Quantum simulations, adiabatic transformations,

and

resonating valence bond states

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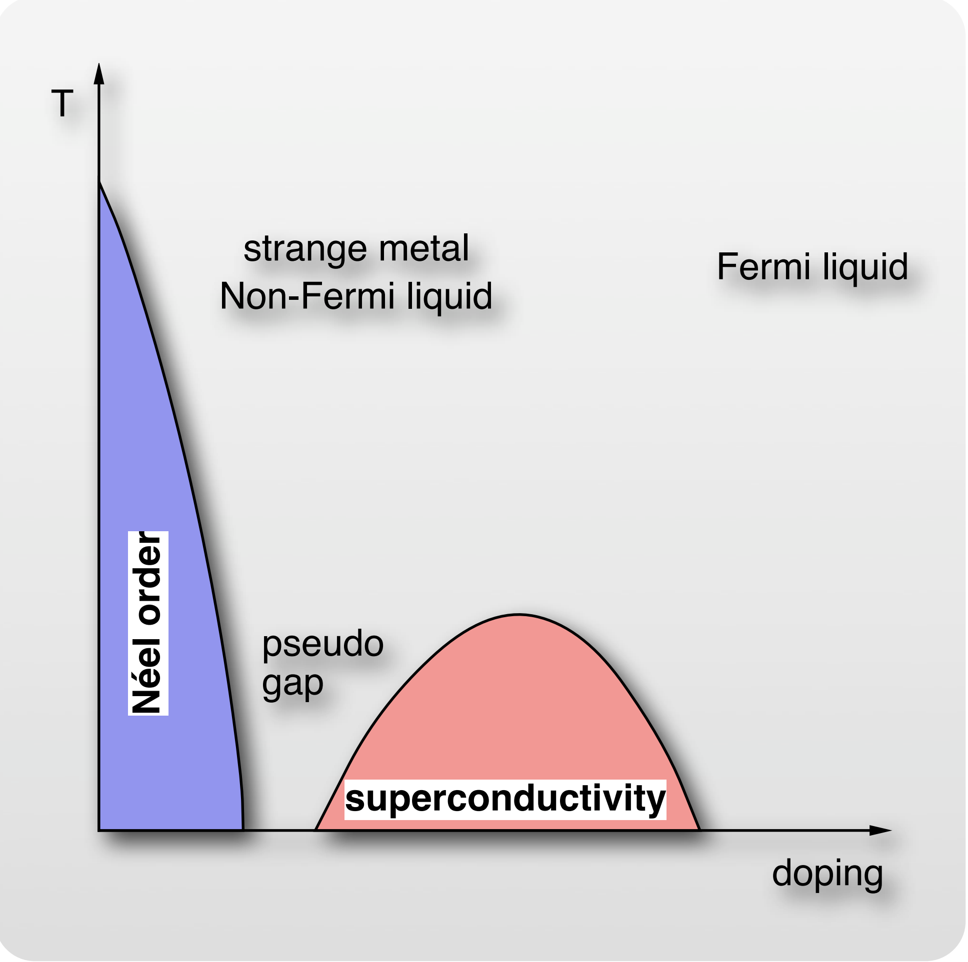
High temperature superconductivity

- 1986: Experimental discovery by Bednorz and Müller
  J. G. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986)

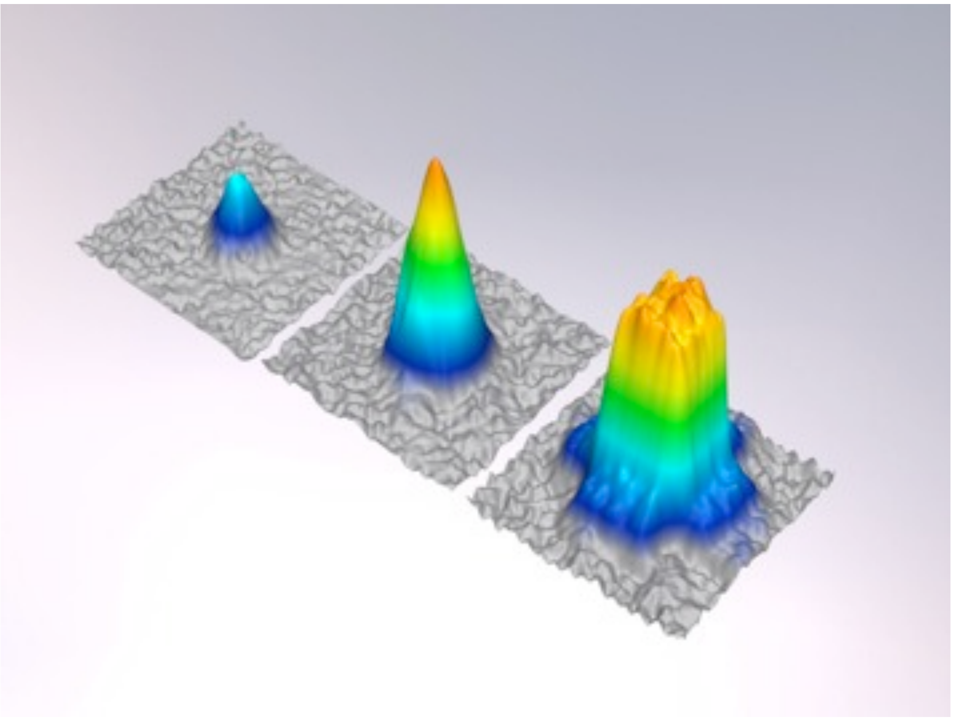
- 1987: RVB theory by Anderson
  P. W. Anderson, Science **235**, 1196 (1987)

- But it is not over yet
  - hard to get clean sample
  - hard to do good experiments
  - hard to solve the theoretical models
  - hard to simulate the theoretical models



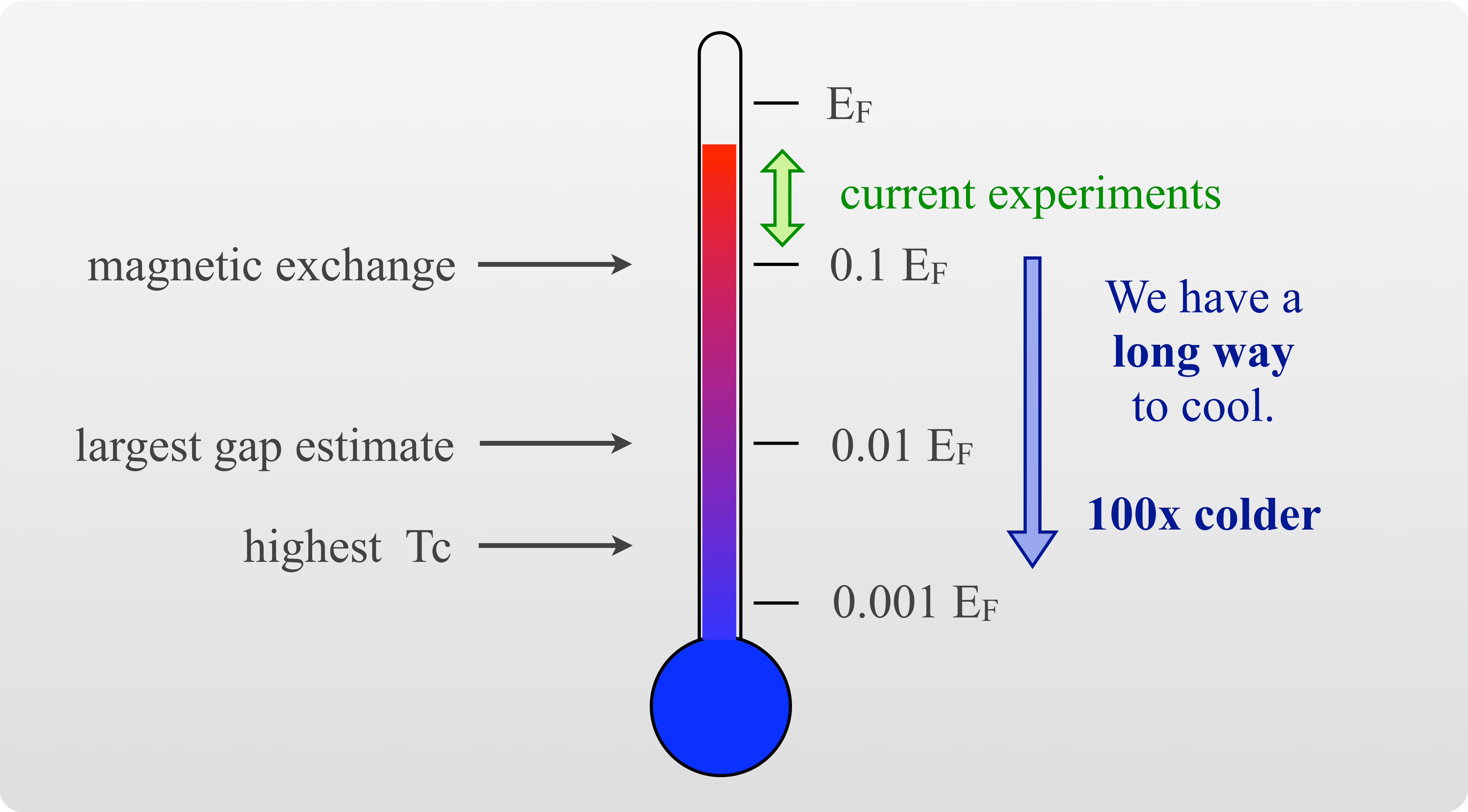
Quantum simulations with lattice fermions

- **Ultracold atomic gases** in optical lattices allow quantum simulation of the **fermionic Hubbard model**
  - **Attractive case:** high-temperature superfluidity
    W. Hofstetter *et al,* PRL **89**, 220407 (2002)
  - **Repulsive case:** hard to simulate on classical computers
    - is the ground state superconducting?
    - is the ground state an RVB state?
    - can it explain high-T_c superconductivity?
- Detection of Fermi surface in ^{40}K
  M. Köhl *et al*, PRL **94**, 080403 (2005)



What are the temperature scales?

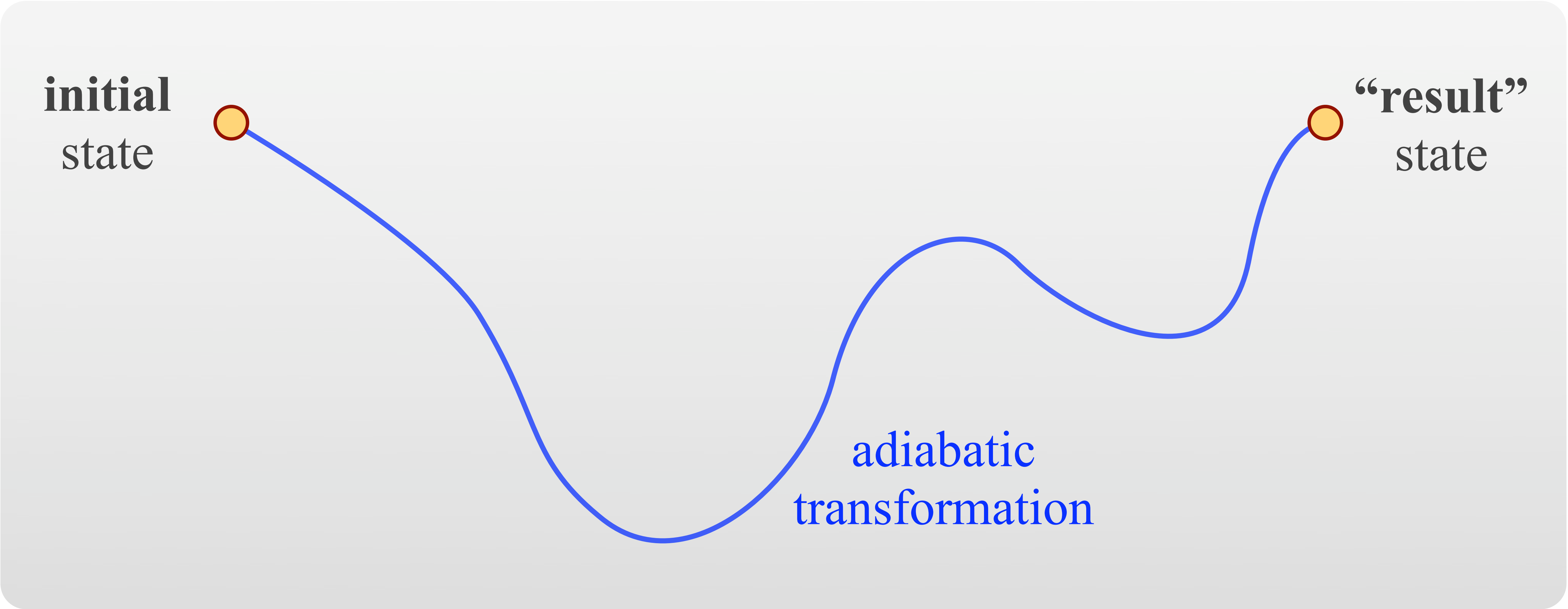
Guidance from experiments on high-T_c cuprates, simulations of 1d systems.



One way out:
Adiabatic quantum simulation

Adiabatic quantum simulation

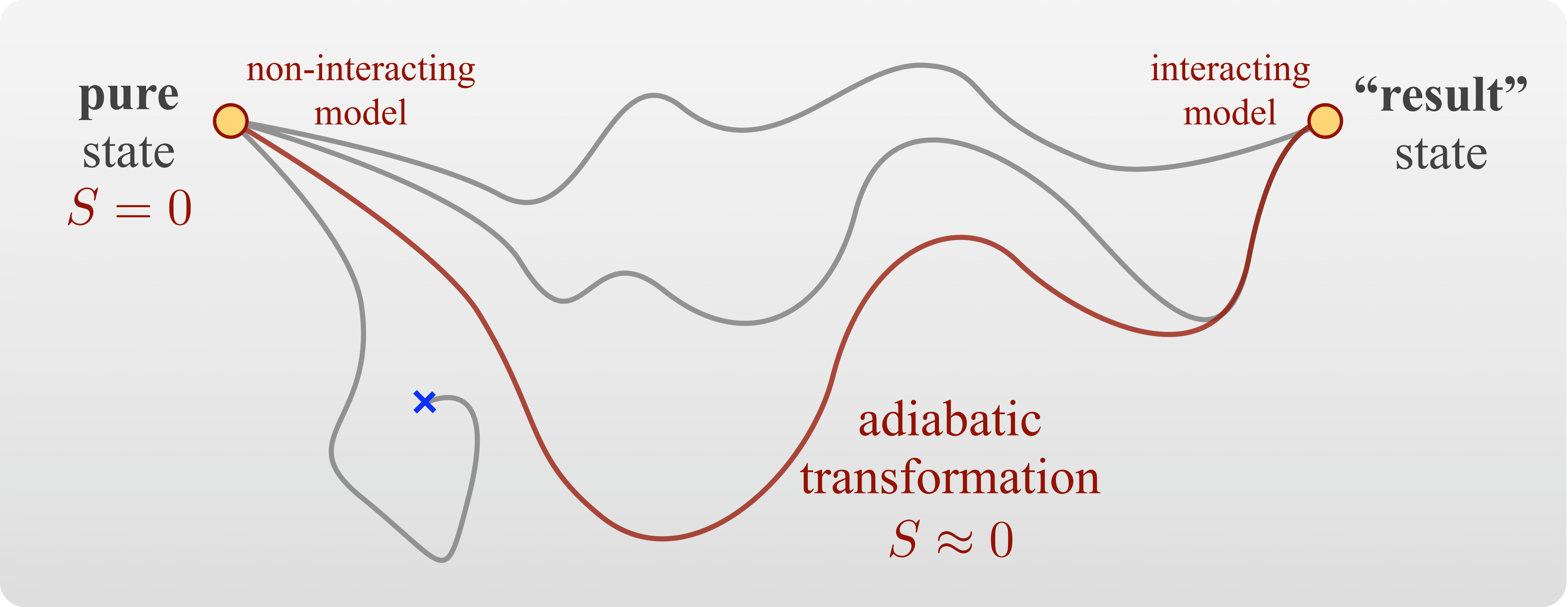
- **Adiabatic quantum computing**
  - Adiabatically transform initial state into the result of a hard computation
    E. Farhi *et al,* Science **292**, 472 (2001)



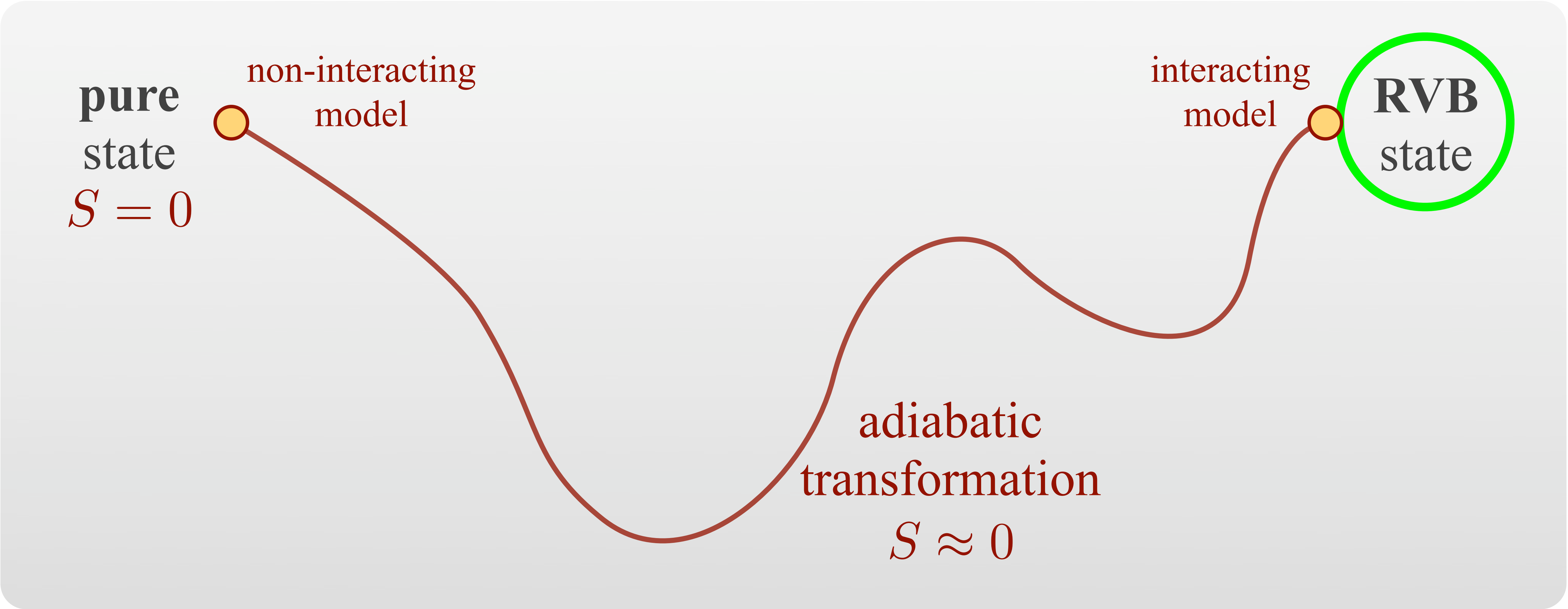
Adiabatic quantum simulation

- **Adiabatic quantum simulation**
  - Adiabatically transform **pure** initial state of non-interacting model into unknown ground state of interacting model

    ST, U. Schollwöck, M. Troyer, P. Zoller, PRL **96**, 250402 (2006)



Example
RVB states

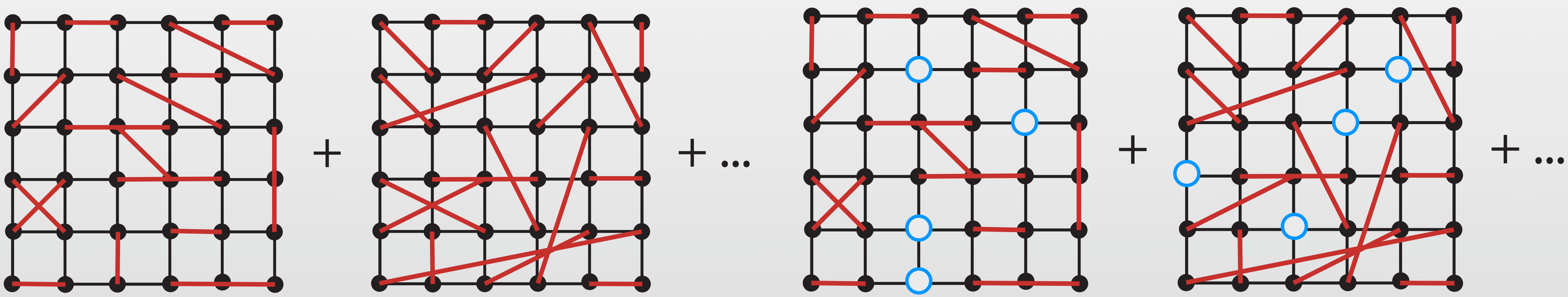


Resonating valence bond states

- Anderson conjectured that high temperature superconductors might be doped **resonating valence bond** (RVB) states

half-filling (parent compounds):
superposition of singlet coverings

hole-doping:
pairs are mobile and condense (BCS)



Gutzwiller-projected BCS wave function: eliminates double occupancies

$$|\Phi\rangle = P_G \prod_k \left(u_k + v_k c^\dagger_{k,\uparrow} c^\dagger_{-k,\downarrow} \right) |0\rangle \rightarrow |\Phi\rangle = P_G \left(\sum_{ij} a(i-j) c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow} \right)^{N/2} |0\rangle$$

Which pairing symmetry?

- Anderson originally conjectured ***s*-wave symmetry**

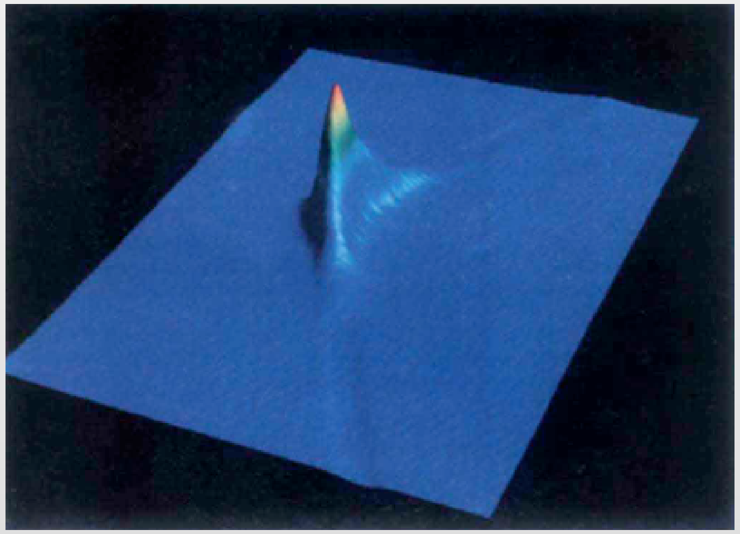
- **d-wave symmetry** proposed soon thereafter
  C. Gros, Phys. Rev. B **38**, 931 (1988)
  G. Kotliar and J. Liu, Phys. Rev. B **38**, 5142 (1988)
  ...

- **Experimental tests** for d-wave symmetry

  - Proposal:
    M. Sigrist and T.M. Rice, J. Phys. Soc. Jpn. **61**, 4283 (1992)

  - Experiments:
    C.C. Tsuei *et al*, Nature **373**, 225 (1995)
    C.C. Tsuei *et al*, Nature **387**, 481 (1997)
    ...

$$\Delta\left(k_x,k_y\right) \propto \left\langle c_{\vec{k}\uparrow} c_{-\vec{k}\downarrow} \right\rangle \propto \Delta\cdot(\cos k_x - \cos k_y)$$

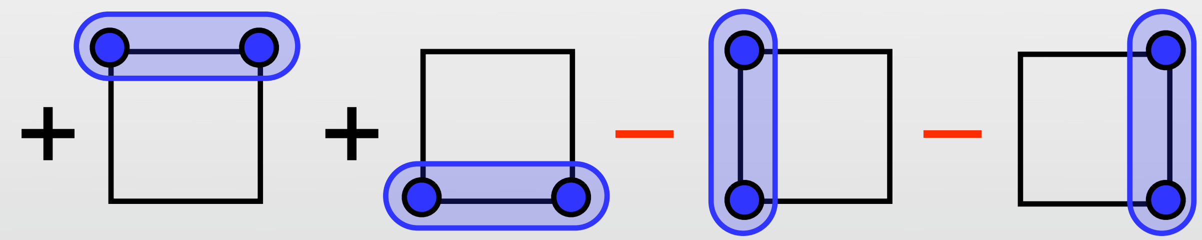


RVB state on 4-site plaquettes

- **d-wave RVB pairs**

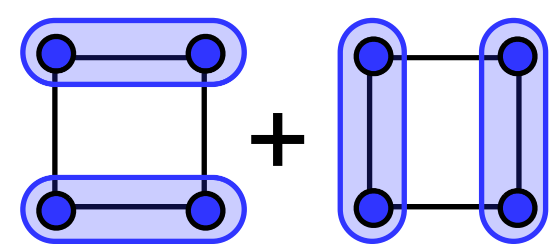
$$s_{i,j} = \frac{1}{\sqrt{2}}\left(c_{i,\uparrow}c_{j,\downarrow} - c_{i,\downarrow}c_{j,\uparrow}\right)$$

$$\Delta_d \approx \frac{1}{2}\left(s_{1,2} + s_{3,4} - s_{1,3} - s_{2,4}\right)$$



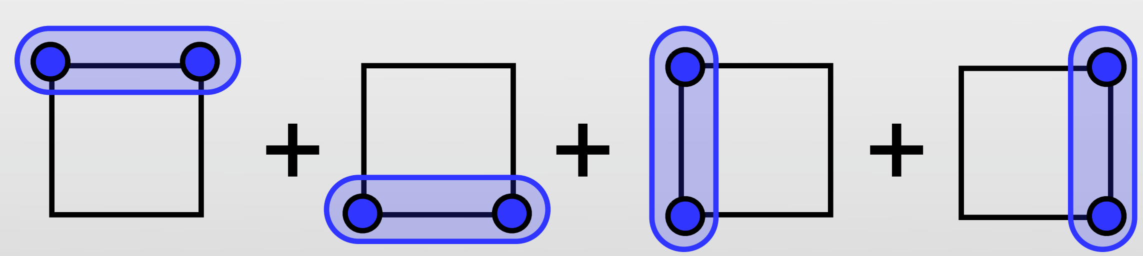
- **4 fermions on a plaquette: d-wave RVB state**

$$|4\rangle \approx s^{\dagger}_{1,2}s^{\dagger}_{3,4} + s^{\dagger}_{1,3}s^{\dagger}_{2,4}$$

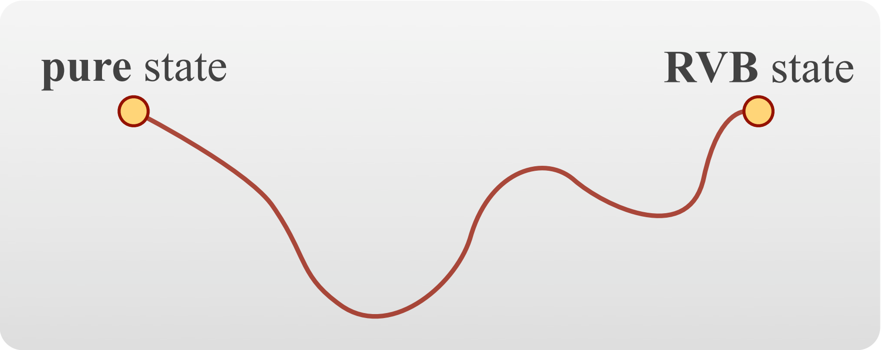


- **removing a d-wave pair gives ground state of 2 fermions**

$$|2\rangle \approx \Delta_d|4\rangle$$

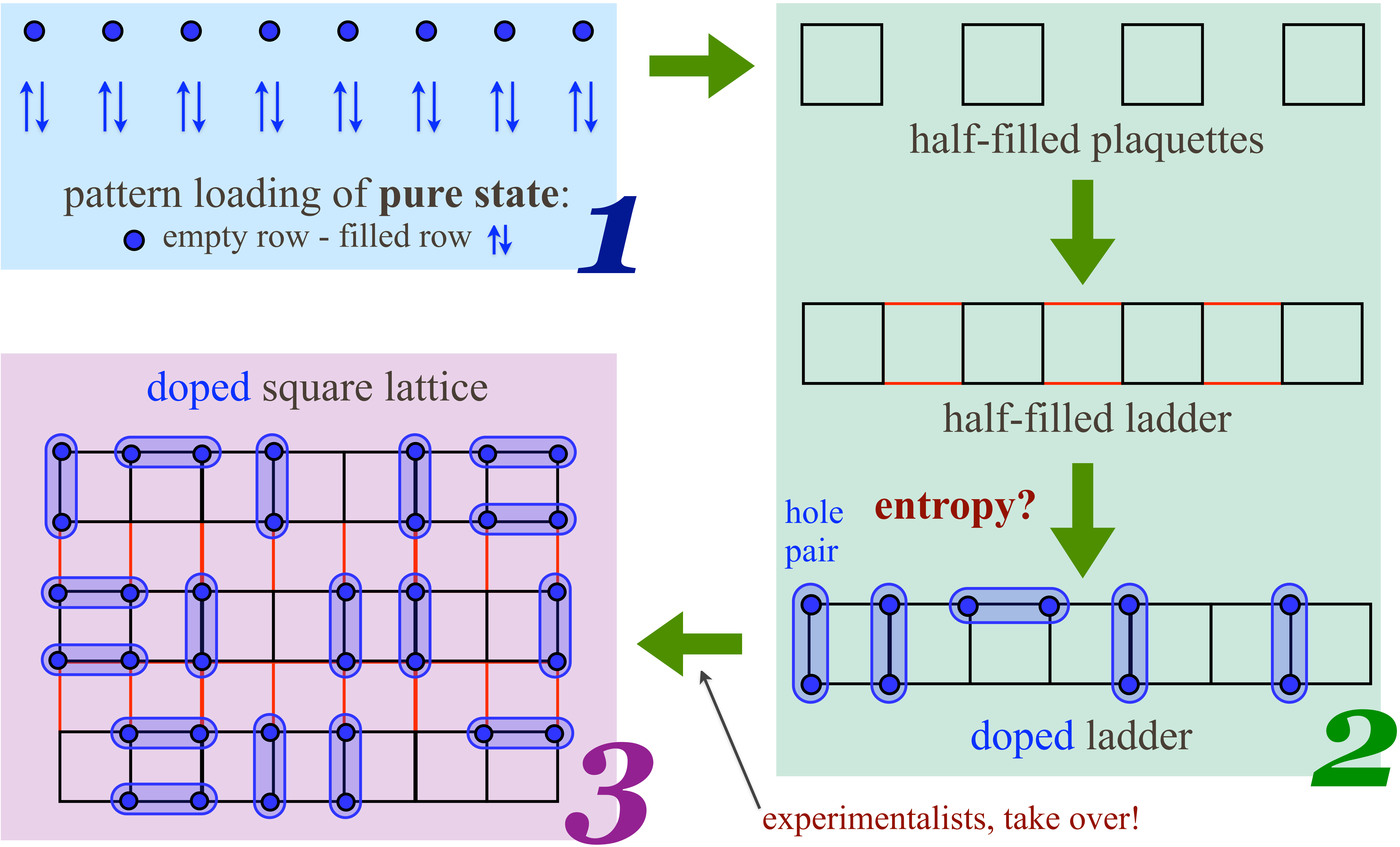


The adiabatic quantum simulation



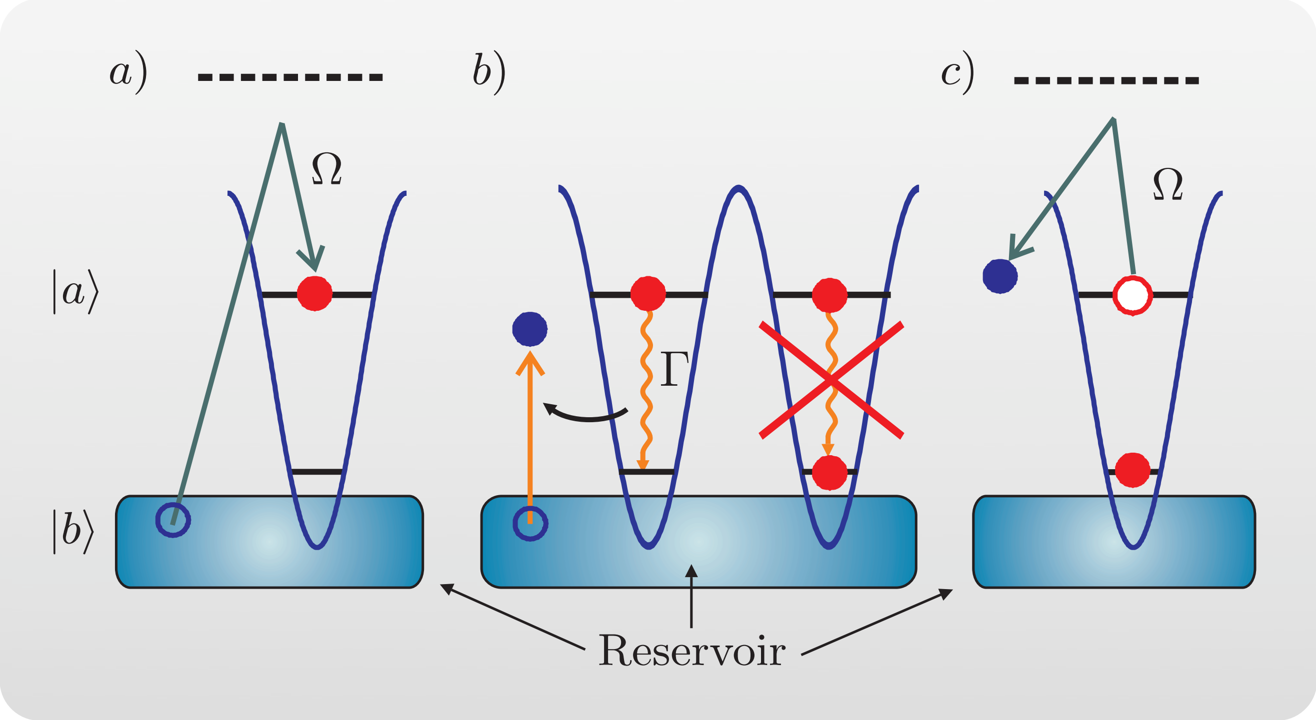
- **Outline**
  - cool Fermi gas as far as possible
  - load a **pure initial ground state** into an optical lattice
  - **adiabatically tune** the model to desired interaction parameters
  - **test the progress** at (several) solvable intermediate points
- **Watch out:**
  - try to keep a **large gap** to excited states
  - avoid **level crossings**
  - avoid mixing in states from wrong **symmetry sectors**

The adiabatic path

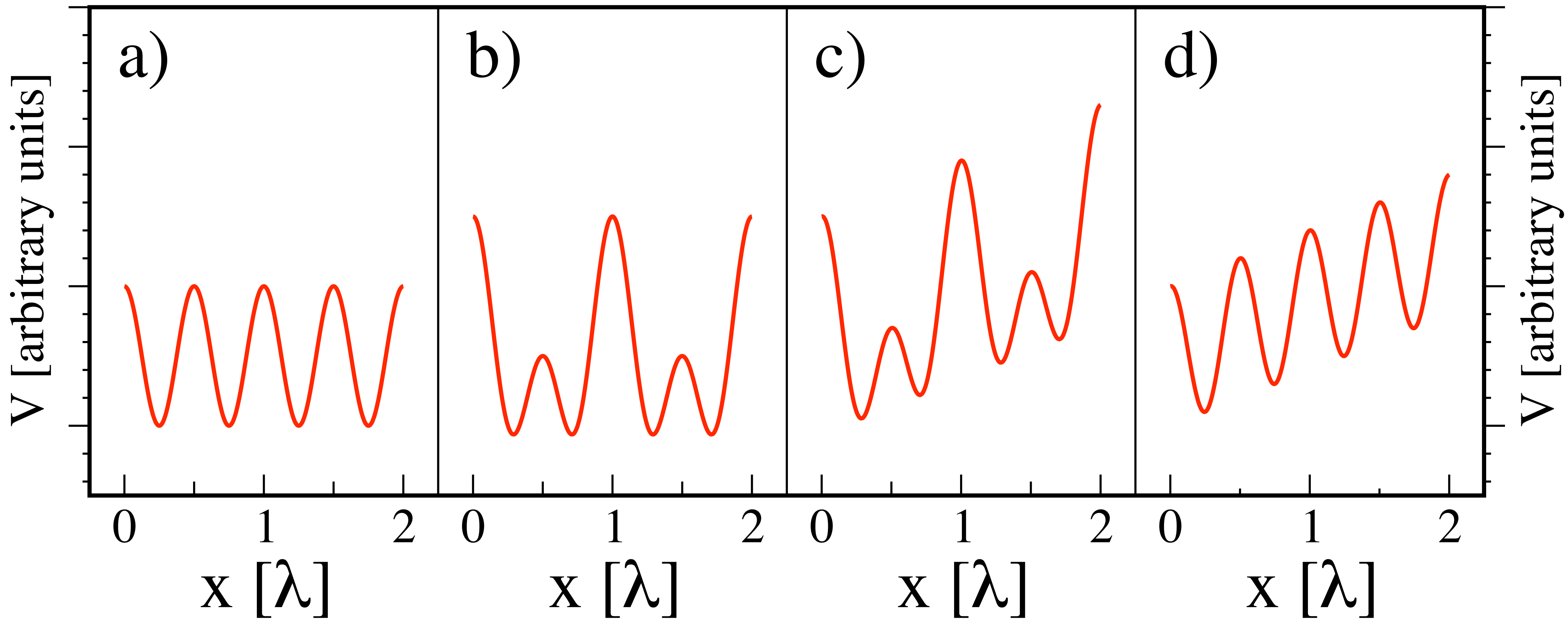


AMO toolbox I: pattern loading

- **Pattern loading:**
  - pump into exited state of a lattice
  - atoms relax to ground state in lattice well
  - pump more atoms
  - then get rid of excited atoms
  - Rabl *et al*, PRL **91**, 110403 (2003)
    Griessner *et al*, PRA **72**, 032332 (2005)



AMO toolbox II: superlattice and ramps



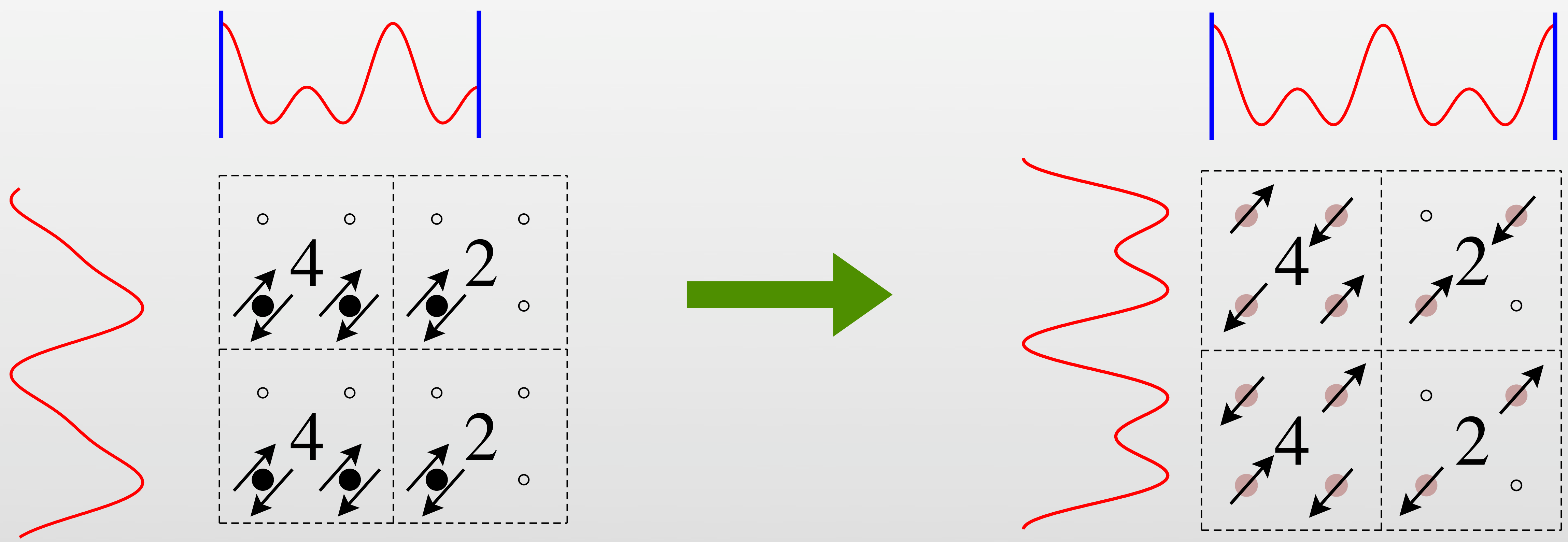
a. One-dimensional optical lattice

b. Superlattice

c.+ d. Linear ramp

$$V(x) = V_x' \sin(k'x + \Phi)$$
$$k' = k\cos(\theta)$$

Adiabatically preparing plaquette RVB states



Pattern loaded isolated plaquettes

- every other chain empty: $\mu_\perp >> t_\perp$
- zero horizontal hopping: $t = 0$

true plaquettes

- full interactions
- ground states for 4, 2 atoms

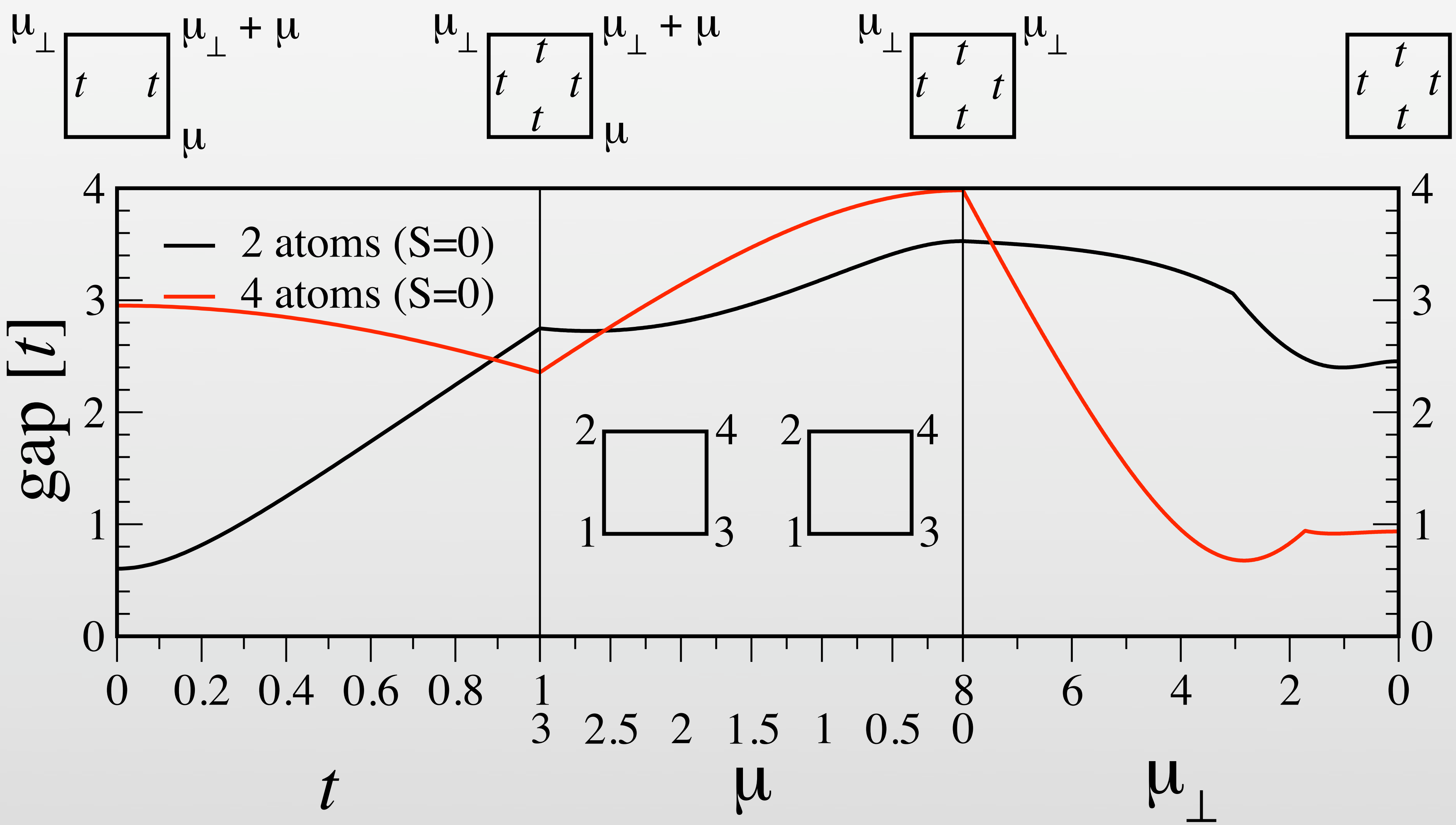
Which path to use to tune μ, $\mu_\perp$, $t_\perp$?

Preparing d-wave RVB states

1 ramp up hoppings t

2 ramp down in-chain μ

3 ramp down intra-chain $\mu_\perp$



Protected by gaps: fidelity > 99% for times ~ 50/t

Watch out: other routes can give s-wave states.

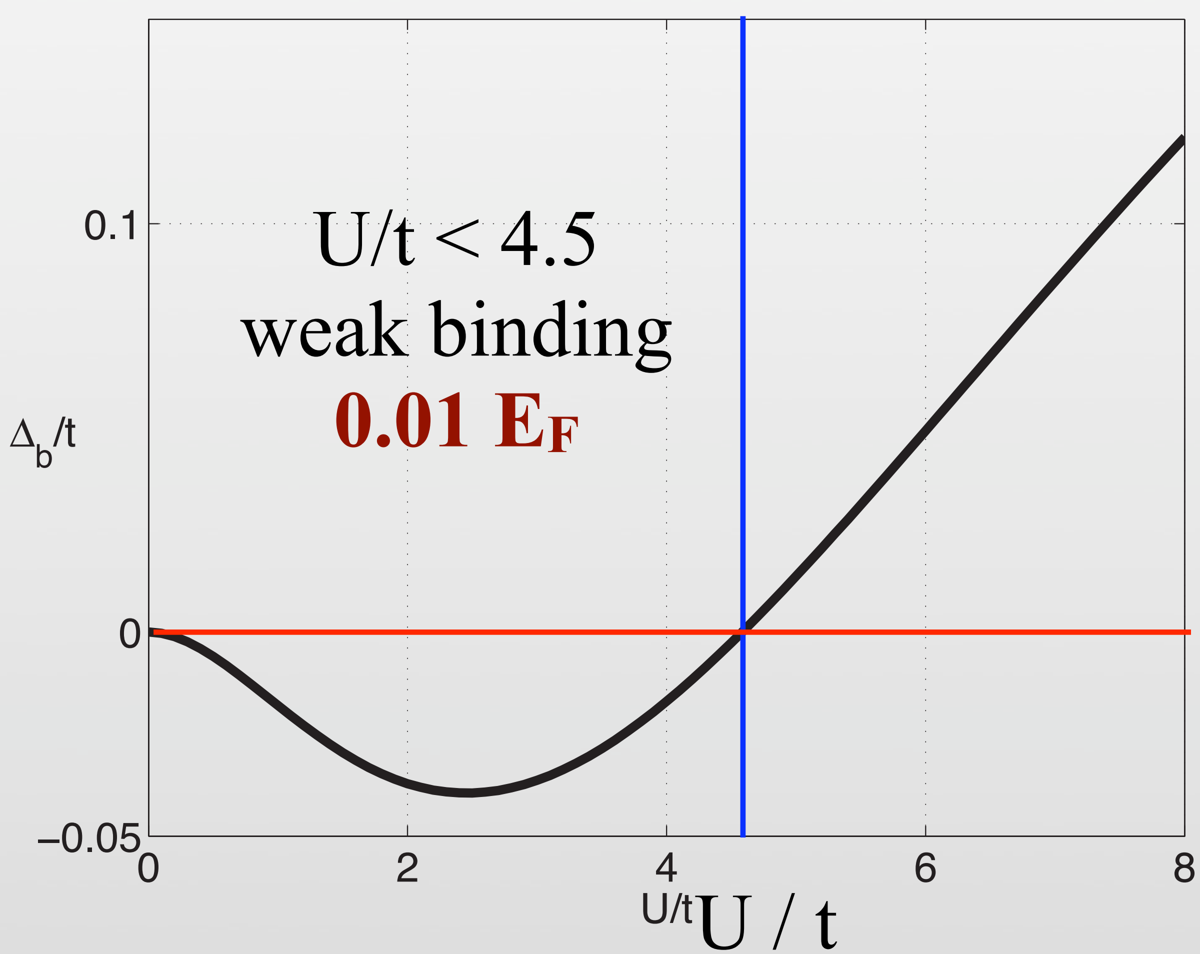
How can we experimentally test the successful generation of the plaquette RVB state?

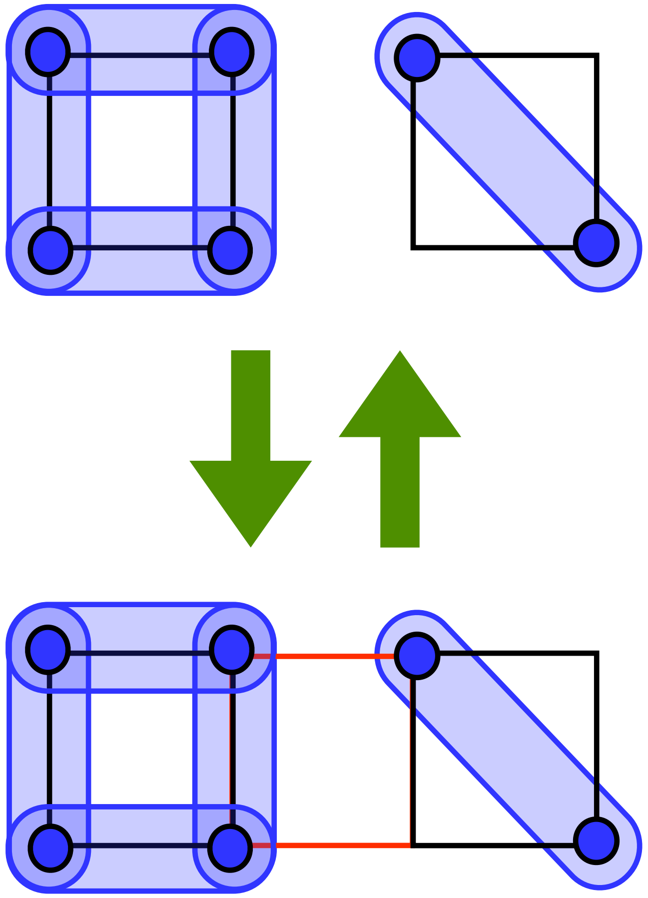
Binding energy of two holes

- Will two holes remain bound when we couple and decouple plaquettes?

$$\Delta_{\text{bind}} = E(N) + E(N-2) - 2E(N-1)$$

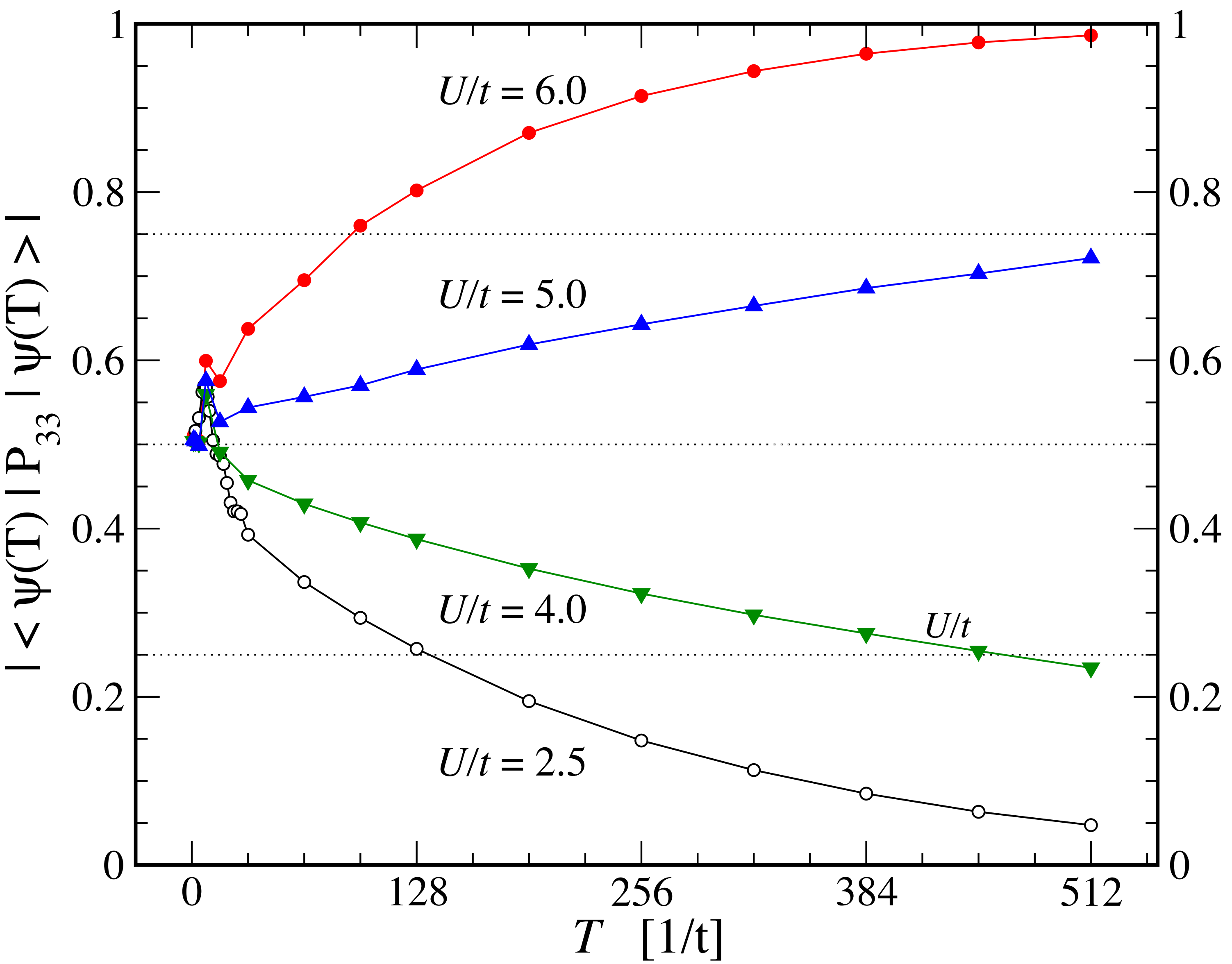
Altmann & Auerbach, PRB **65,** 104508 (2002)





Decoupling two plaquettes

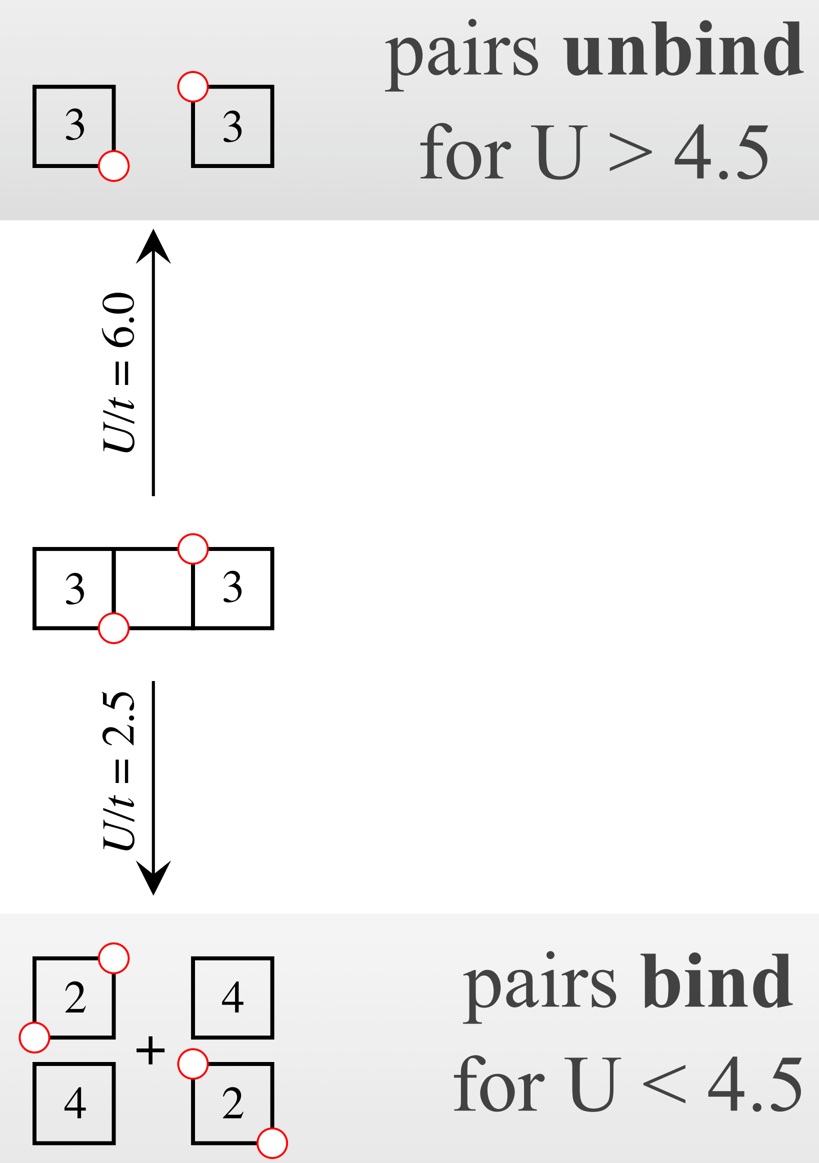
Pair breaking probability



Decoupling time (non-linear ramping)

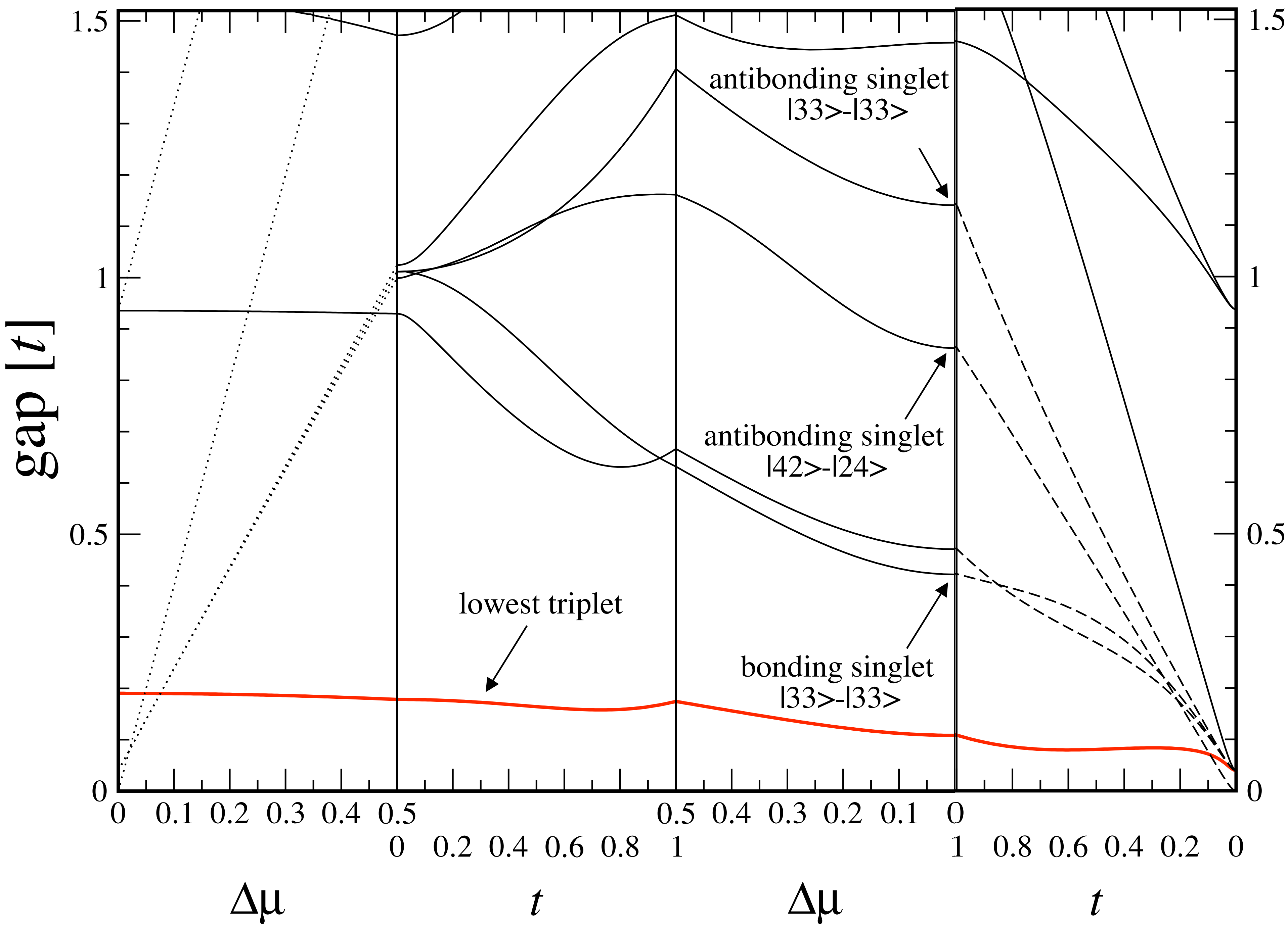
~500 1/t (about 500 ms)

can form 2 molecules



can form 3 molecules

All details can be calculated



binding energy:

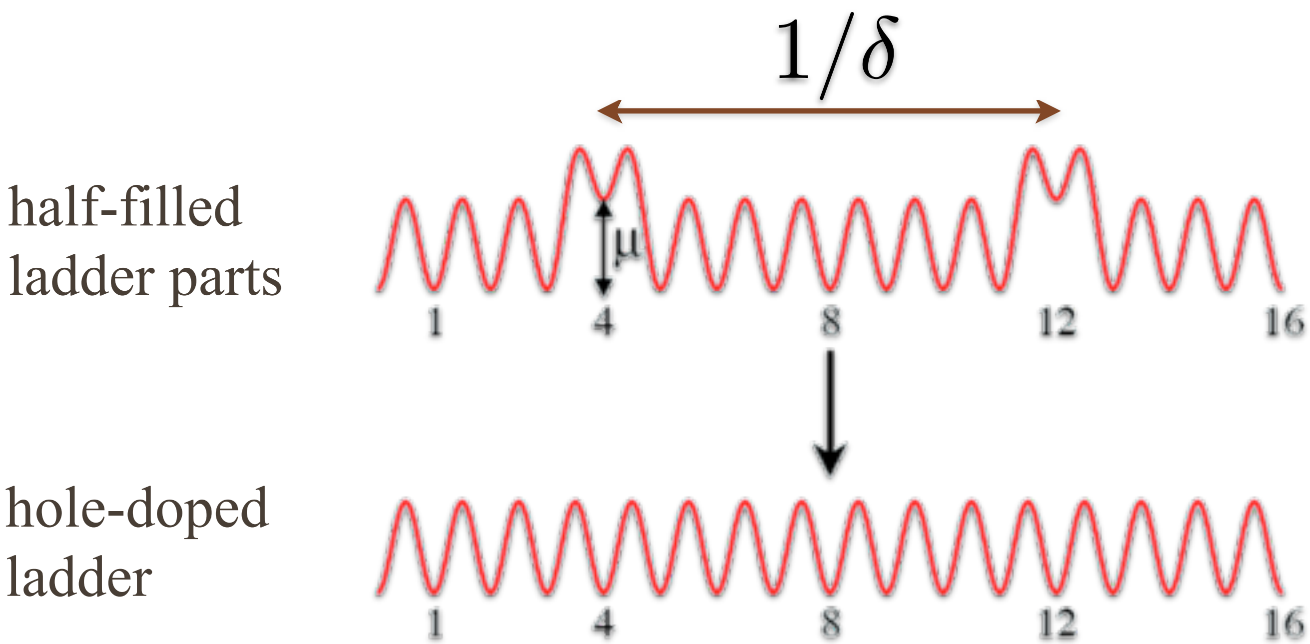
tiny gap!

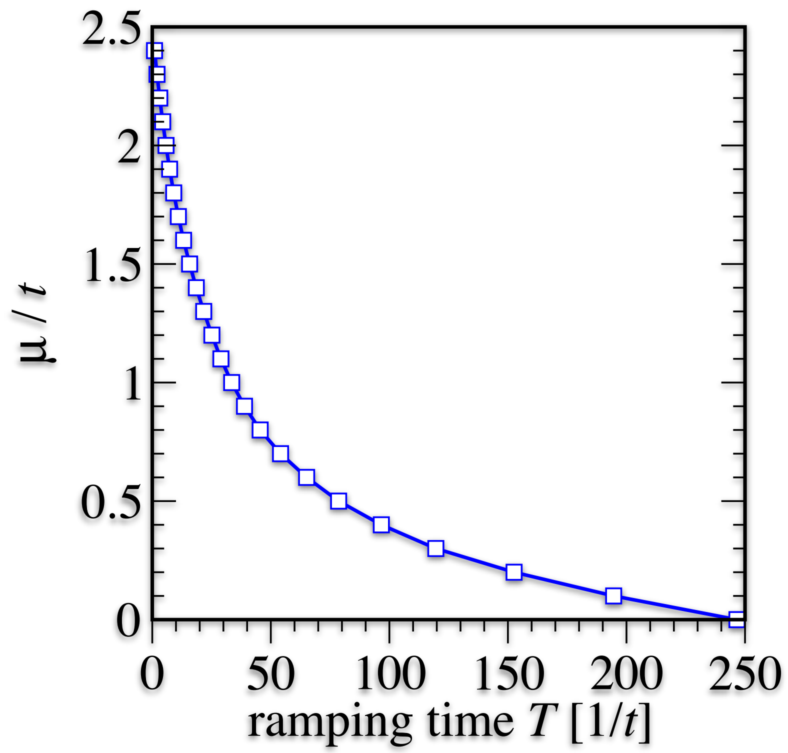
requires long time scales for decoupling

Going beyond plaquettes

Doping a half-filled ladder

- doping δ: hole pairs “crystallize” $a_P = 1/\delta$
- prepare ladder segments separated by empty rungs
  - empty rungs at preferential hole locations
- reduce chemical potential
  - holes appear minimal particle motion
  - phase coherence between ladder parts
- DMRG, 2x32 ladder, 56 particles
  - ramping-down speed must decrease
  - 99% fidelity in 1/2 s

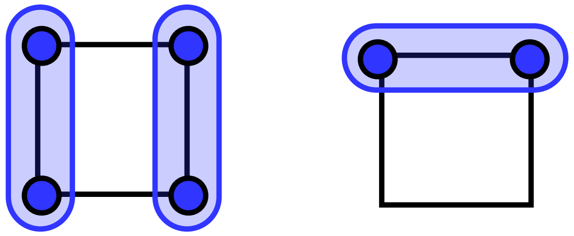




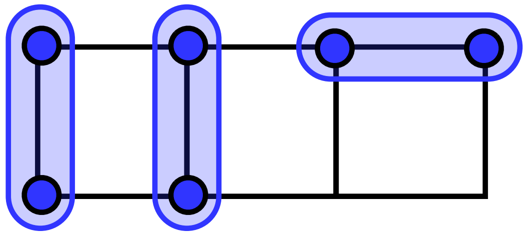
From plaquettes to ladders

Assuming hopping of 1 kHz

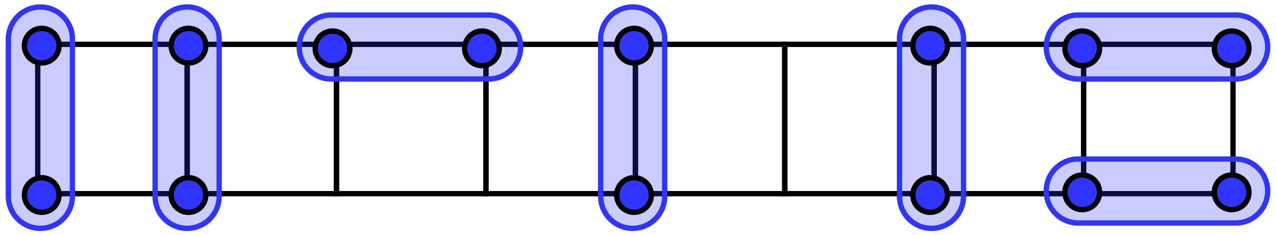
- Creating isolated RVB plaquettes: **50 ms**



- Coupling plaquettes: **100 ms**



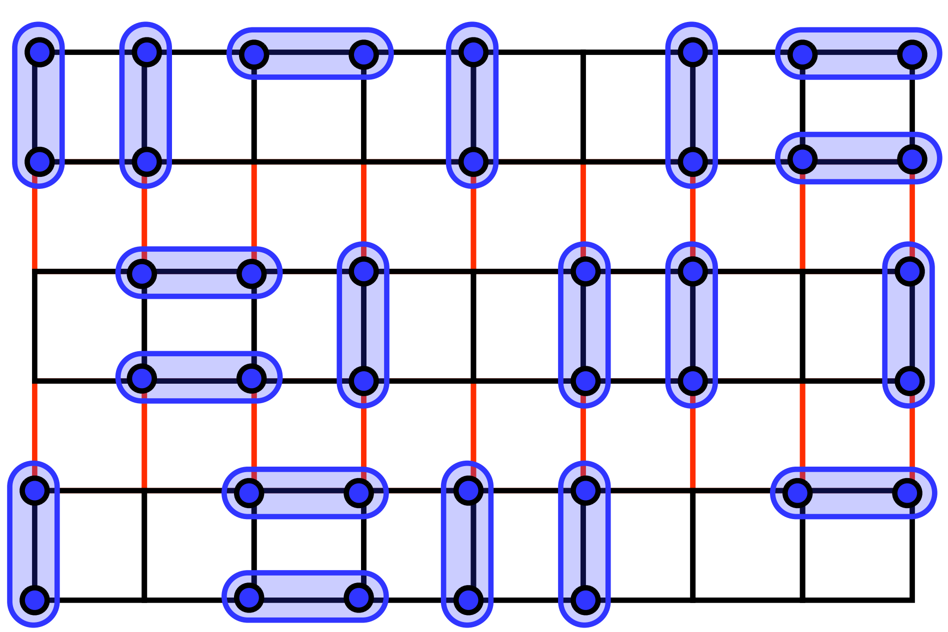
- Pairing on plaquettes can be measured in **500 ms**
- Creating ladders: **250 ms**

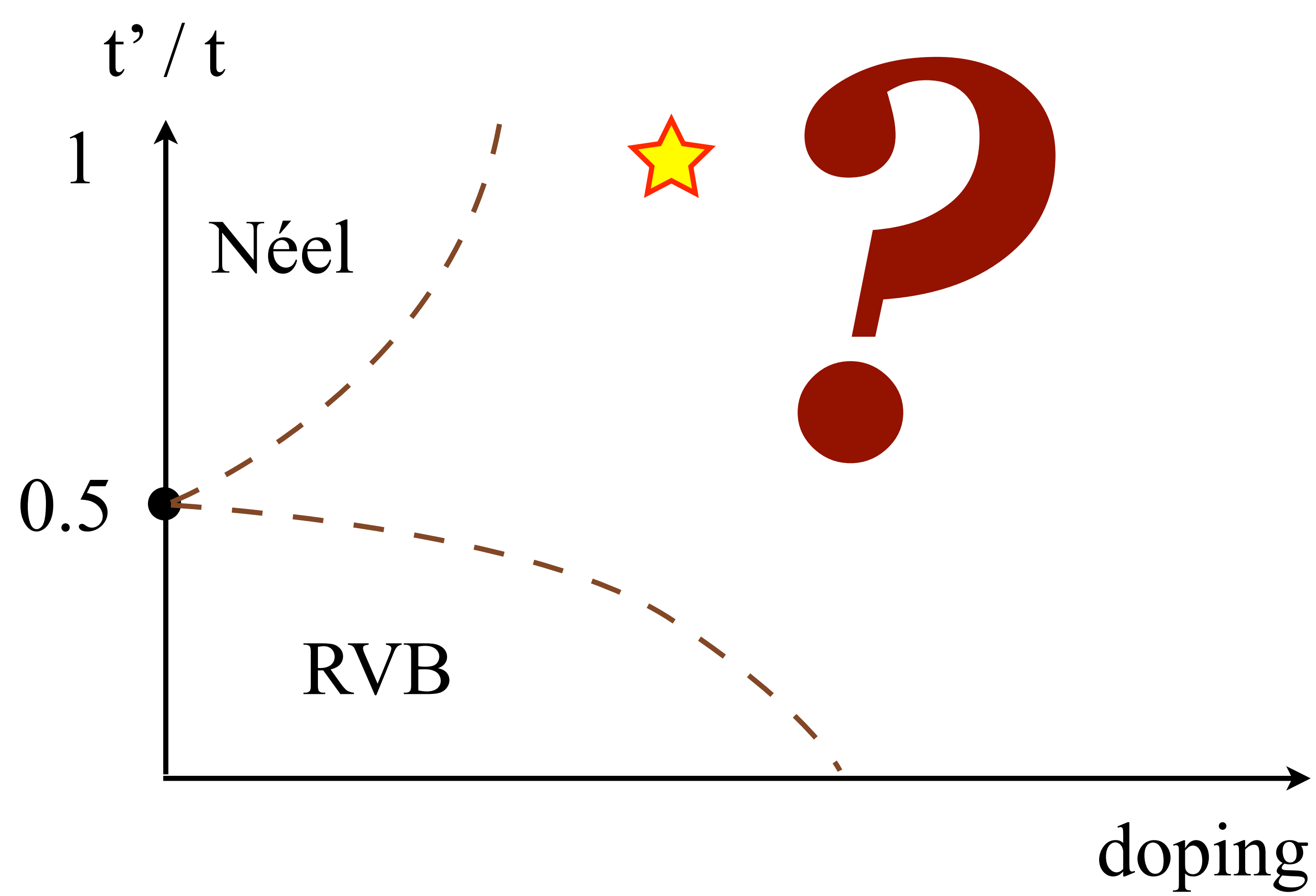


- **All time scales are smaller than decoherence time**

Quantum simulation of RVB states

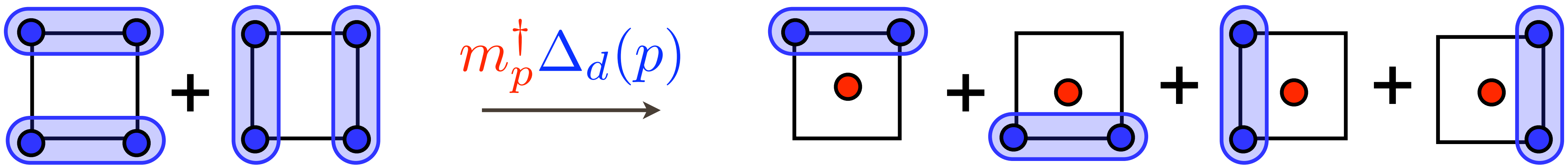
- Experiment can go further than we could and test whether
  - the **RVB states survives** when going to the uniform square lattice?
  - or is there a **phase transition** to a new phase?
- *Either* answer would be important.





Measuring d-wave pairing

- How can we measure d-wave pairing?
  - convert d-wave pair on a plaquette into a molecule in the center of the plaquette $m_p^\dagger \Delta_d(p)$
  - state-selective molecule formation by laser-induced Raman transition
  - then measure phase coherence of the bosonic molecules



Summary

- **Adiabatic quantum simulation**
  - Start from a pure initial state of a noninteracting model
  - Adiabatically transform it into the unknown ground state of an interacting model
  - Can reach ground states of interacting quantum systems in short times

- **Example: RVB states in Hubbard models**
  - Weakly coupled plaquettes and ladders have d-wave RVB ground states
  - Energy scales are very low (gap less than $0.01E_F$)
  - Seems impossible to reach these states by simple cooling
  - But can be prepared adiabatically in very short times from filled bands!
  - Ground state of 2D fermionic Hubbard model can be tested

ST, U. Schollwöck, M. Troyer, P. Zoller, PRL **96**, 250402 (2006)