Monitored Quantum Criticality

Nishimori physics, quantum measurements, and non-unitary dynamics

CRC183 ENTANGLED STATES OF MATTER

Simon Trebst
University of Cologne

ML4Q MATTER AND LIGHT FOR QUANTUM COMPUTING

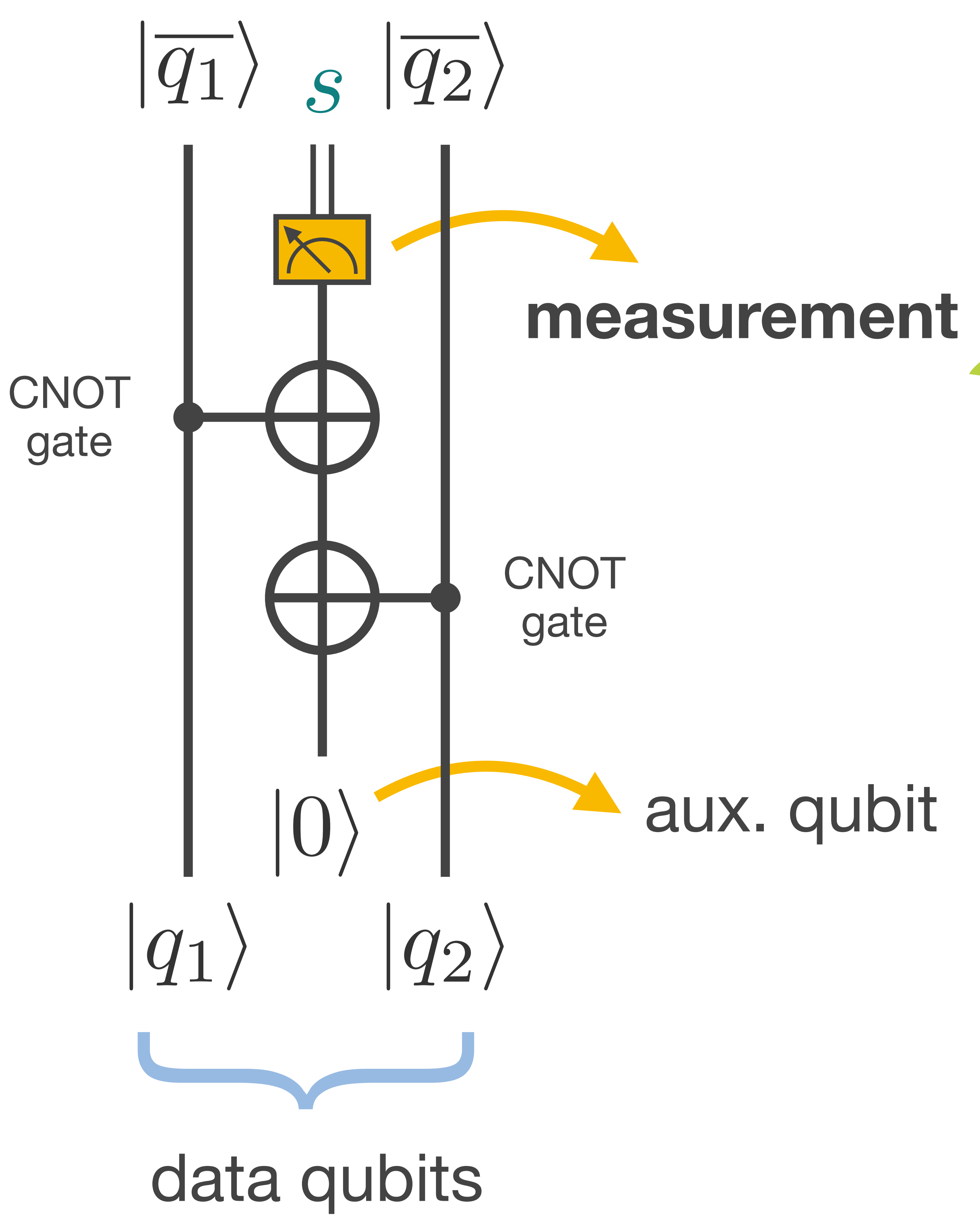
Quantum Interactive Dynamics (3rd edition)

Oxford, June 2025

quantum measurements

joint measurements & entanglement

quantum circuit for 2-qubit
joint measurement



extract information

$\|q_1\rangle\|q_2\rangle$	$\|\overline{q_1}\rangle\|\overline{q_2}\rangle$	s
$\|0\,0\rangle$	$\|0\,0\rangle$	0
$\|0\,1\rangle$	$\|0\,1\rangle$	1
$\|1\,0\rangle$	$\|1\,0\rangle$	1
$\|1\,1\rangle$	$\|1\,1\rangle$	0

syndrome measurement

ZZ **parity check**

shape entanglement

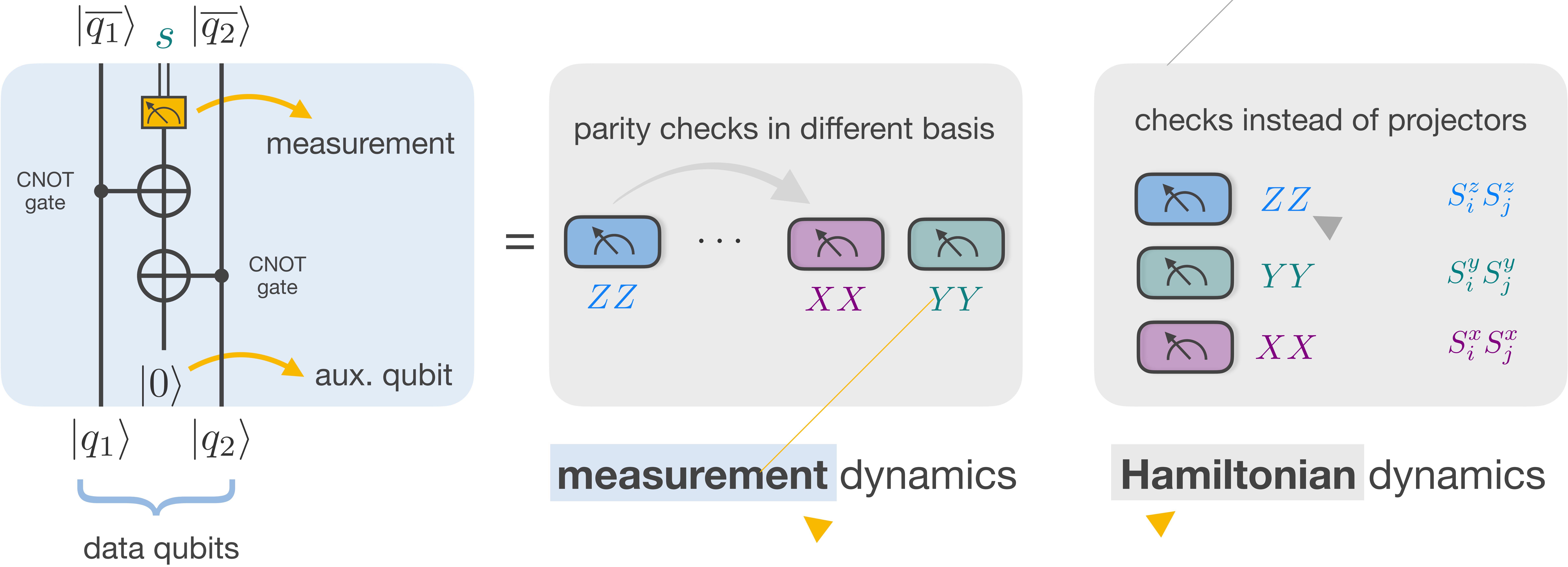
product state

$\|q_1\rangle\|q_2\rangle$	$\|\overline{q_1}\rangle\|\overline{q_2}\rangle$	s
$\|++\rangle$	p = 1/2 $(\|00\rangle + \|11\rangle)/\sqrt{2}$	0
	p = 1/2 $(\|01\rangle + \|10\rangle)/\sqrt{2}$	1

Bell state

$$|++\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

joint measurements & entanglement

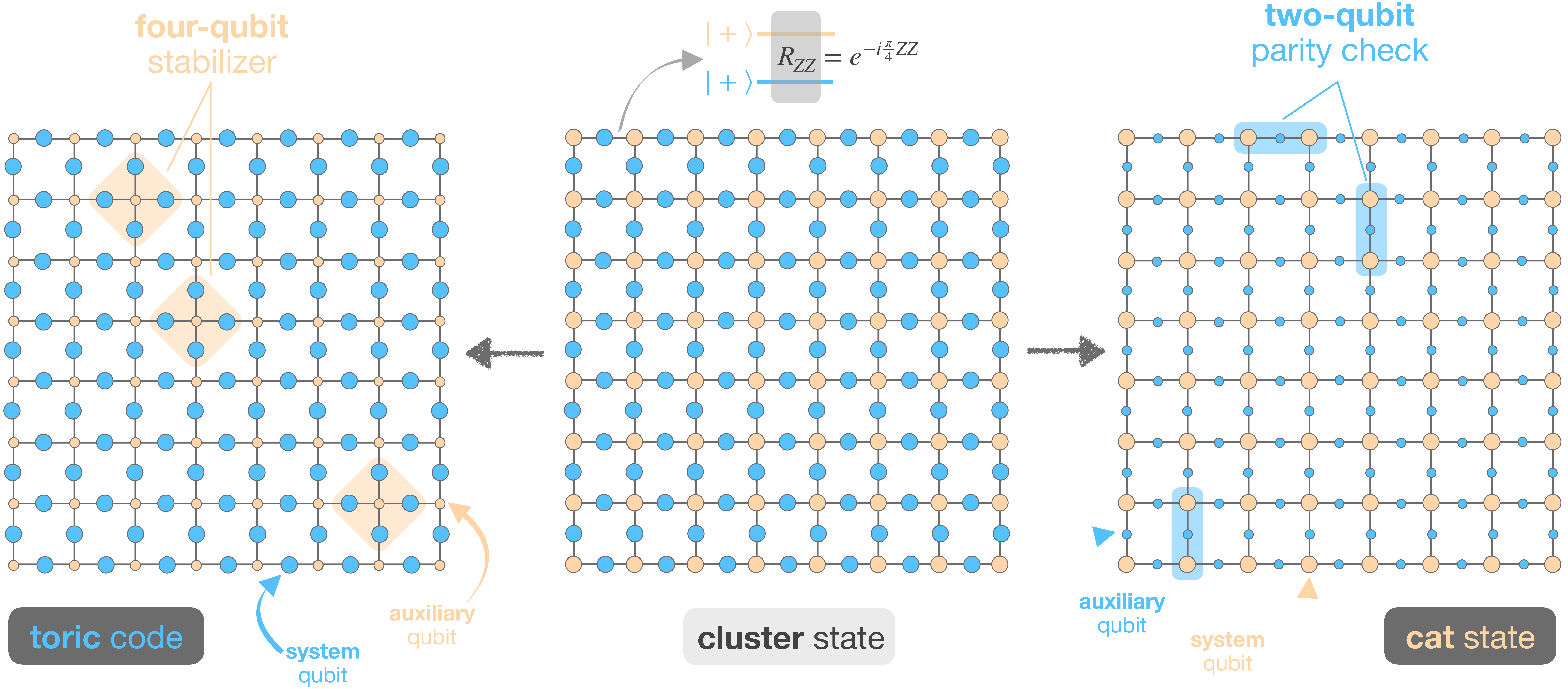


measurement dynamics

Hamiltonian dynamics

new toolbox for quantum many-body physics

joint measurements & entanglement



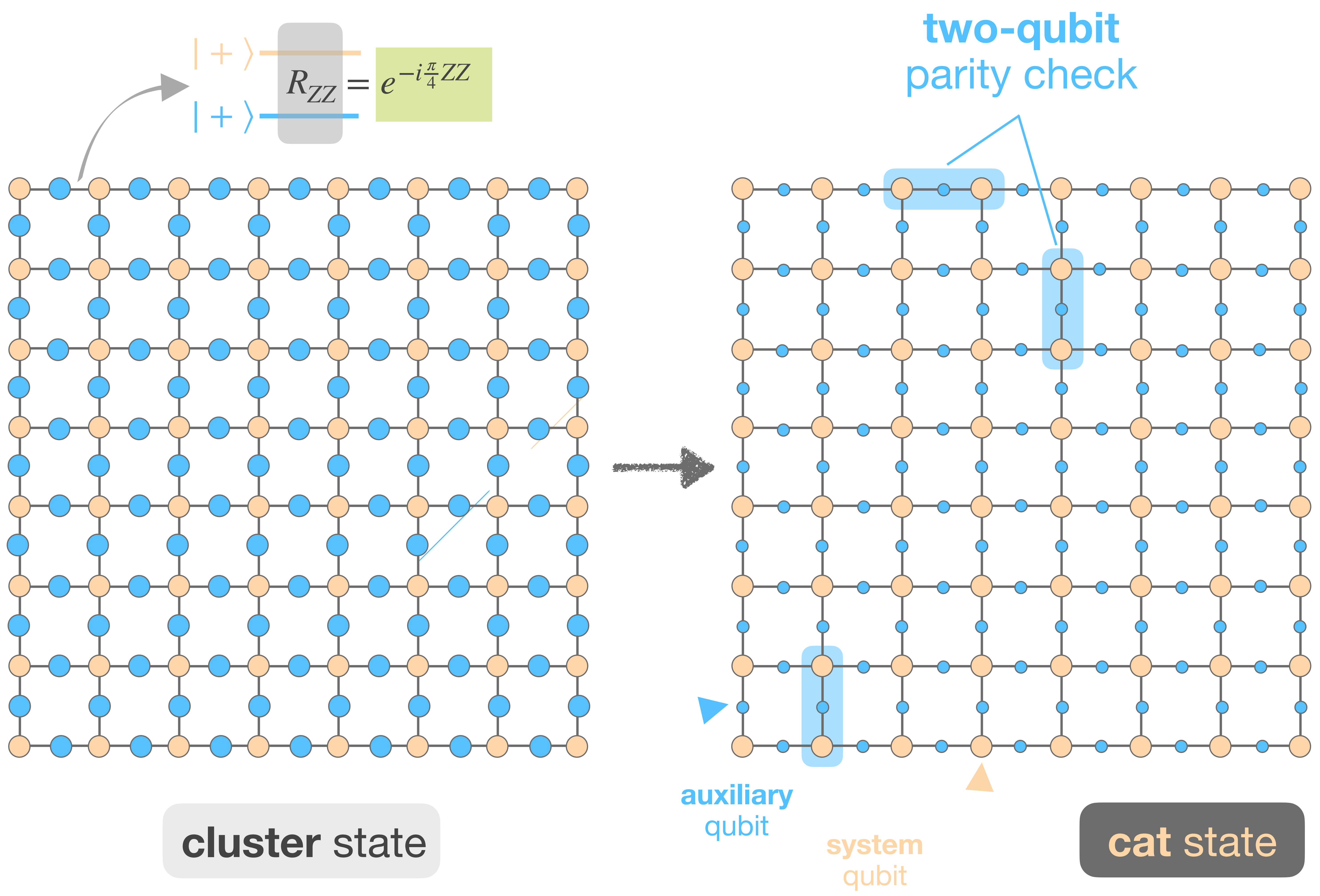
measurement-induced phase transitions

What happens if our **resource** — the cluster state — is **imperfectly prepared**?

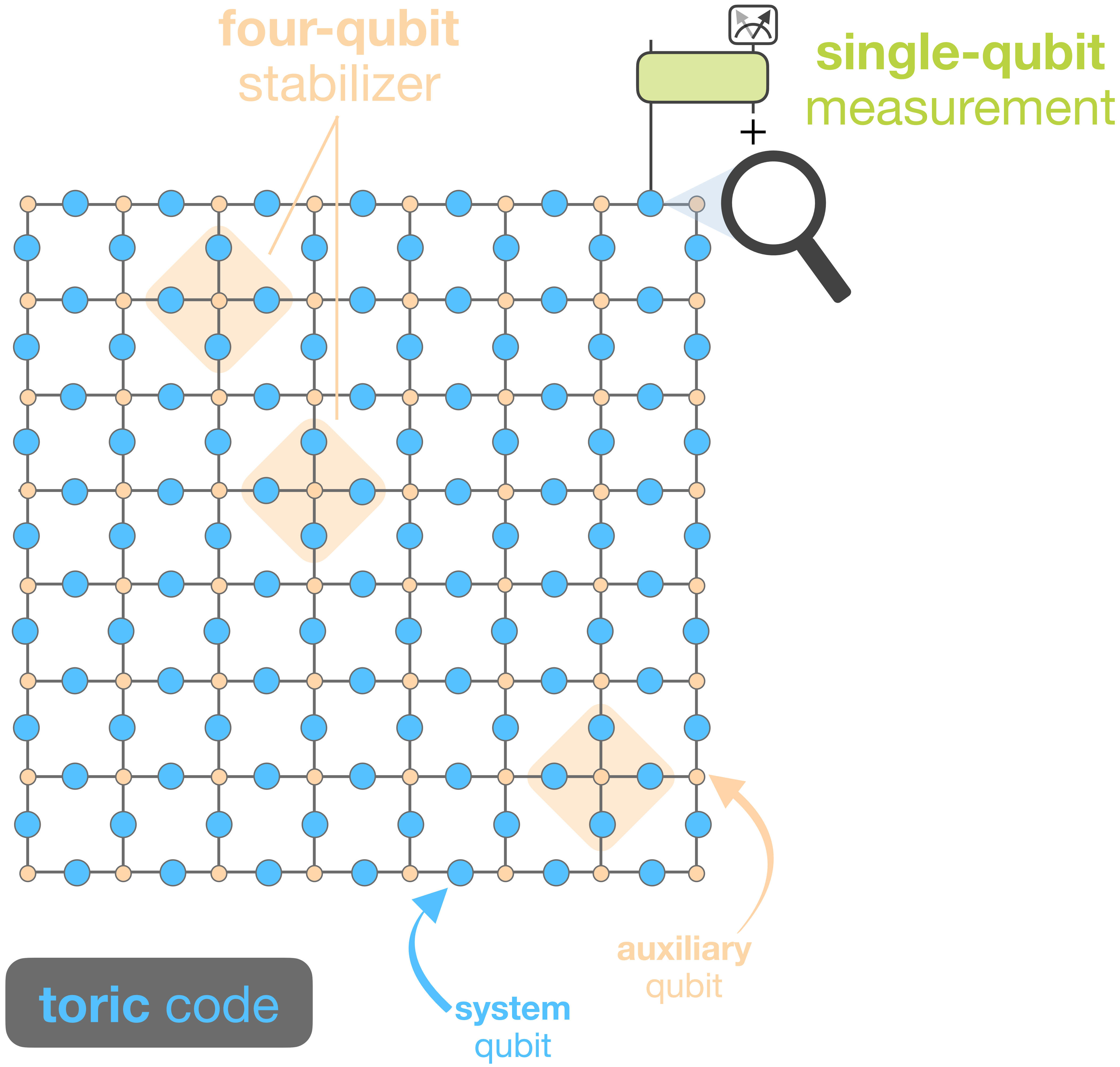
$$0 \leq t \leq \frac{\pi}{4}$$

Is the prepared **cat state stable** to this coherent deformation? Is there a **threshold**?

Nishimori transition



measurement-induced phase transitions



What happens if, for the toric code, we weakly **monitor all system qubits**?

$$0 \leq t \leq \frac{\pi}{4}$$

Is the monitored **toric code stable** to this coherent deformation? Is there a **threshold**?

Nishimori transition

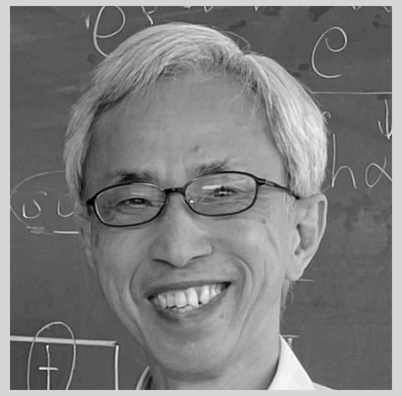
Nishimori physics

Phys. Rev. Lett. 131, 200201 (2023)

G-Y. Zhu, N. Tantivasadakarn, A. Vishwanath, R. Verresen



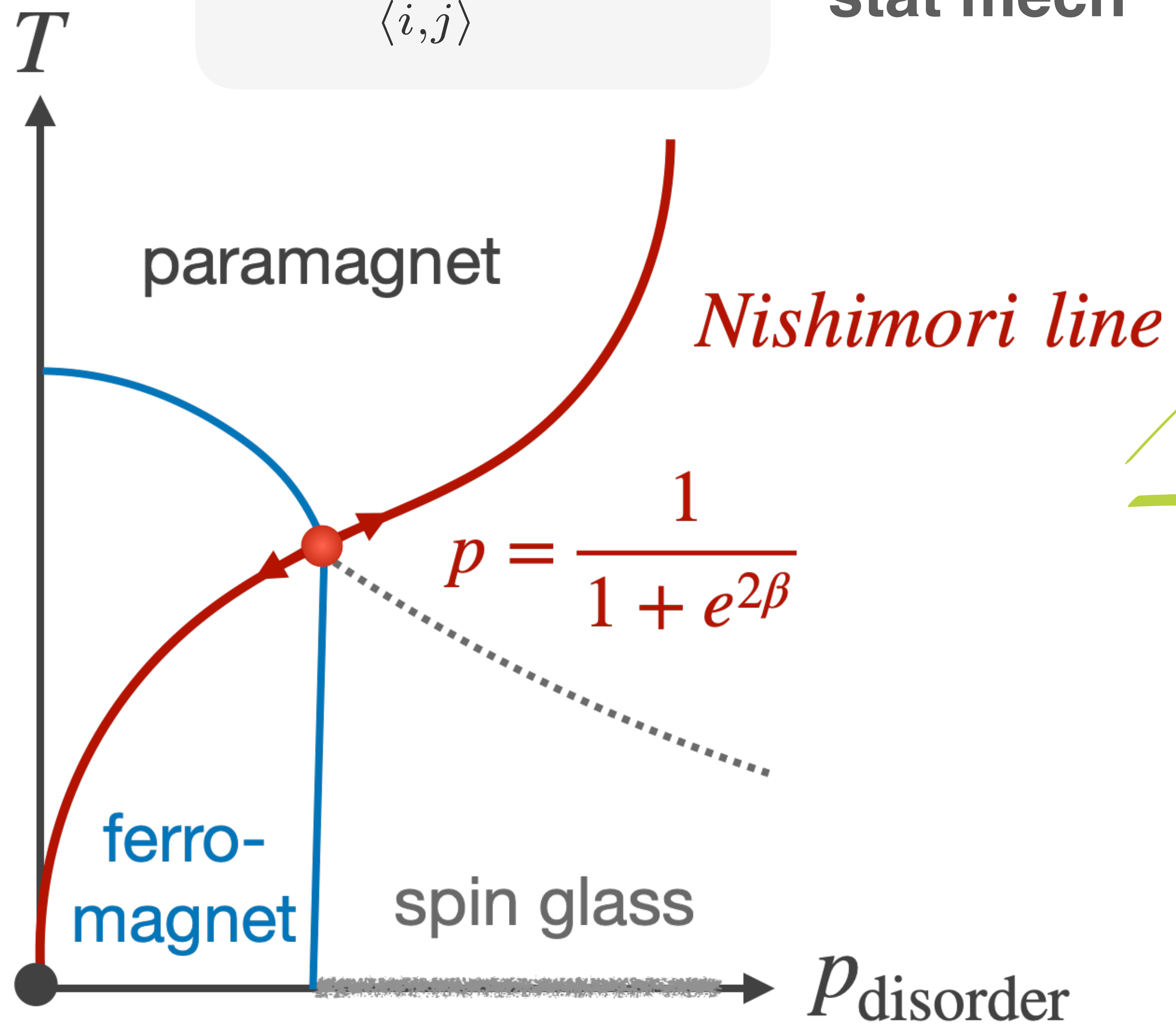

Nishimori physics



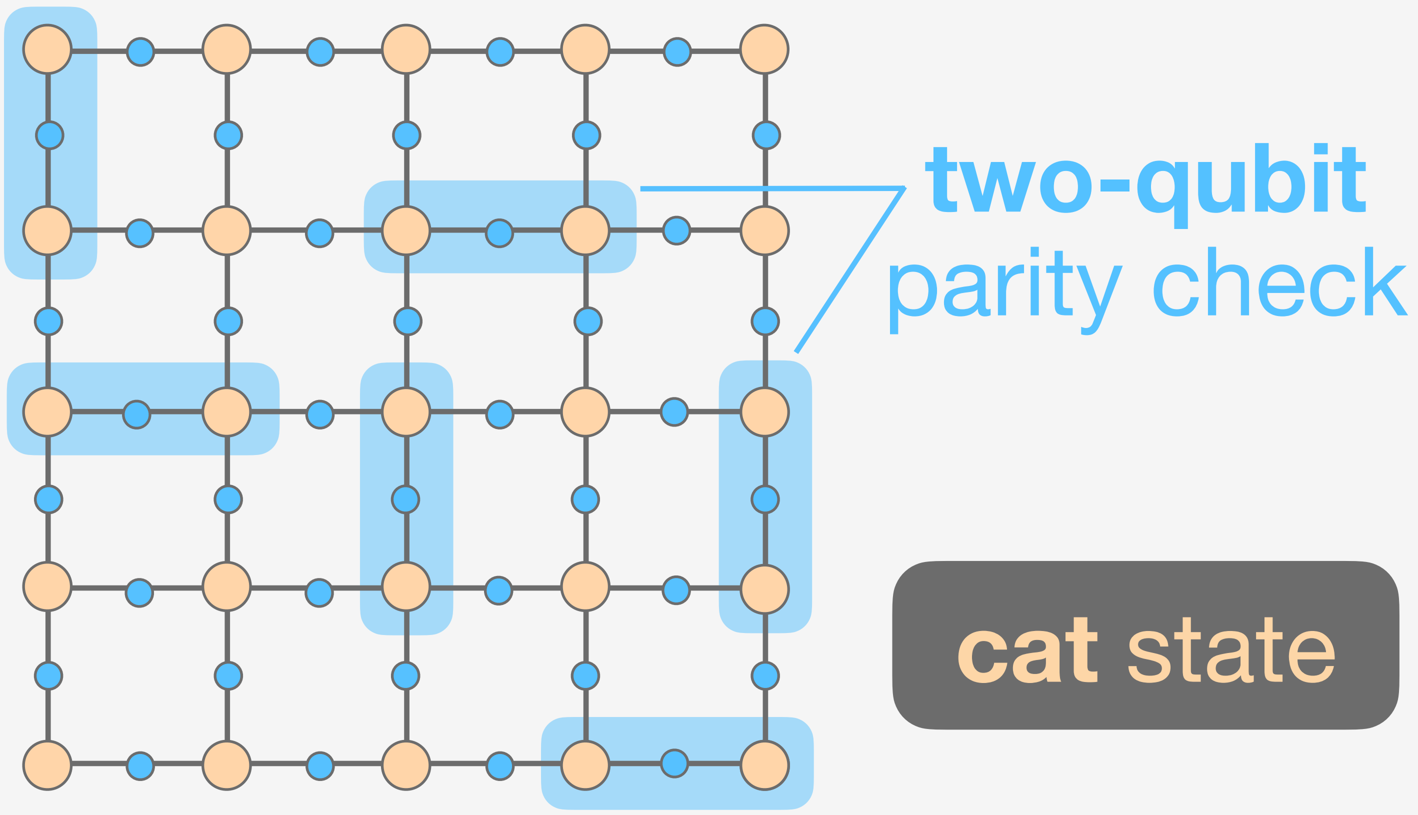
Nishimori (1981)

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \; S_i^z S_j^z$$

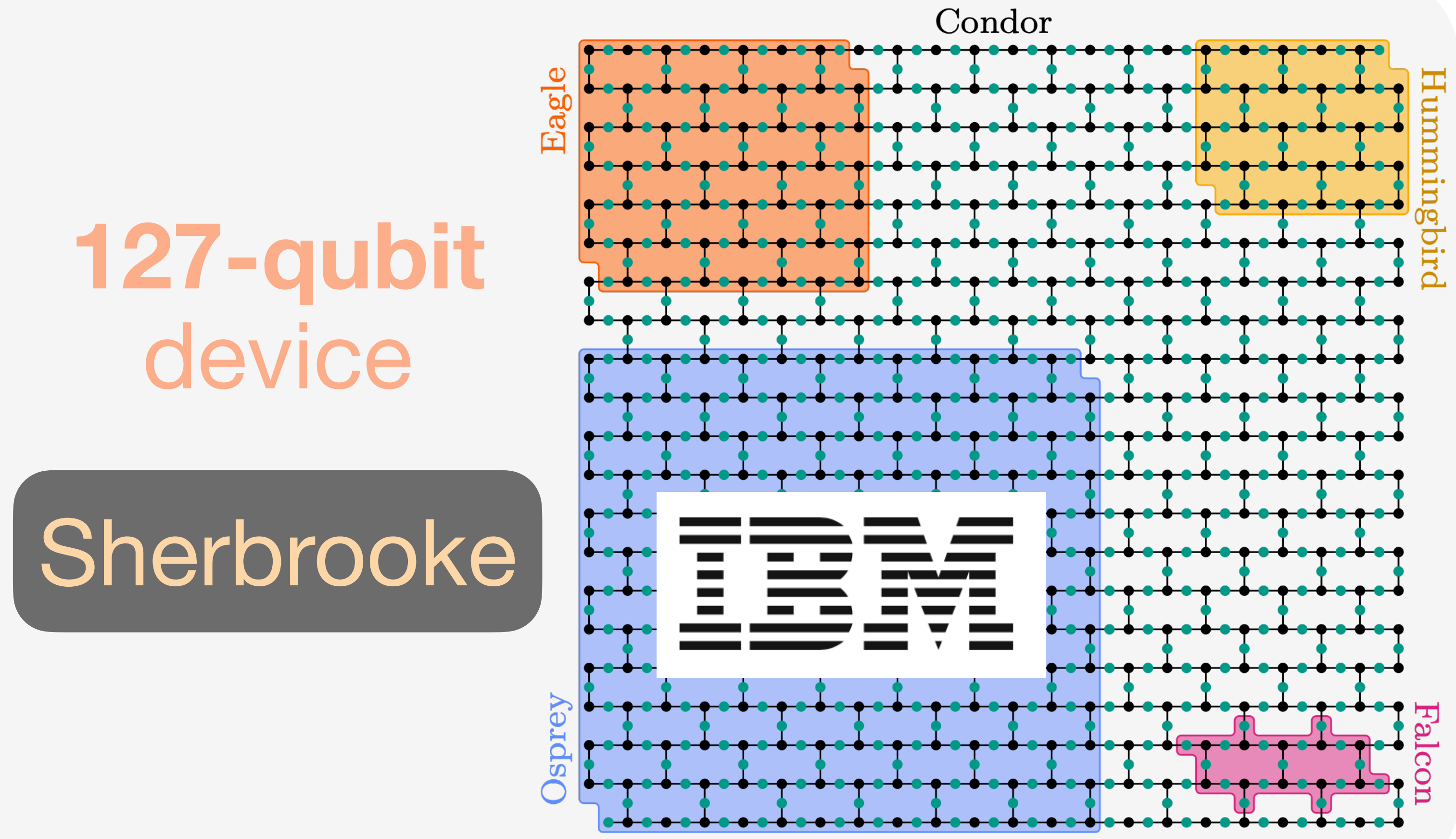
classical
stat mech



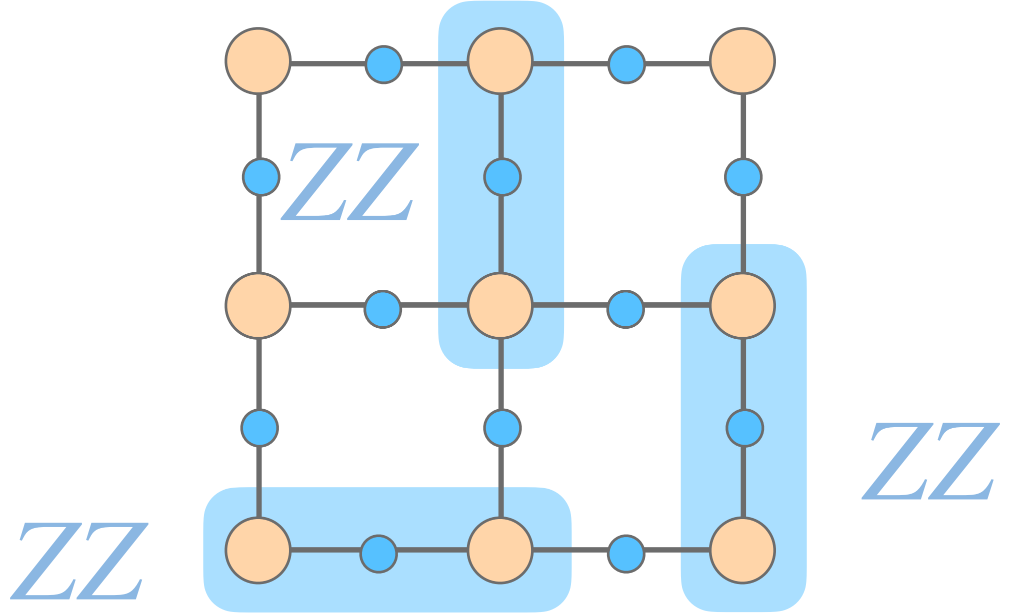
monitored
circuits



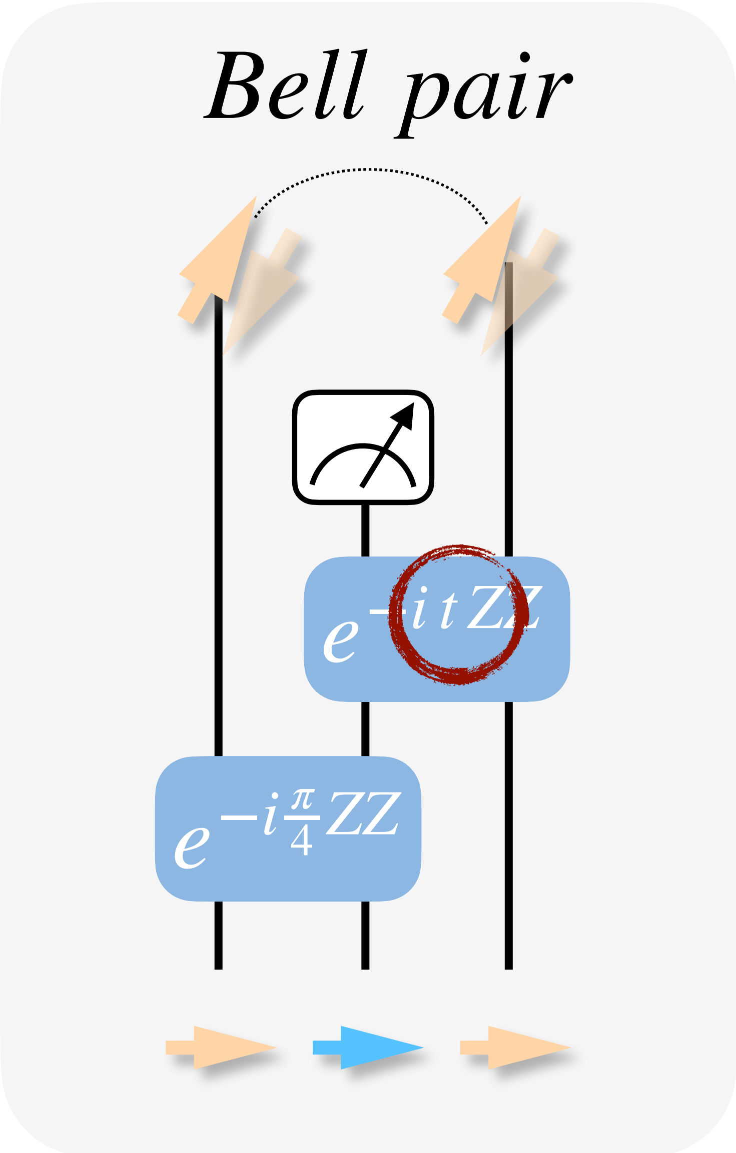
NISQ
devices



Nishimori's cat



Nishimori's cat



$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}}\sigma_i^z\sigma_j^z}\,|+\rangle^{\otimes N}$$

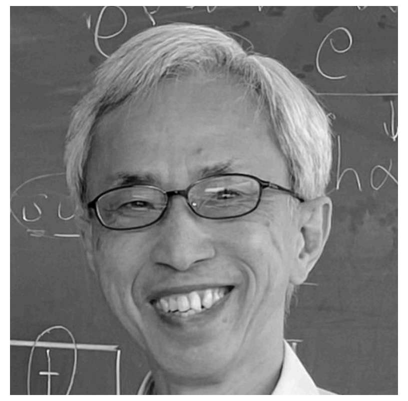
Born's rule

interpret as
classical
stat mech model

$$\tanh\frac{\beta J_\pm}{2} = \pm\tan t$$

thermal **fluctuations** and
disorder are **locked**

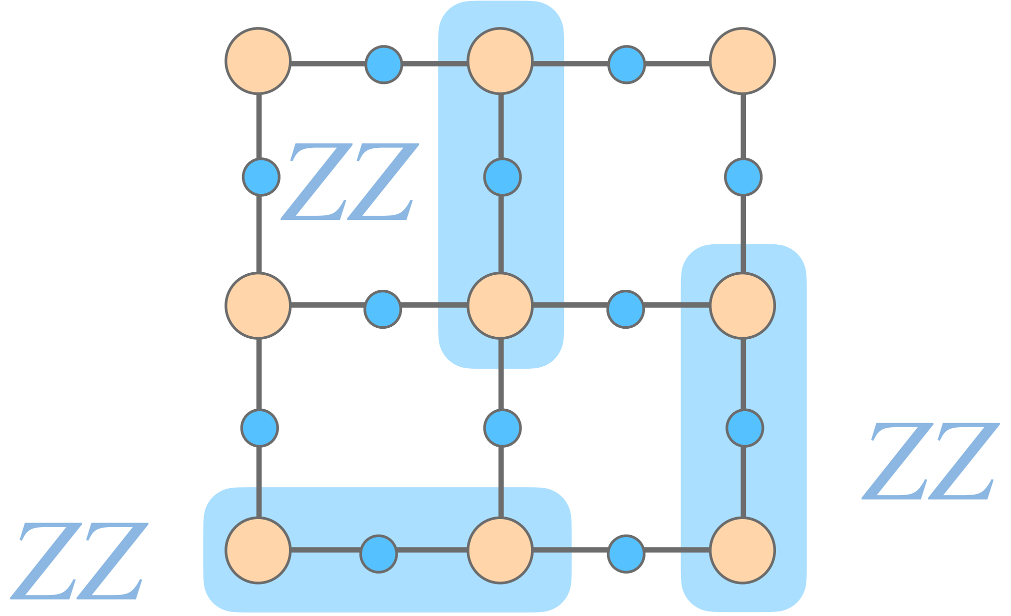
$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta\sum_{ij} J_{s_{ij}}\sigma_i\sigma_j}$$



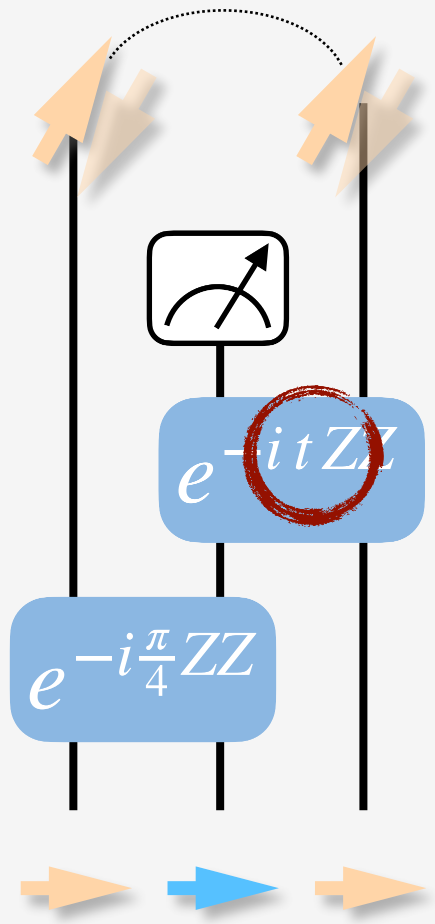
Nishimori (1981)

random bond Ising model

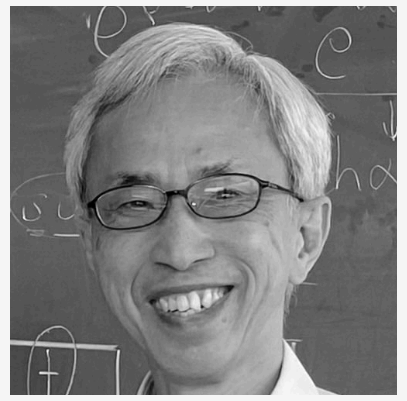
Nishimori's cat



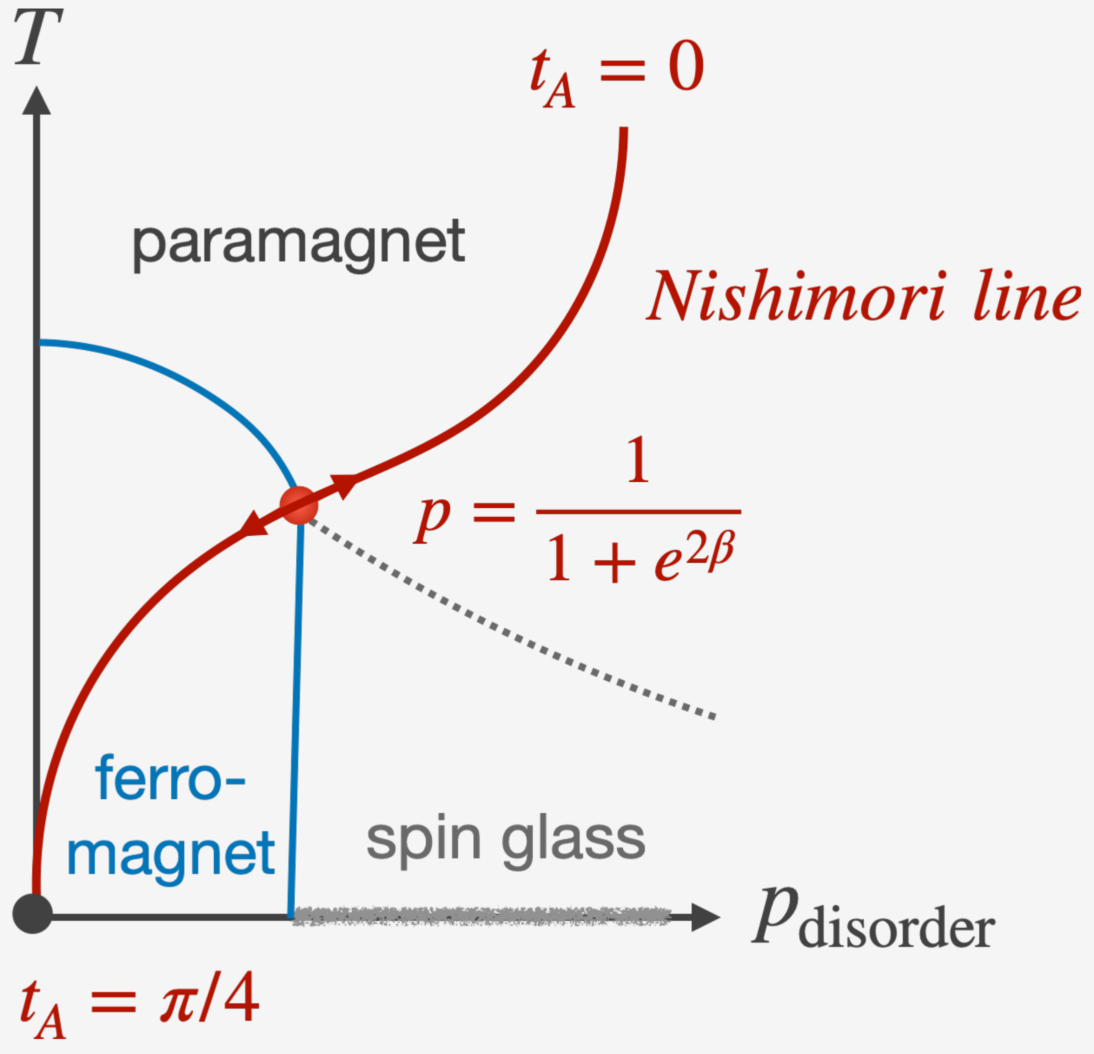
Nishimori's cat

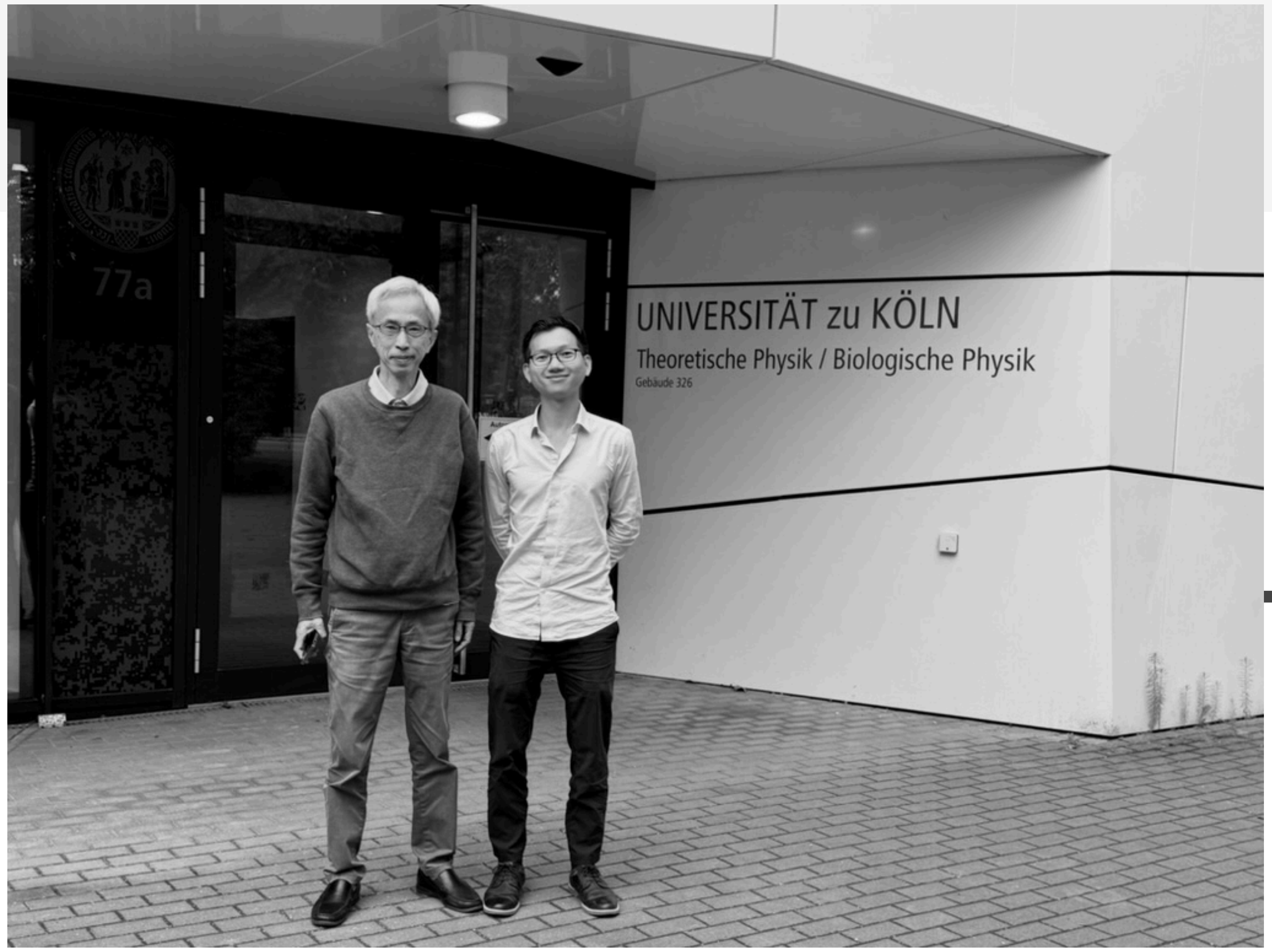


thermal fluctuations and disorder are **locked**

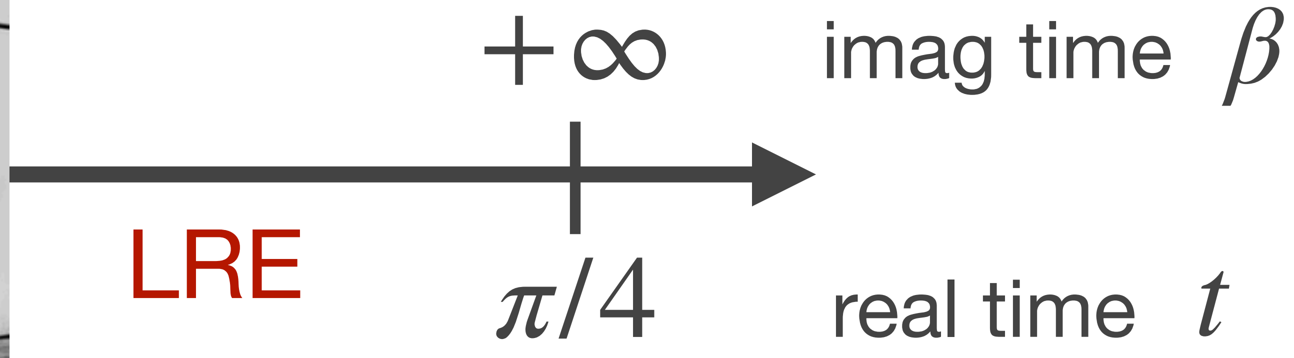


Nishimori (1981)

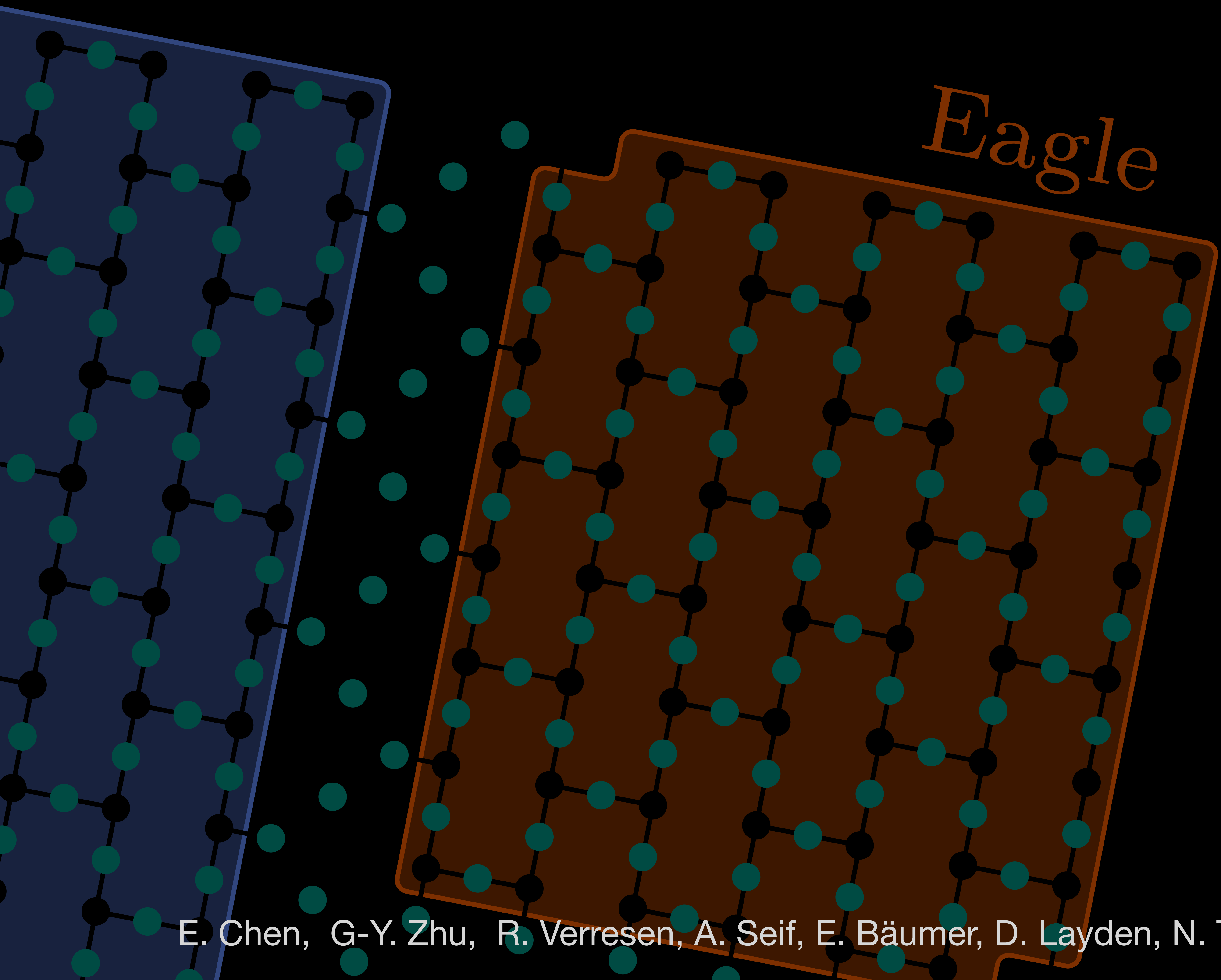




"low temperature"



strong measurement

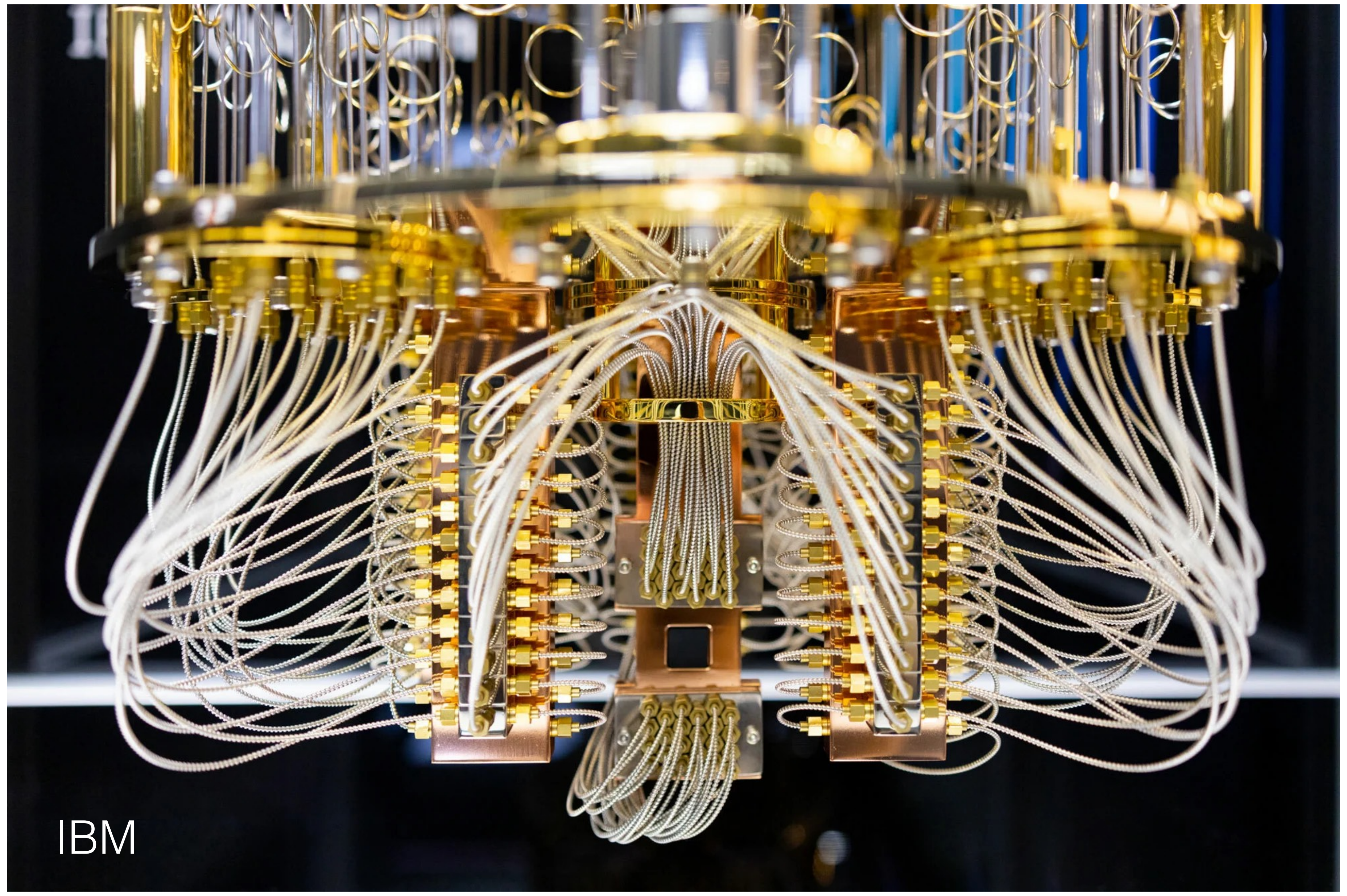


experiment

Nature Physics 21, 161 (2025)

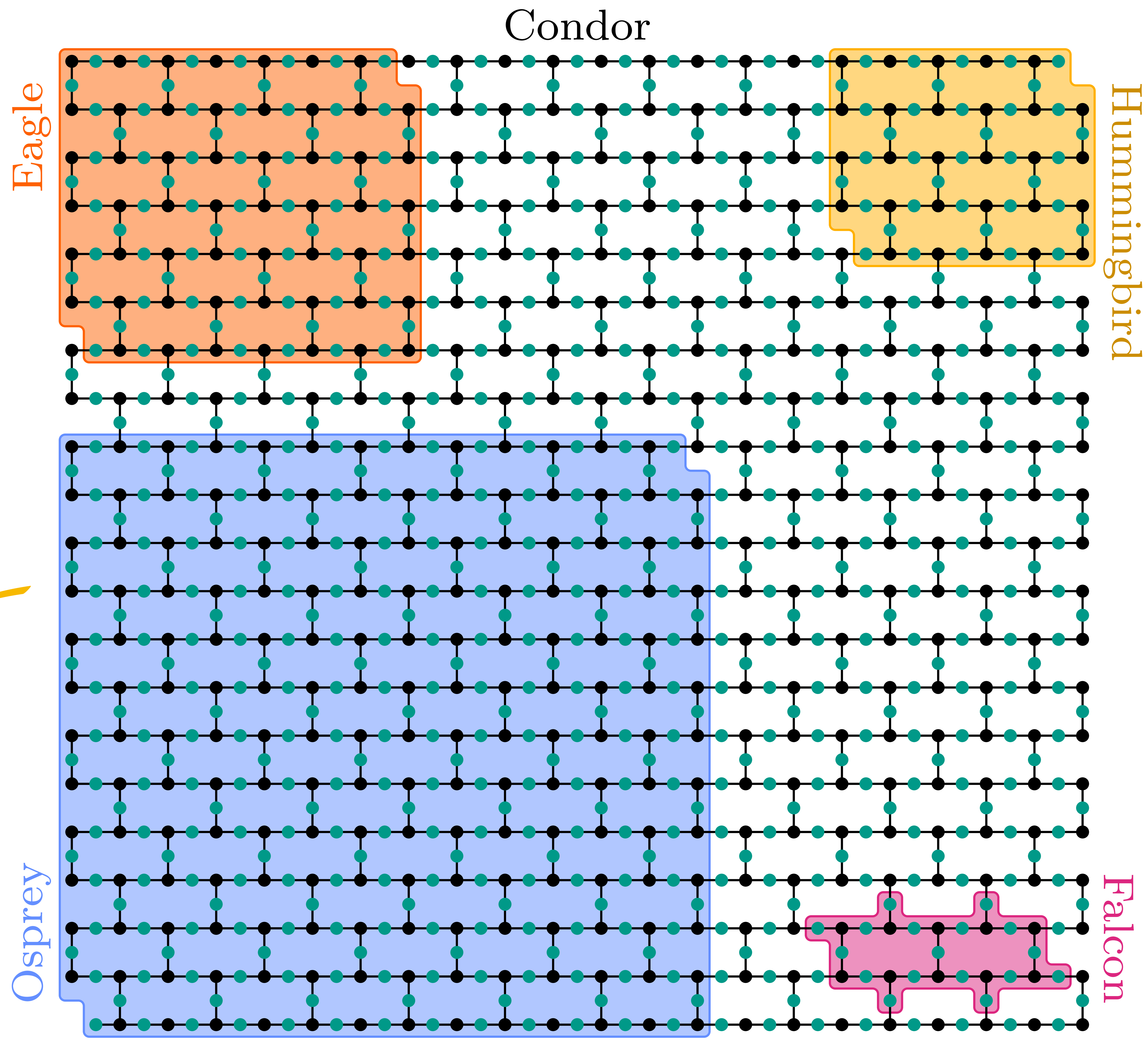
E. Chen, G-Y. Zhu, R. Verresen, A. Seif, E. Bäumer, D. Layden, N. Tantivasadakarn, G. Zhu, S. Sheldon, A. Vishwanath, A. Kandala

IBM quantum cloud devices



NISQ devices built on transmon qubits

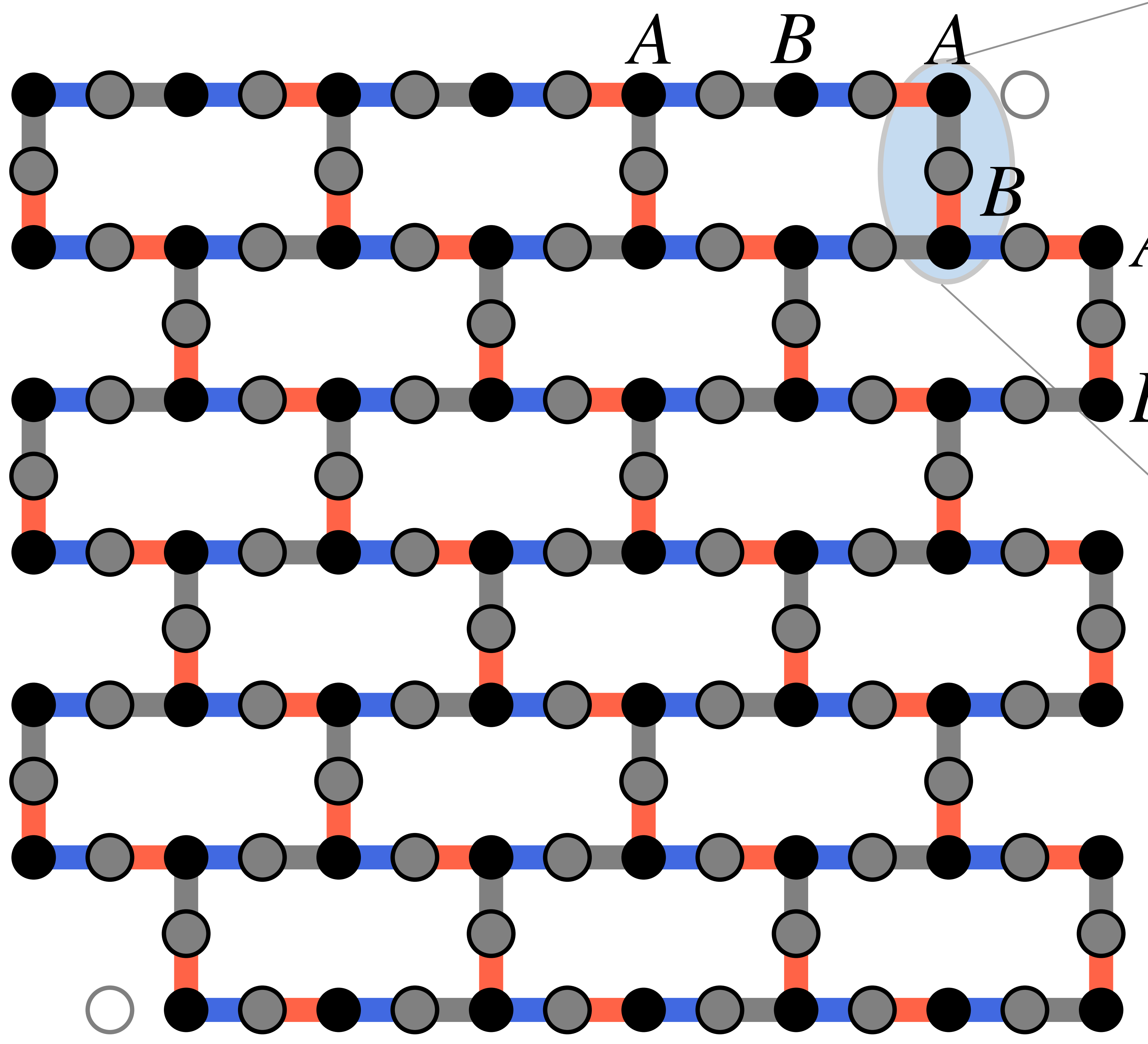
noisy intermediate scale quantum devices

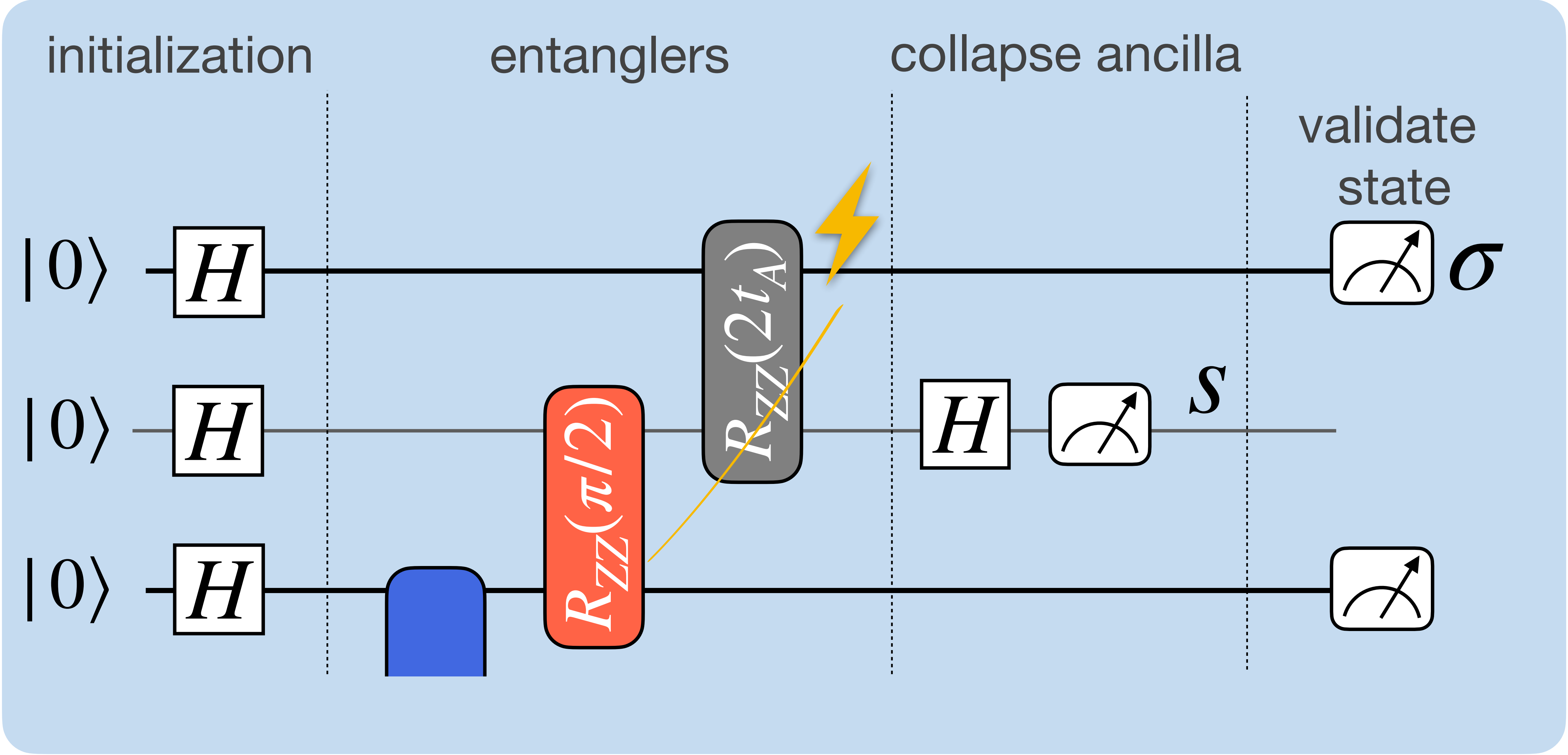


heavy-hexagon geometry
+
Ising evolution gates

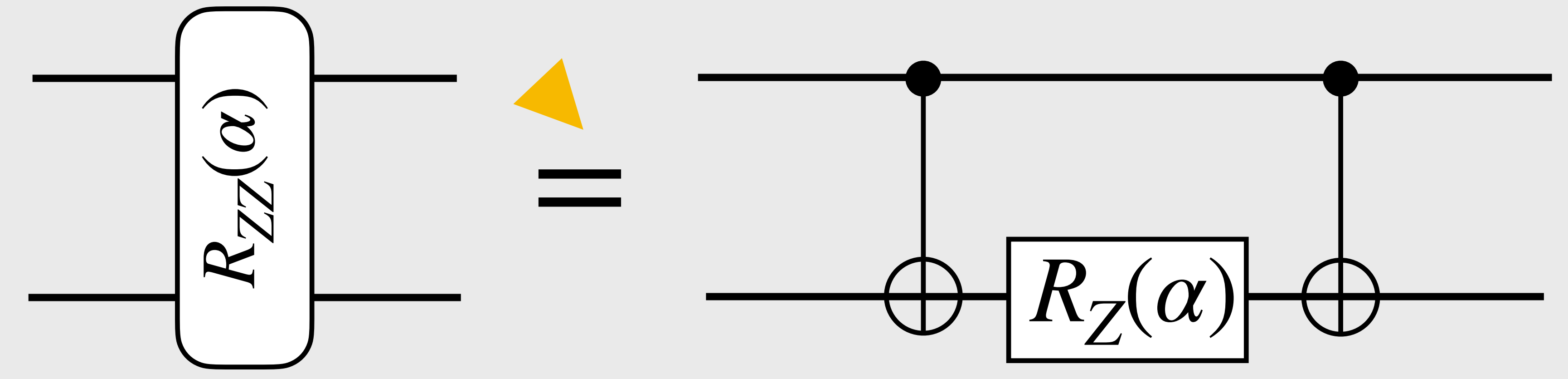
protocol on IBM heavy-hexagon lattice

IBM_Sherbrooke





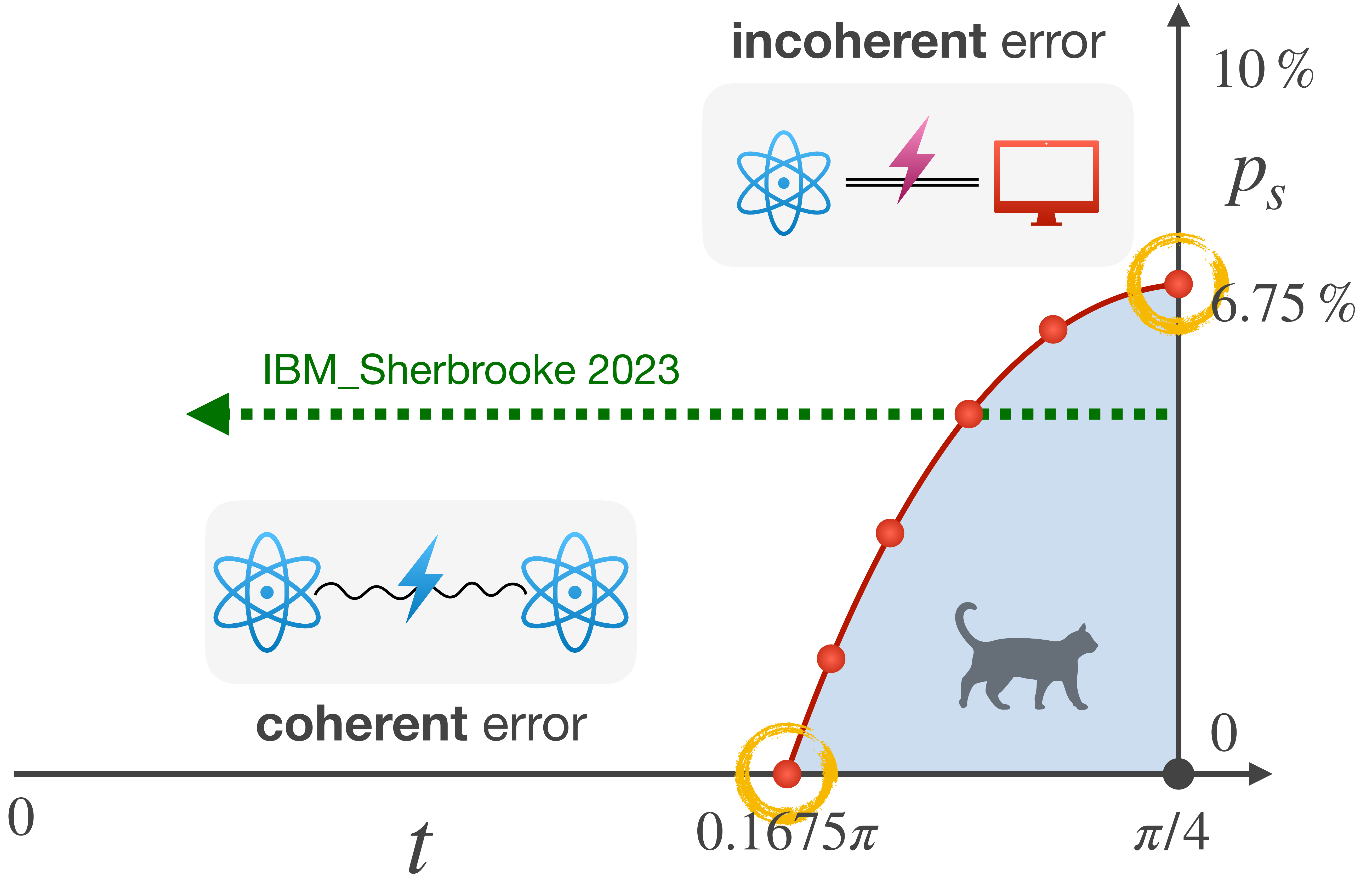
- **system** qubit: site of honeycomb lattice
- **auxiliary** qubit: bond of honeycomb lattice

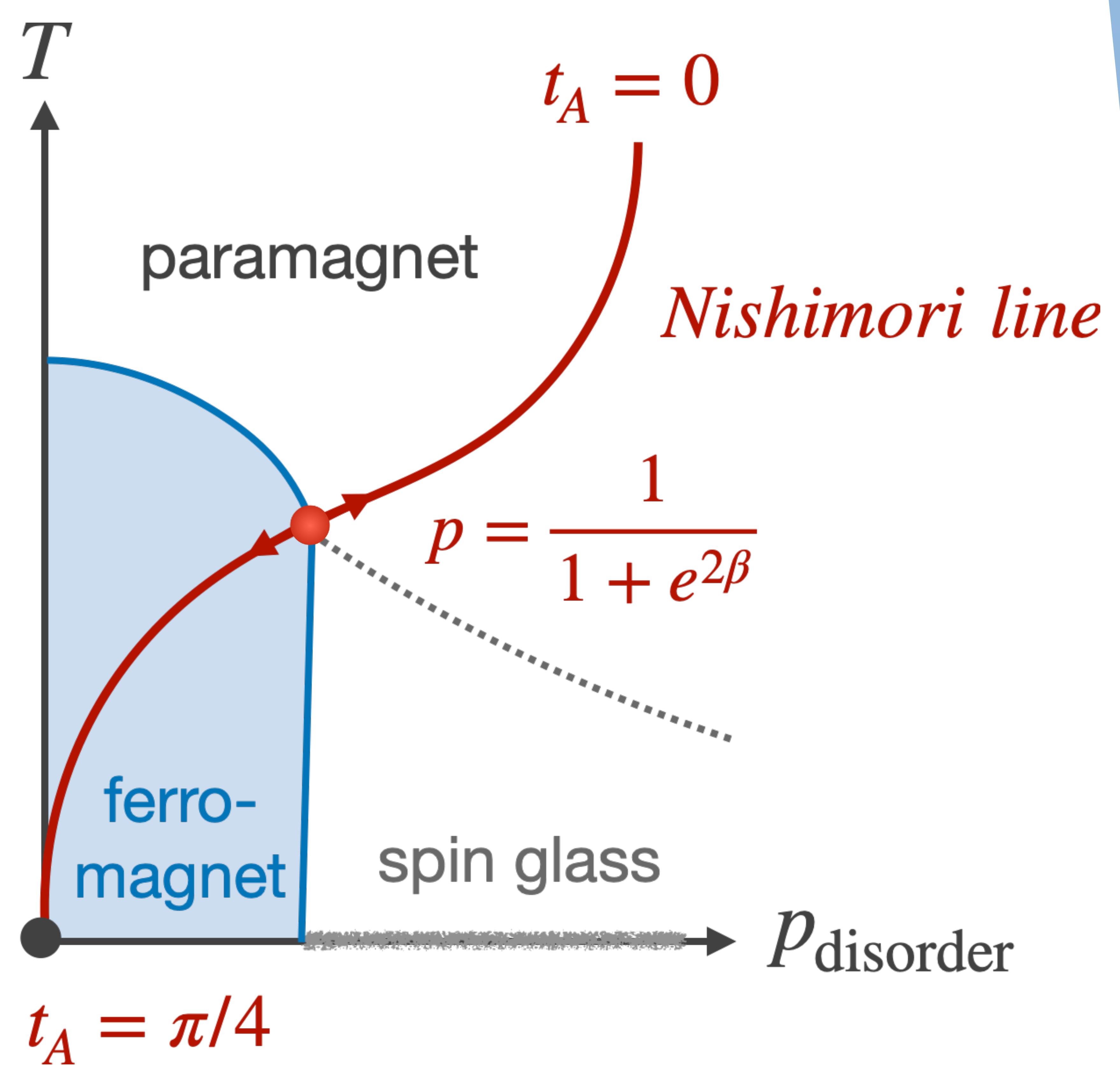


- $\alpha < \pi/4$ injects **tunable coherent error**
- generically a **non-Clifford** gate

incoherent & coherent errors

Dennis, Kitaev, Landahl, Preskill 2002

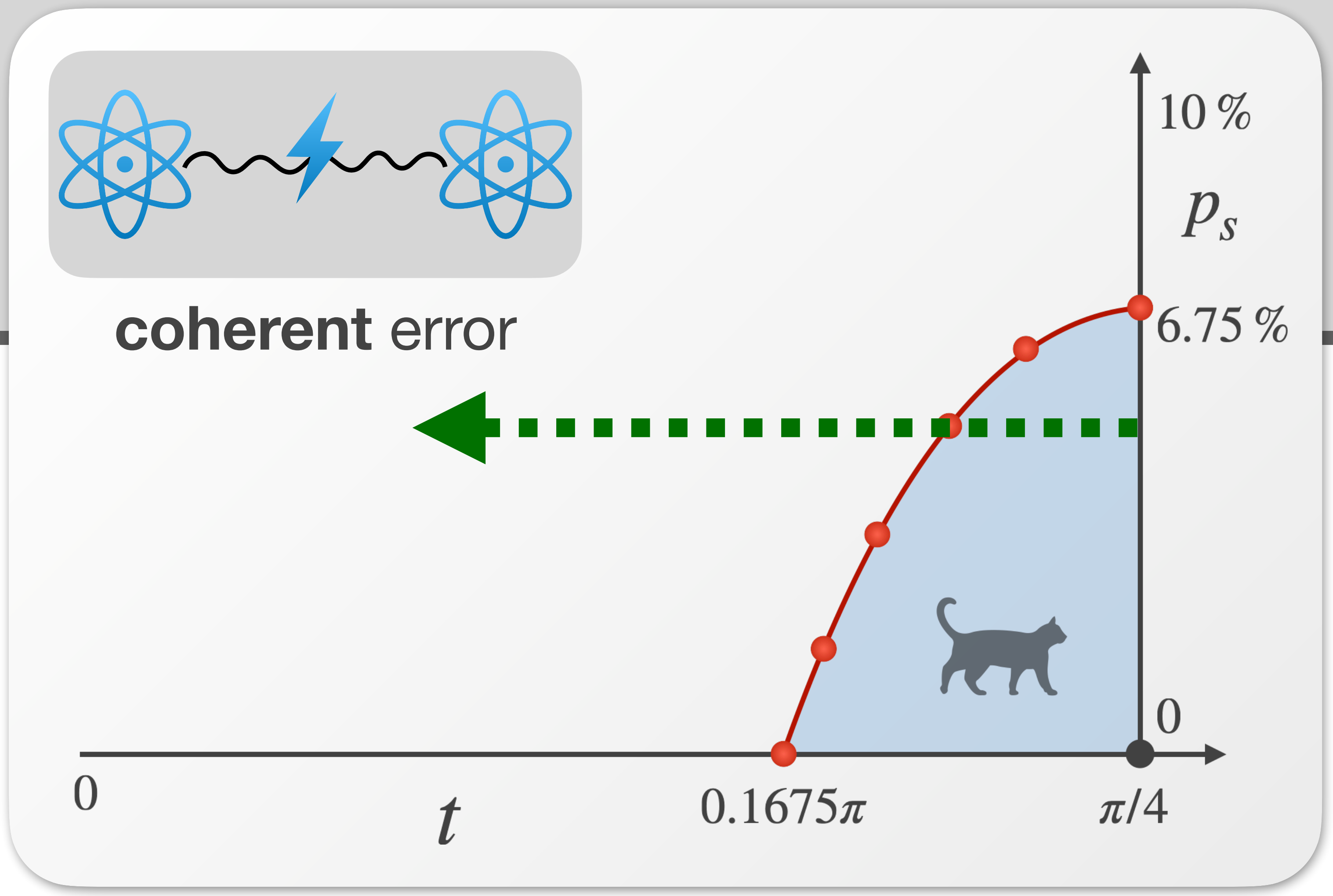


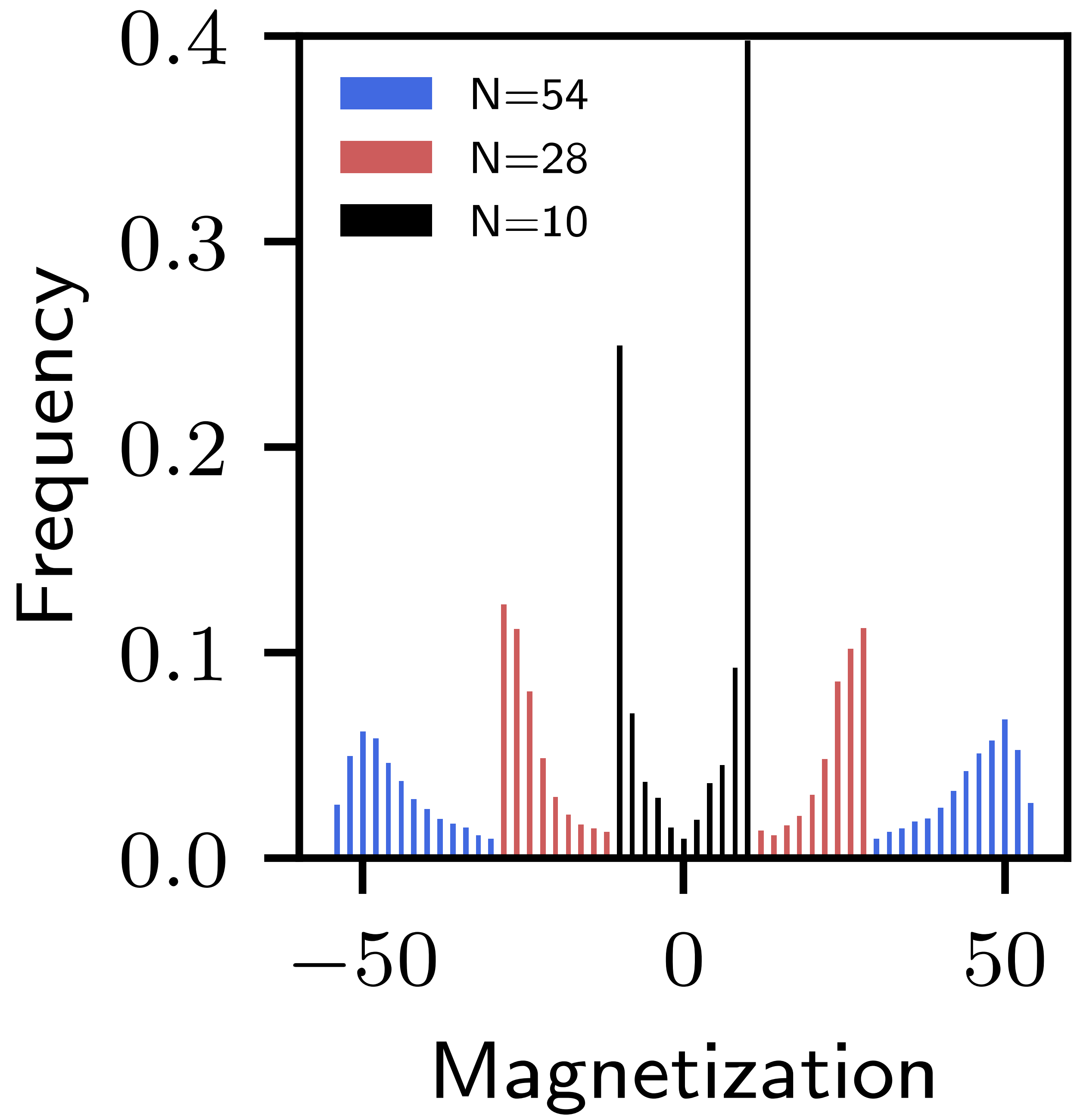


entire phase diagram is mapped to **Nishimori line**

$$\tilde{p} = \frac{1-(1-2p_s)\sin(2t_A)}{2}$$

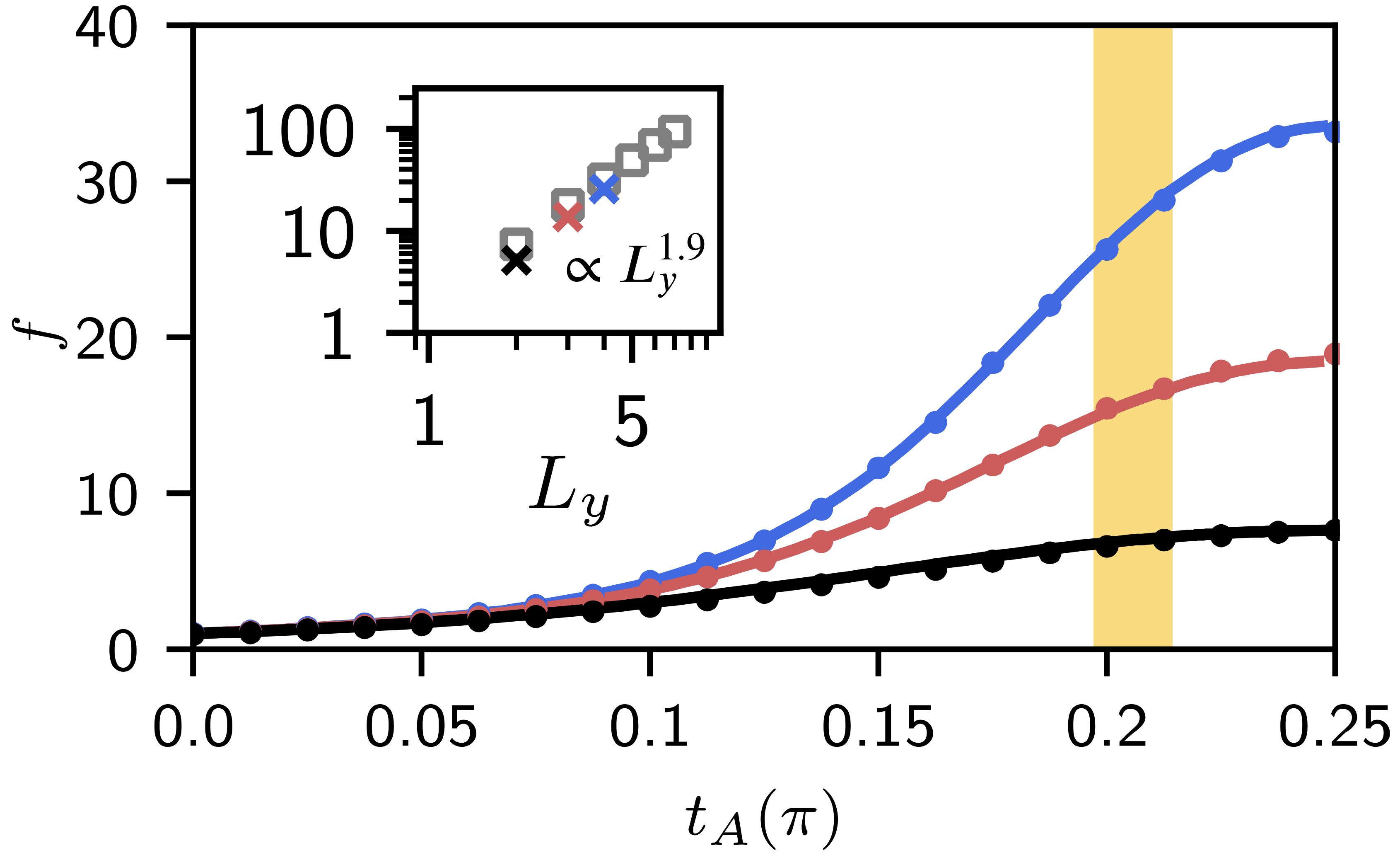
coherent error transition





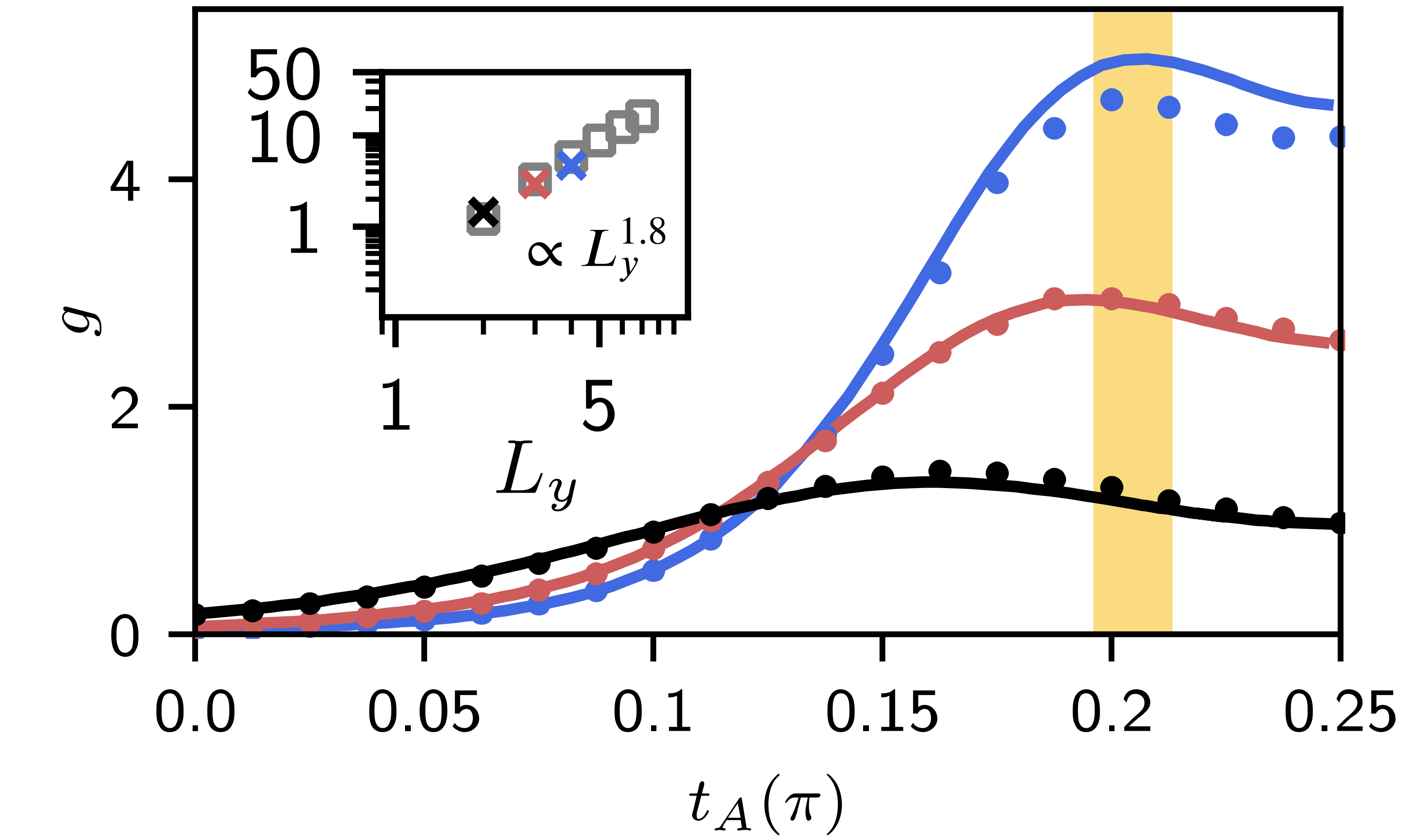
$$f = \frac{1}{N}\left(\langle M^2\rangle - \langle M\rangle^2\right)$$

classical
correlations

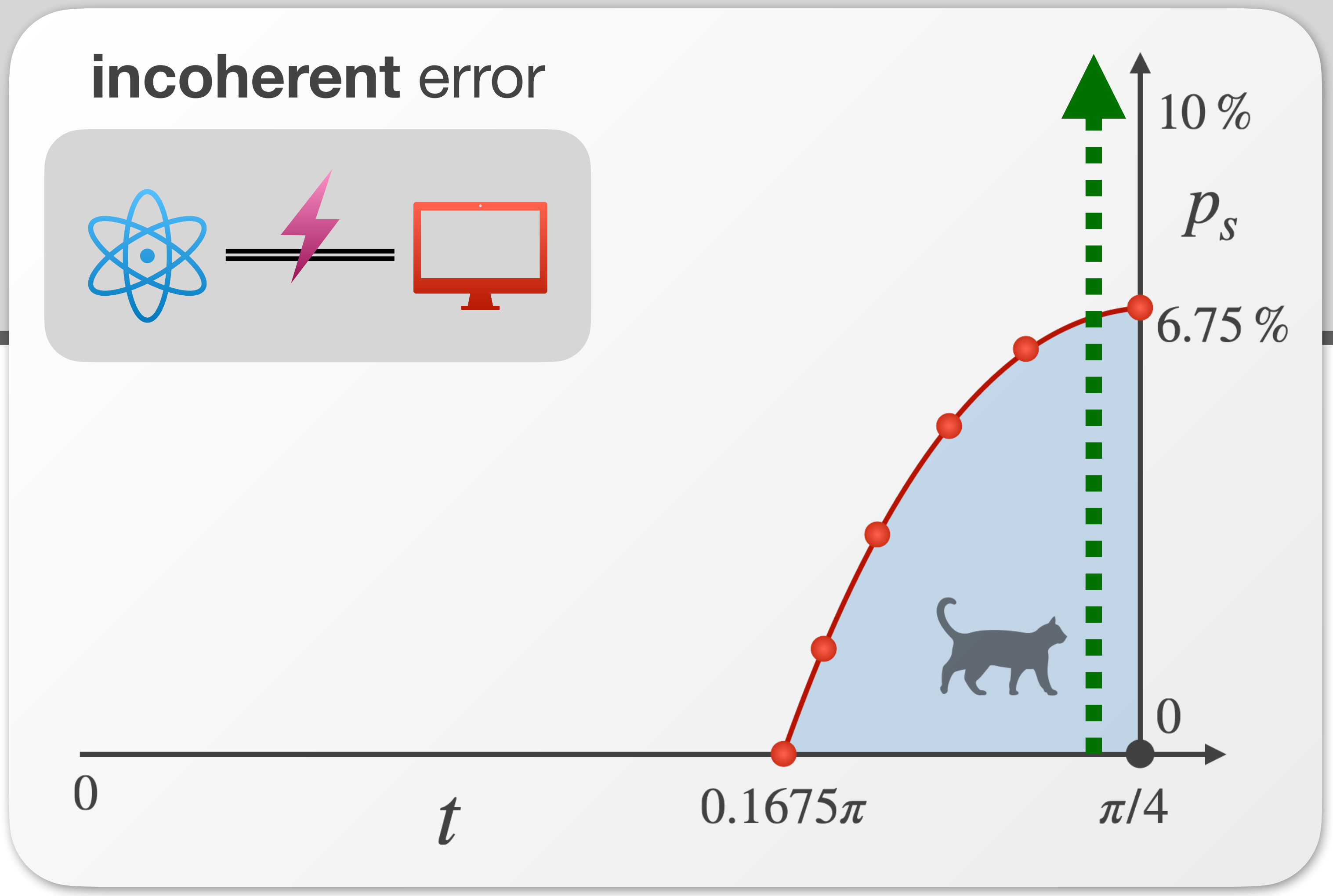


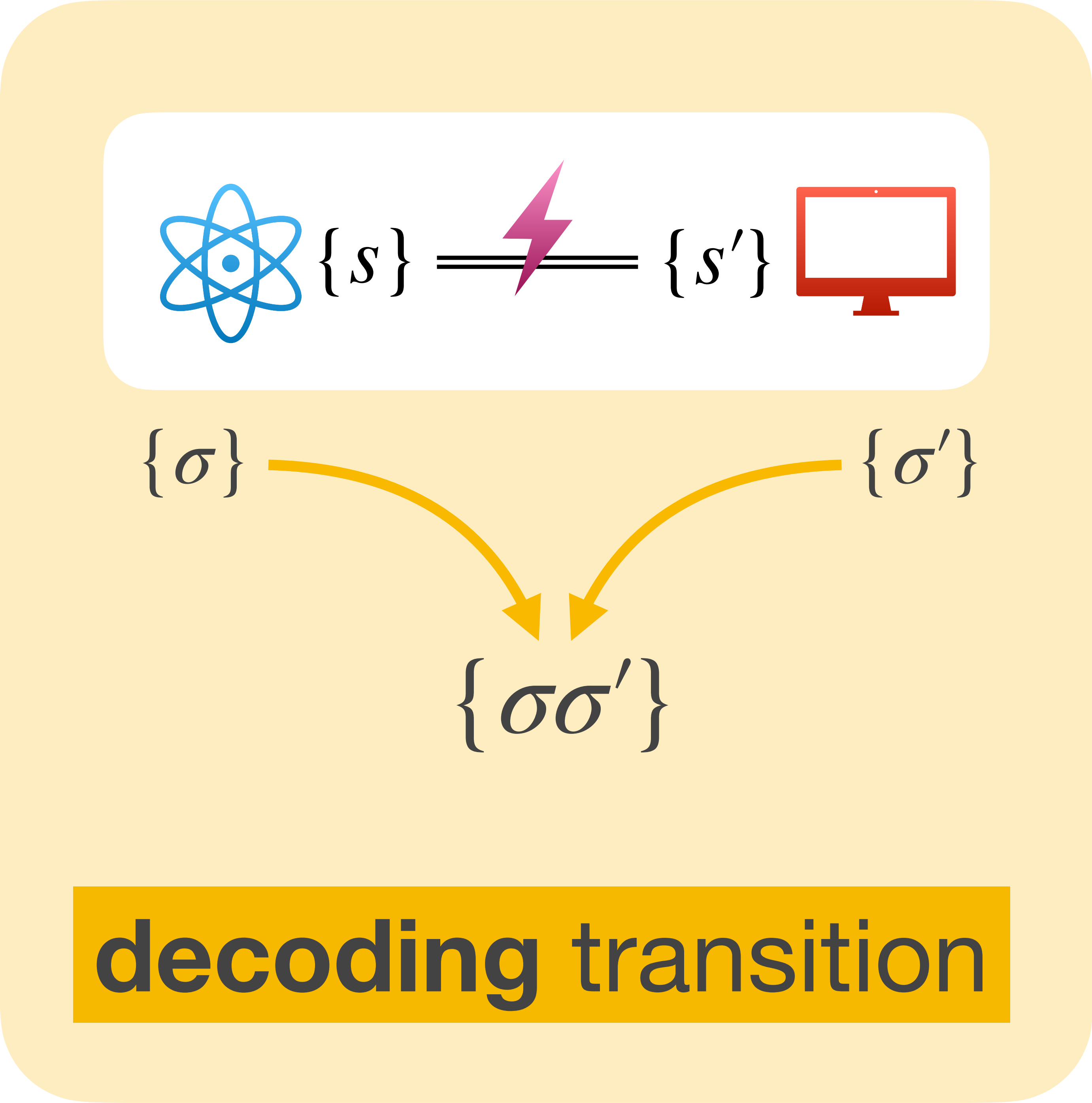
normalized
variance

$$g = \frac{1}{N^3}\left(\langle M^4\rangle - \langle M^2\rangle^2\right)$$

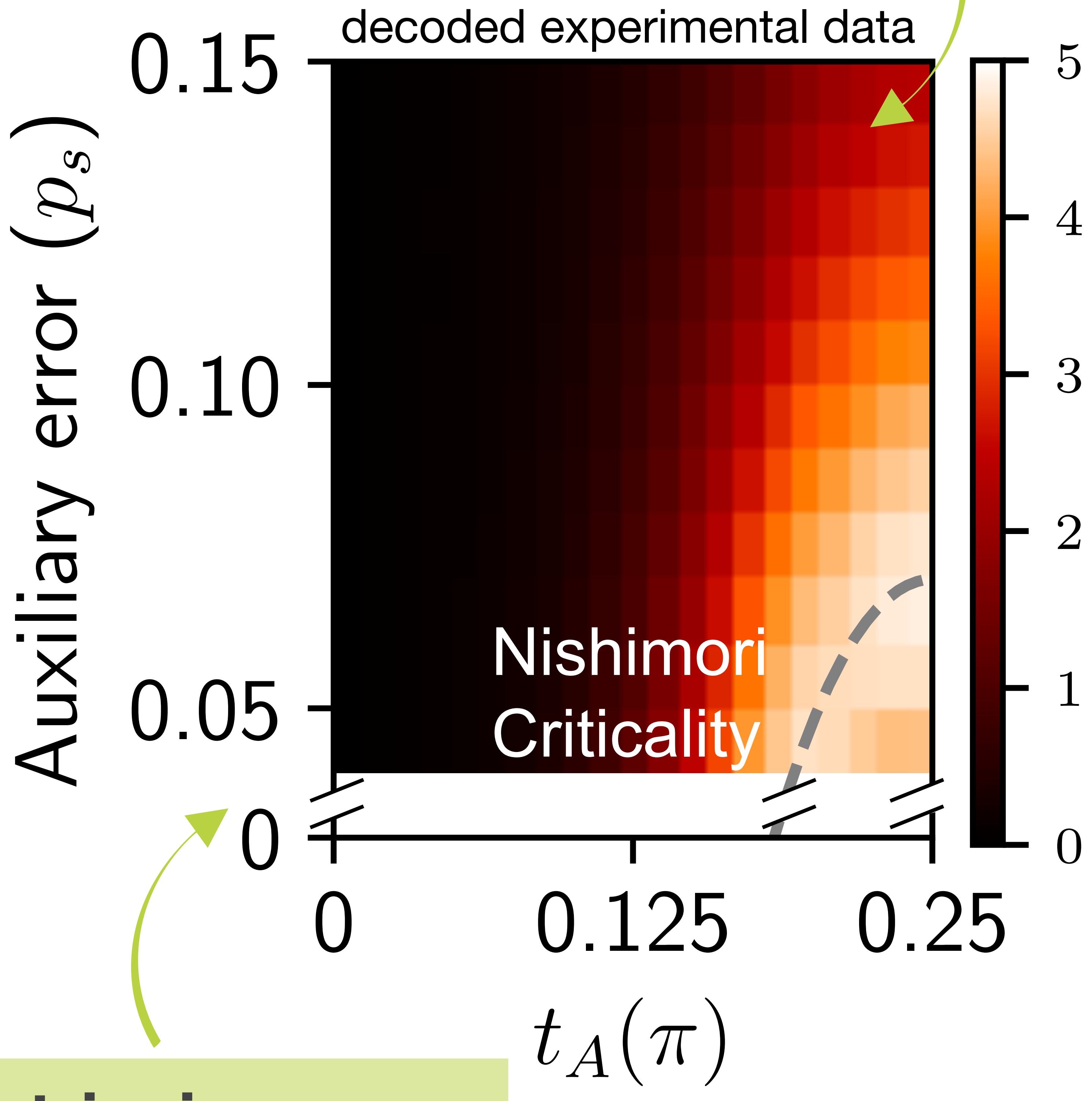


incoherent error transition

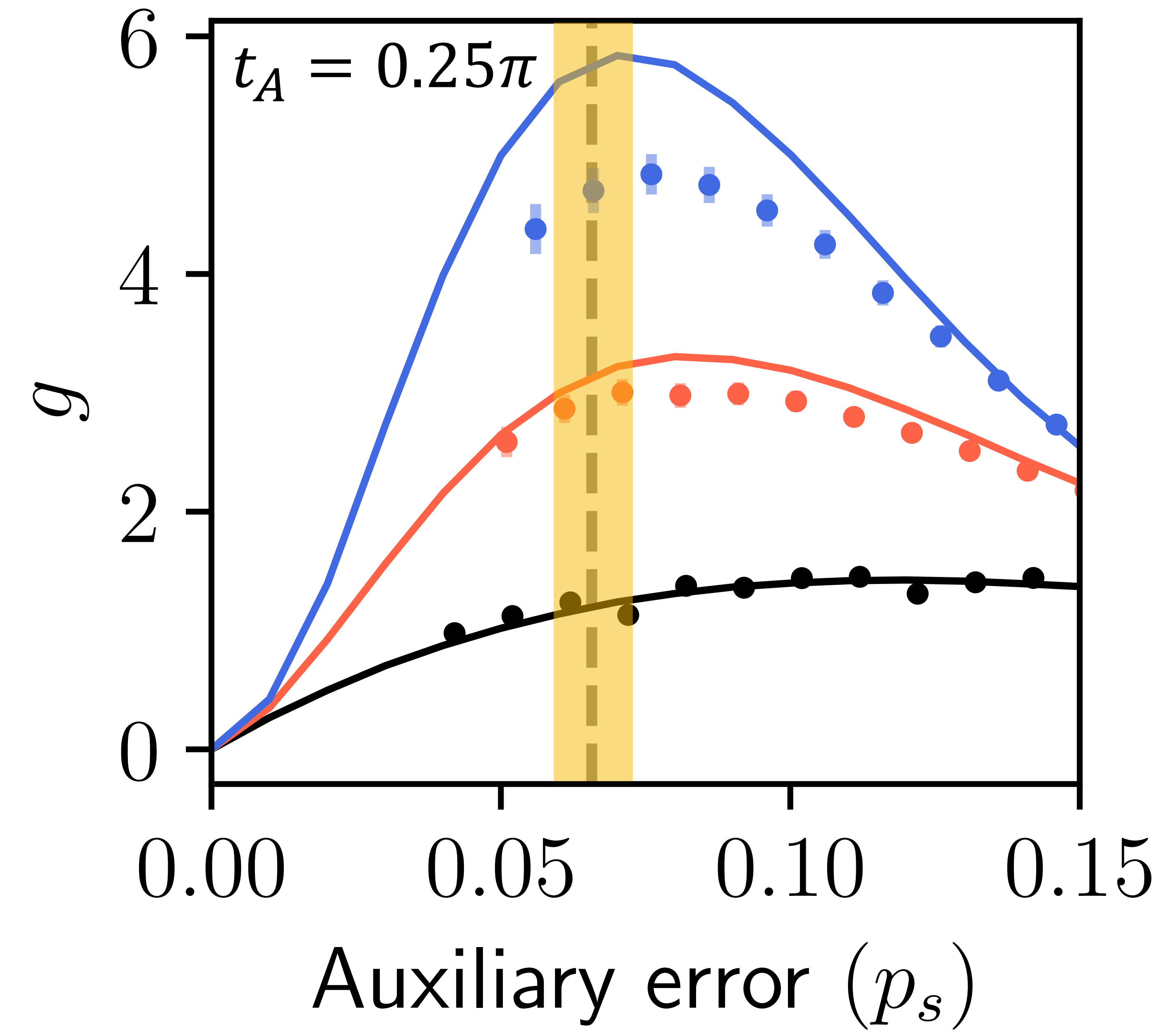




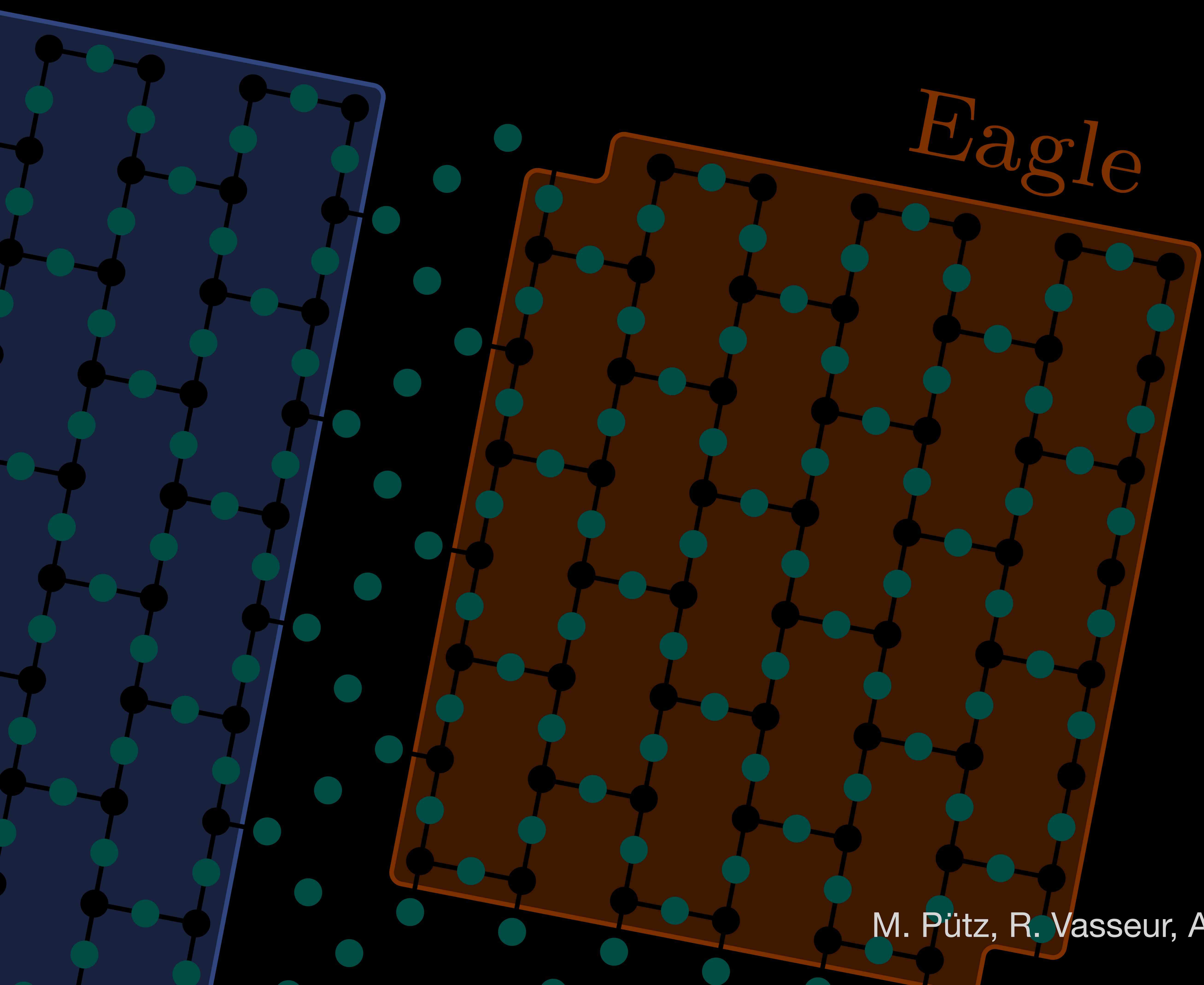
injected error
into classical computer



intrinsic error
of quantum device



··· experimental data on IBM Sherbrooke
— theoretical benchmark with noise



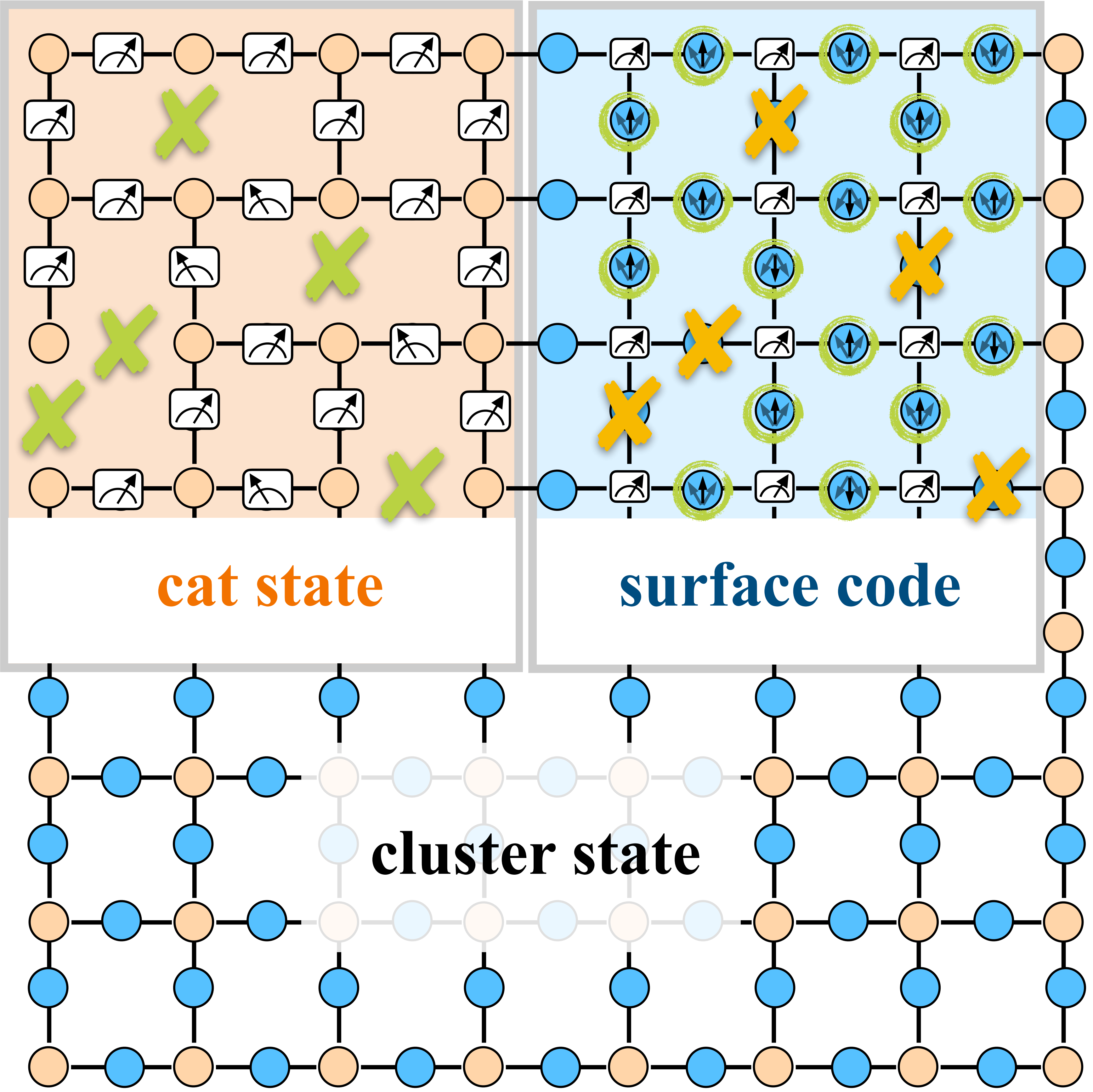
qubit loss

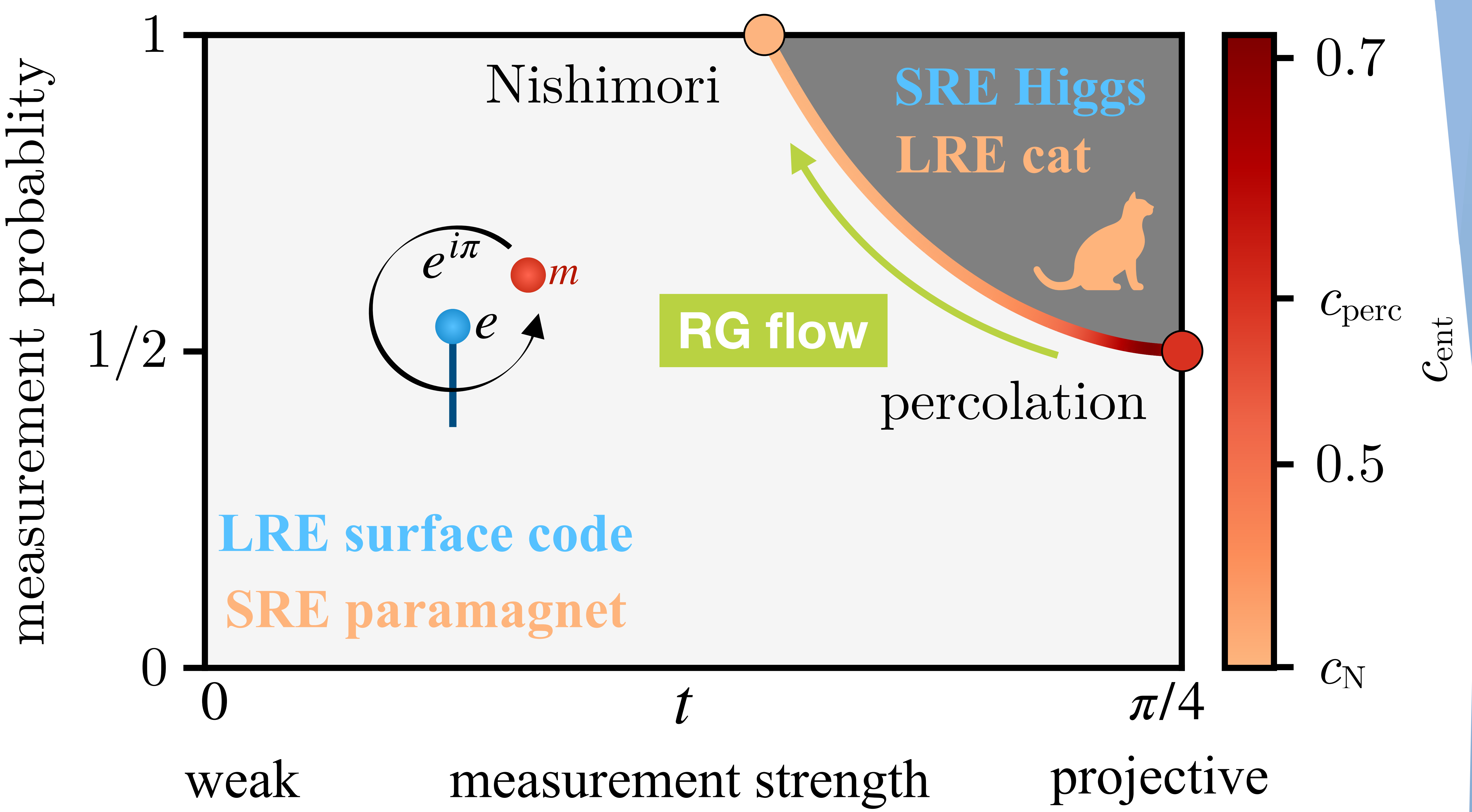
arXiv:2505.22720

M. Pütz, R. Vasseur, A. Ludwig, G-Y. Zhu

Nishimori / percolation & **RG flows**

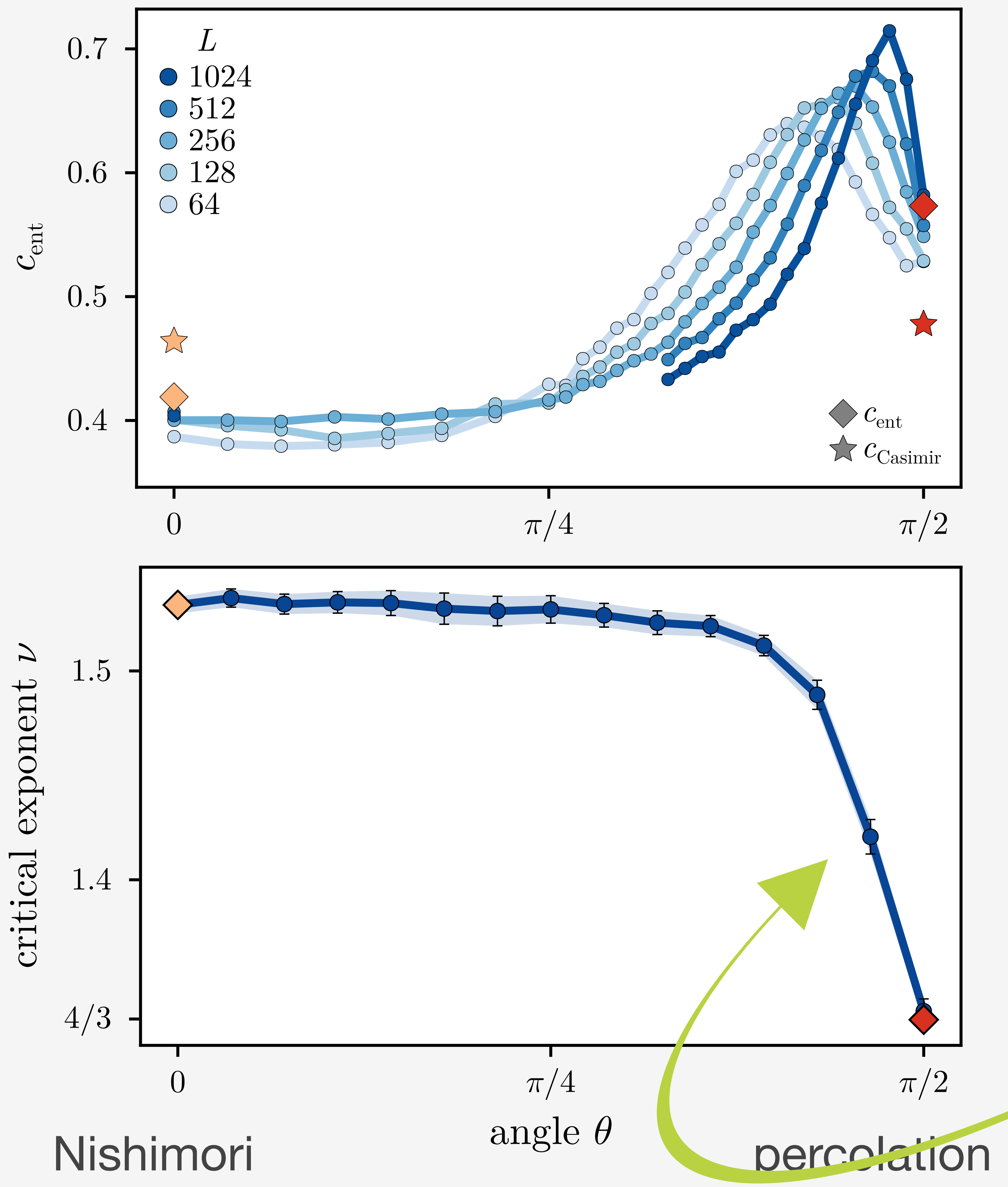
qubit loss

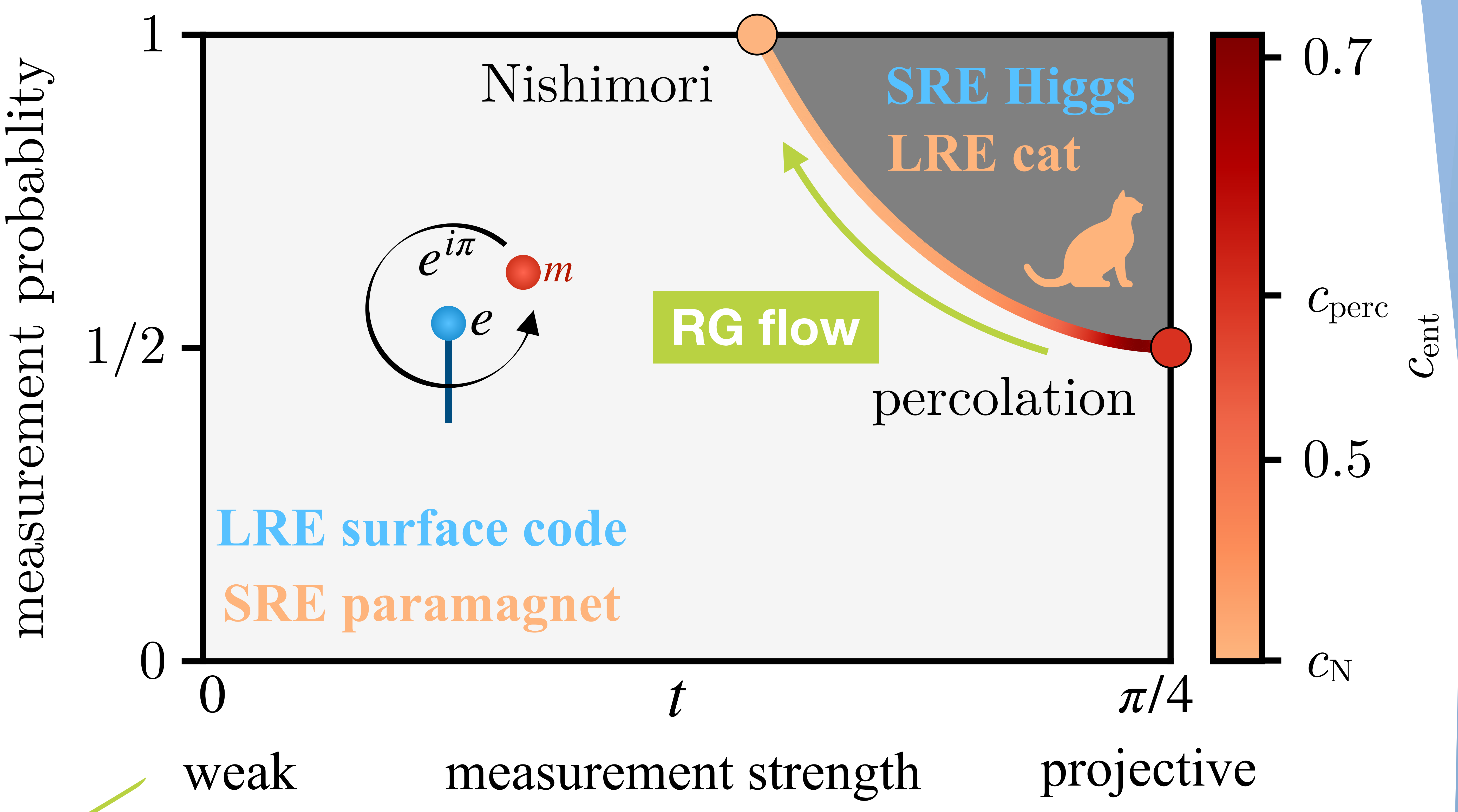




RG flow from percolation **to Nishimori** criticality

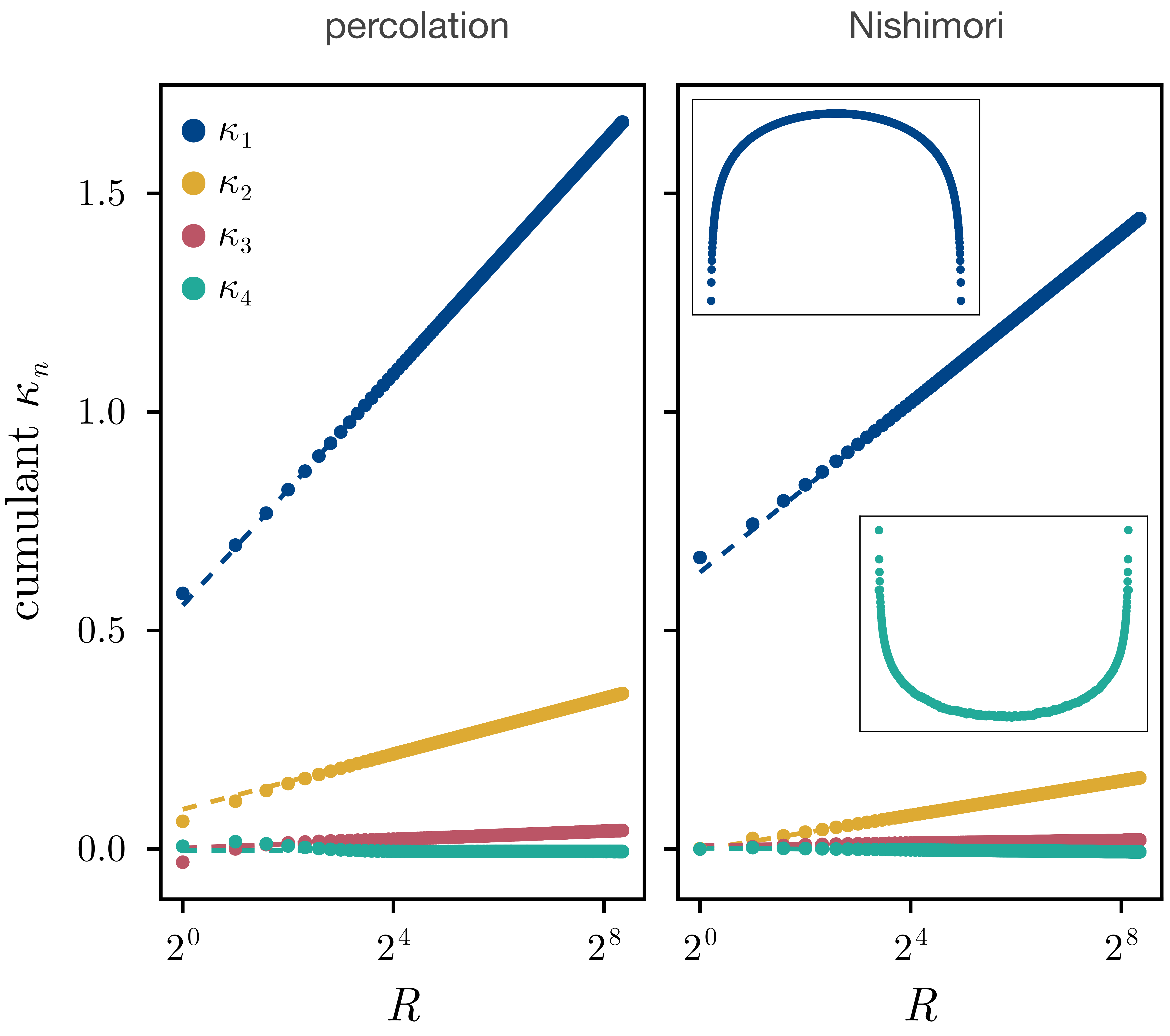
Nishimori / percolation & **RG flows**





RG flow from percolation
to Nishimori criticality

Nishimori / percolation & **multifractality**



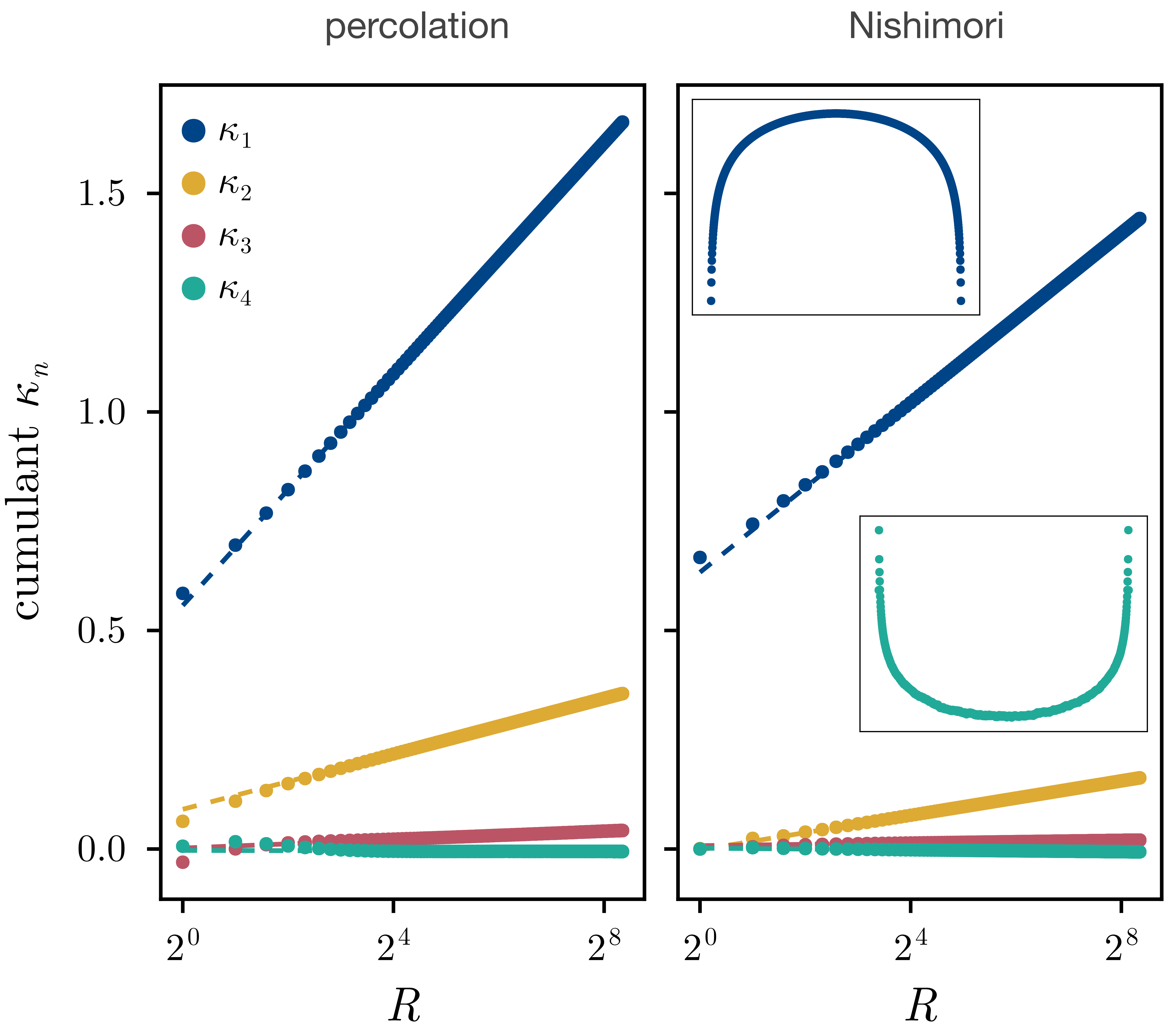
Both percolation and Nishimori criticality exhibit **multifractal spectra** of scaling dimensions.

Different moments of correlations functions scale with distinct **non-trivial exponents.**

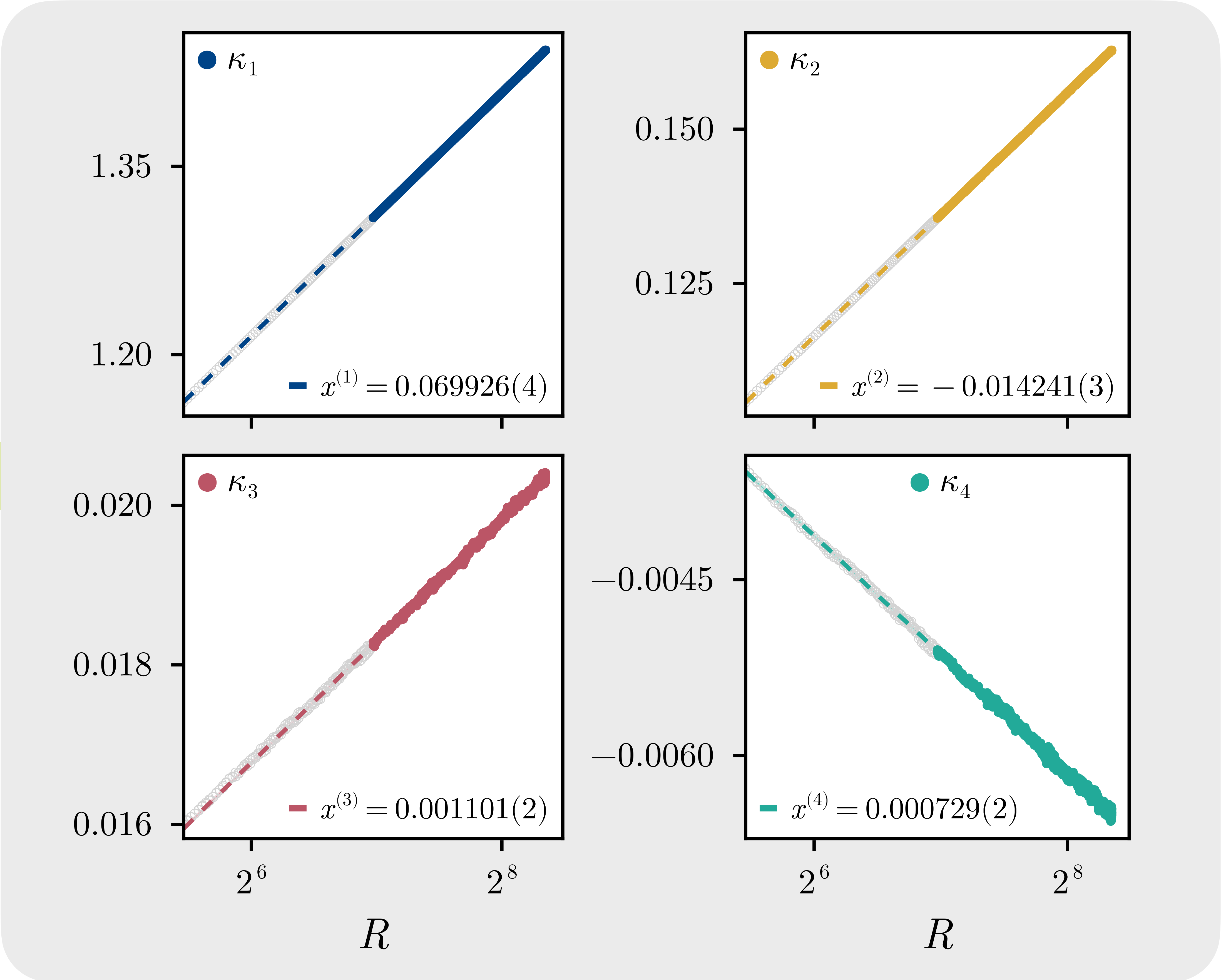
The multifractal scaling quantifies the **statistical fluctuations** of entanglement entropies.

Scaling behavior of the boundary condition changing (boundary) **twist operators.**

Nishimori / percolation & **multifractality**



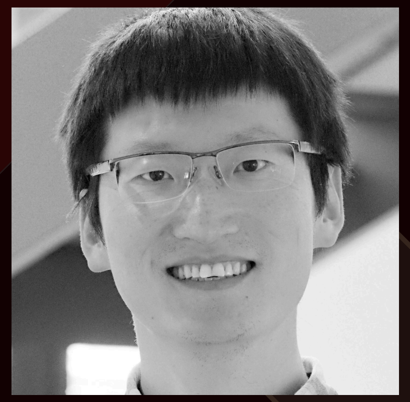
Both percolation and Nishimori criticality exhibit **multifractal spectra** of scaling dimensions.



monitored toric codes

arXiv:2502.14034
PRX Quantum **5**, 040313 (2024)

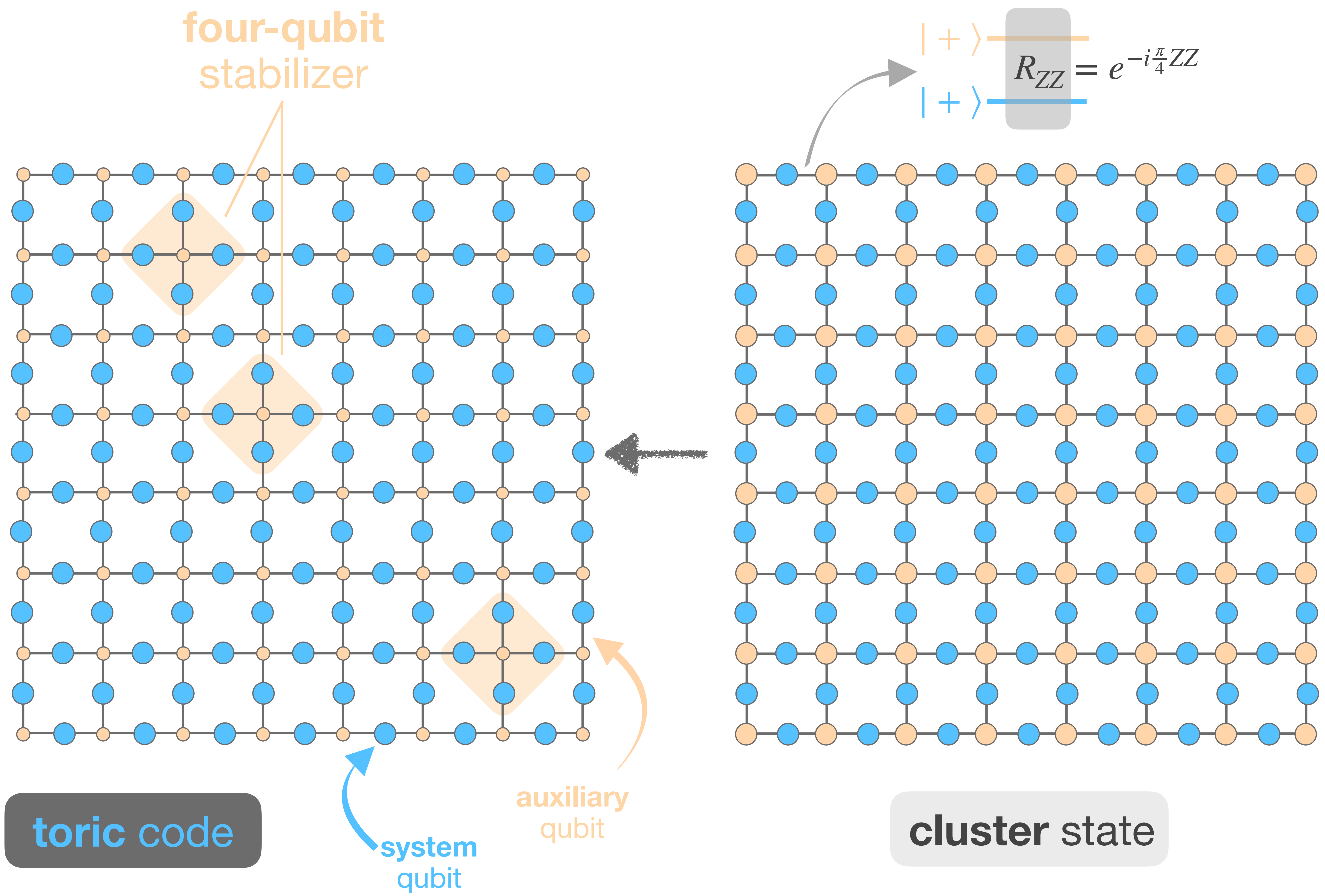
F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu



entanglement by measurement: **surface code**

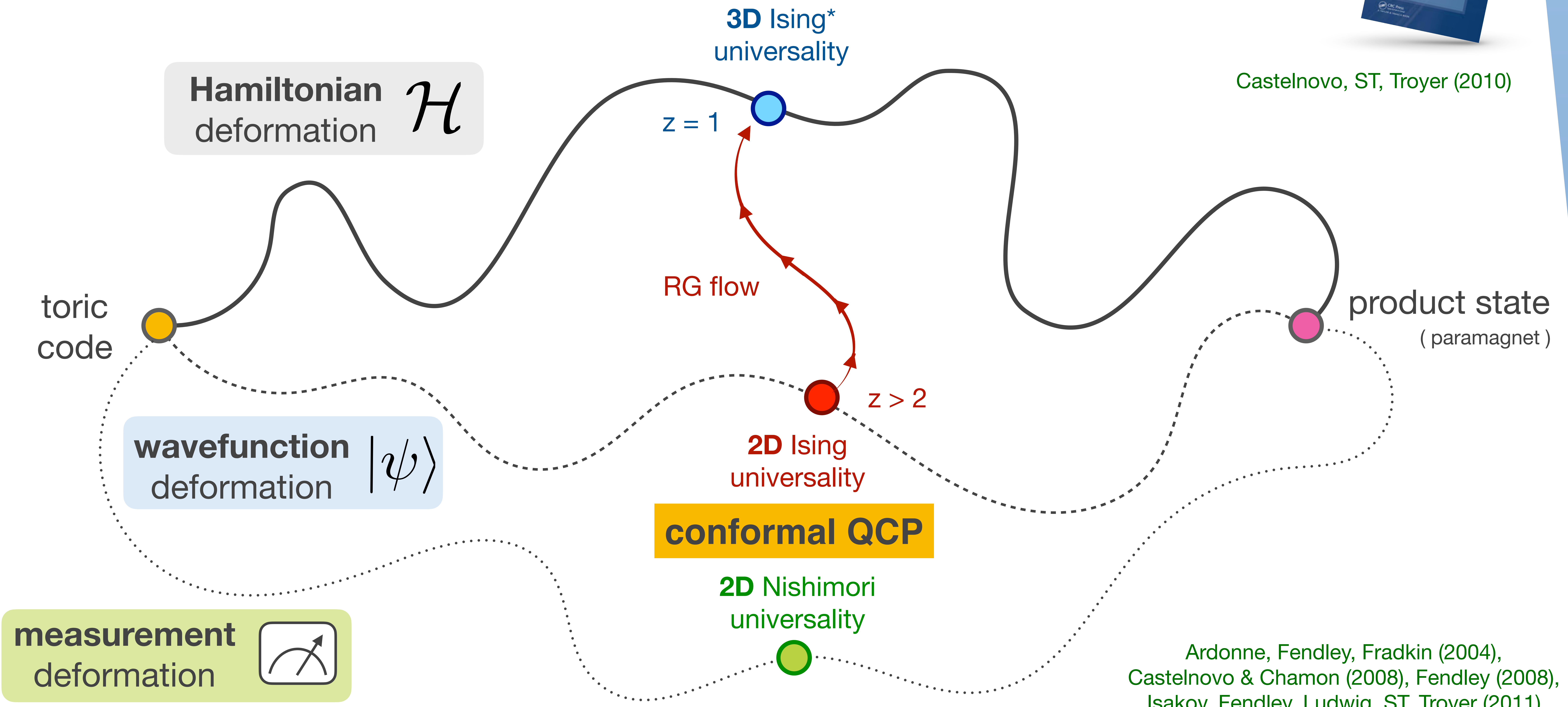
Kitaev (1997)



The toric/surface code is an ingenious **measurement protocol.**

The toric/surface code is an exactly solvable **Hamiltonian model.**

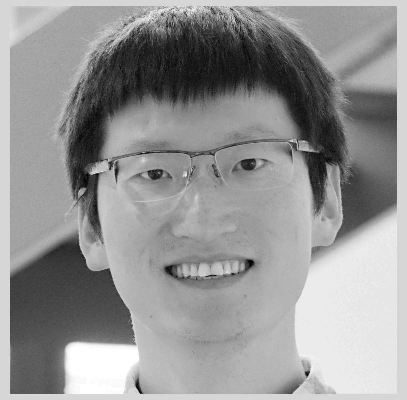
phase transitions & deformations

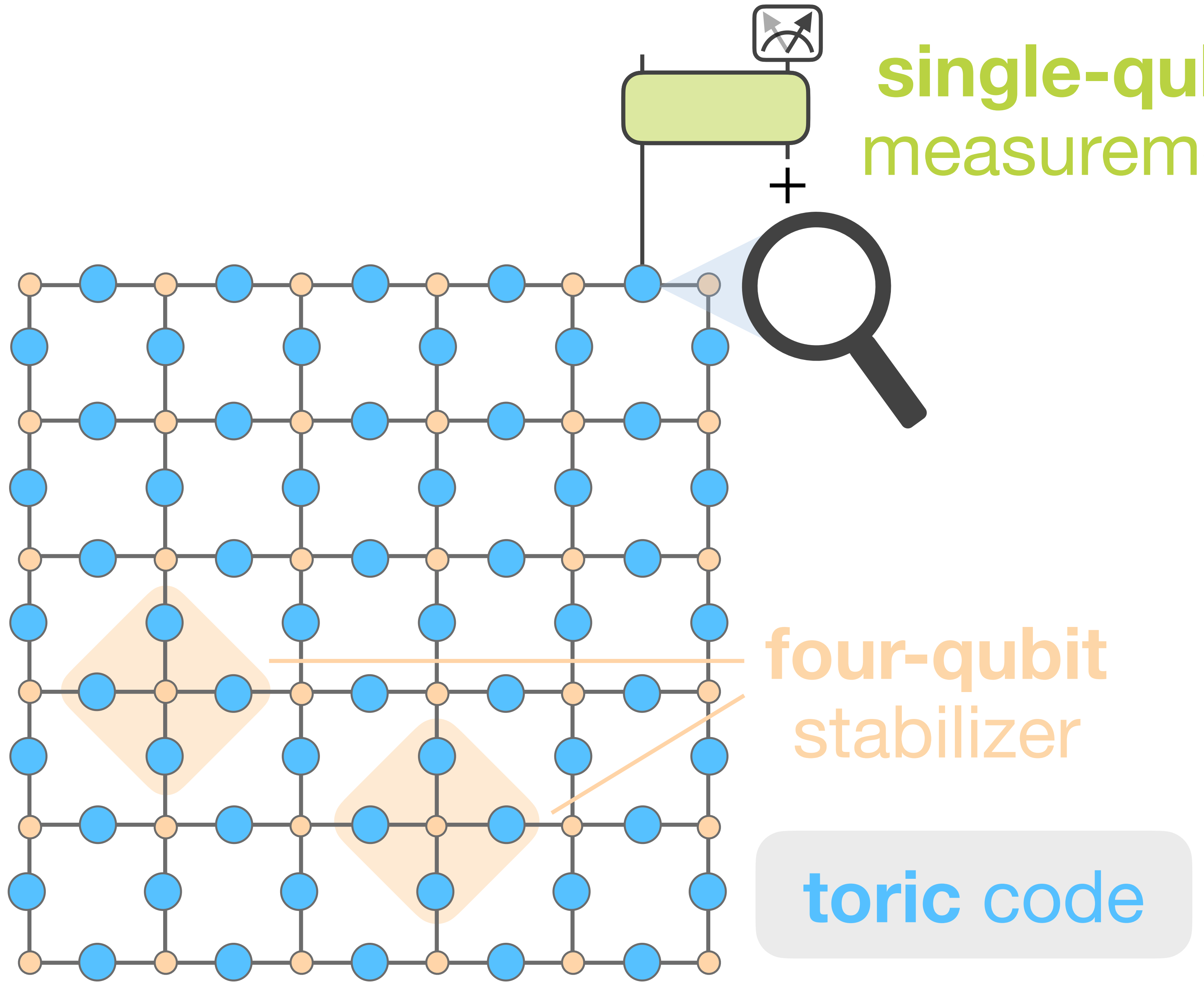


Castelnovo, ST, Troyer (2010)

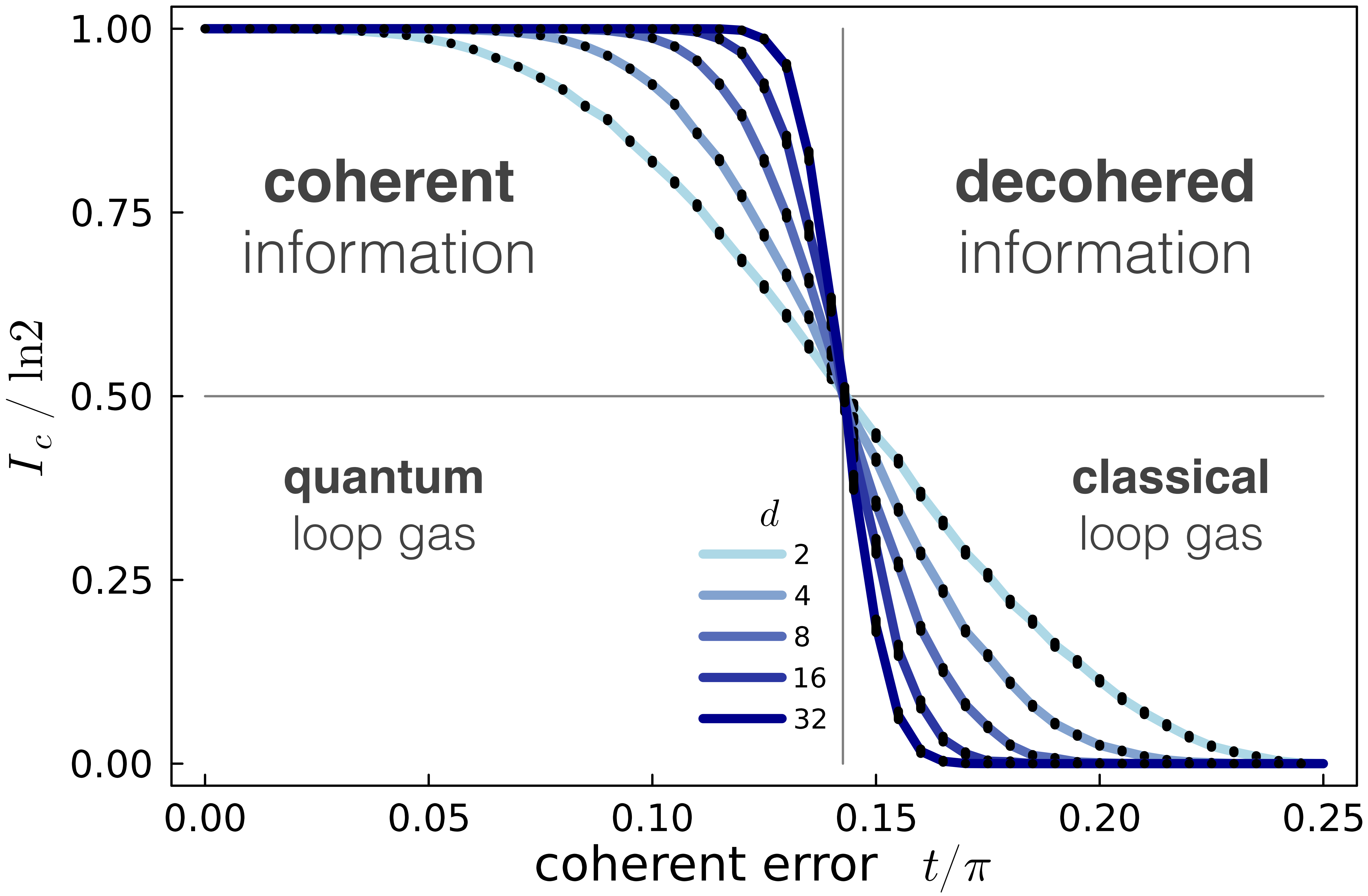
Ardonne, Fendley, Fradkin (2004), Castelnovo & Chamon (2008), Fendley (2008), Isakov, Fendley, Ludwig, ST, Troyer (2011)

Toric code under weak measurement





$$|\psi\rangle \mapsto \begin{cases} \left(1 + \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle \\ \left(1 - \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle \end{cases}$$



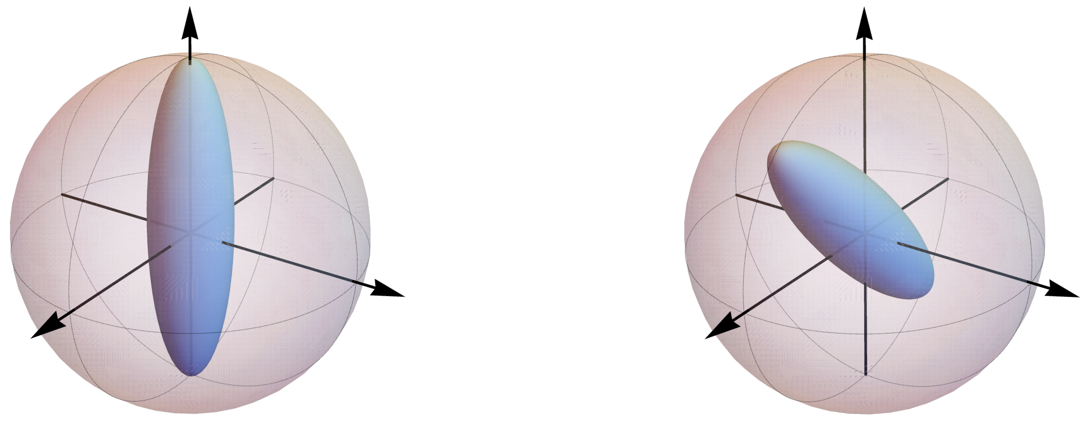
Nishimori transition

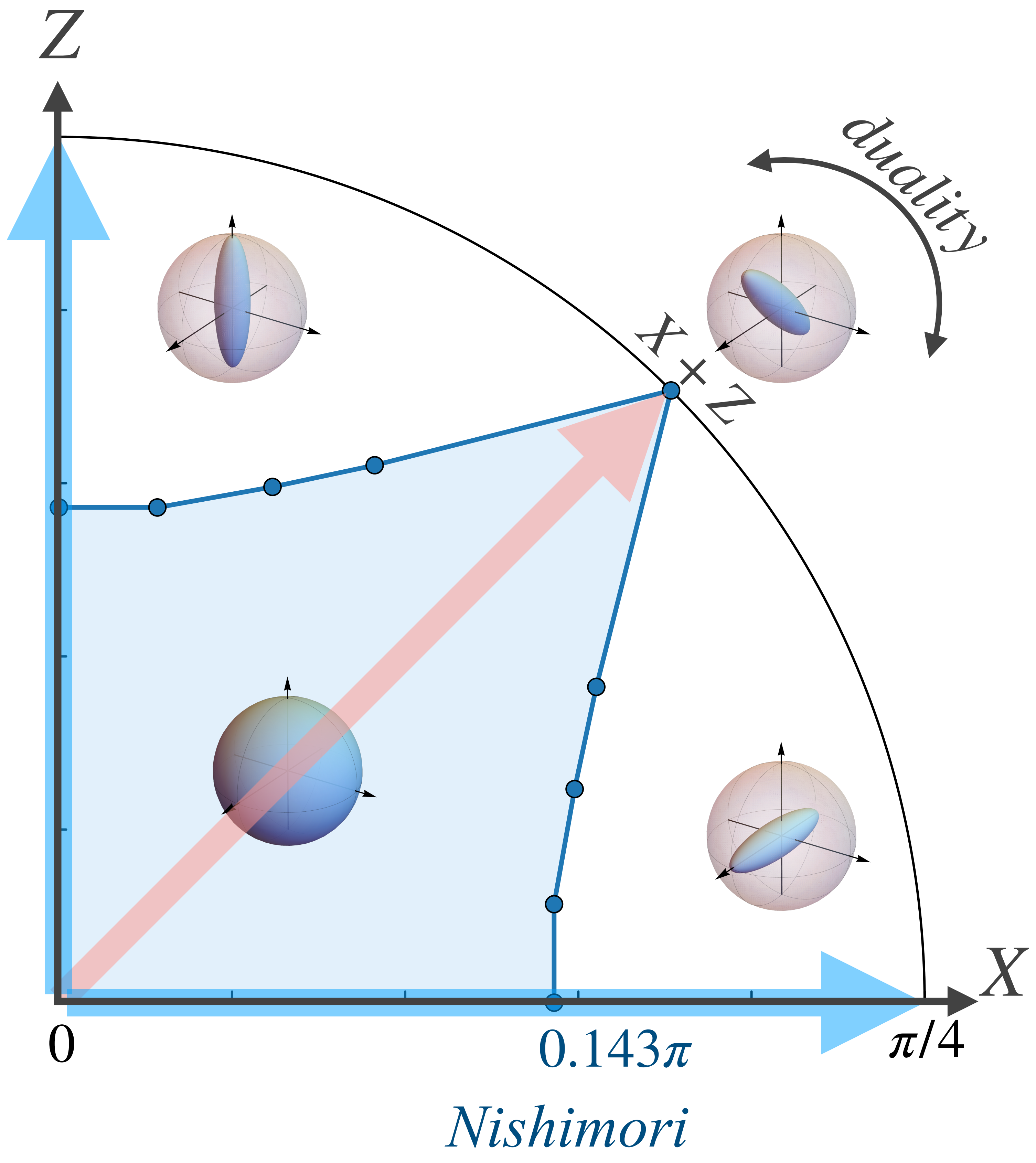
F. Eckstein, B. Han, ST, G.Y. Zhu
PRX Quantum **5**, 040313 (2024)

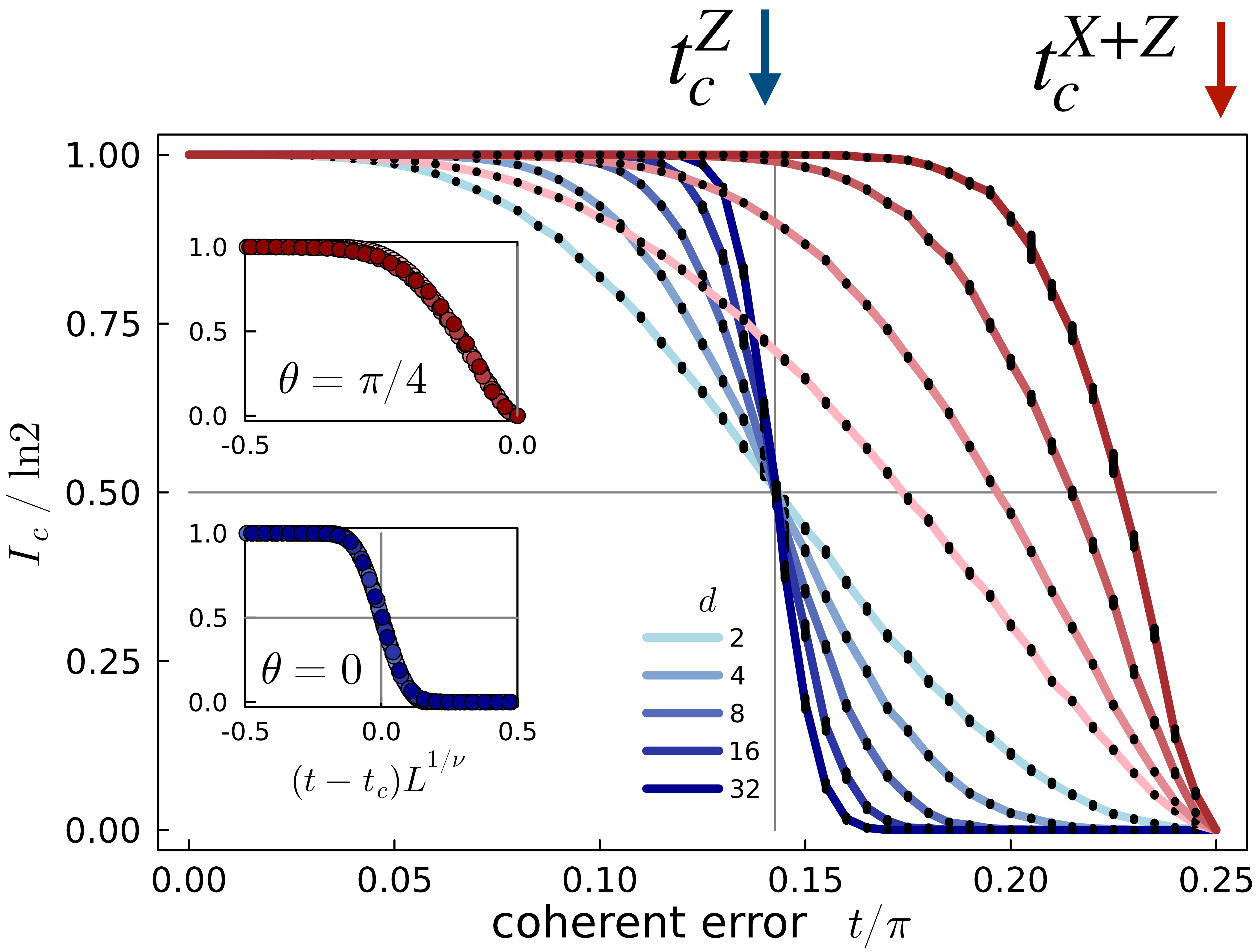
Self-dual criticality

$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**

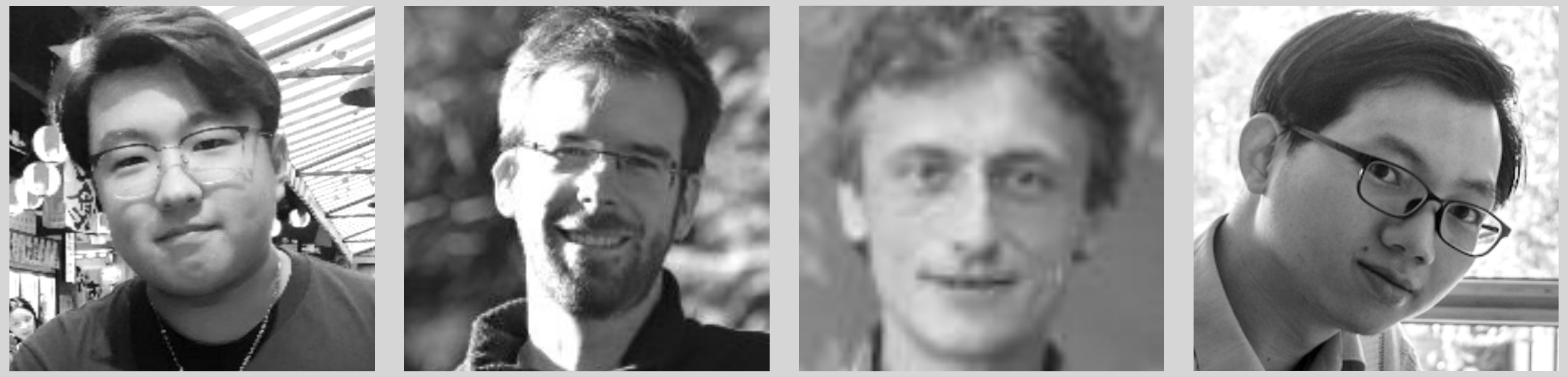






$\nu = 1.6(1)$ $\nu = 1.8(1)$

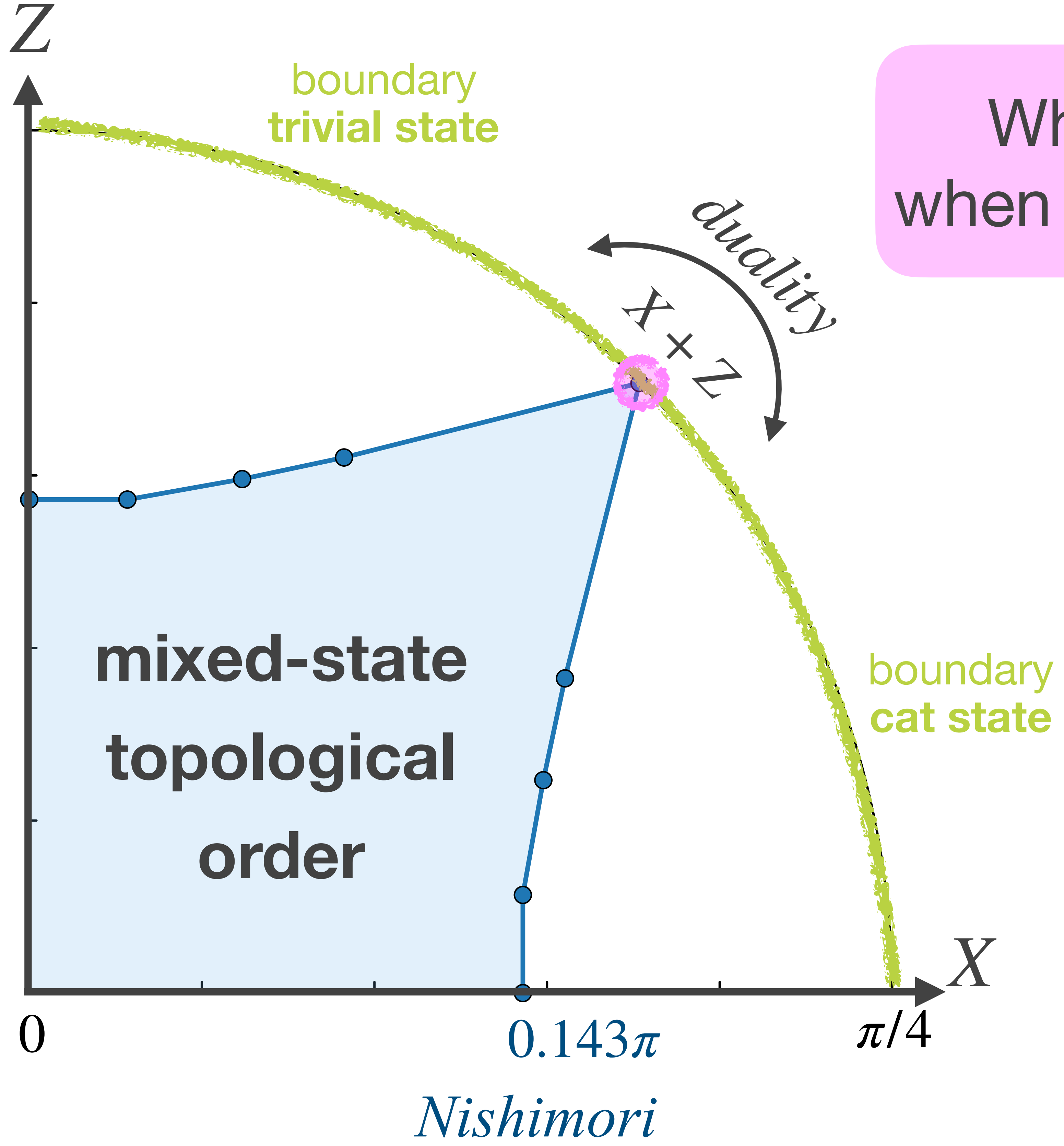
Self-dual criticality



$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

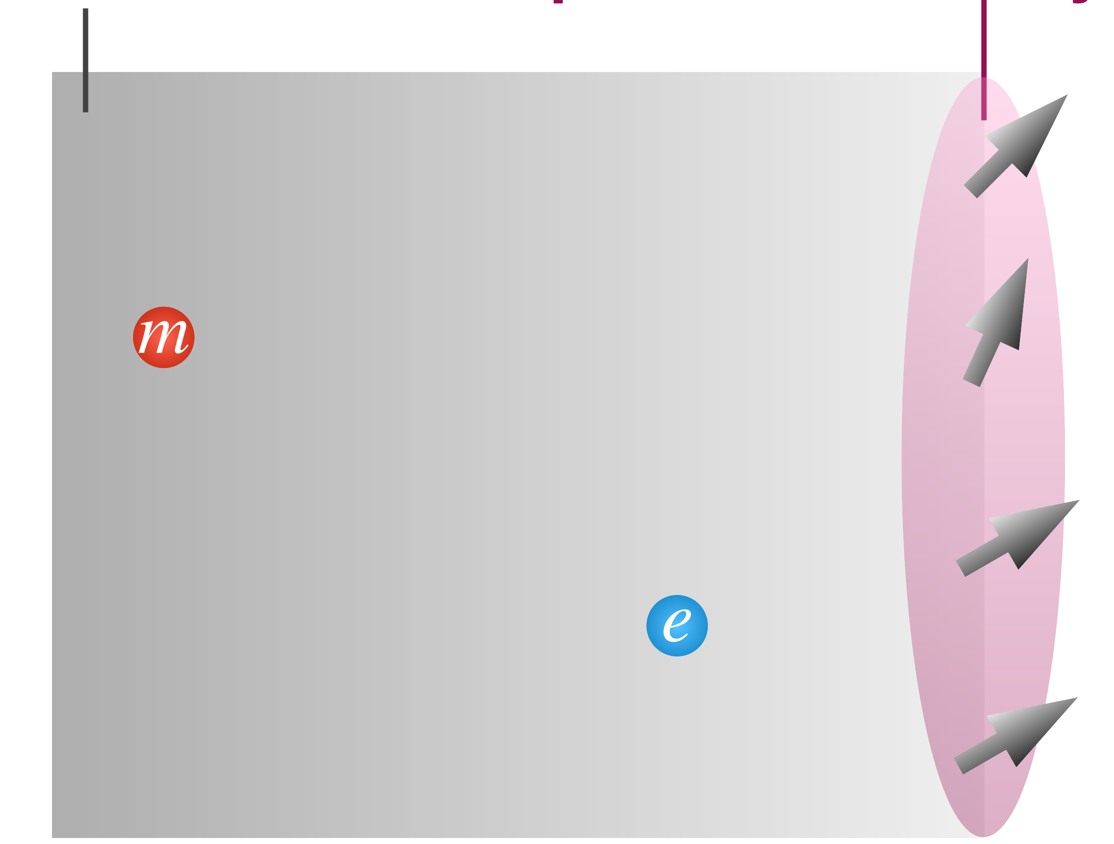
measurement **angle**

Self-duality makes the toric code absolutely **stable** against decoherence.



Why is there a **critical point** when all the bulk qubits **collapse**?

$$\rho = \sum_{em} |em\rangle\langle em|_C \otimes |\psi(em)\rangle\langle\psi(em)|_Q$$



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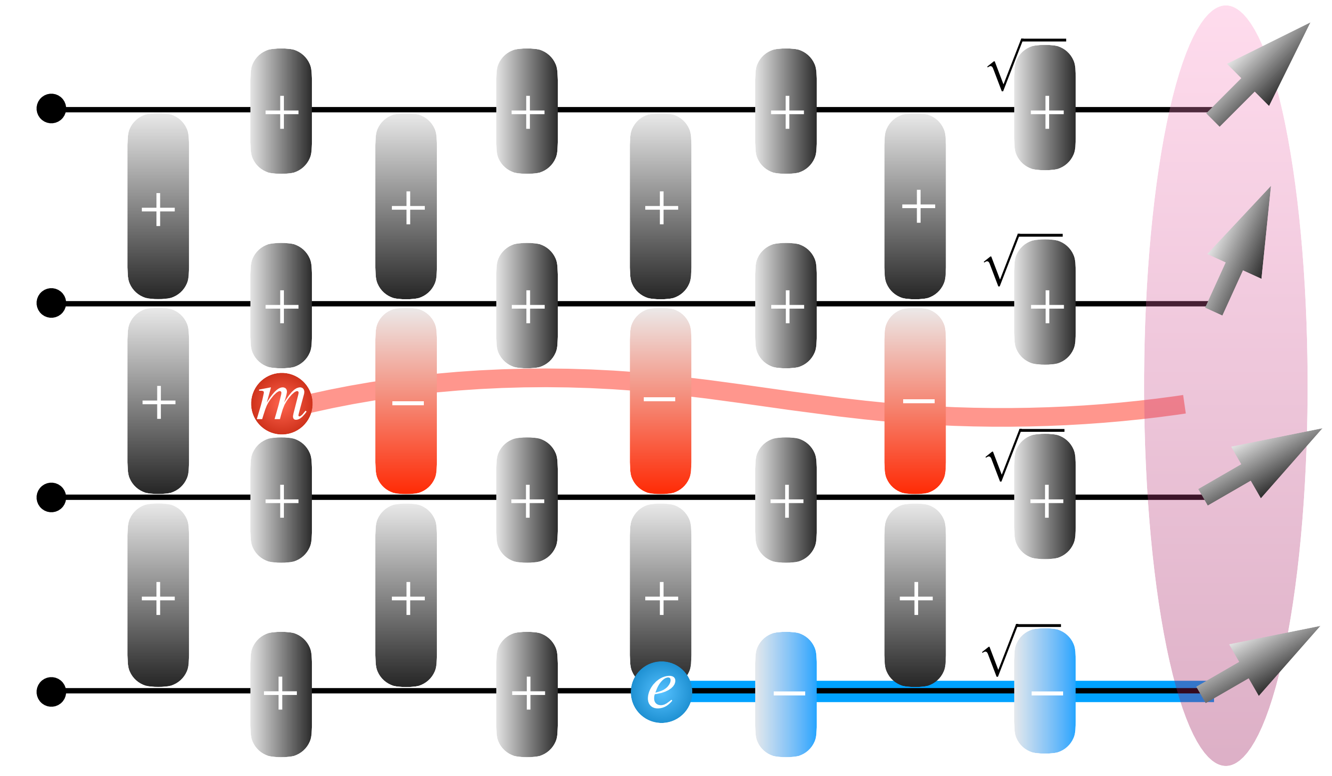
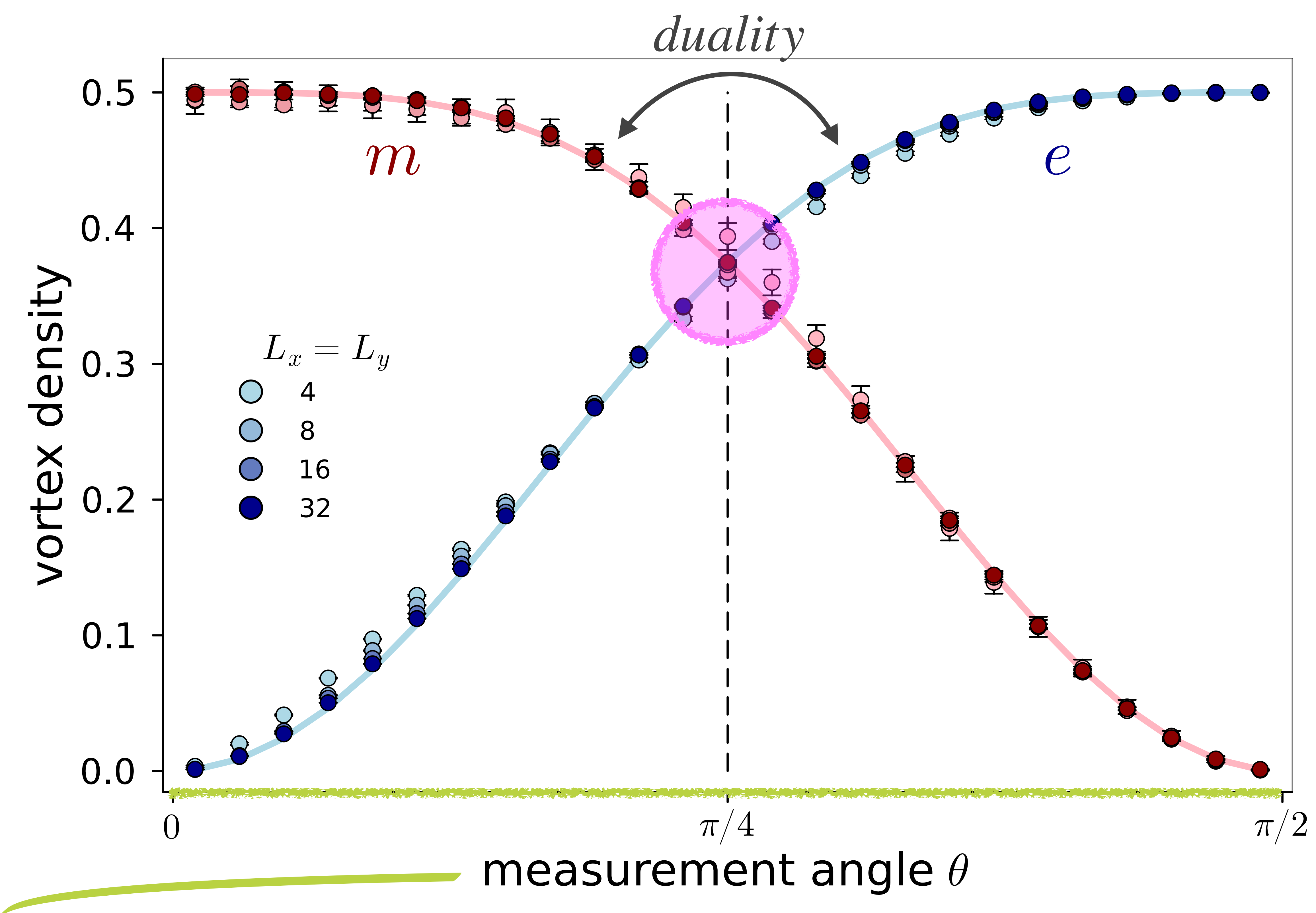


bulk: mixed-state phase transition

boundary: measurement-induced phase transition

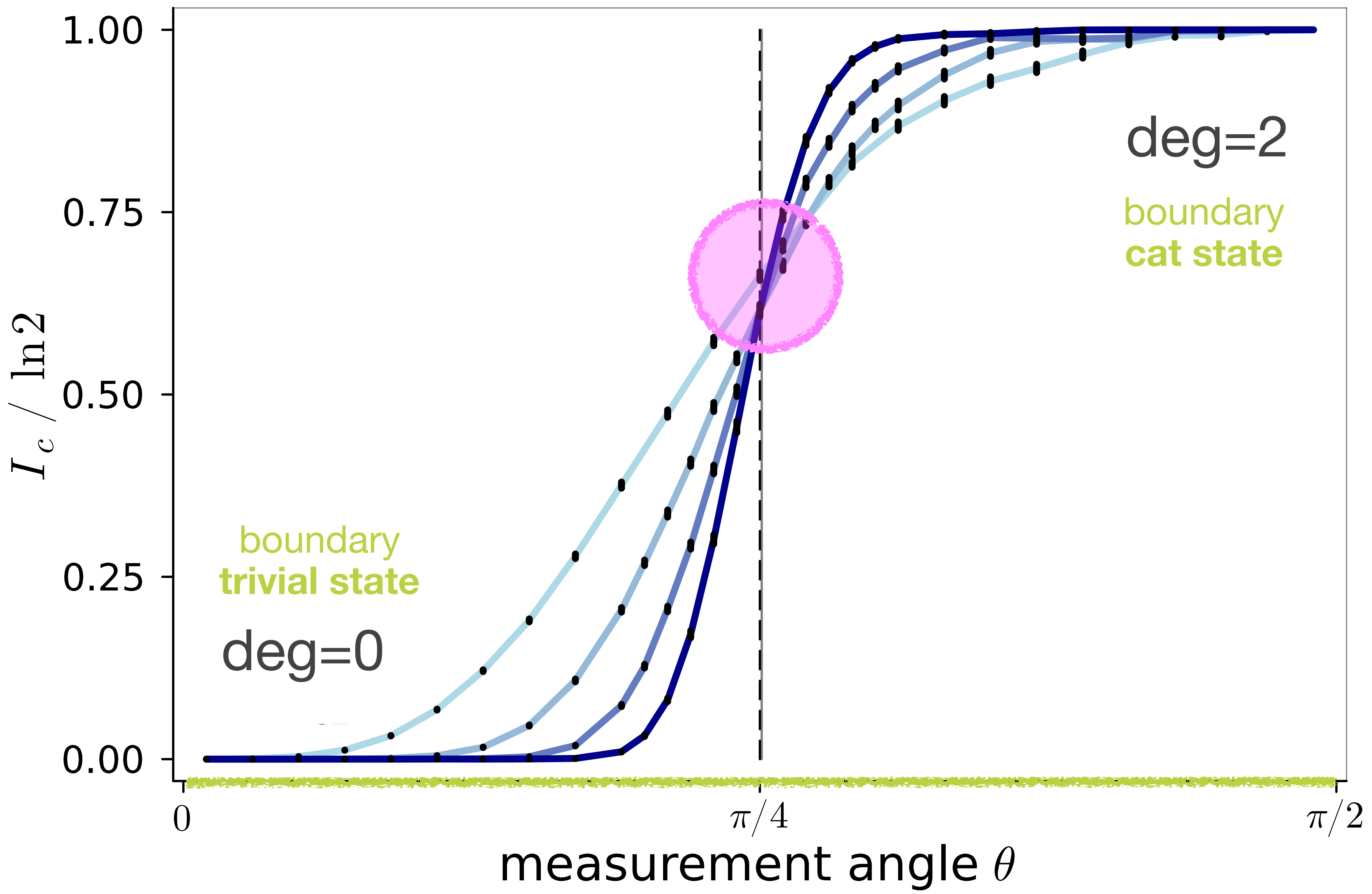
Self-dual criticality

Criticality is dictated by **self-duality**

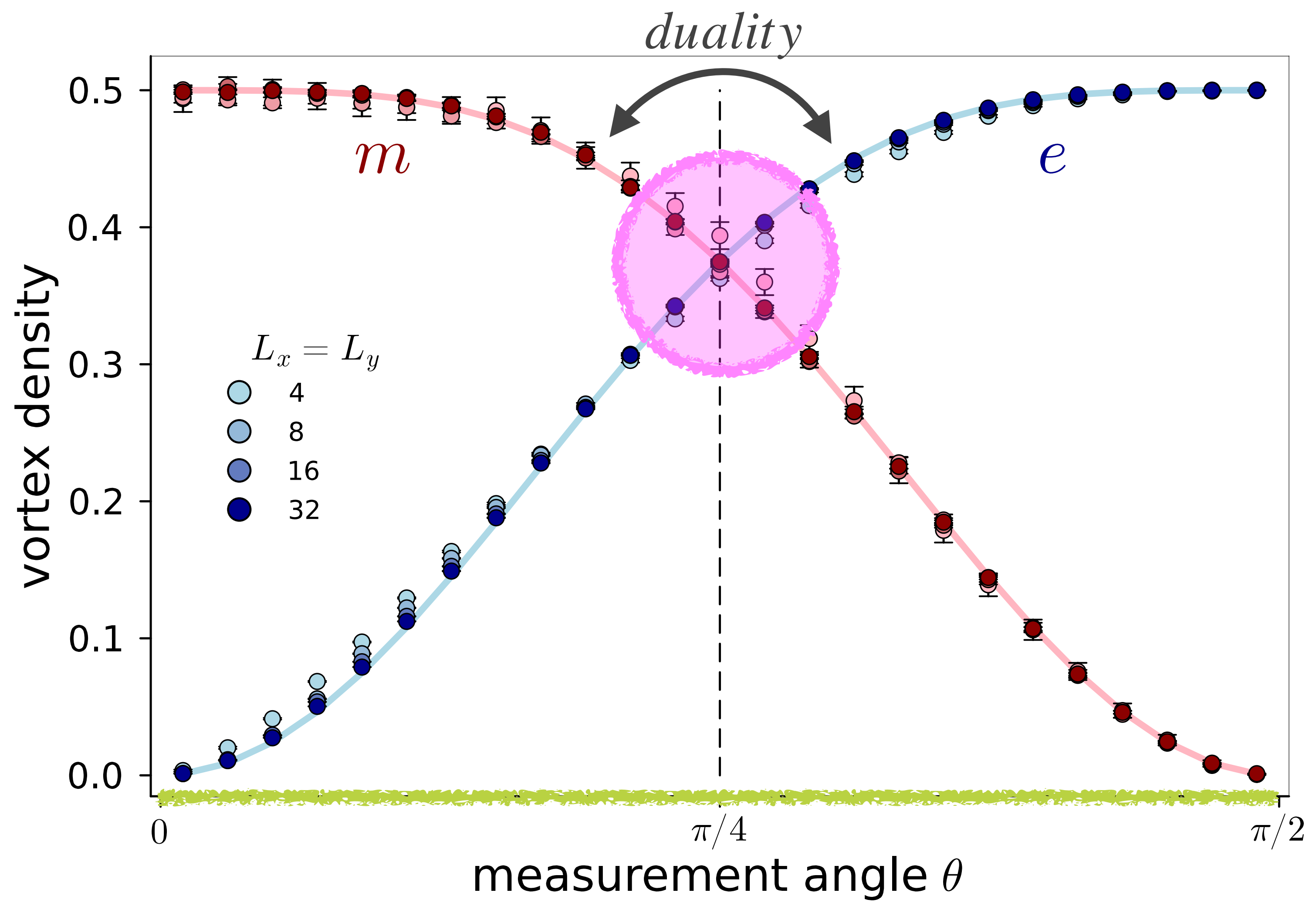


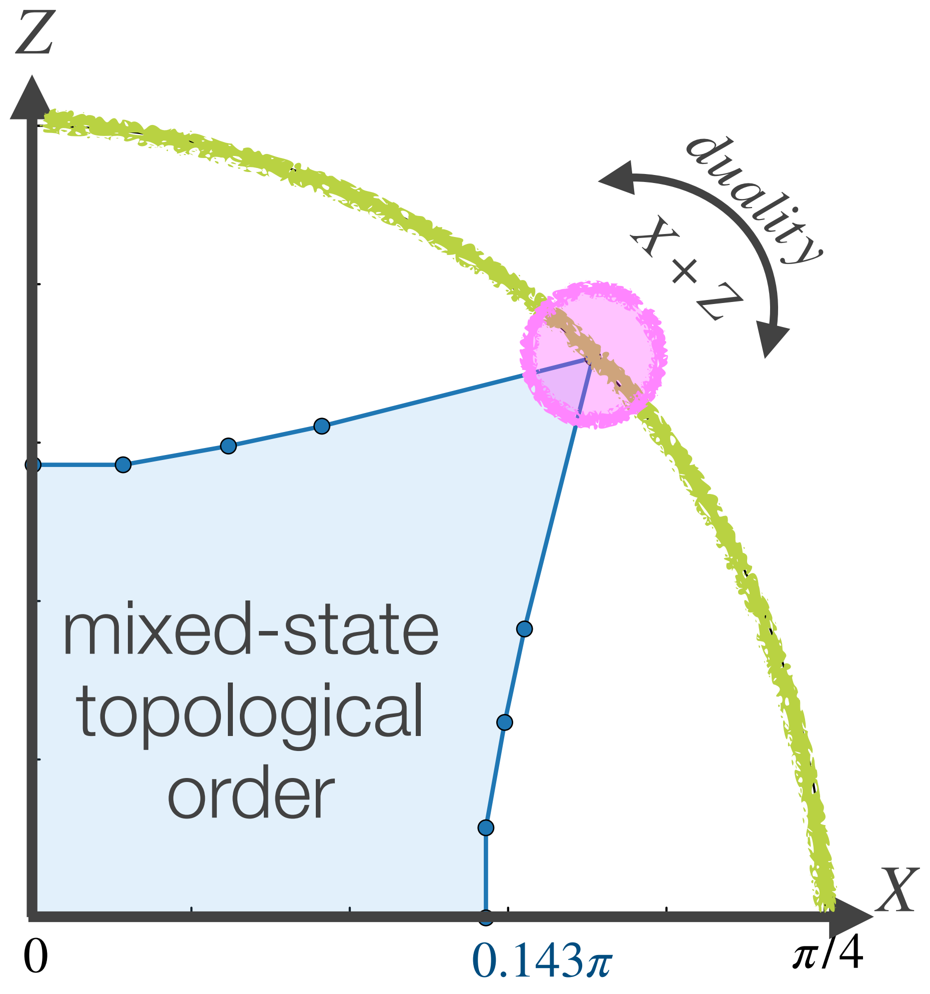
tunes the **relative fluctuations of e & m** in toric code

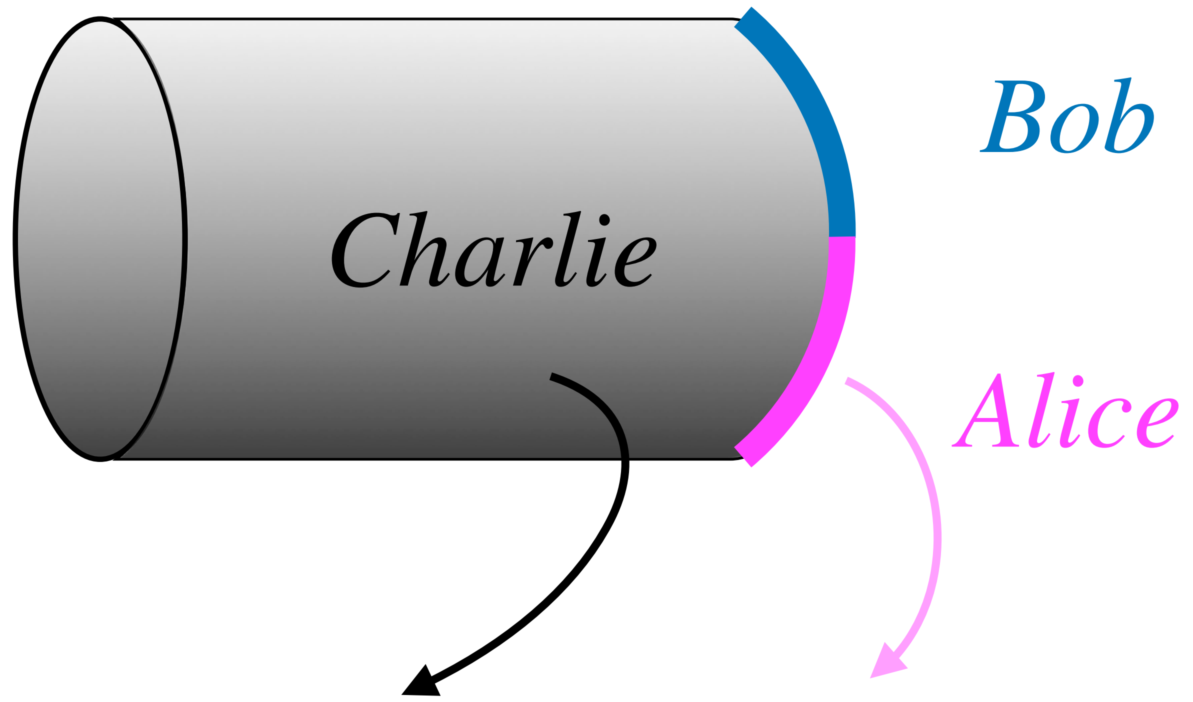
Coherent information evaluates the size of the **degenerate space** of the boundary state.



Mixed critical state: conformal scaling







$$S_{AC} = S_C + S_{A|C}$$

$$= -\frac{\pi c_{\text{Casimir}} L_y}{6L_x} + \frac{c_{\text{ent}}}{3} \ln\left(\frac{L_x}{\pi} \sin \frac{\pi l}{L_x}\right) + \ldots$$

bulk

mixed-state phase transition

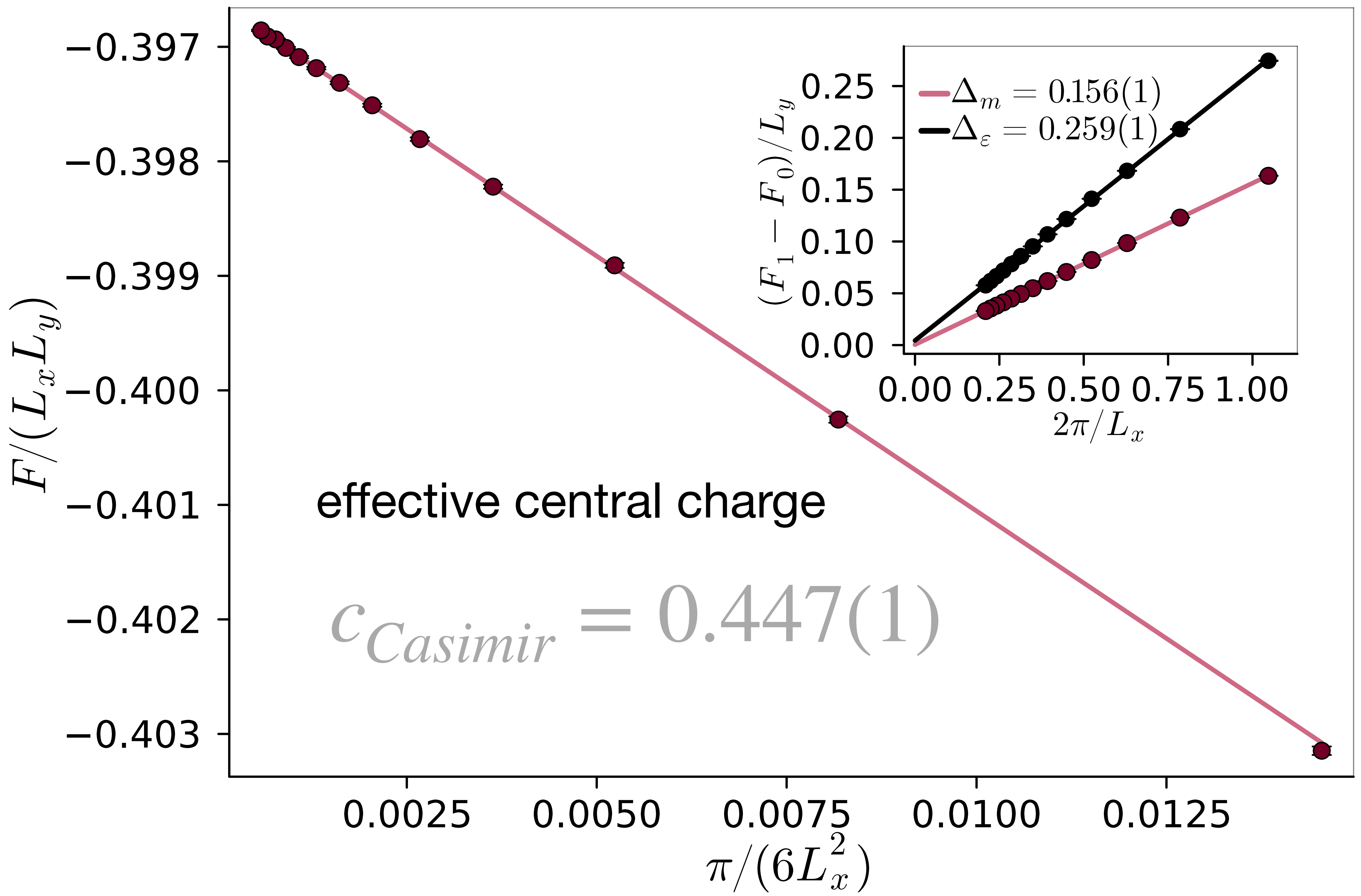
boundary

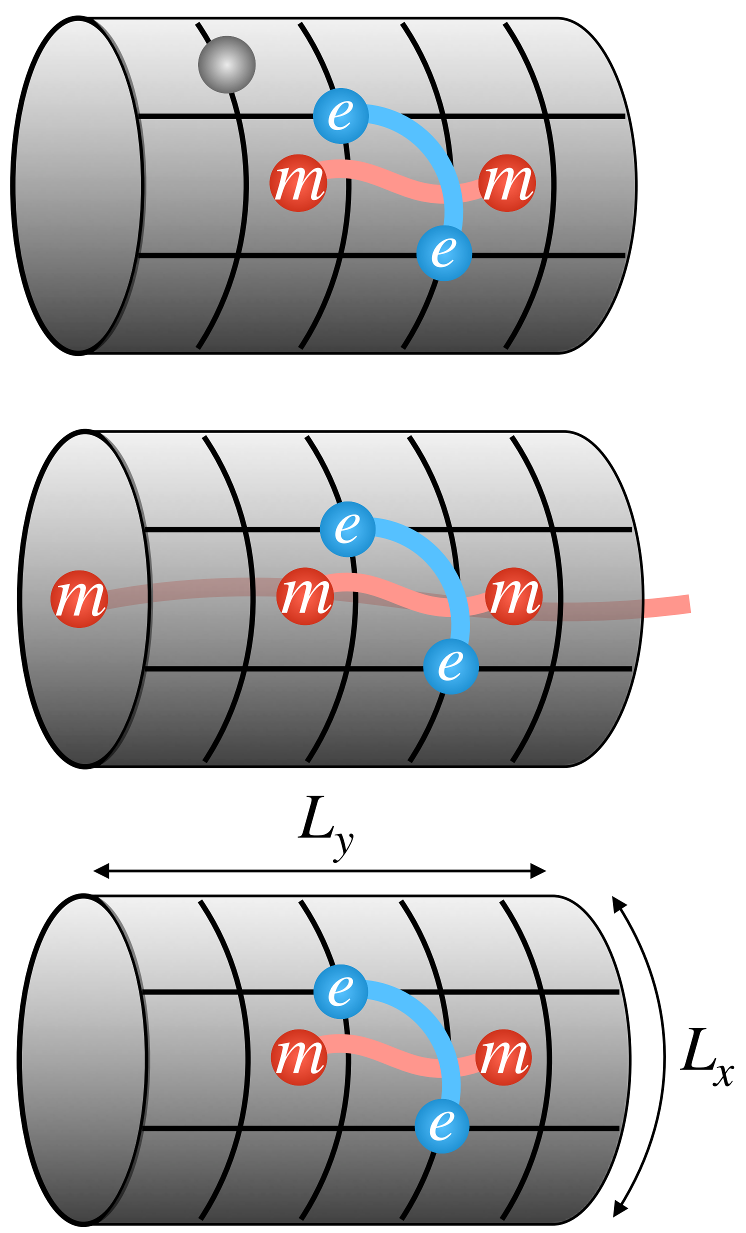
measurement-induced phase transition

bulk conformal data

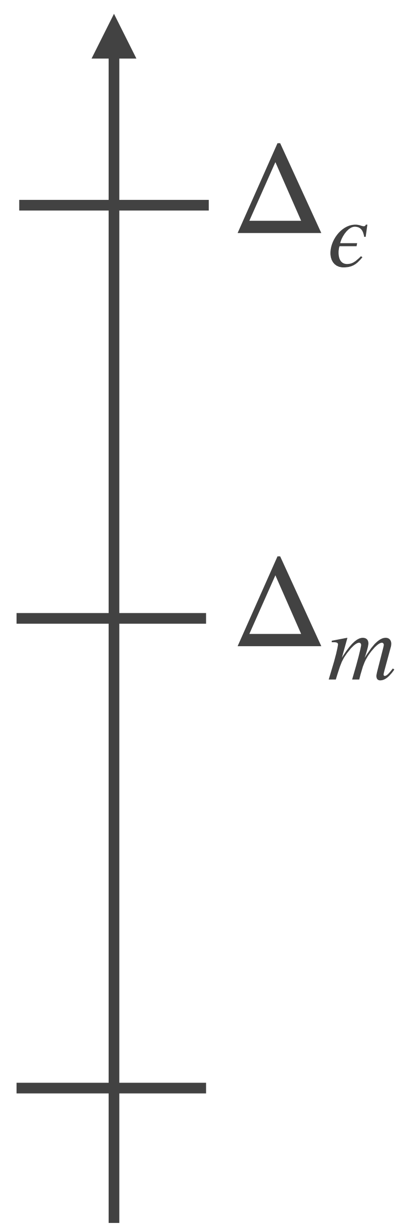
Shannon entropy of the mixed state = quenched average free energy of the random statistical model

$$S_C = F = -\sum_{em} P(em)\ln P(em) = \text{const}\cdot L_x L_y - c_{\text{Casimir}} \cdot \frac{\pi}{6}\frac{L_y}{L_x} + \ldots$$



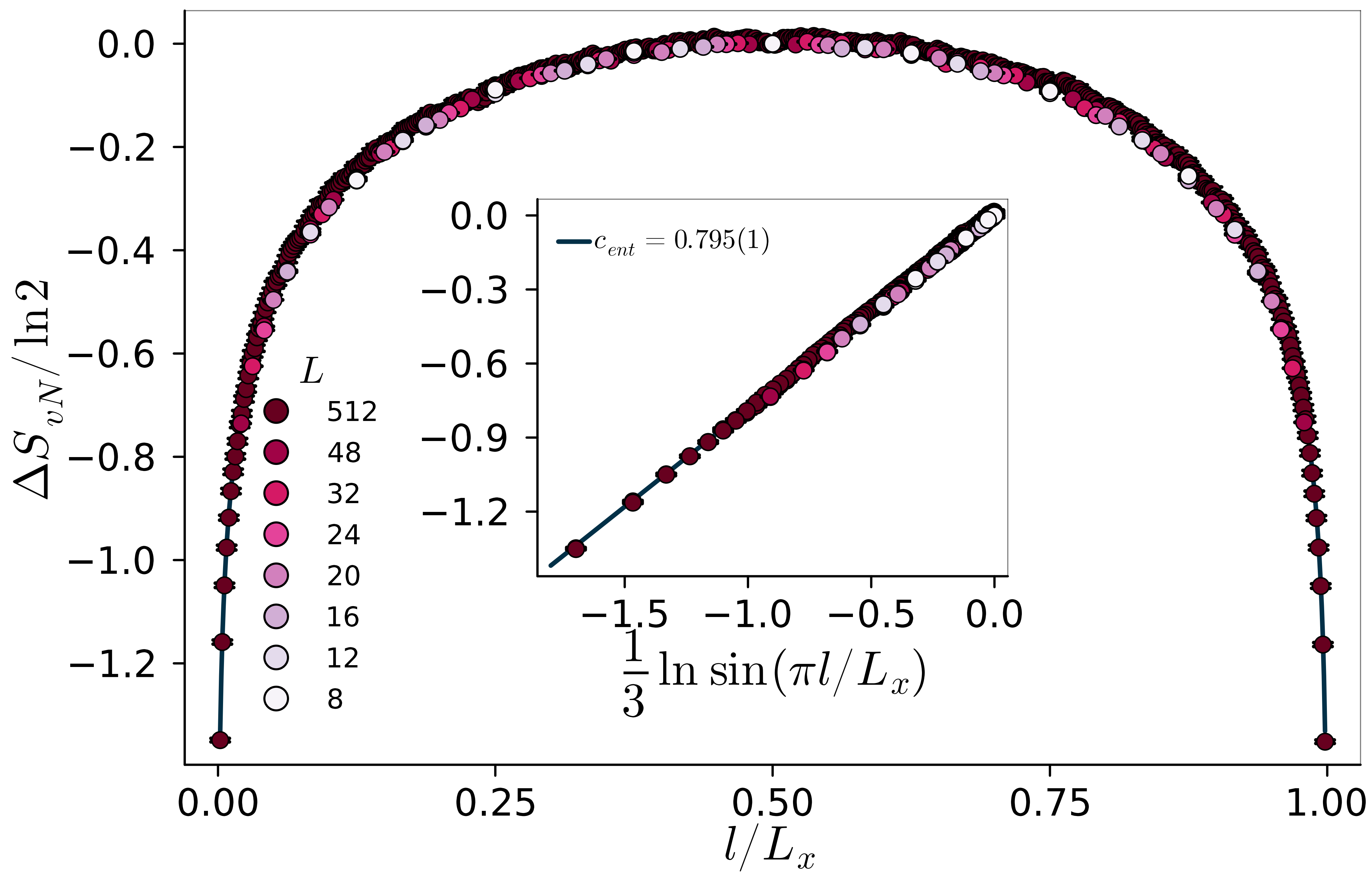


“energy” - scaling dims

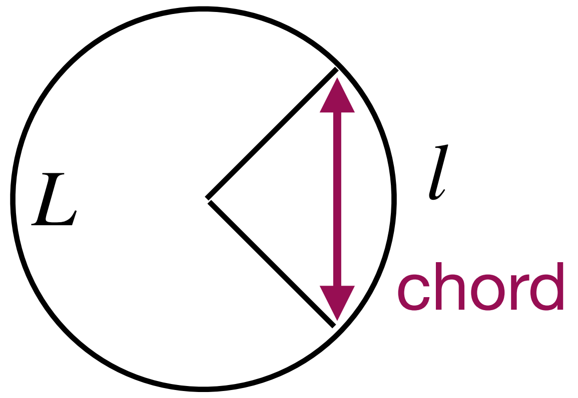


boundary conformal data

von Neumann **entanglement entropy**



c_{ent} - scaling dimension of twist operator for "boundary condition changing"



$$S_{A|C} = \frac{c_{ent}^{vN}}{3} \ln\left(\frac{L}{\pi}\sin\frac{\pi l}{L}\right) + \cdots$$

$$c_{\text{ent}}^{\text{vN}} = 0.795(1) \neq c_{\text{Casimir}} = 0.447(1)$$

for **non-unitary** conformal field theories

c_{ent} and c_{Casimir} generally differ

Universality classes

Ising	Nishimori	weak self-dual
no disorder	Born rule disorder (1-replica)	Born rule disorder (1-replica)
Z_2 **SSB** spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking
Kramers-Wannier duality	**broken** duality	**weak** duality
$\mathrm{KW}\rho \propto \rho$		$\mathrm{KW}\,\rho\,\mathrm{KW} = \rho$

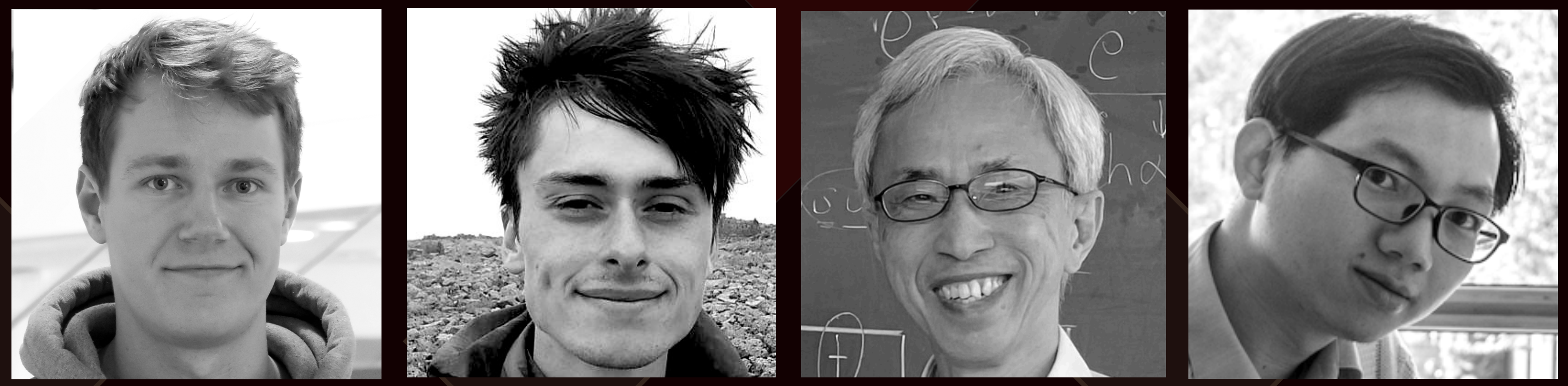
Universality classes

Ising	Nishimori	weak self-dual	tricritical
no disorder	Born rule disorder (1-replica)	Born rule disorder (1-replica)	Born rule disorder (1-replica)
Z_2 **SSB** spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	**strong/weak/broken** Z_2 symmetry phases meet
Kramers-Wannier duality	**broken** duality	**weak** duality	learning transitions
$\mathrm{KW}\rho \propto \rho$		$\mathrm{KW}\,\rho\,\mathrm{KW} = \rho$	

learning transitions

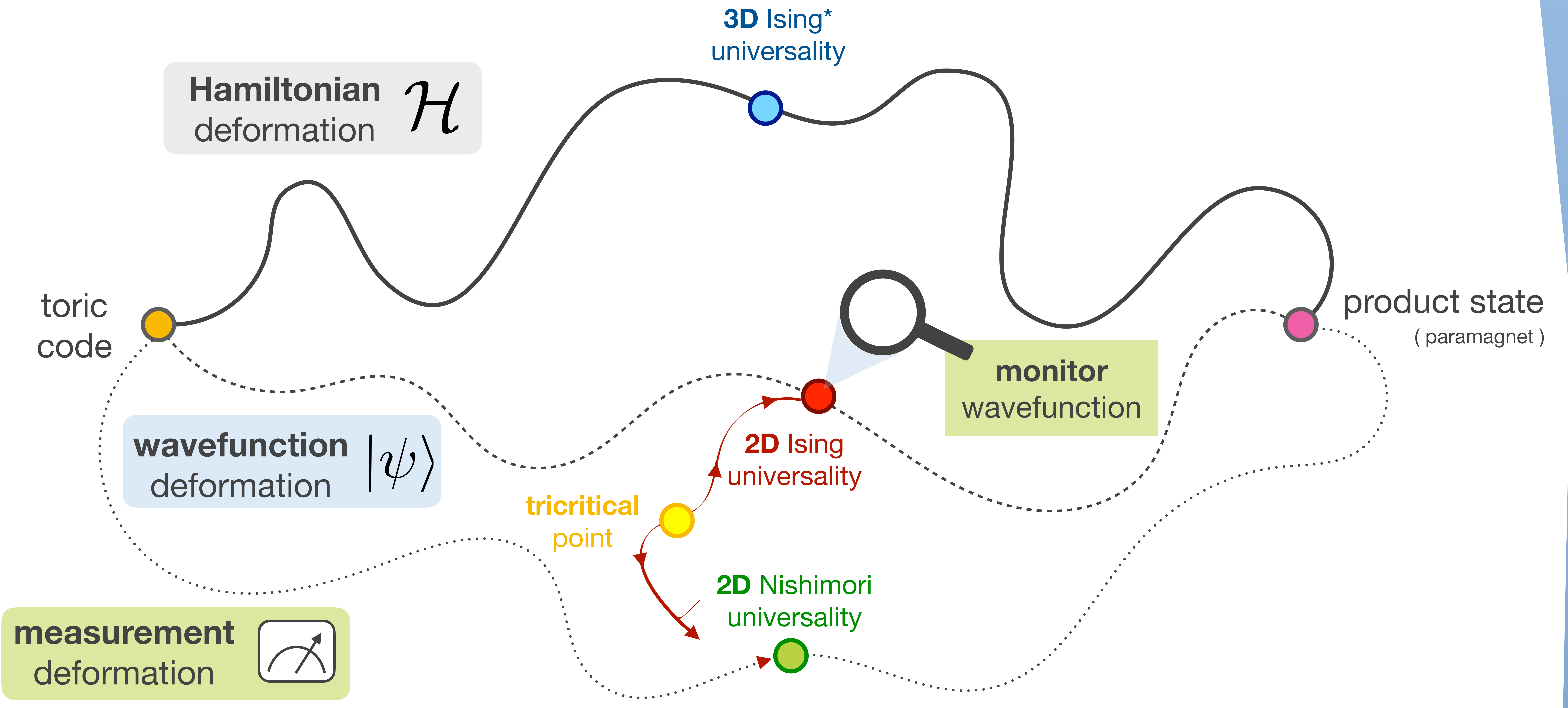
arXiv:2504.12385

M. Pütz, S. Garratt, H. Nishimori, G-Y. Zhu



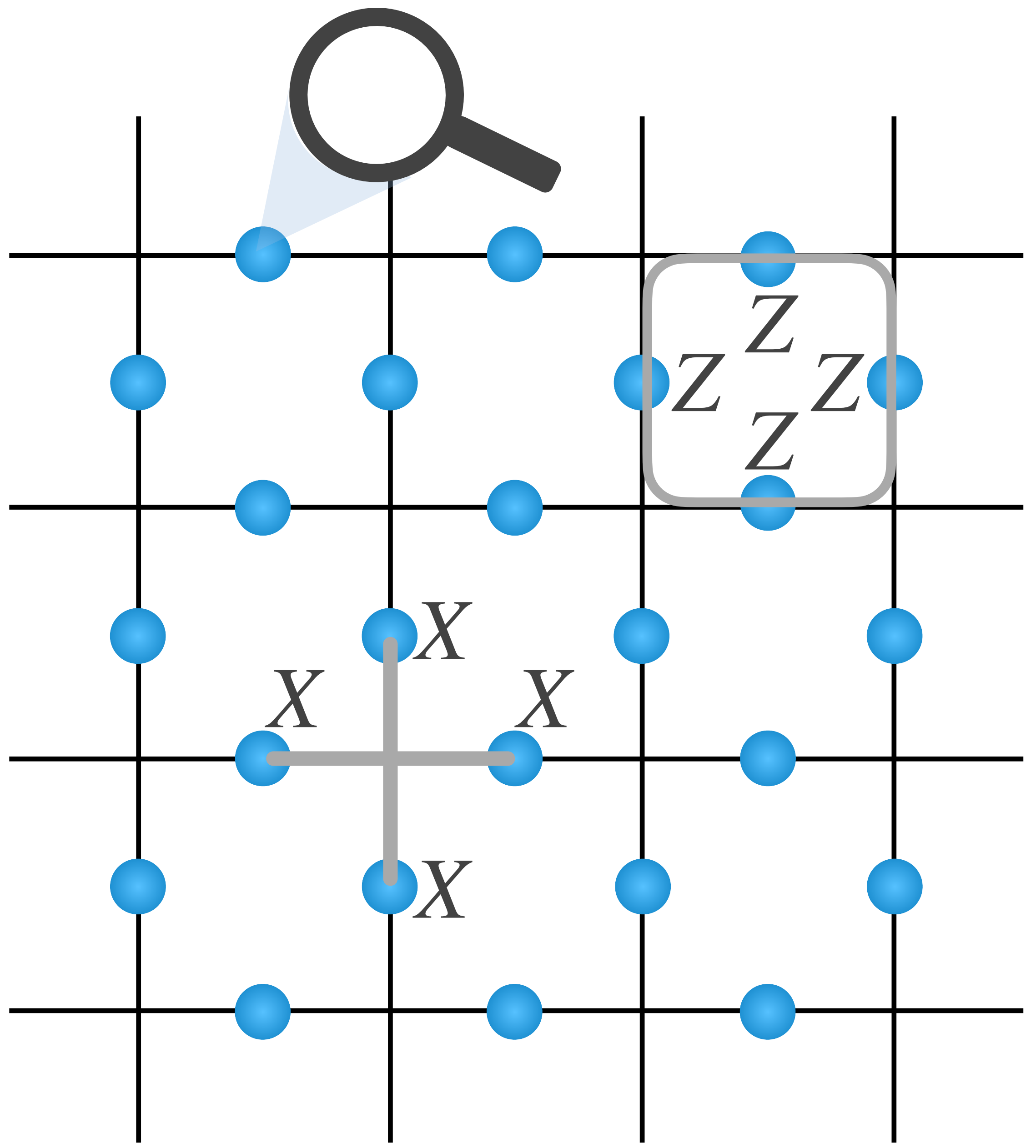
See also
Nahum & Jacobsen, arXiv:2504.01264
Kim, von Keyserlink & Lamacraft, arXiv:2504.08888

Monitored quantum criticality

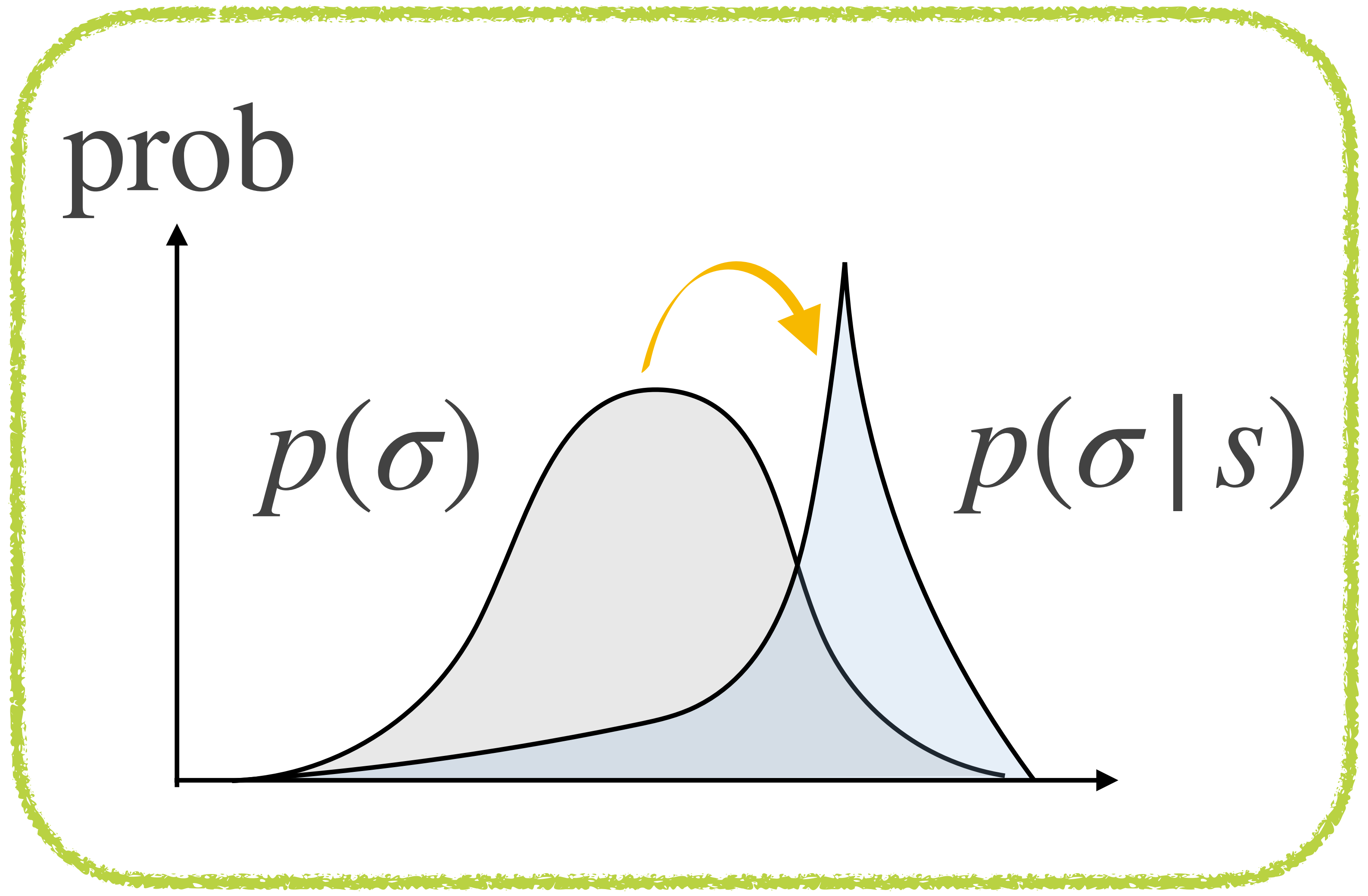


quantum monitoring & classical inference

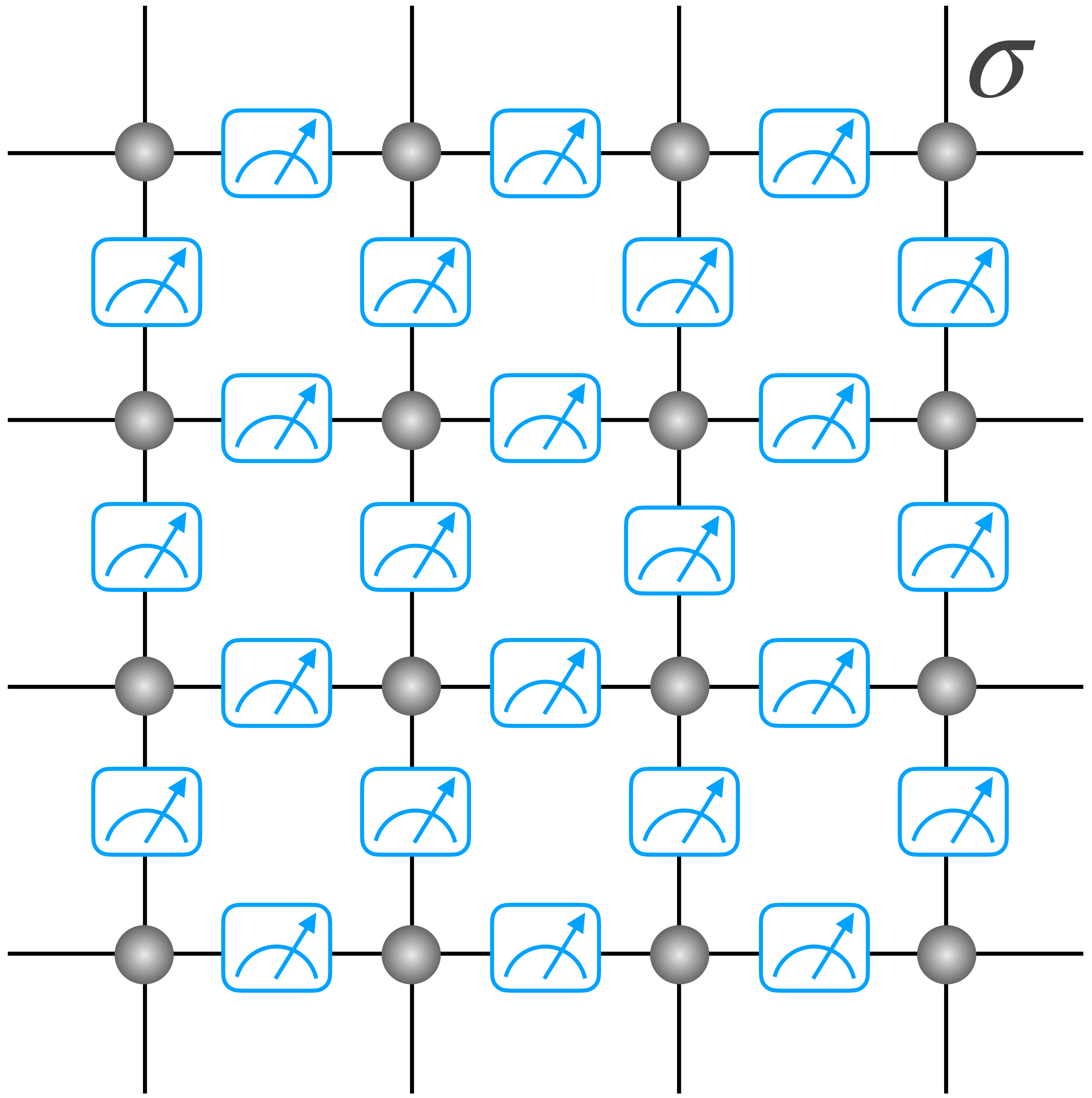
deformed **toric code**



$$H_{\mathrm{RK}} = H_{\mathrm{TC}} + \sum_{+} e^{-\beta \sum_{j \in \partial +} Z_j}$$



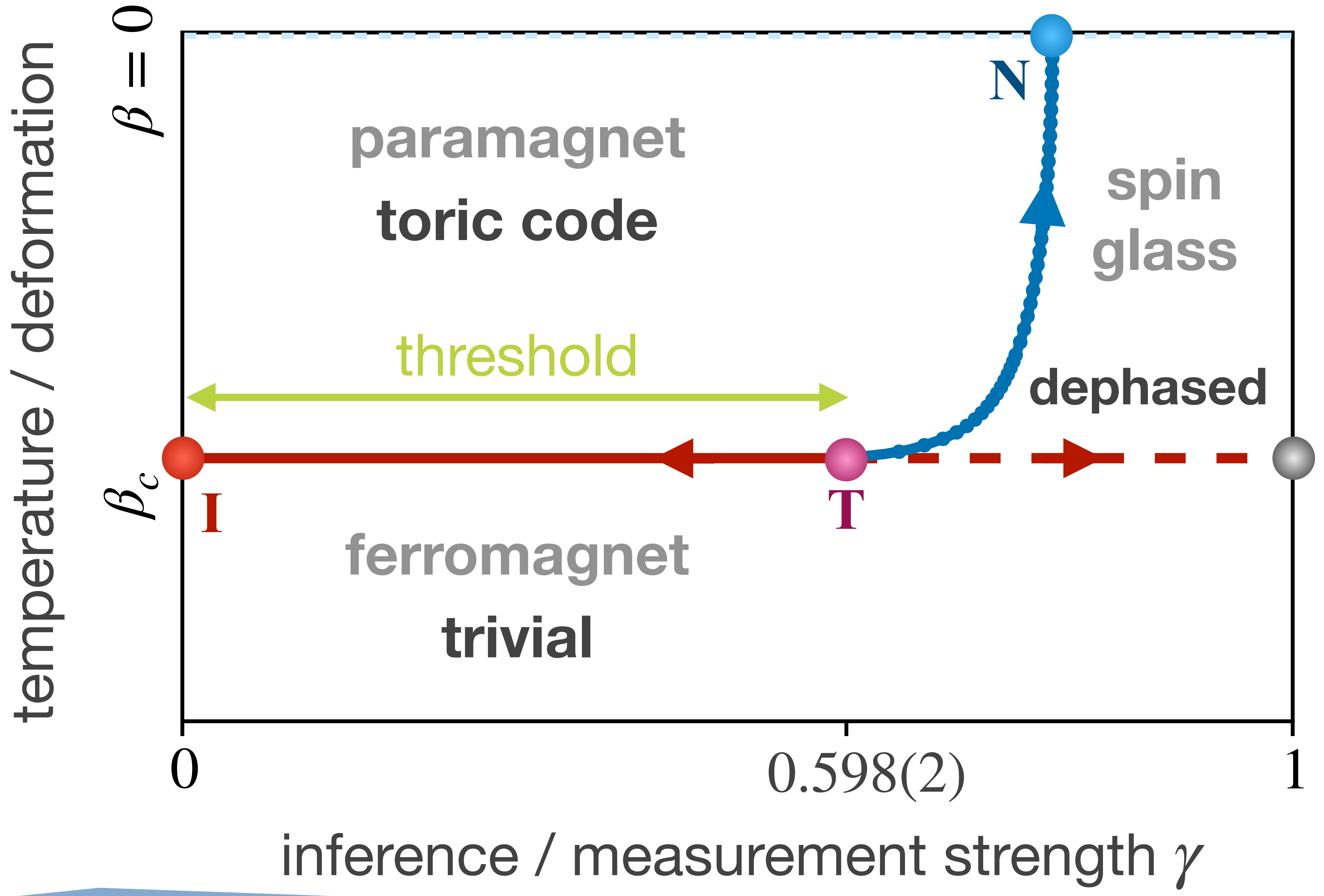
critical **Ising model**



Born's rule → **conditional distribution** ← **Bayesian inference**

learning transitions

Bayesian / Born phase diagram

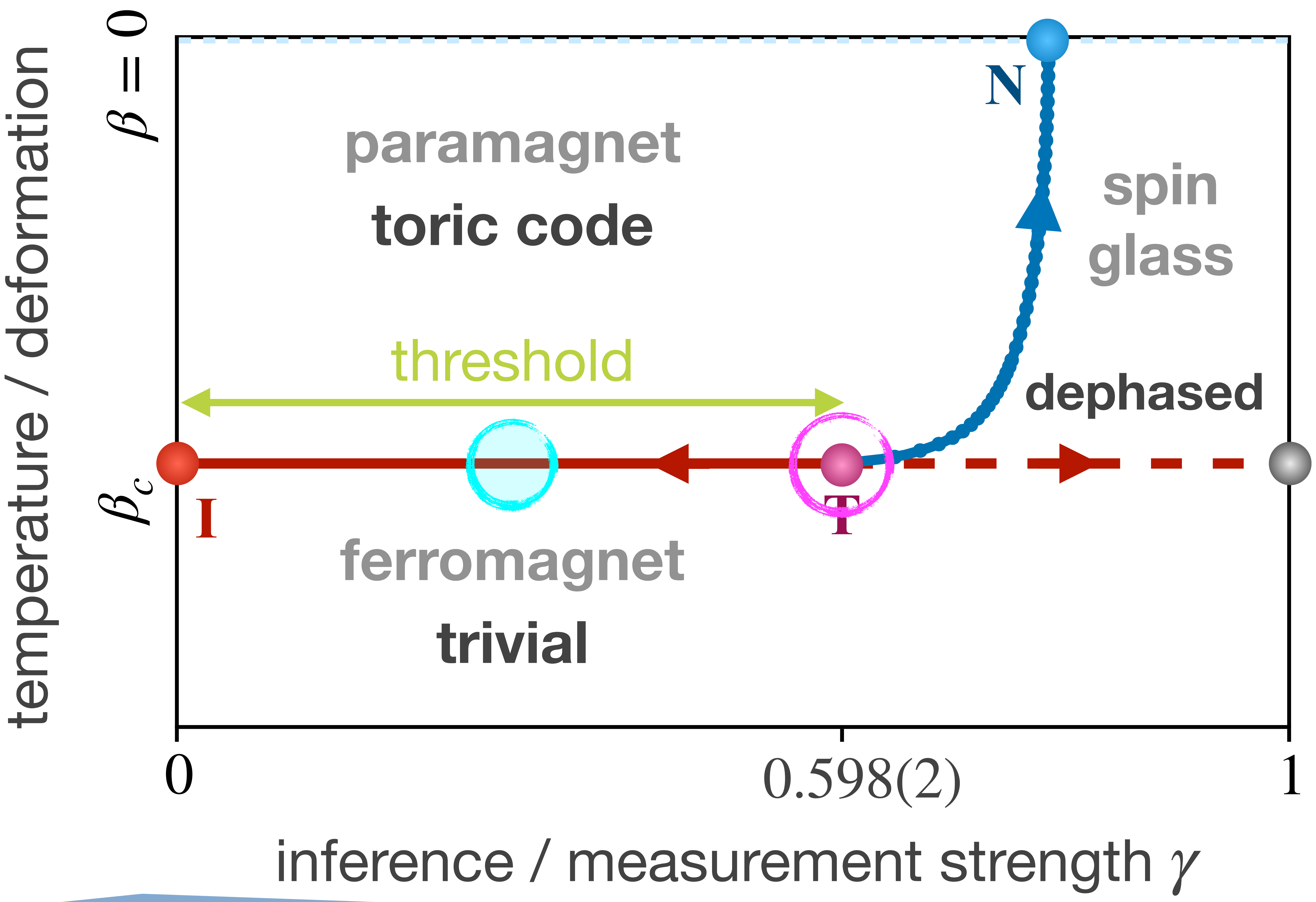


dictionary of phases

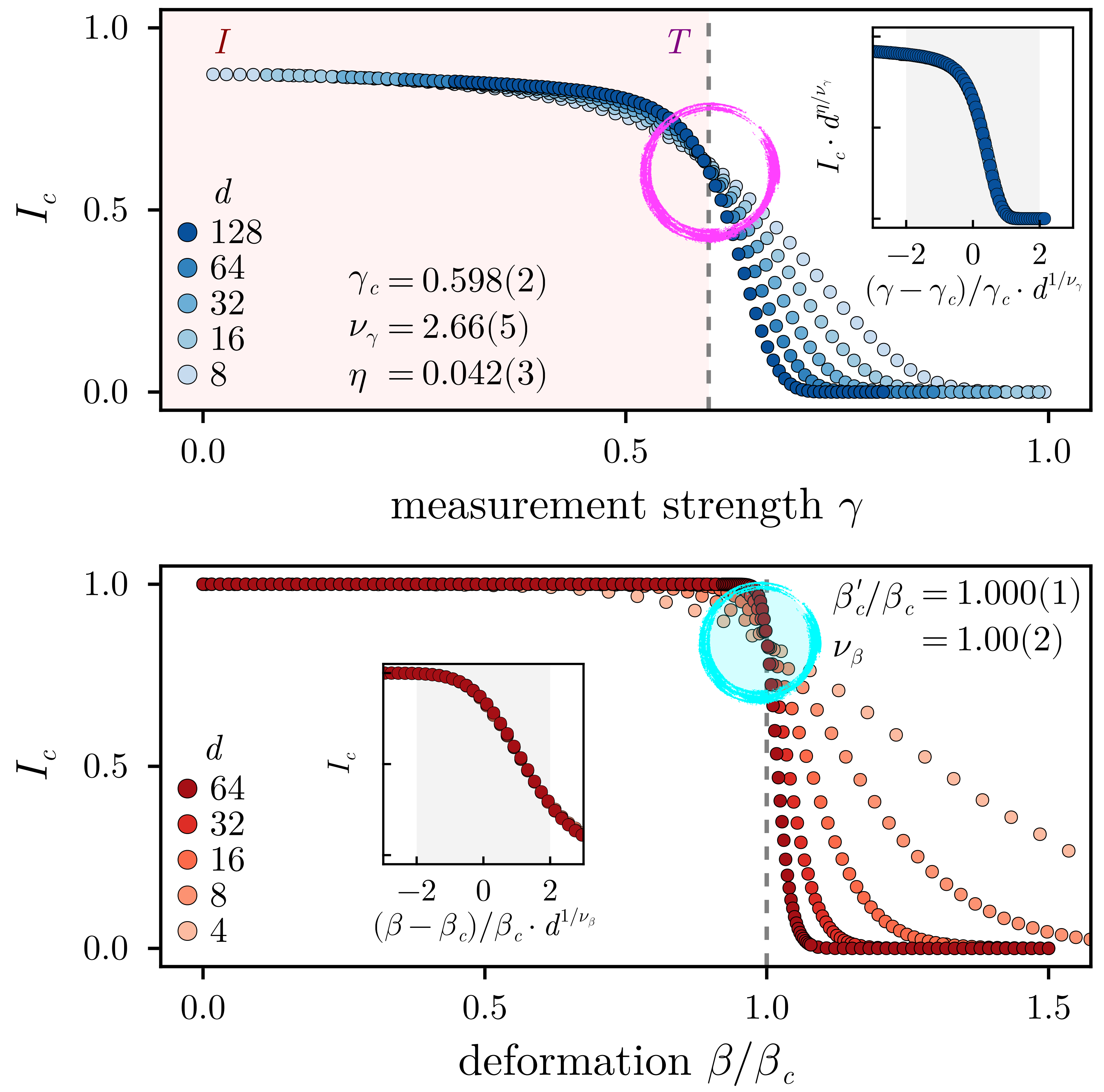
classical Ising	measured toric code	$\mathbb{Z}_2$ symmetry
paramagnet	quantum memory	strong
spin glass	classical memory	weak
ferromagnet	lost memory	broken

learning transitions

Bayesian / Born phase diagram



coherent information **cuts**



learning transitions

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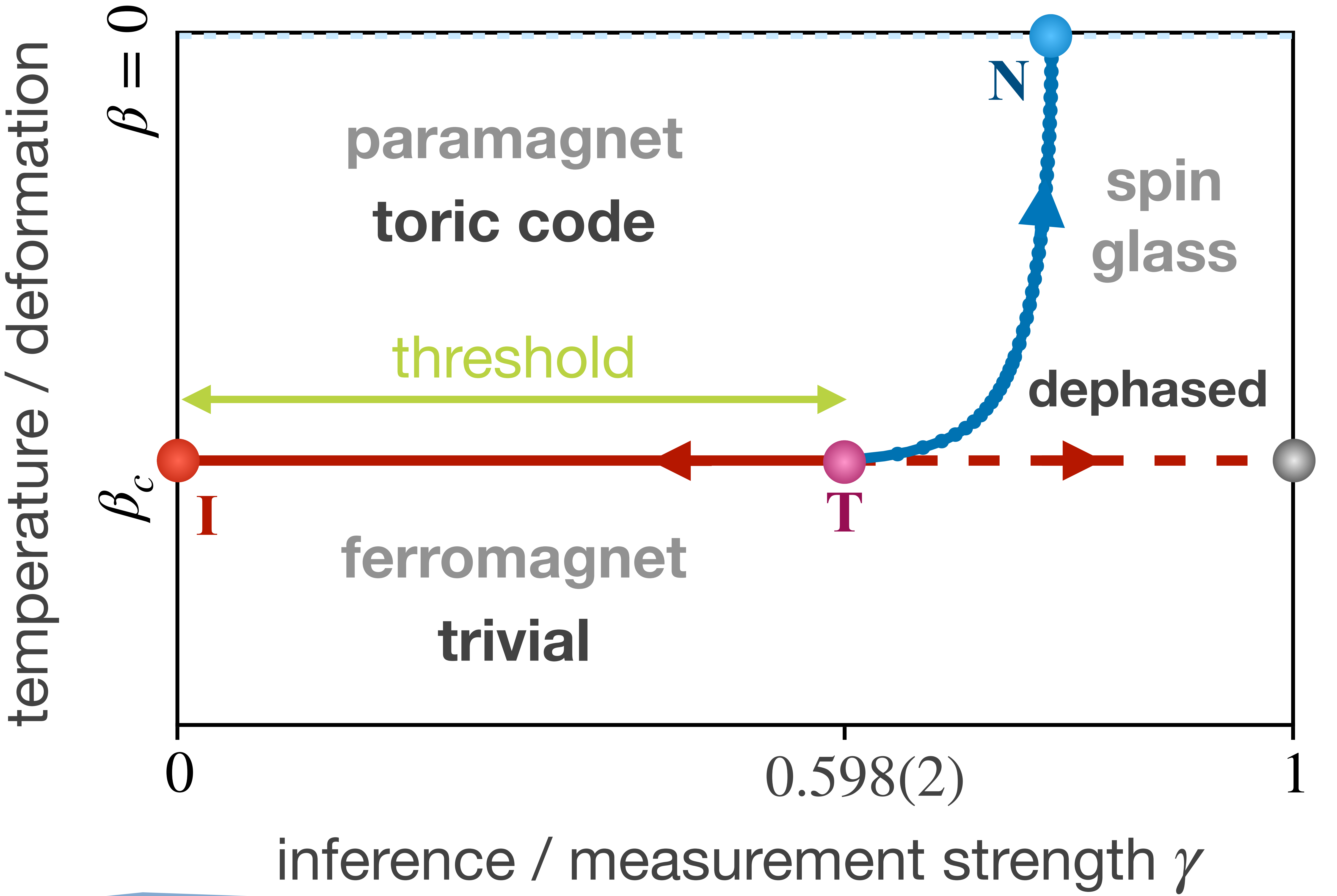
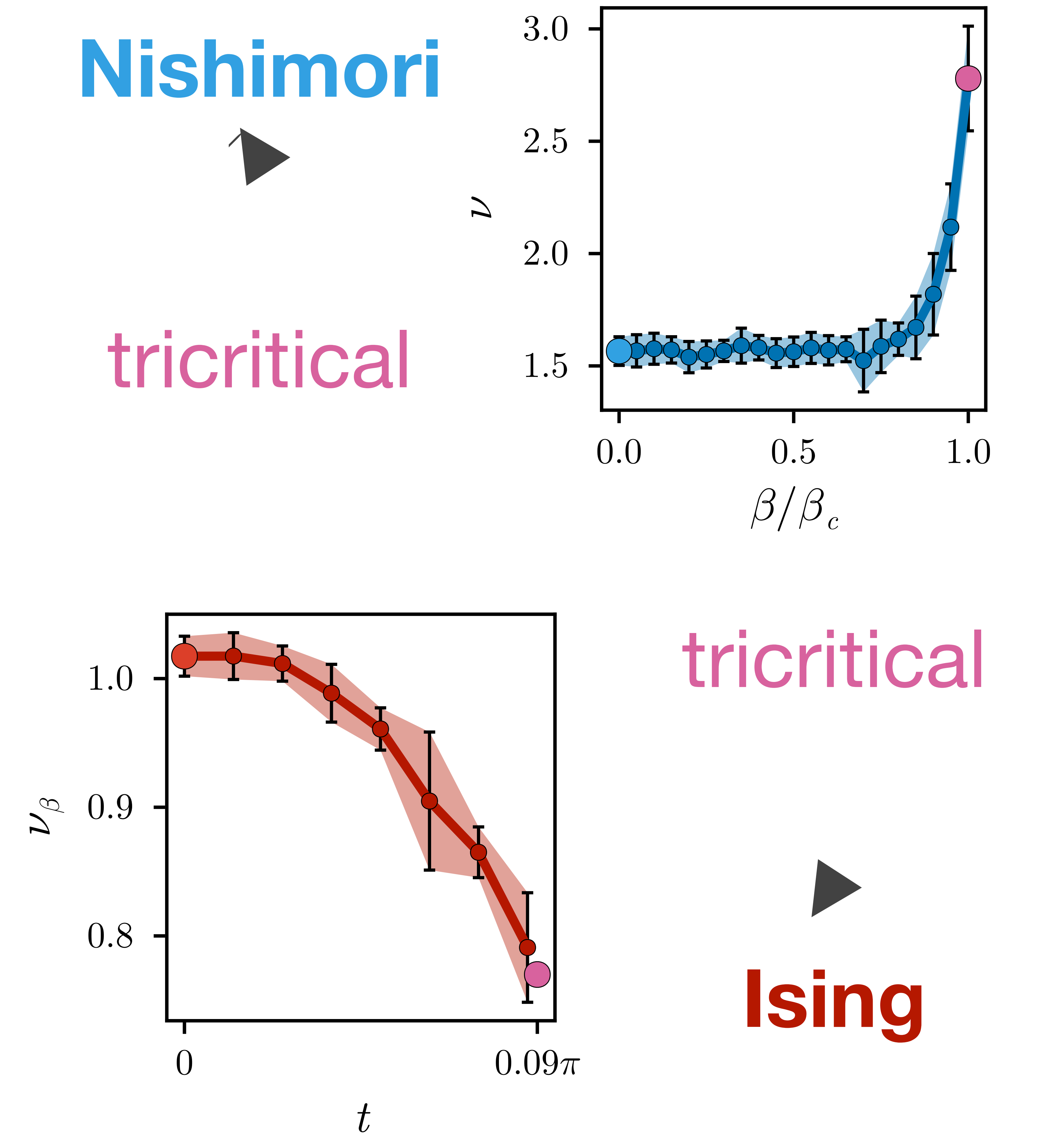


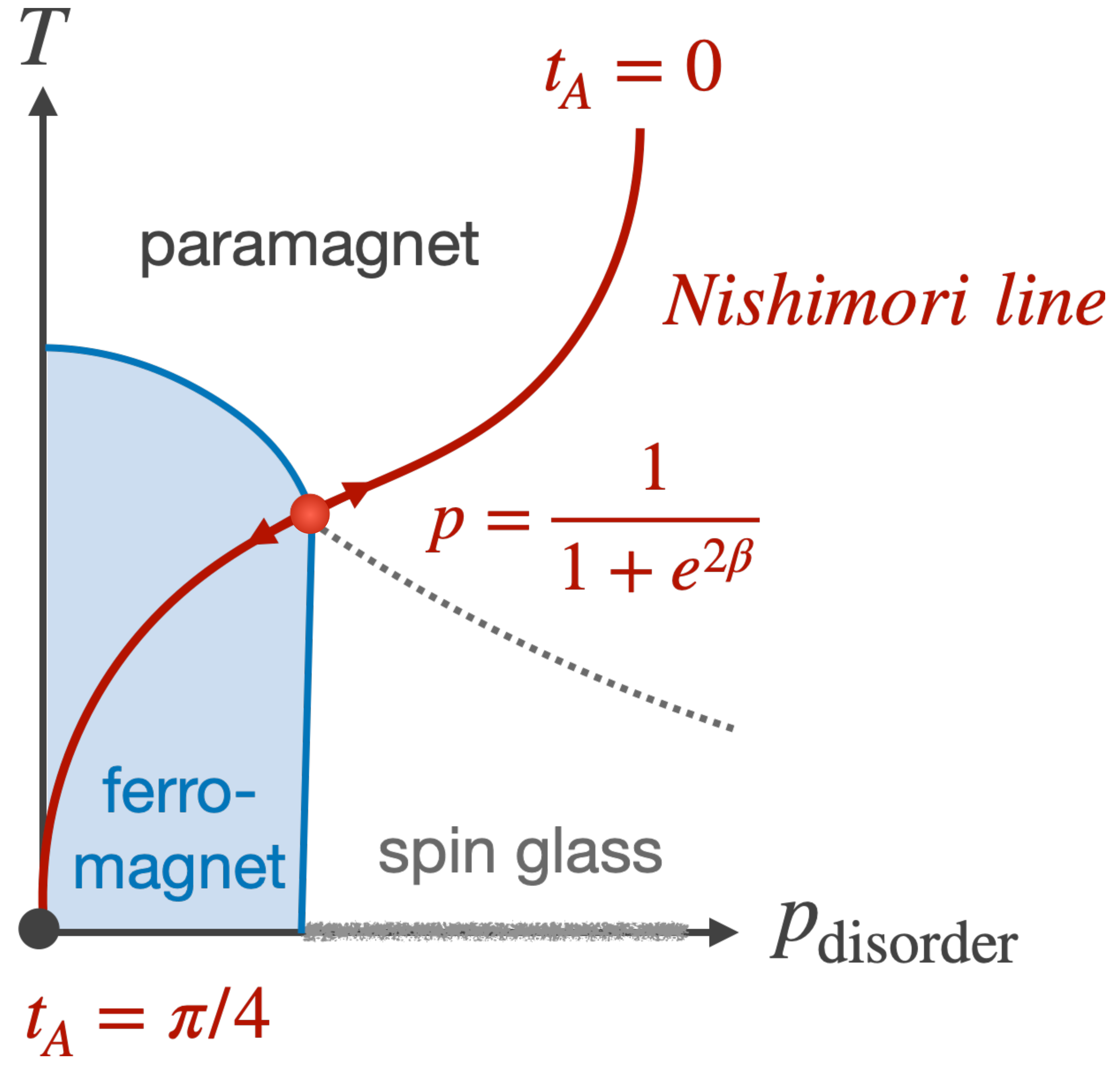
RG flows



perspectives

Nishimori physics

- a staple of classical **statistical physics**
- but **ubiquitous in quantum** physics
  - enforced by **Born's rule**
  - induced by **coherent** and **incoherent** errors
  - **RG flow** form percolation, self-dual, tricriticality
  - an emerging **fixed point universality class** for **non-unitary** conformal QCPs?



theory – Phys. Rev. Lett. **131**, 200201 (2023)
experiment (IBM) – Nature Physics **21**, 161 (2025)

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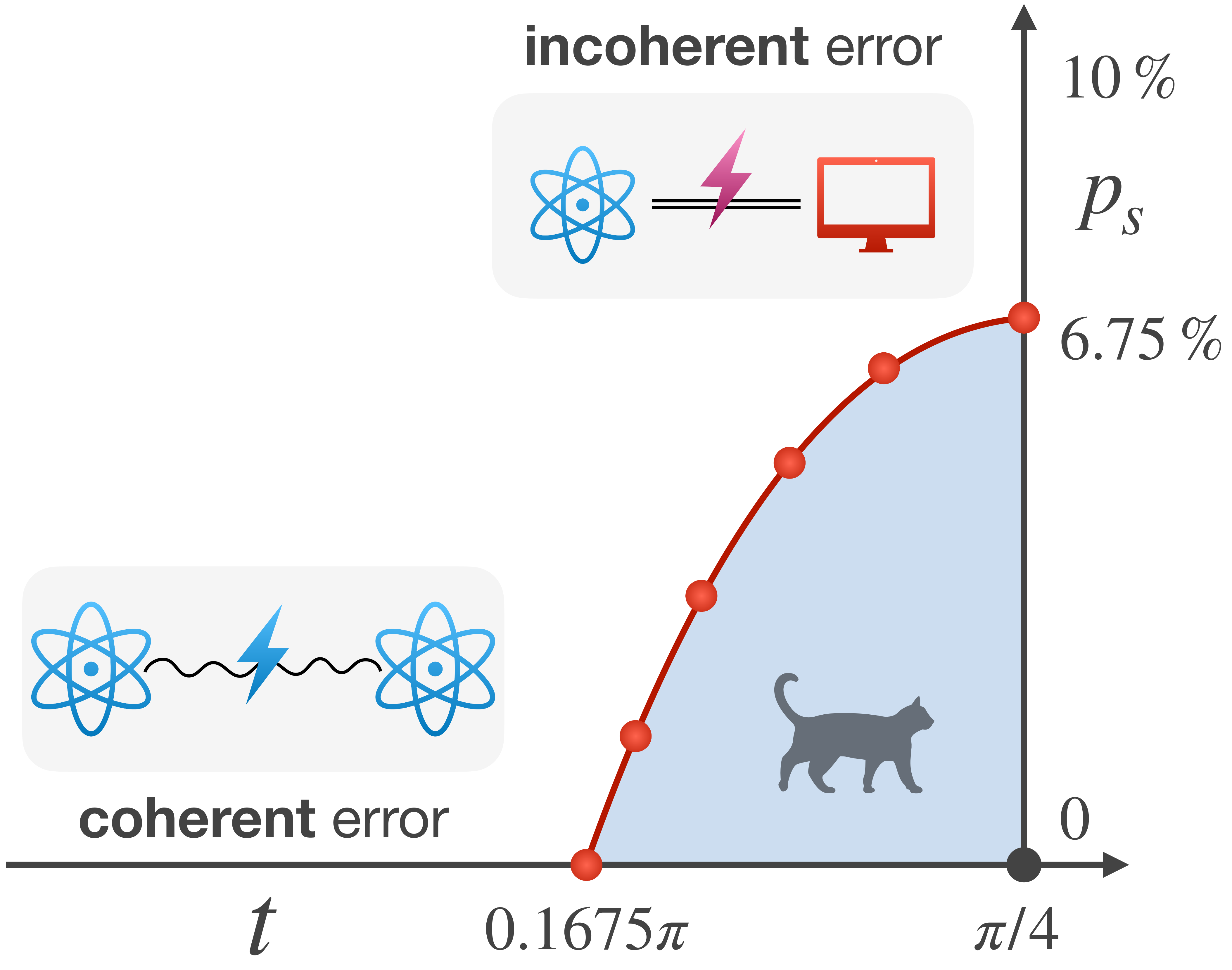

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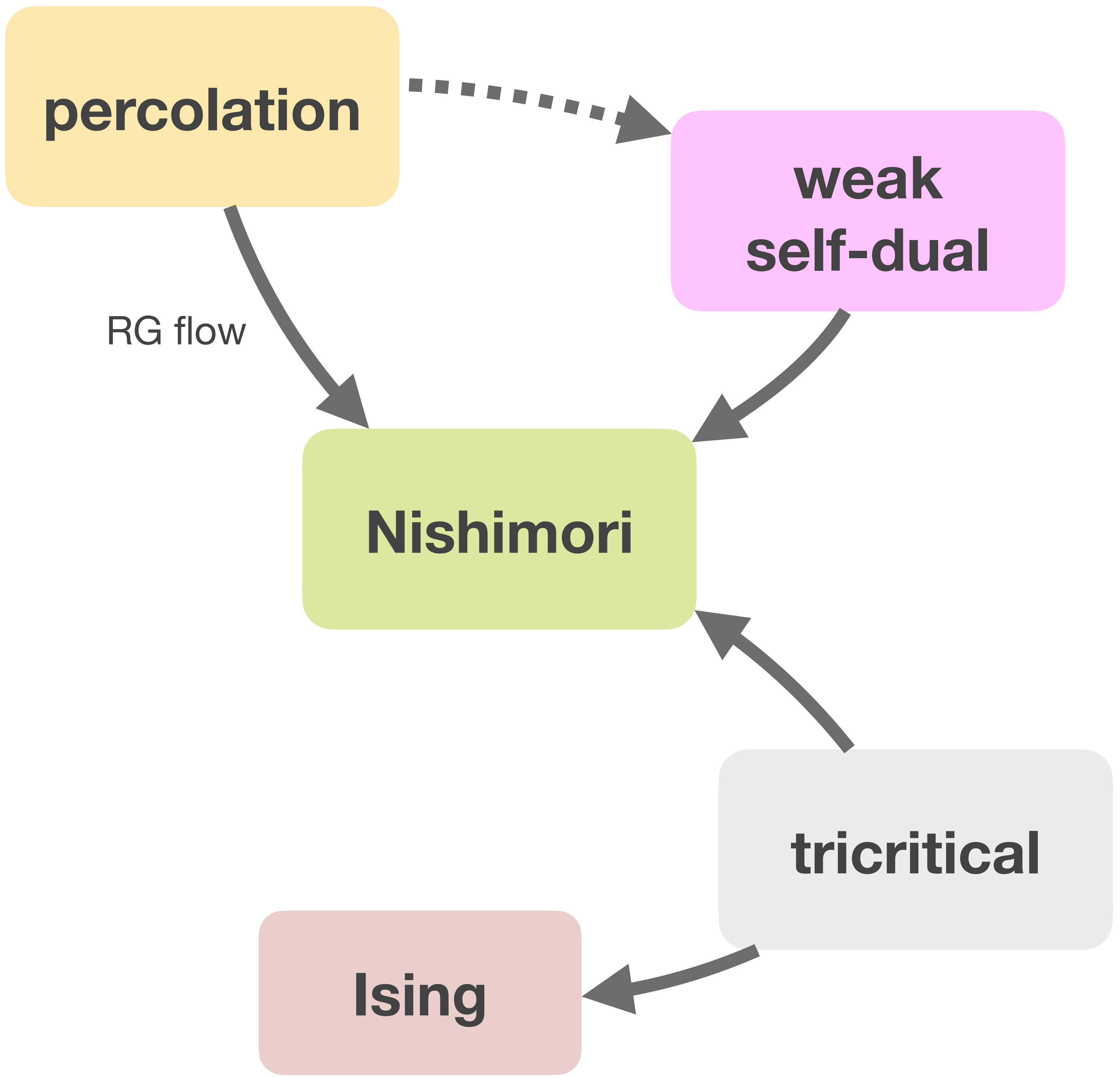

theory – Phys. Rev. Lett. **131**, 200201 (2023)
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percolation – arXiv:2505.22720
self-dual – arXiv:2502.14034, PRX Quantum 5, 040313 (2024)
tricritical – arXiv:2504.12385

Thanks!