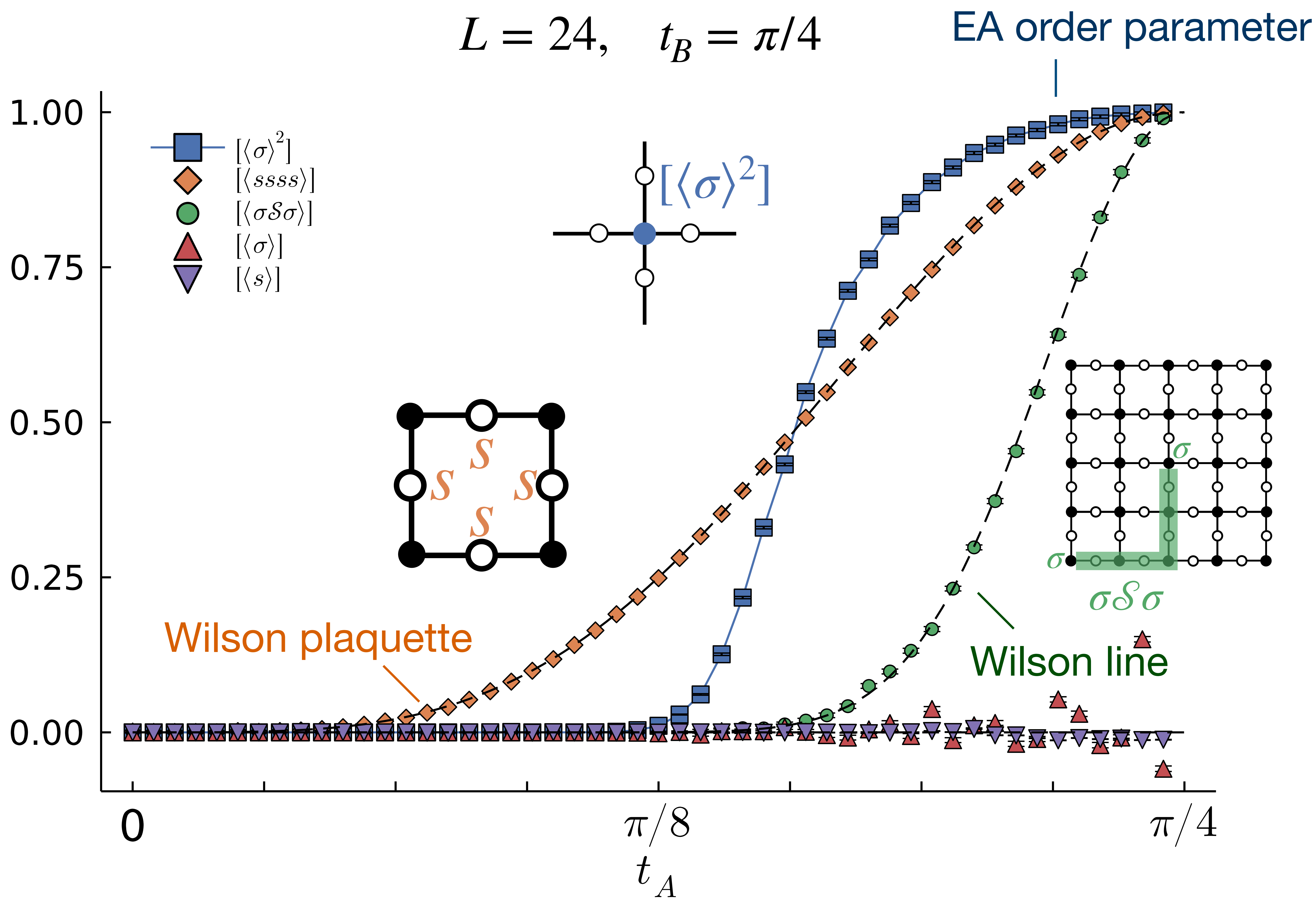
$p_{\{s\}} = \cdots$  $\cdots$

finite disordered TN

$\frac{p_{\{s'\}}}{p_{\{s\}}} =$ 

$\langle \sigma_j \rangle =$ 

Nishimori line



$\beta = \ln|\tan(t_A + \pi/4)|$

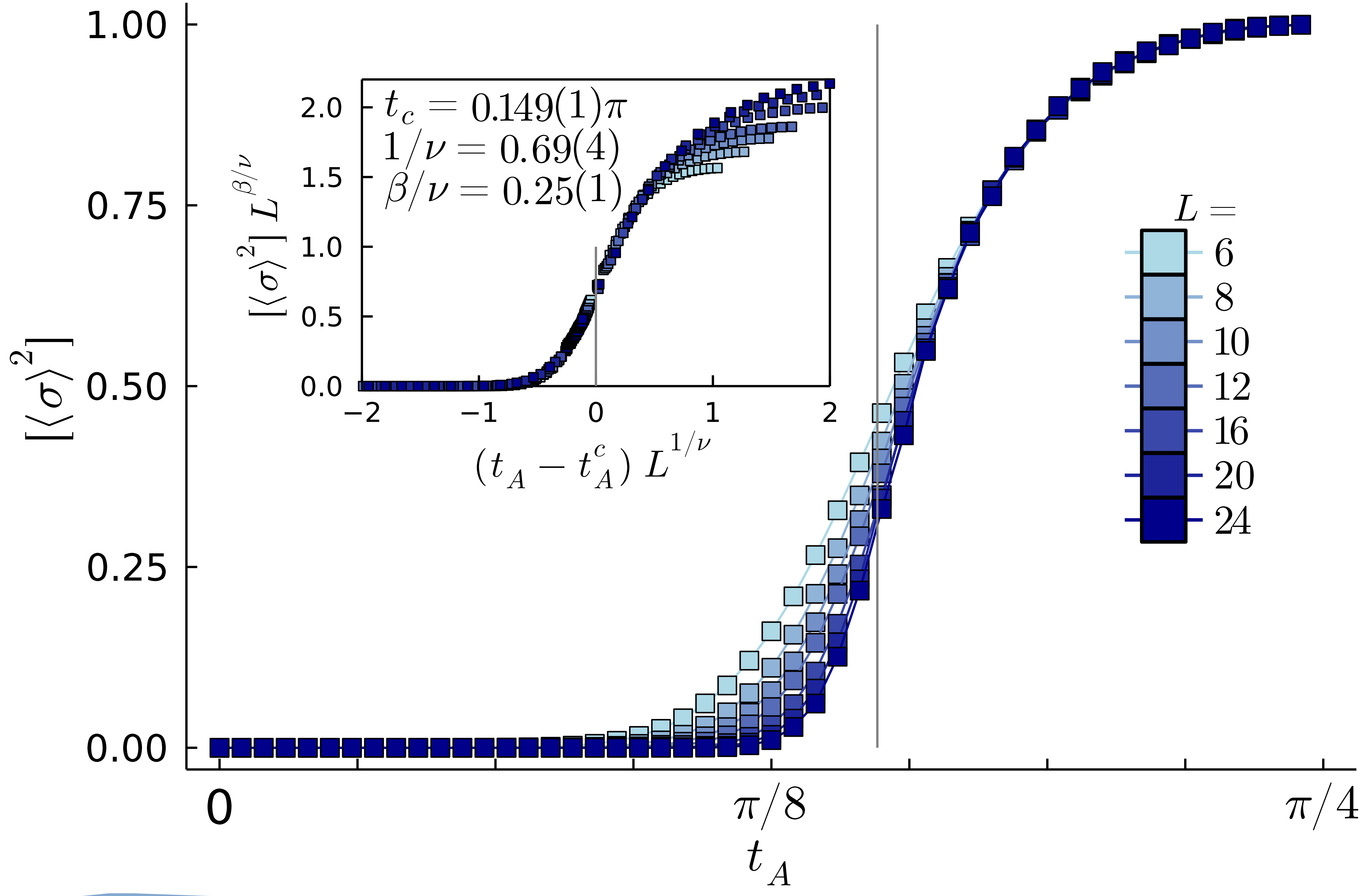
$[\langle s \rangle] = 0$

$[\langle ssss \rangle] = \sin^4(2t_A)$

$[\langle \sigma \mathcal{S} \sigma \rangle] = \sin(2t_A)^{2L}$

gauge invariant quantities

finite-size scaling



established RBIM numerics

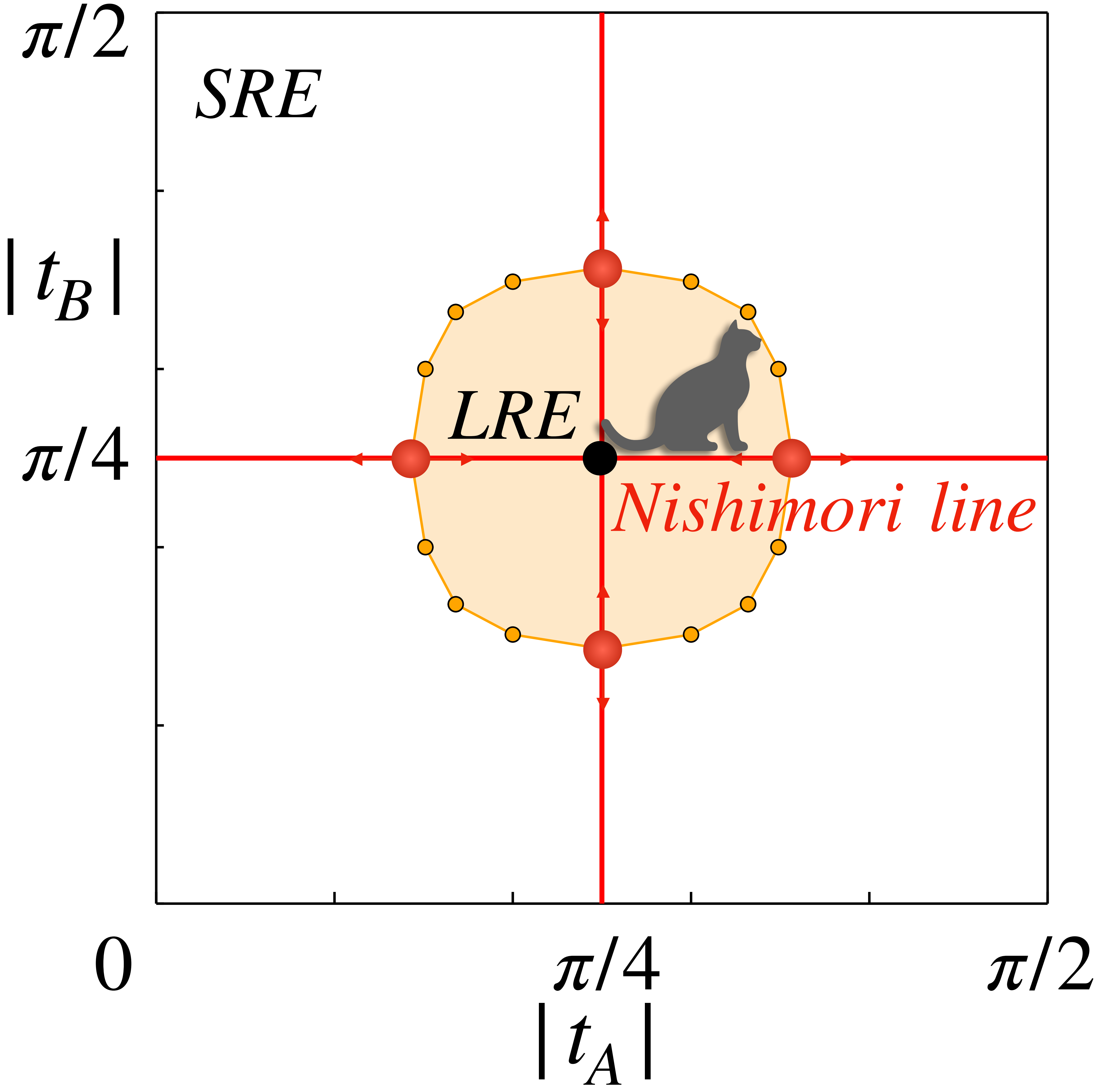
$p_c \approx 0.109$

$t_A^c \approx 0.143\pi$

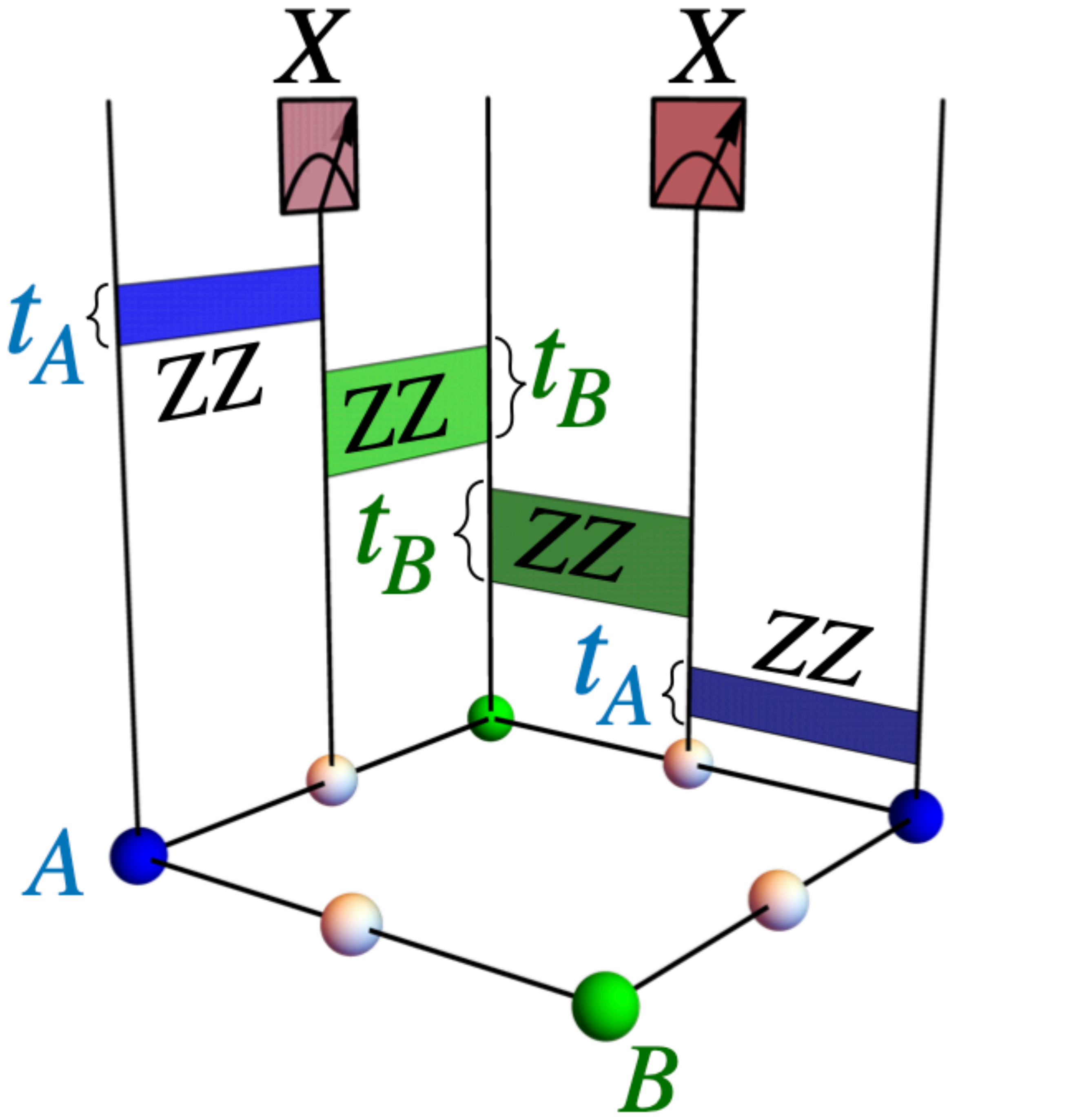
$1/\nu \approx 3/4$

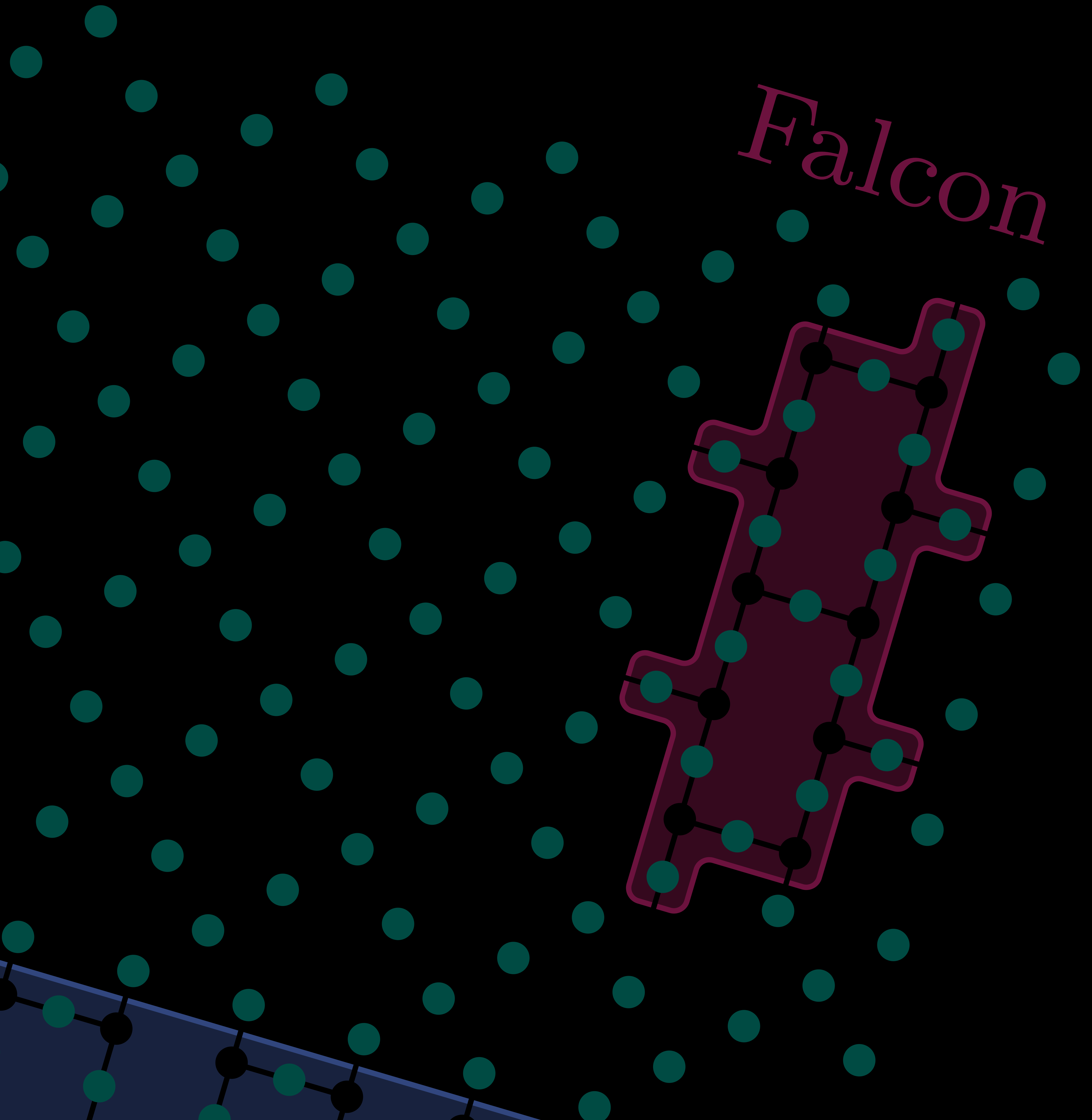
Adler '97
Fisher '97
Harris '88
Pujol '01
Chalker '02
Hartmann '04
...

phase diagram



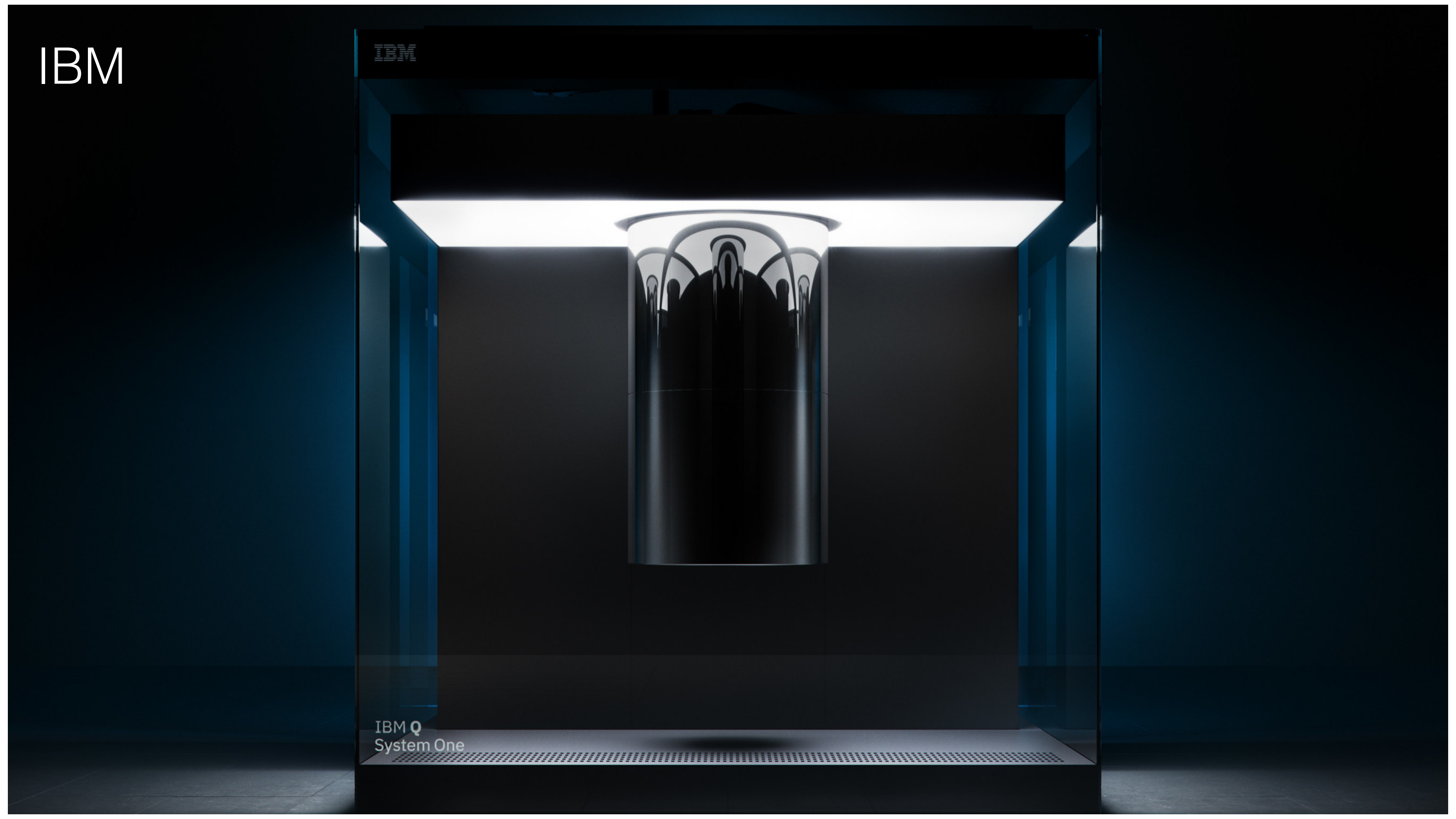
$$[\langle\sigma\rangle^2] \equiv \sum_{\{s\}} p_{\{s\}} \langle\sigma\rangle^2_{\{s\}}$$





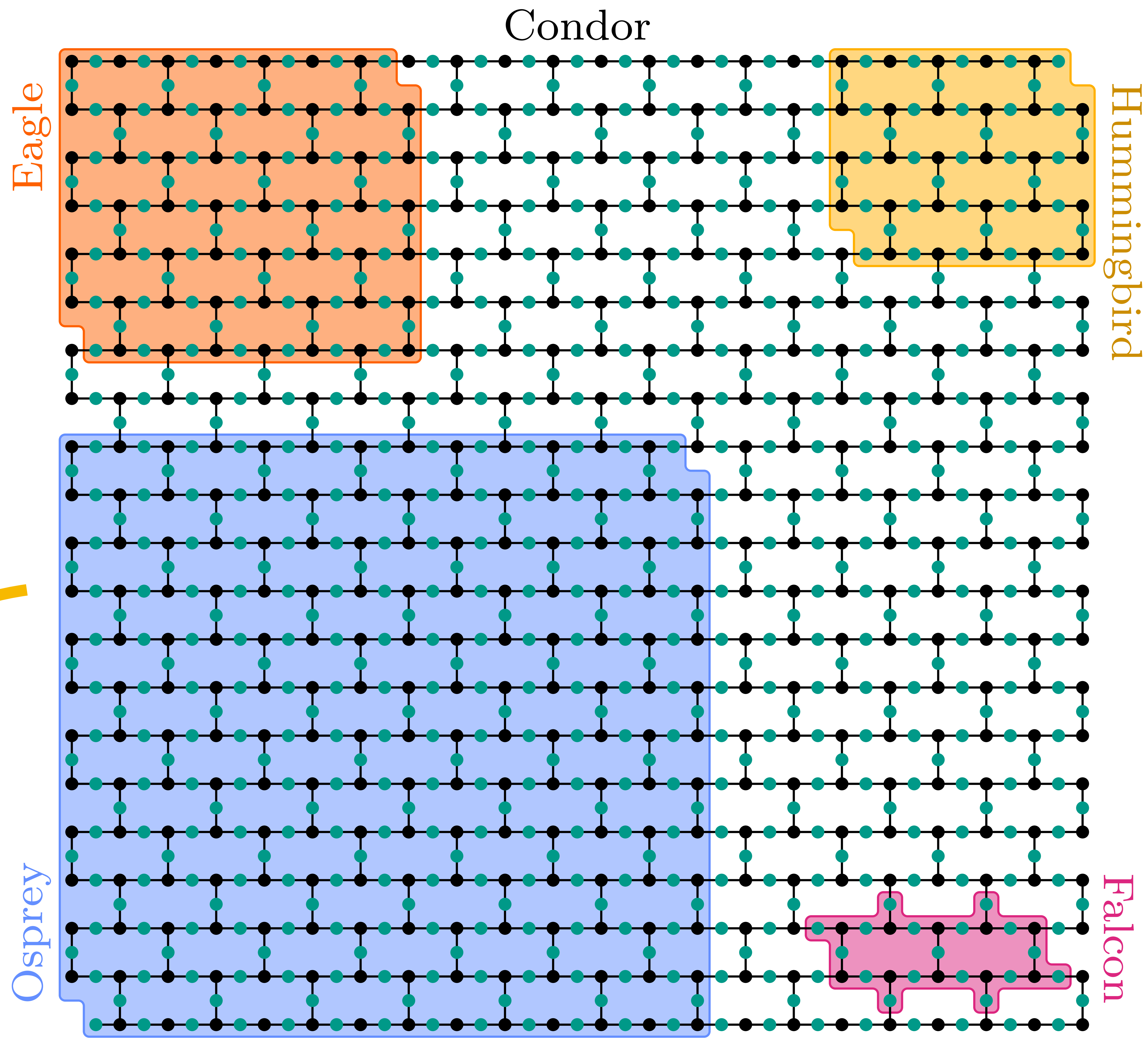
experiment

IBM quantum cloud



NISQ devices built on transmon qubits

noisy intermediate scale quantum devices



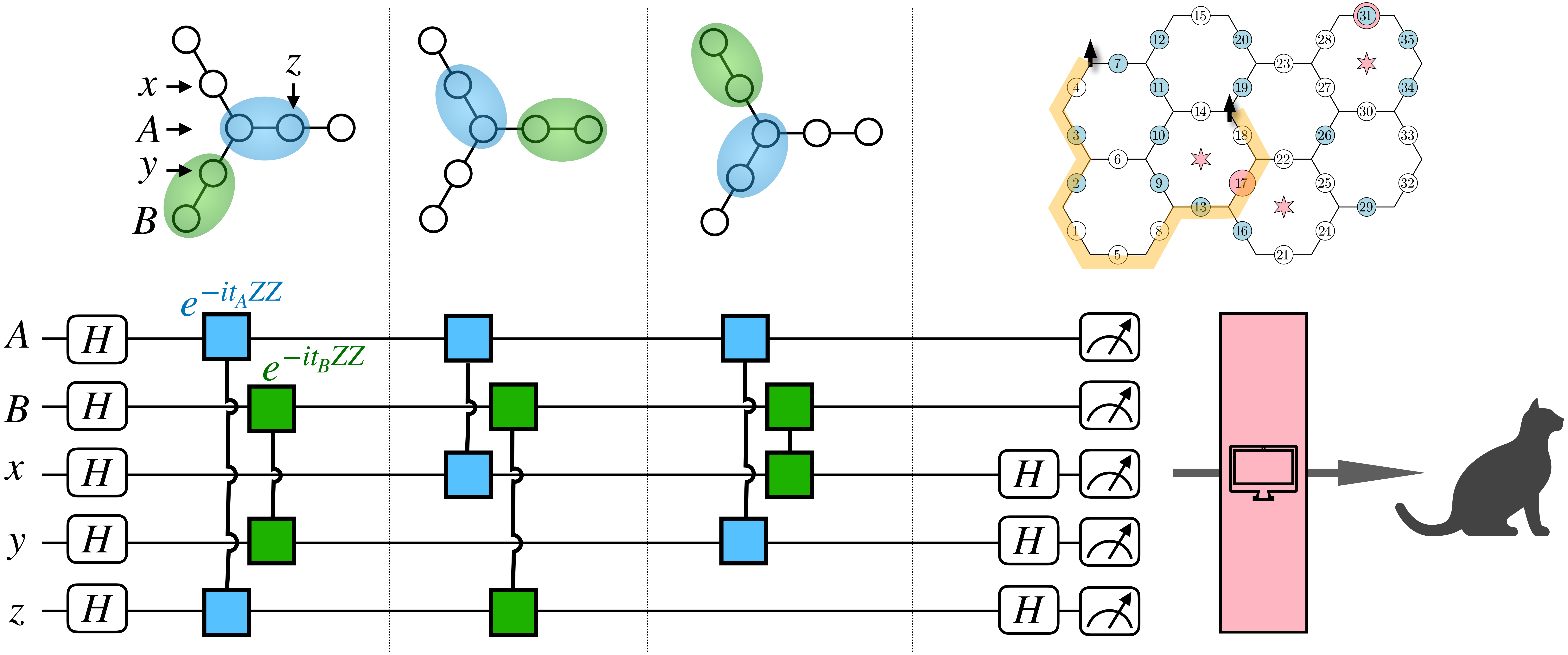
heavy-hexagon geometry

Lieb lattice

+

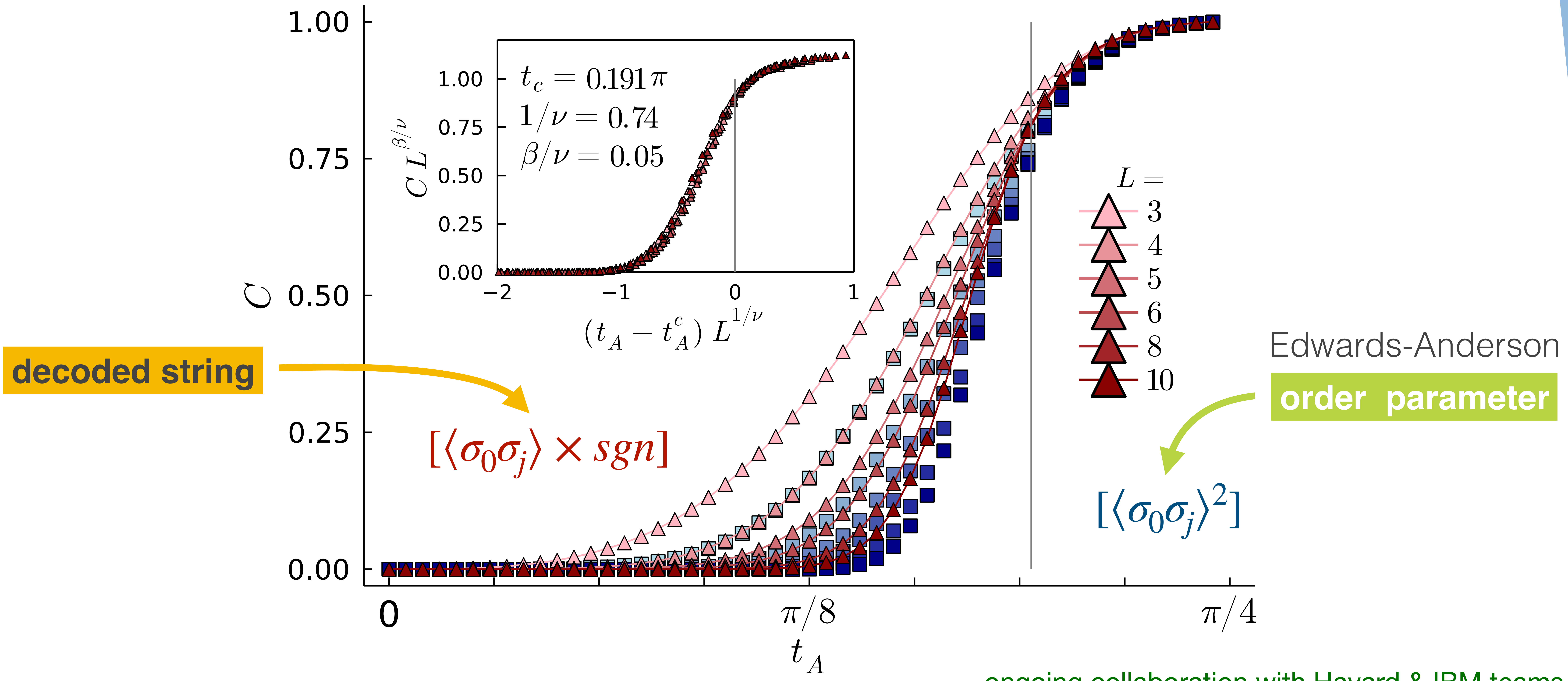
Ising evolution gates

depth-3 quantum circuit



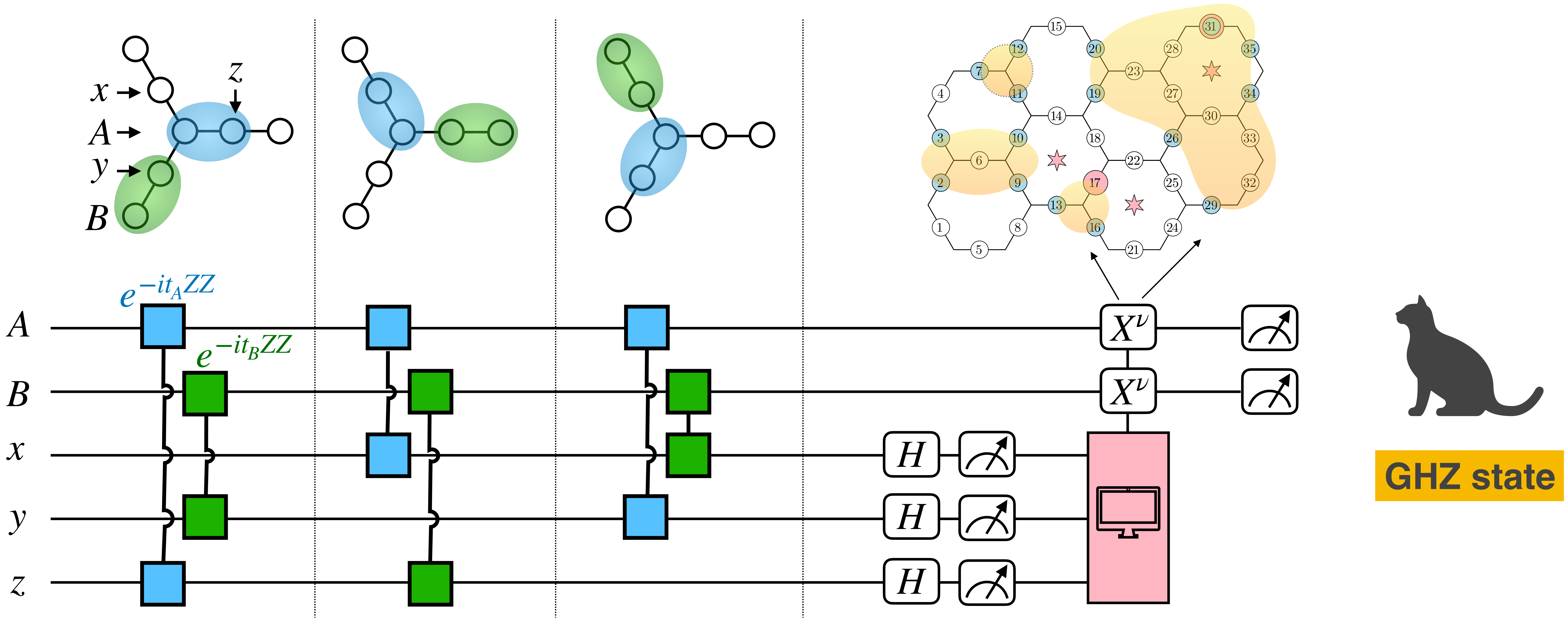
ongoing collaboration with Havard & IBM teams

decoding versus order parameter



ongoing collaboration with Havard & IBM teams

Nishimori's cat decoded



ongoing collaboration with Havard & IBM teams

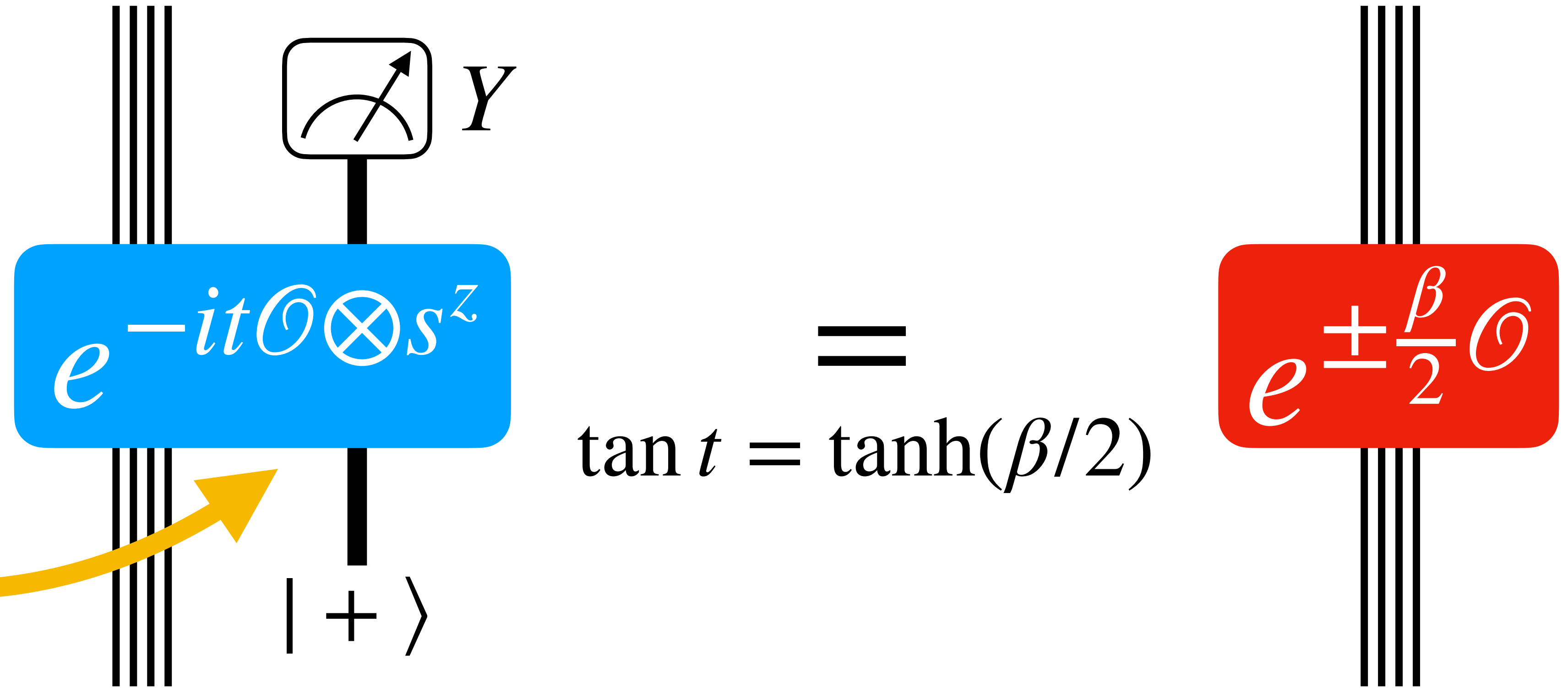
stabilizer codes

measuring stabilizers

unitaries + measurements = non-unitaries + randomness

strategy

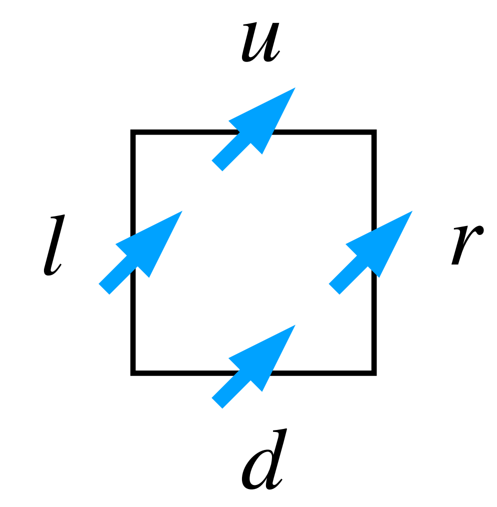
entangle single-site measurement with an effective measurement of operator



real time evolution

imaginary time evolution

e.g.



$$\mathcal{O} = B_p = \sigma_l^z \sigma_r^z \sigma_u^z \sigma_d^z$$

post-measurement states

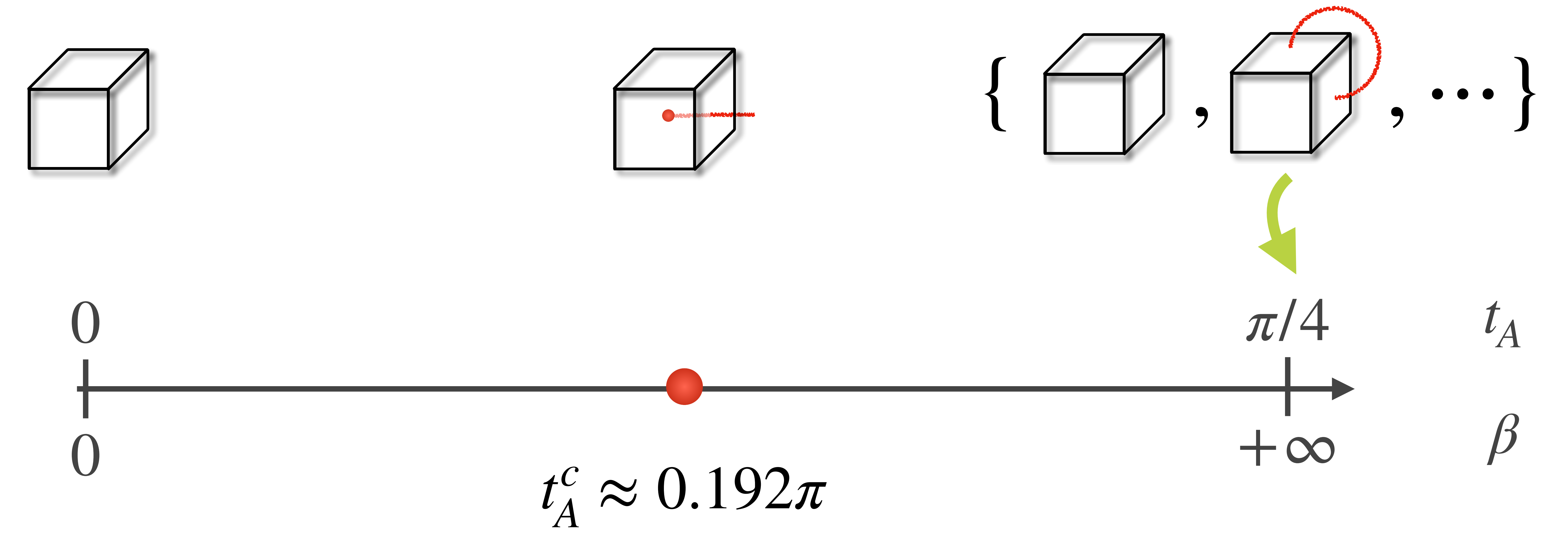
$$\langle \{s\} | \psi \rangle = e^{-\frac{1}{2}\beta \sum_j s_j \mathcal{O}_j} | + \rangle^{\otimes N}$$

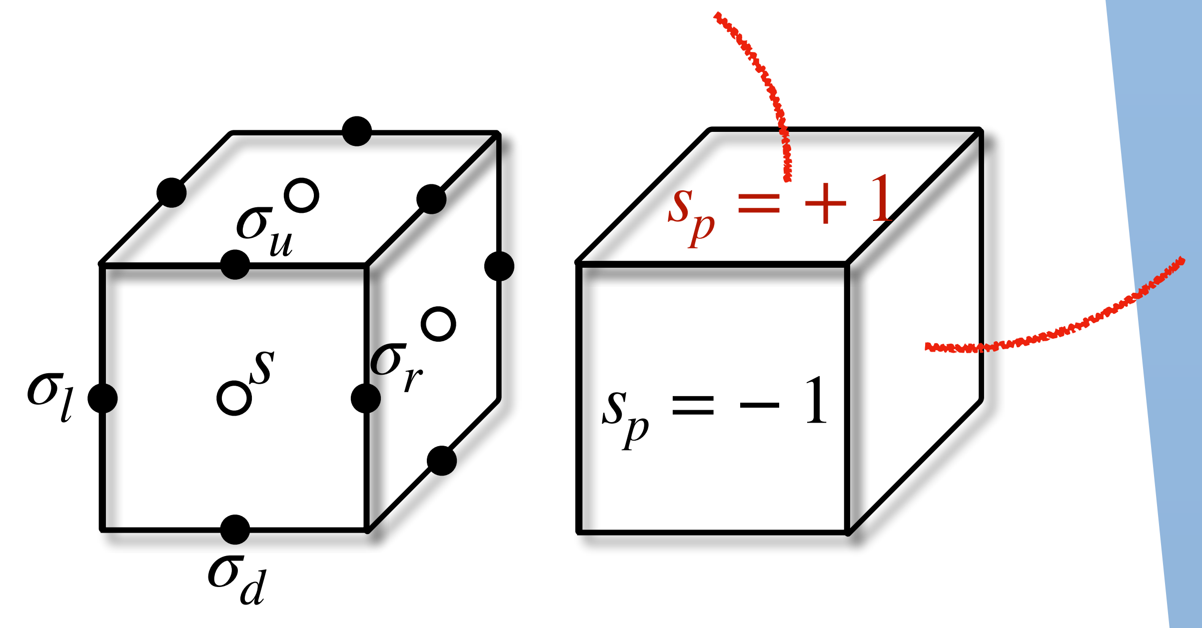
$$\mathcal{O}^2 = 1$$

measuring stabilizers

frustration
monopole disorder

static magnetic
flux loop defects





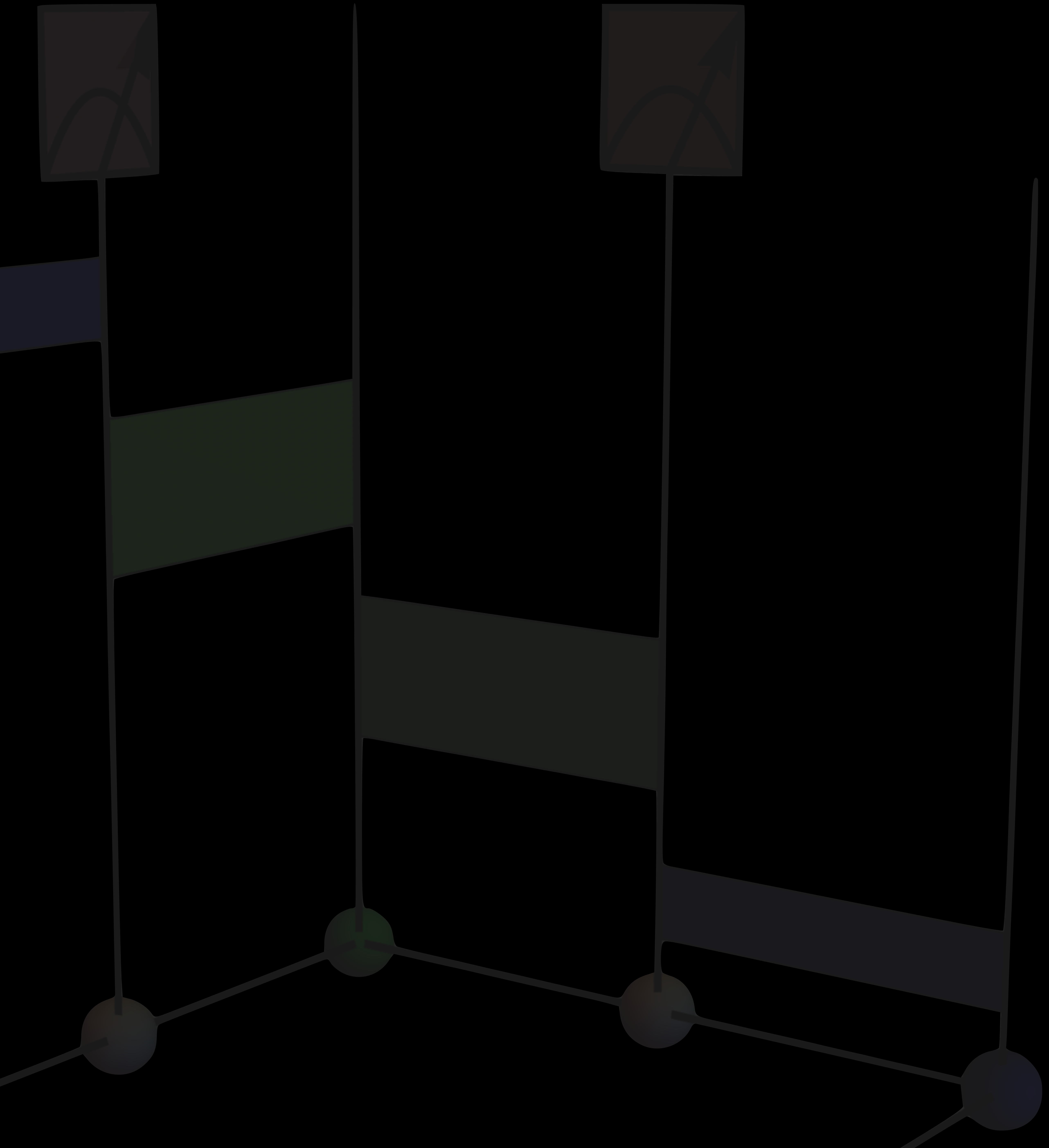
random plaquette Ising gauge model on **Nishimori line**

$$p_{\{s\}} \propto Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_p s_p B_p}$$

uncorrelated RPGM

$p_c \approx 0.033$

Dennis, Kitaev, Landahl, Preskill 2002;
Ohno, Arakawa, Ichinose, Matsui 2004



summary

summary

arXiv:2208.11136

- **shallow** deterministic quantum circuits

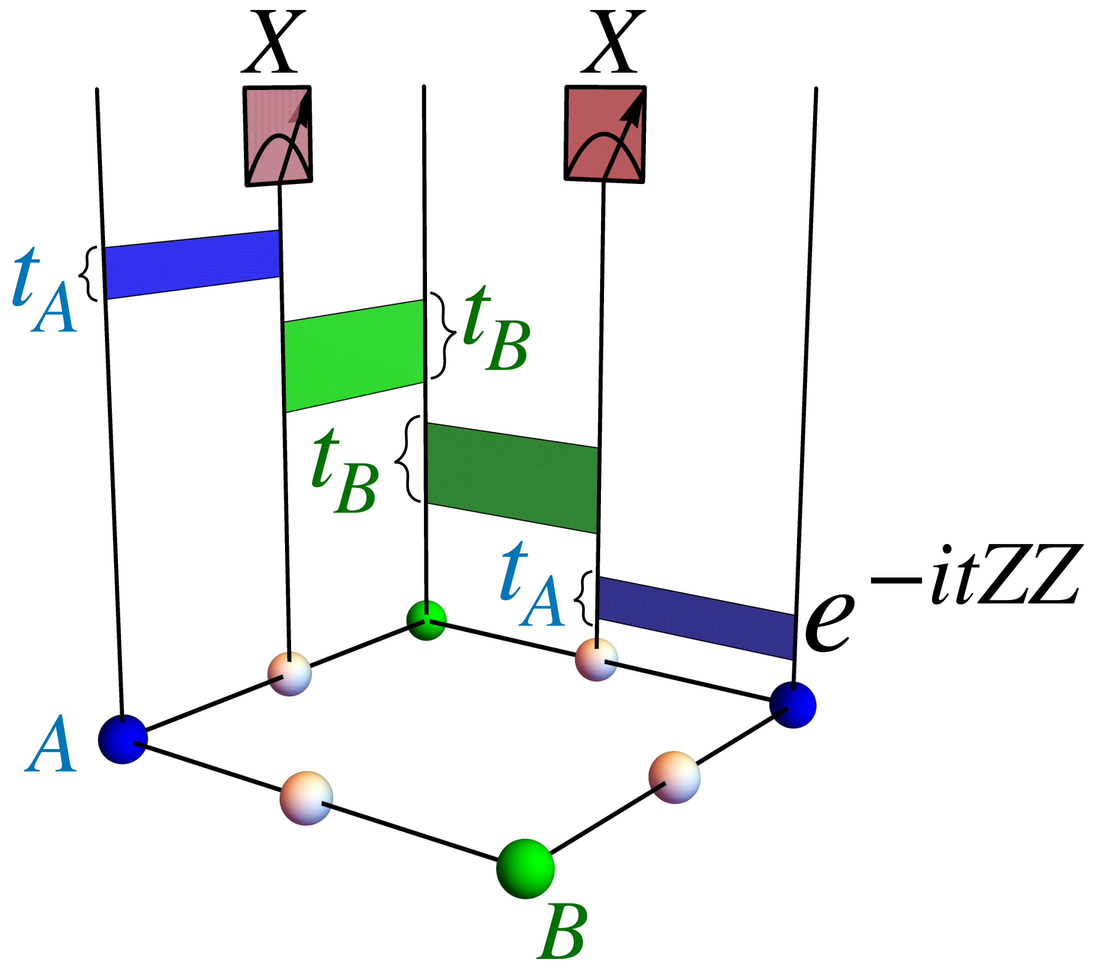
  stable **long range entanglement** and **quantum criticality**

- **analytical** solution

  Lieb lattice geometry **Nishimori cat**

- **experimental** realization

  go-to: IBM's heavy-hexagon transmon platform



- Outlook
  - **topological orders** (twisted, non-Abelian, fracton, chiral, …)
  - universe of **conformal quantum critical points** – unitary and non-unitary
  - **Floquet codes**

Guo-Yi's talk

Guo-Yi Zhu


∇·B=0
S = ln(Ω)
iħ∂tΨ = HΨ
dE = TdS − pdV
Rμν − 1/2 Rgμν = 8πGTμν
found by Guo-Yi