Interactions and disorder in topological quantum matter

February 5th, 2013

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University of Cologne

Overview

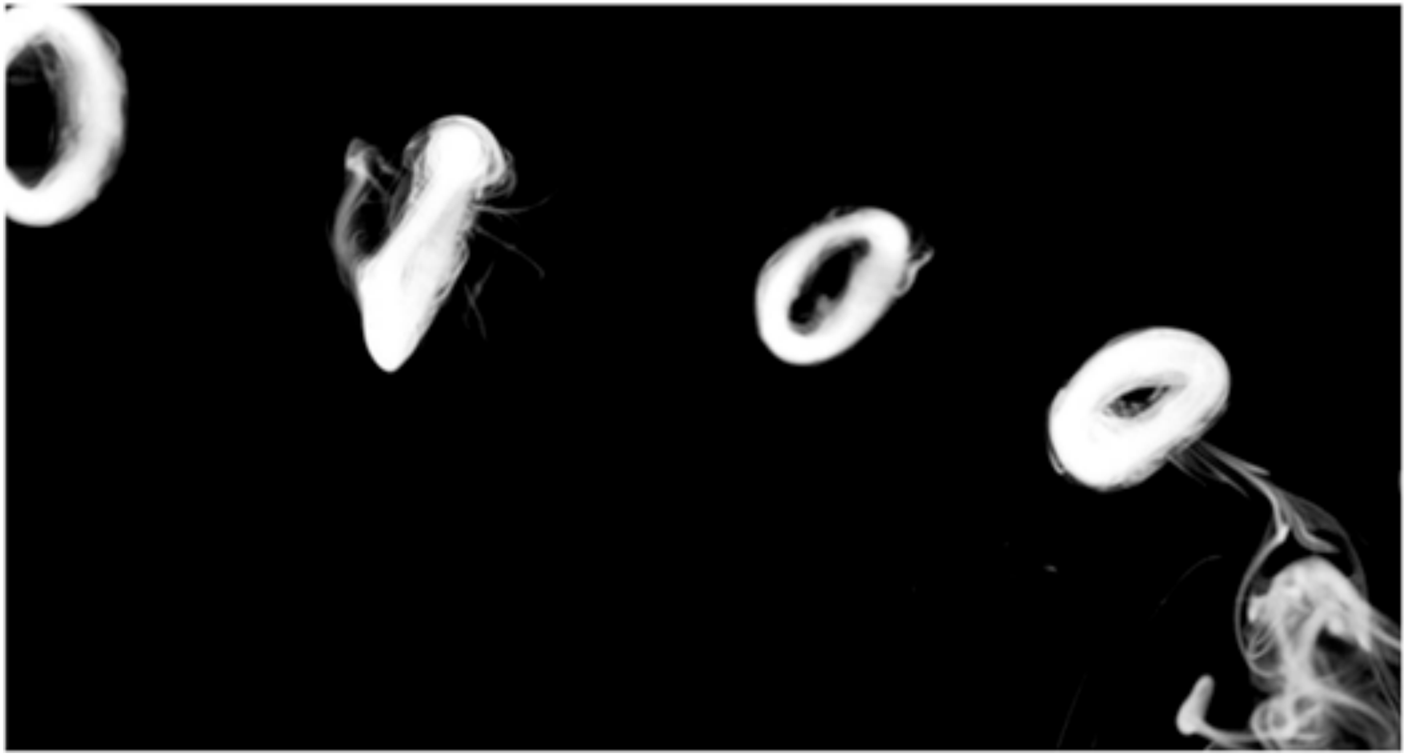
Topological quantum matter

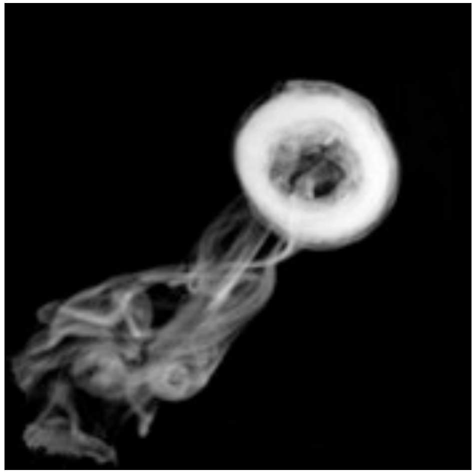
Interactions

Disorder

Experiments

Topological quantum matter



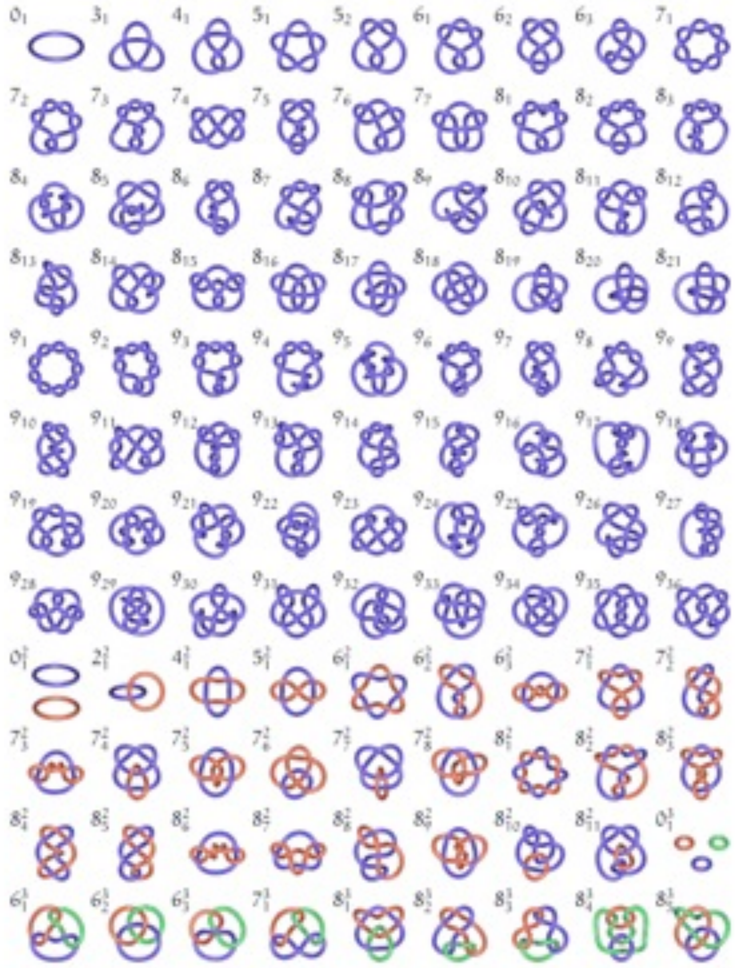


Topological quantum matter

- 1867: **Lord Kelvin**
  atoms = knotted tubes of ether
  Knots might explain stability, variety, vibrations, ...

- **Maxwell**: "It satisfies more of the conditions than any atom hitherto imagined."

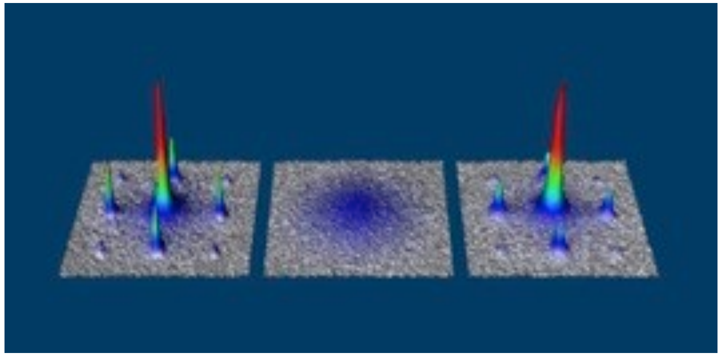
- This inspires the mathematician **Tait** to classify knots.



Non-topological (quantum) matter

water ice

superconductor



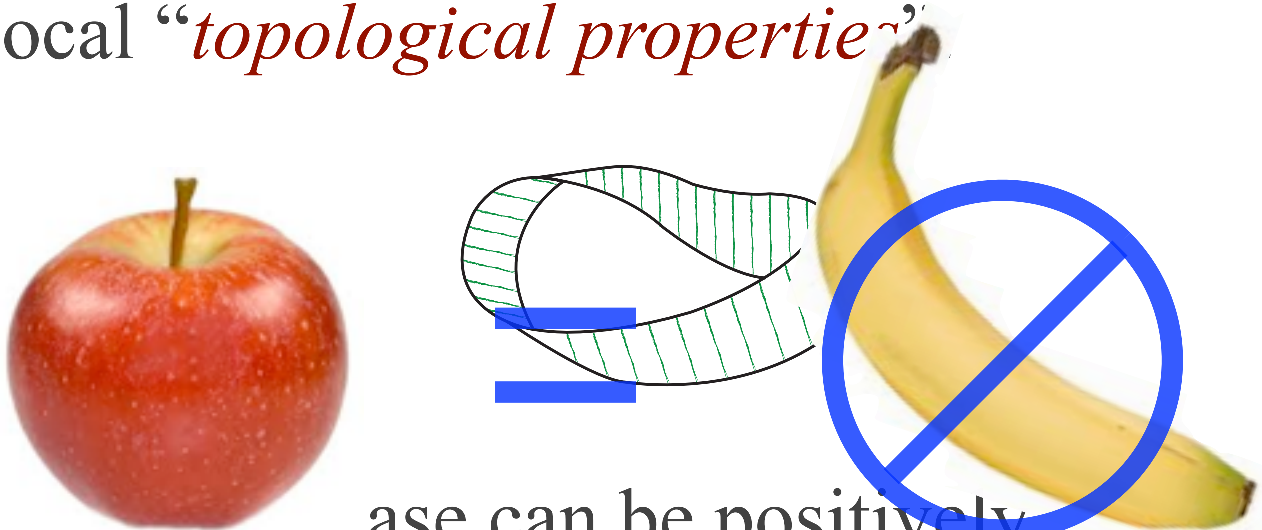
Bose-Einstein condensate

All these states can be described by a local order parameter.

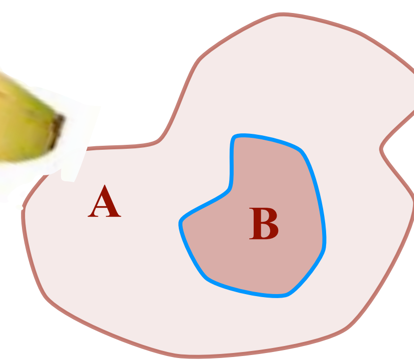
Topological quantum matter

- Xiao-Gang Wen: A ground state of a many-body system that *cannot* be fully characterized by a *local* order parameter.

- Often characterized by a variety of non-local "*topological properties*".



- A topological phase can be positively identified by its *entanglement properties*.



Knots & edge states

- Bringing a topological and a conventional state into spatial proximity will result in a *gapless edge state* – literally a *knot* in the wavefunction.

- We know this: "Counterintuitive states"

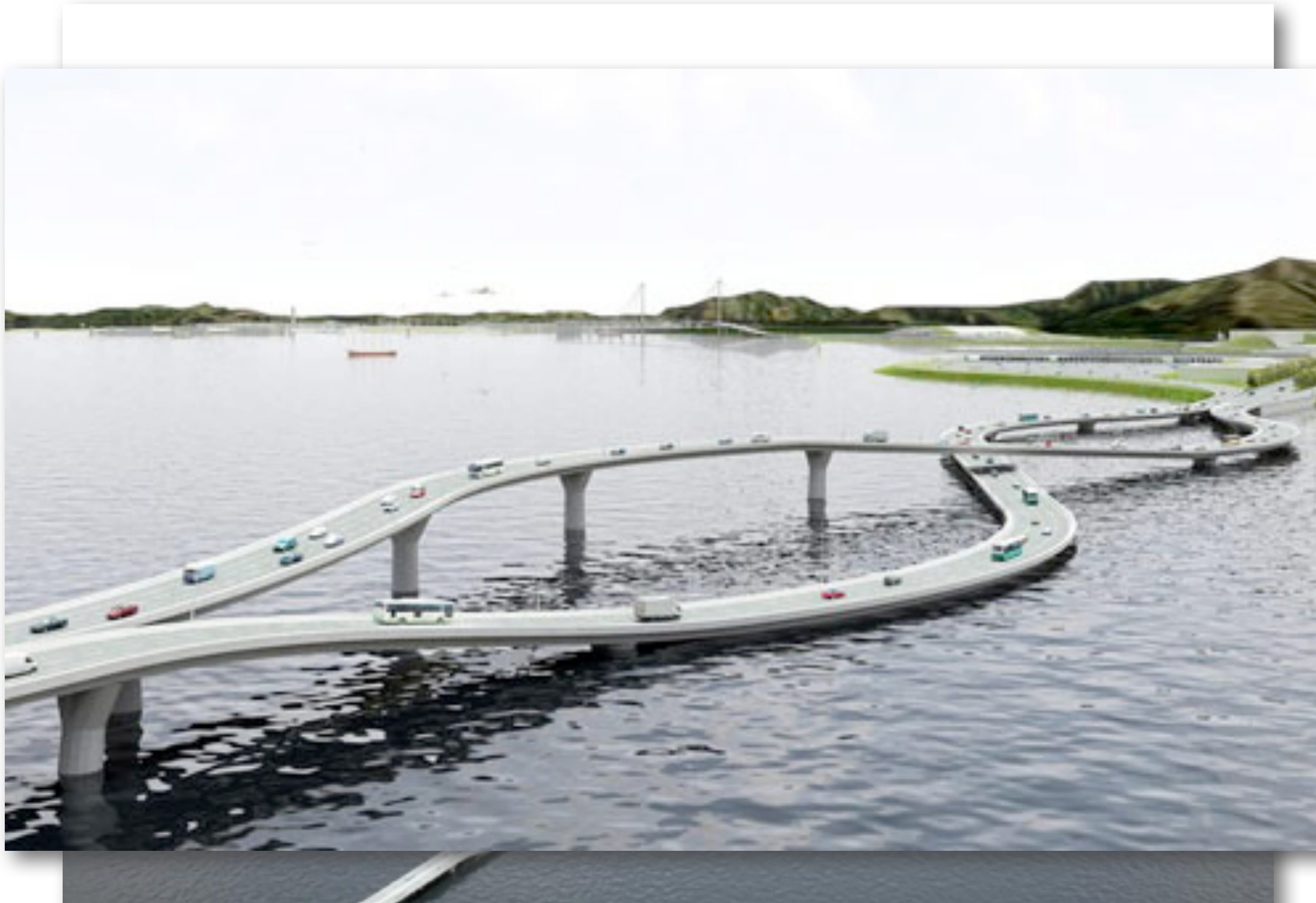
China

UK Ireland Australia Japan Hong Kong

Knots & edge states

China

Hong Kong



Flipper bridge

Topological matter / classification (rough)

Topological order

inherent

Gapped phases that cannot be transformed
– without closing the bulk gap –
to “simple phases”
via any “paths”

quantum Hall states
spin liquids

symmetry protected

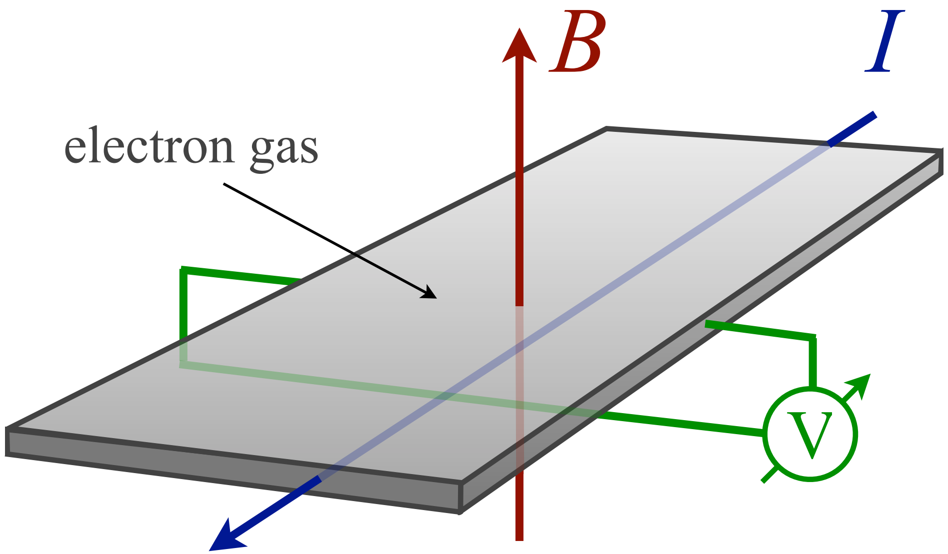
Gapped phases that cannot be transformed
– without closing the bulk gap –
to “simple phases”
via any **symmetry preserving** “paths”.

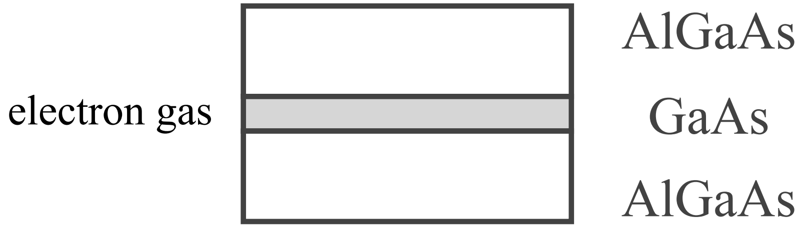
topological band insulators

Quantum Hall effect

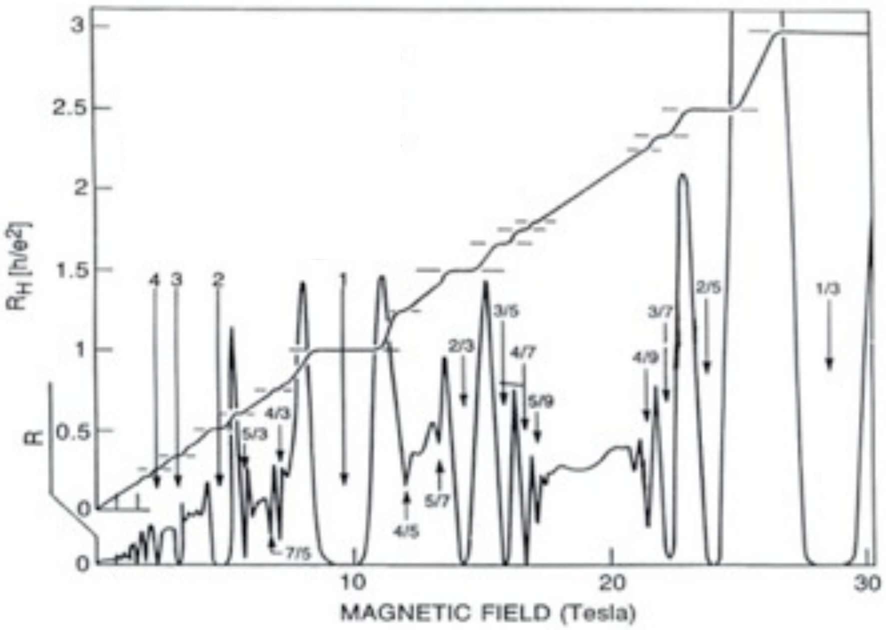
Quantization of Hall conductivity
for a two-dimensional electron gas
at very low temperatures
in a high magnetic field.

$$\sigma = \nu \frac{e^2}{h}$$





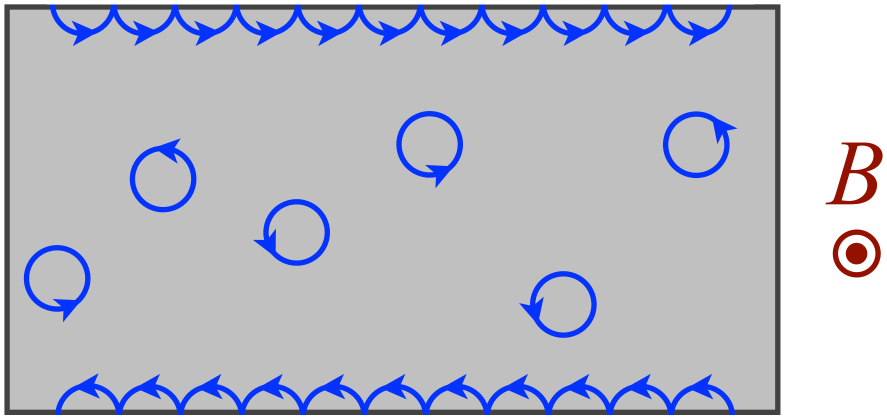
Semiconductor heterostructure confines electron gas to two spatial dimensions.



Quantum Hall states

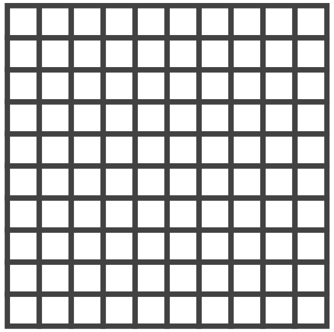
Landau levels

$$E_n = h\frac{eB}{m}\left(n + \frac{1}{2}\right)$$



edge states

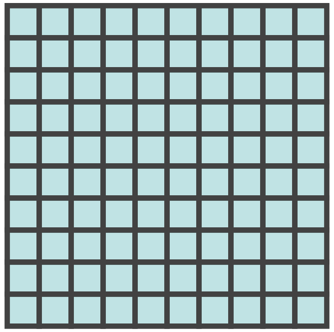
Landau level **degeneracy**



$2\Phi/\Phi_0$

orbital states

integer quantum Hall

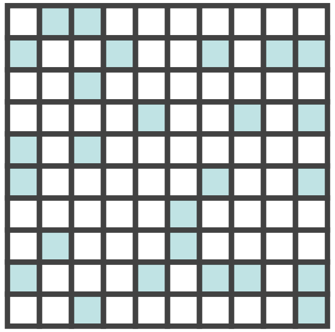


filled level

↓

incompressible liquid

fractional quantum Hall

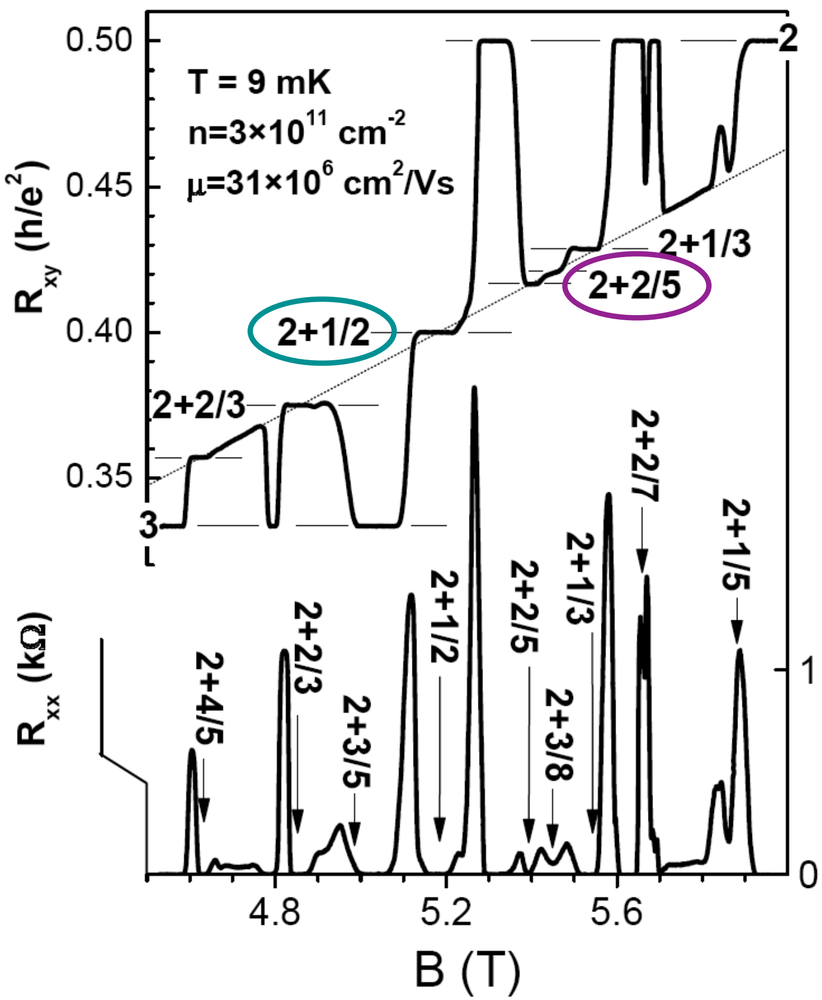


partially filled level

↓ Coulomb repulsion

incompressible liquid

Fractional quantum Hall states



J.S. Xia *et al.*, PRL (2004)

"Pfaffian" state

Moore & Read (1994)

Charge e/4 quasiparticles
Ising anyons

Nayak & Wilczek (1996)

$SU(2)_2$

"Parafermion" state

Read & Rezayi (1999)

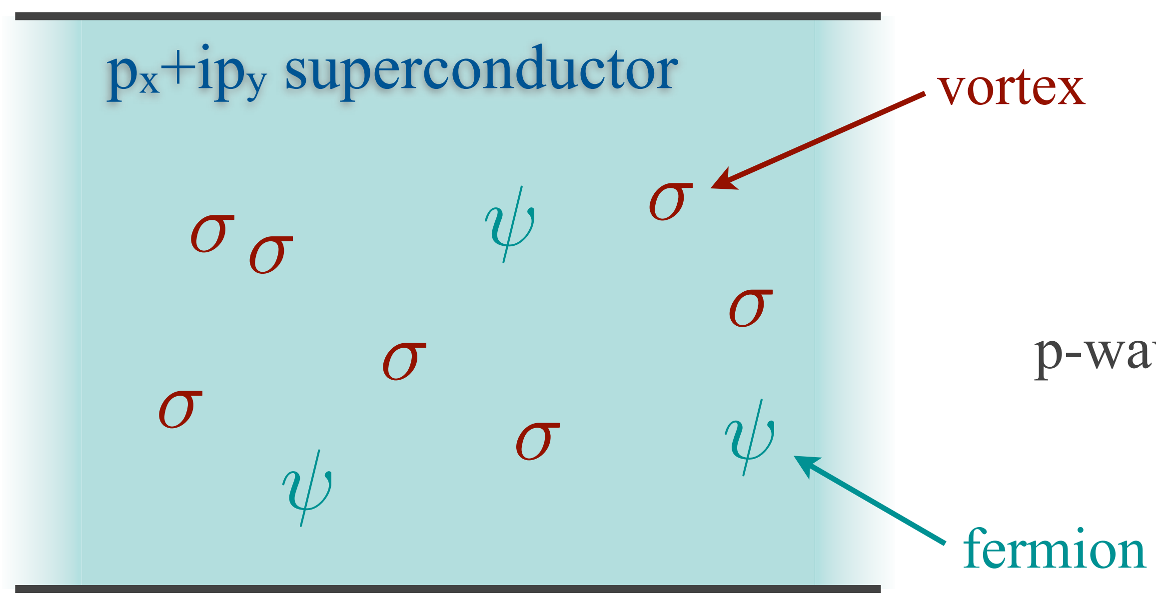
Charge e/5 quasiparticles
Fibonacci anyons

Slingerland & Bais (2001)

$SU(2)_3$

p_x+ip_y superconductors

Read & Green (2000)



possible realizations

Sr_2RuO_4

p-wave superfluid of cold atoms

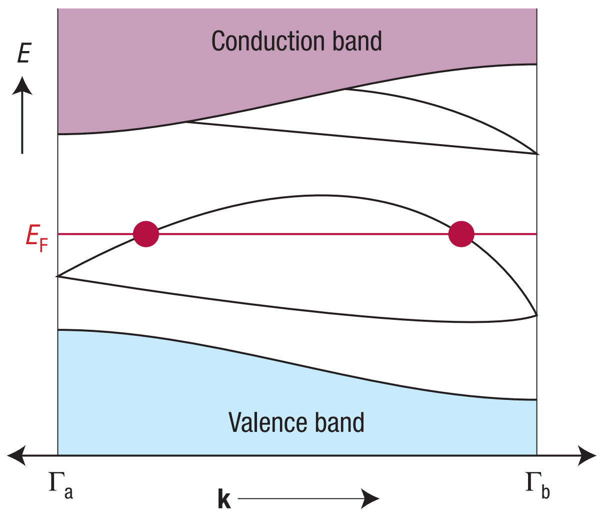
A_1 phase of ^{3}He films

Vortices carry characteristic “**zero mode**”, the so-called **Majorana fermion**.

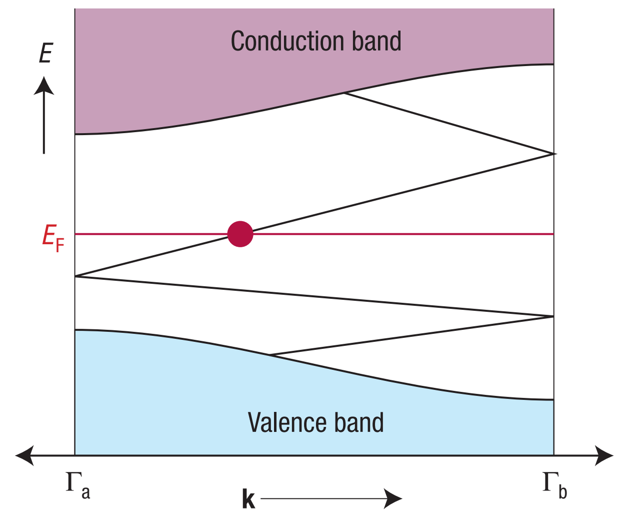
2N vortices give **degeneracy** of 2^N.

Topological insulators

Spin-orbit driven band inversion of a conventional band insulator.

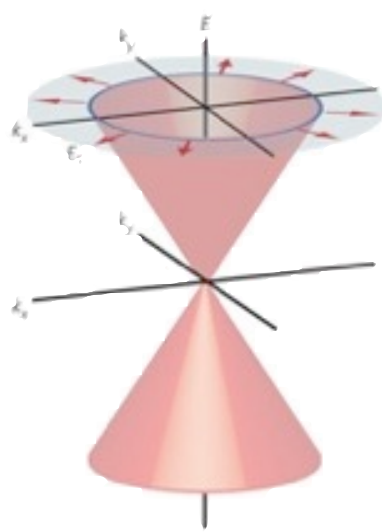


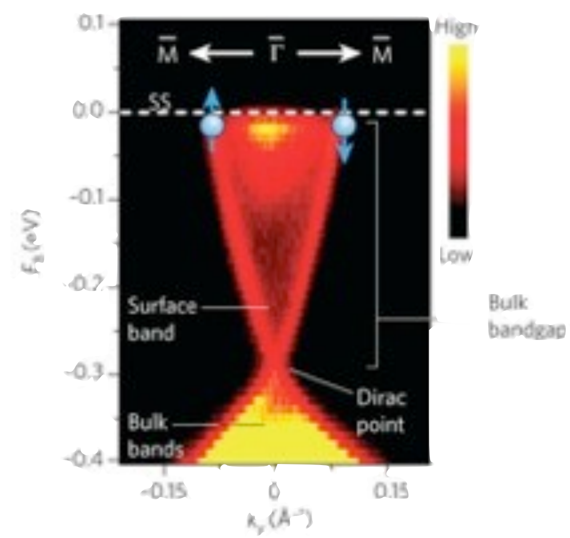
conventional band insulator



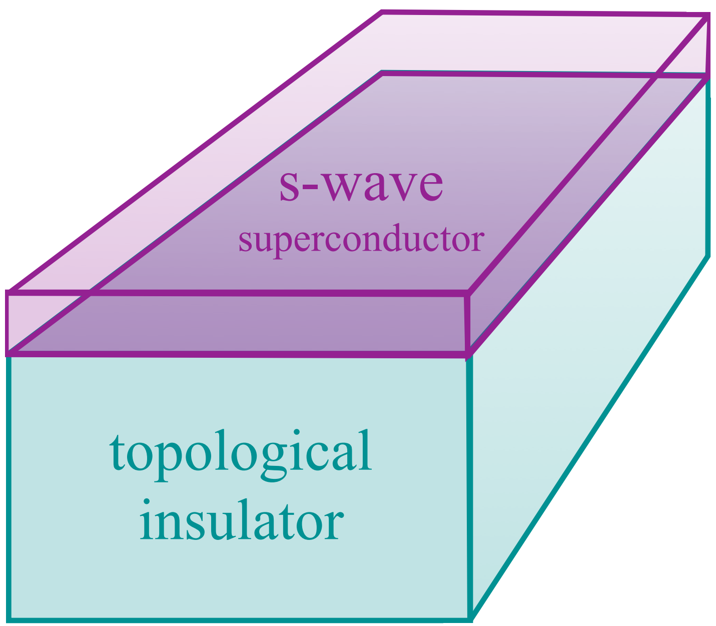
topological band insulator

Bi_2Se_3





Proximity effects / heterostructures



Proximity effect

Proximity effect between an s-wave superconductor and the surface states of a (strong) topological insulator induces exotic vortex statistics in the superconductor.

Spinless p_x+ip_y superconductor where vortices bind a **zero mode**.

Vortices, quasiholes, anyons, ...

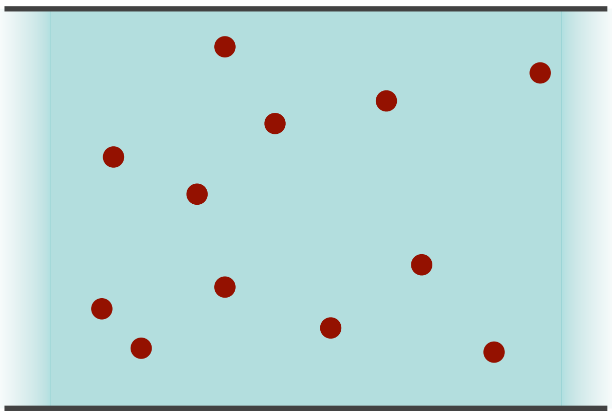
Interactions

Vortices

Abelian vs. non-Abelian vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

Abelian

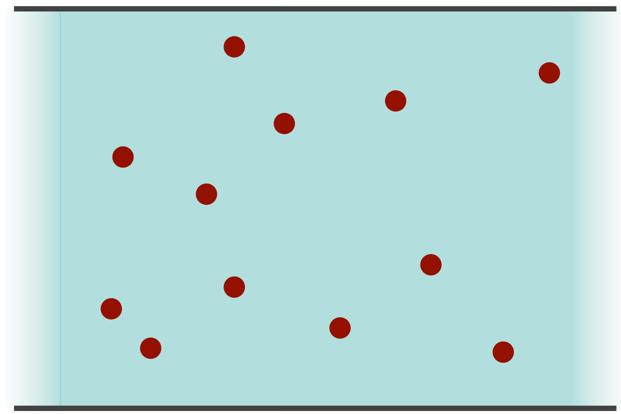


single state

Example:

Laughlin-wavefunction + quasiholes

non-Abelian



(degenerate) **manifold** of states

Manifold of states grows **exponentially** with the number of vortices.

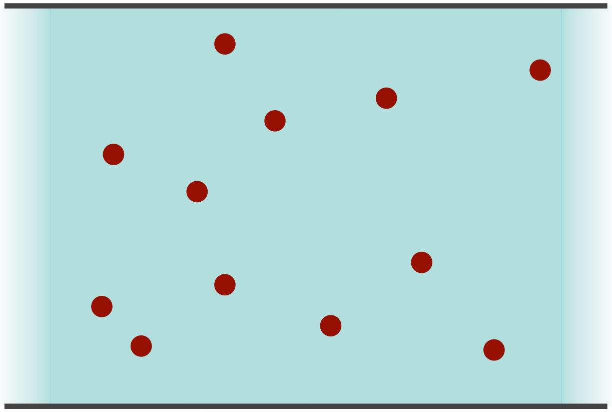
Ising anyons (Majorana fermions) $\sqrt{2}^N$

Fibonacci anyons ϕ^N

Abelian vs. non-Abelian vortices

Consider a set of ‘**pinned**’ vortices at fixed positions.

Abelian

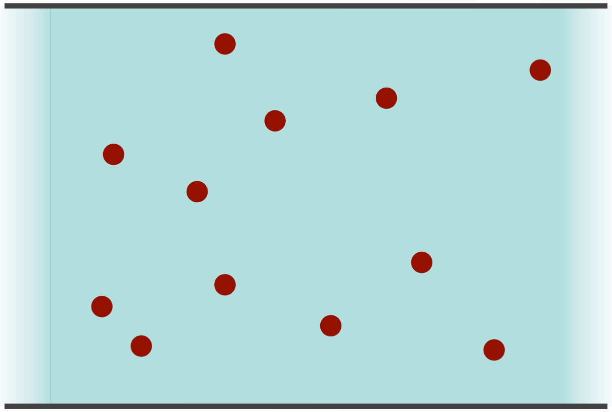


single state

$$\psi(x_2, x_1) = e^{i\pi\theta} \cdot \psi(x_1, x_2)$$

fractional phase

non-Abelian



(degenerate) **manifold** of states

matrix

$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

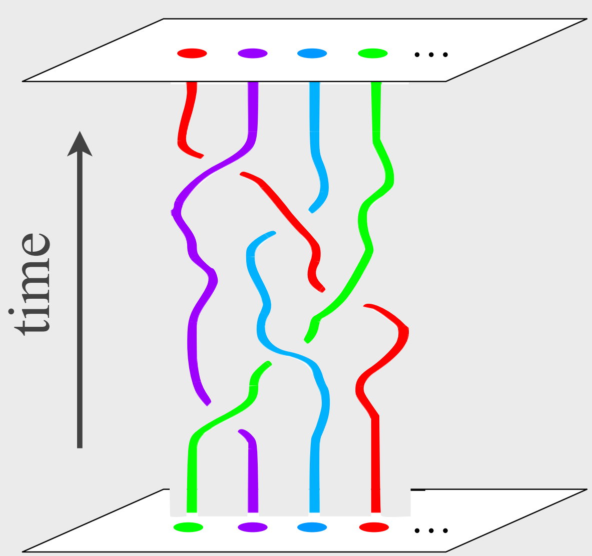
In general M and N do not commute!

Topological quantum computing

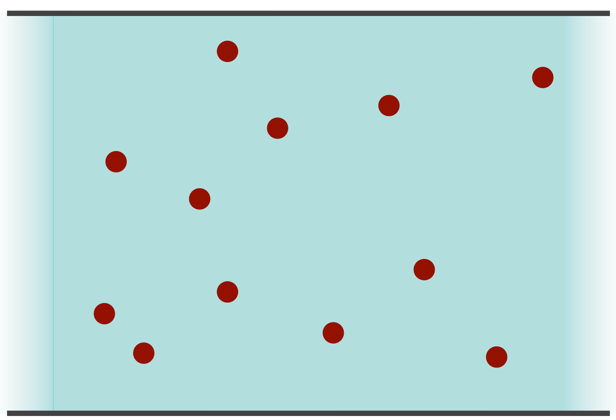
Topological quantum computing

Degenerate manifold = qubit

Employ **braiding** of non-Abelian vortices to perform computing (unitary transformations).



non-Abelian



(degenerate) **manifold** of states

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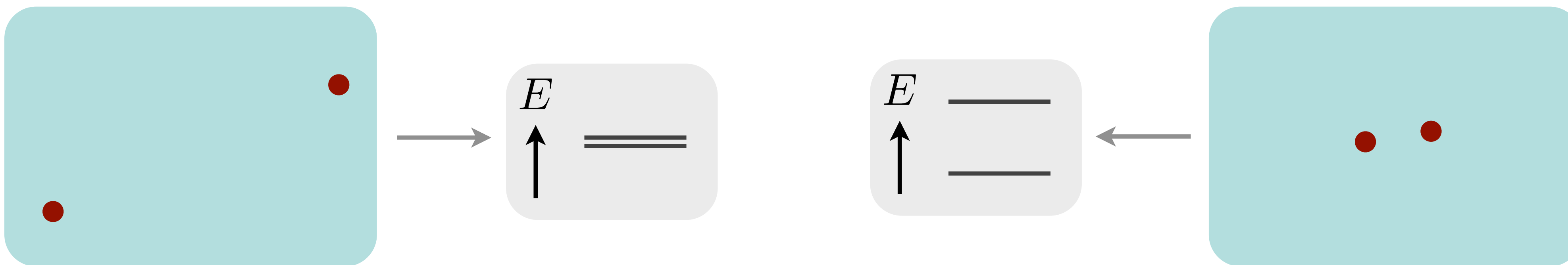
$$\psi(x_1 \leftrightarrow x_3) = M \cdot \psi(x_1, \ldots, x_n)$$

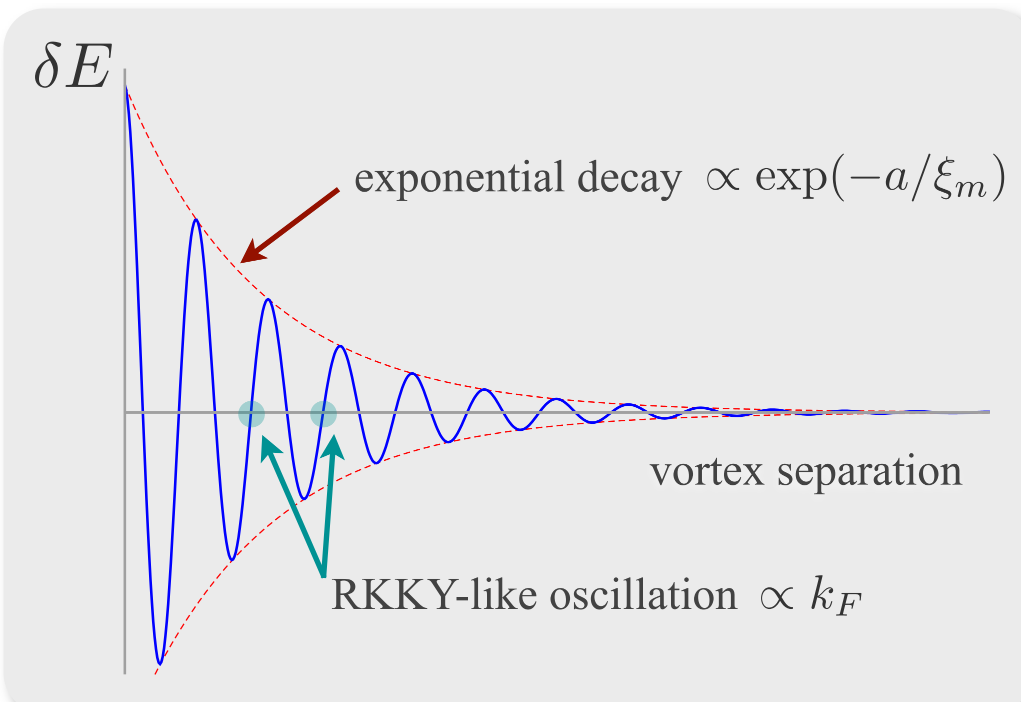
$$\psi(x_2 \leftrightarrow x_3) = N \cdot \psi(x_1, \ldots, x_n)$$

In general *M* and *N* do not commute!

Vortex-vortex interactions

$a \gg \xi_m$ $a \approx \xi_m$





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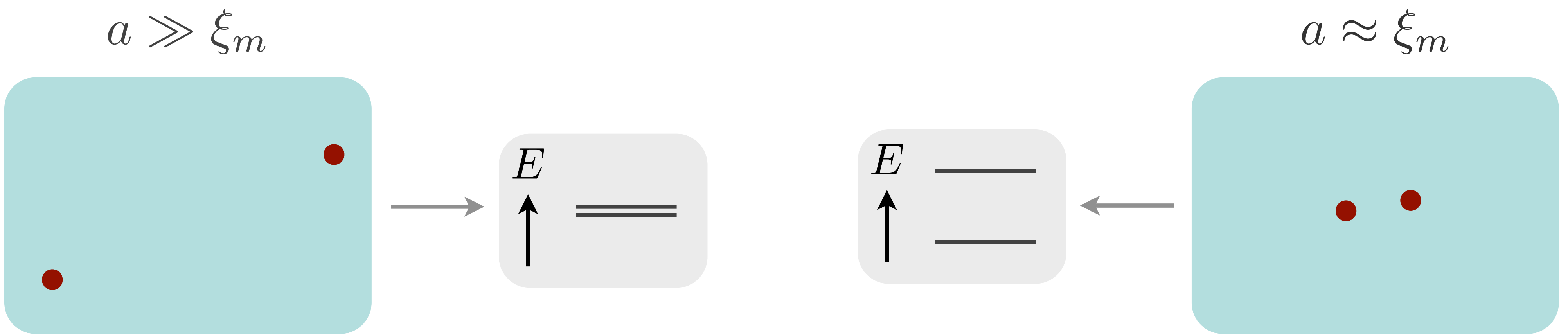
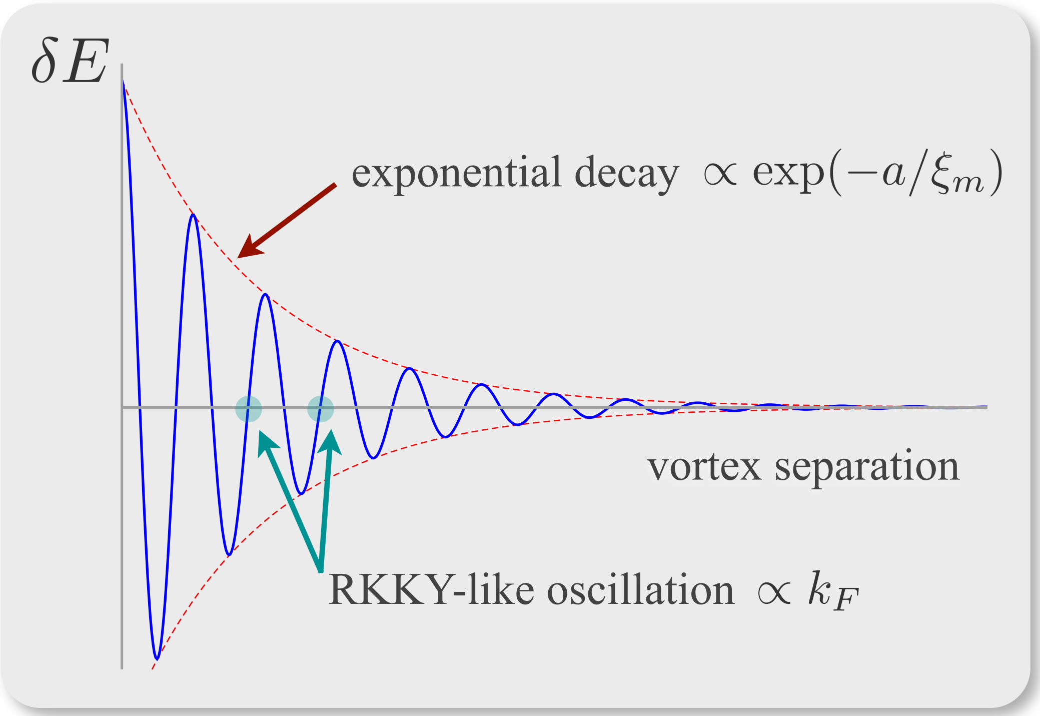
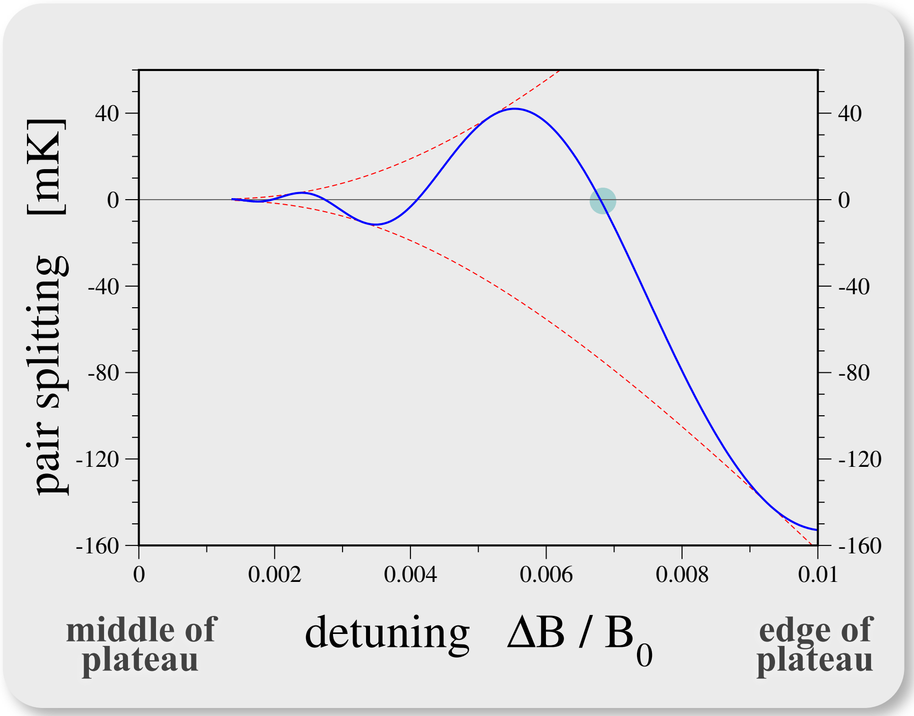






Vortex-vortex interactions

Vortex quantum numbers

$SU(2)_k$ = ‘deformation’ of SU(2)

with finite set of representations

$$0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \frac{k}{2}$$

fusion rules

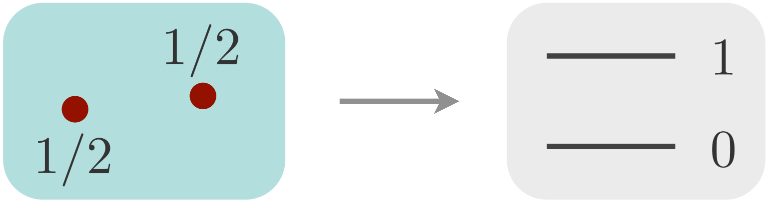
$$j_1 \times j_2 = |j_1 - j_2| + (|j_1 - j_2| + 1) + \ldots + \min(j_1 + j_2, k - j_1 - j_2)$$

example k = 2

$$1/2 \times 1/2 = 0 + 1$$

Vortex pair

$$1/2 \times 1/2 = 0 + 1$$



Energetics for many vortices

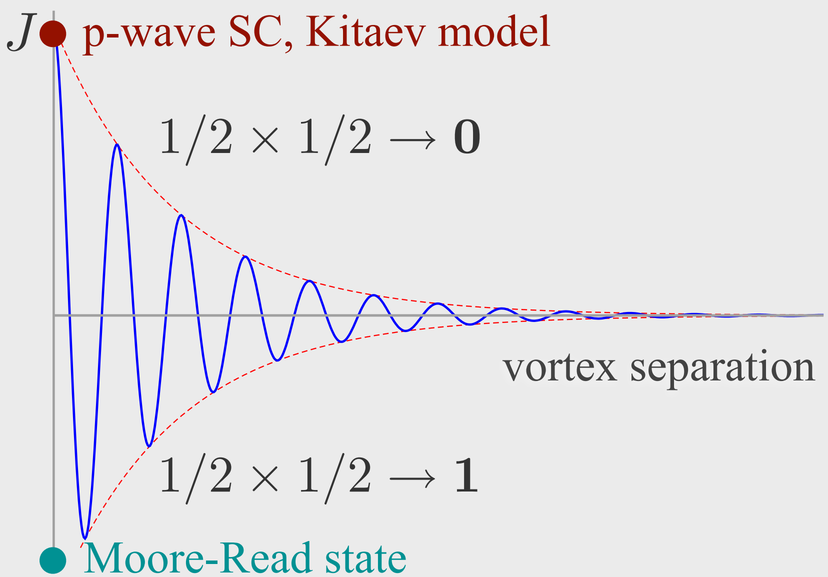
$$H = J \sum_{\langle ij \rangle} \prod_{ij}^{0}$$

“Heisenberg Hamiltonian”
for vortices

Vortex-vortex interactions

Microscopics

Which channel is favored is
not universal, but microscopic detail.



Vortex pair

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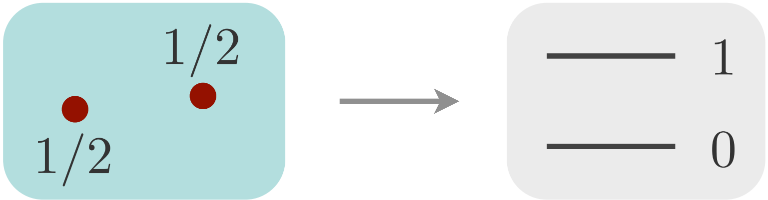

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"Heisenberg Hamiltonian"
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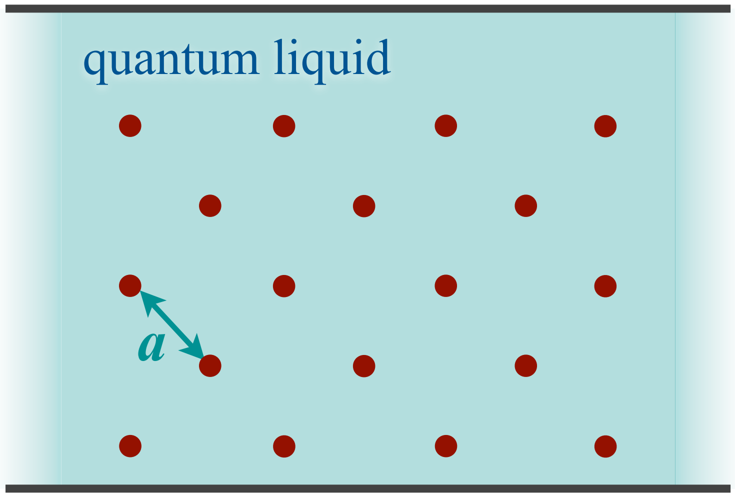
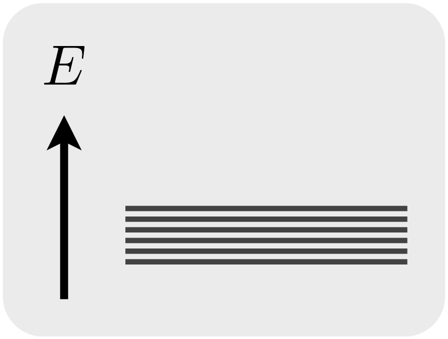
The many-vortex problem



$$a \gg \xi_m$$



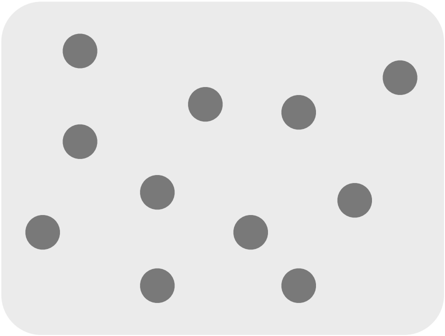
macroscopic degeneracy

vortex-vortex interactions

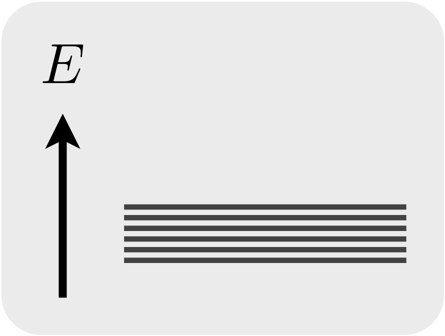
?

A conceptual question in good company

Some of the most intriguing phenomena in condensed matter physics arise from the splitting of ‘accidental’ degeneracies.



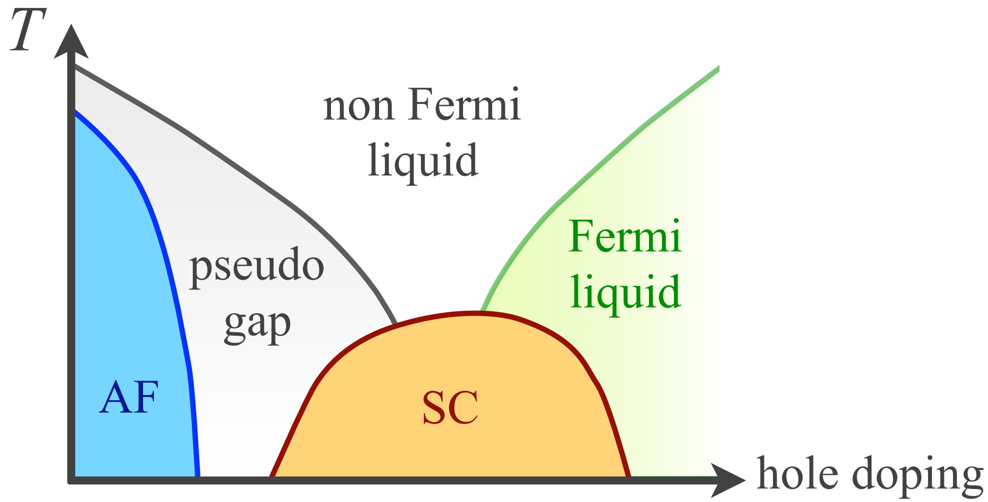
interacting
many-body system



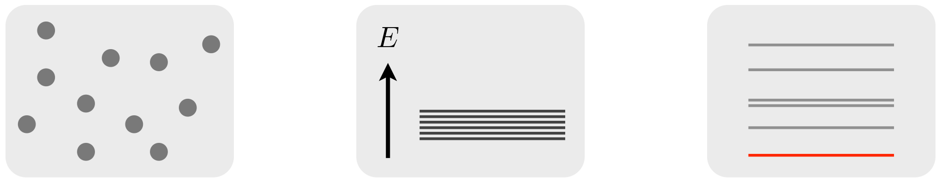
‘accidental’
degeneracy

residual effects
select ground state

phase diagram of
cuprate superconductors



Examples – frustrated magnets



interacting
many-body system

‘accidental’
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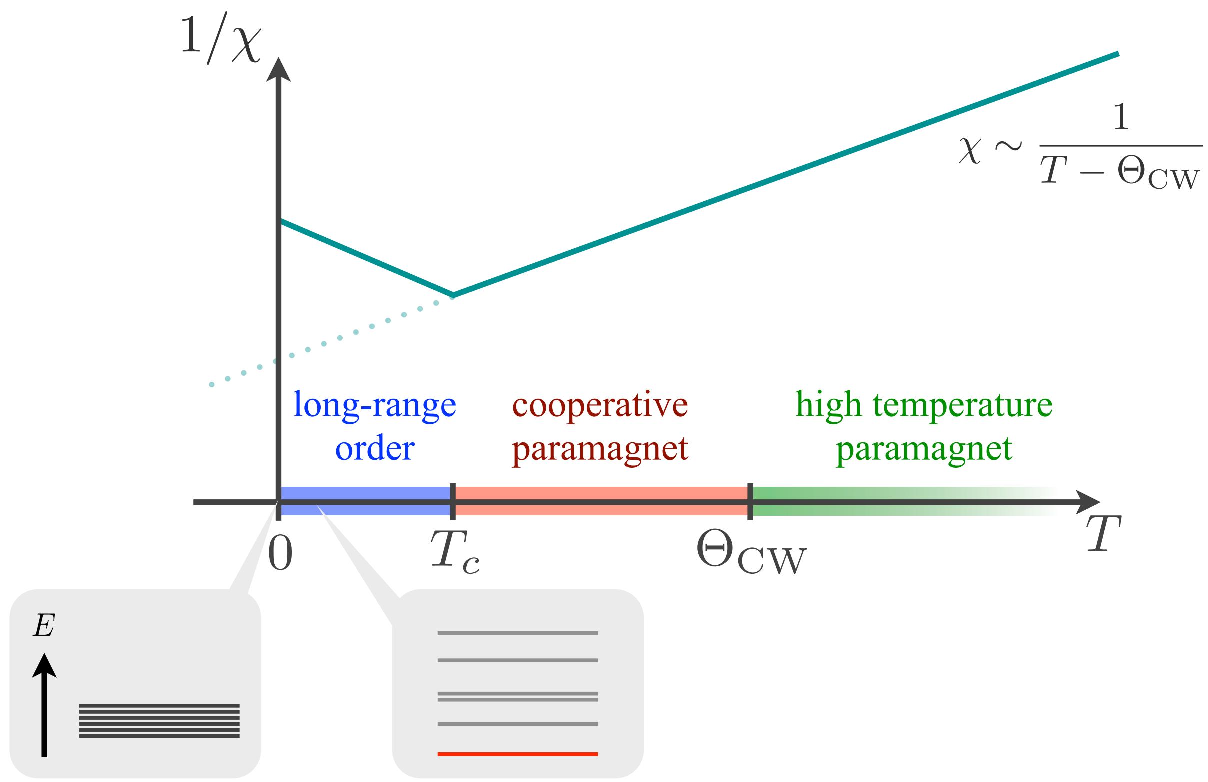
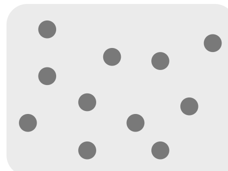
residual effects
select ground state



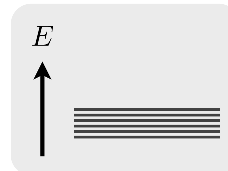
T=0 residual entropy

long-range order

Examples – quantum Hall liquids



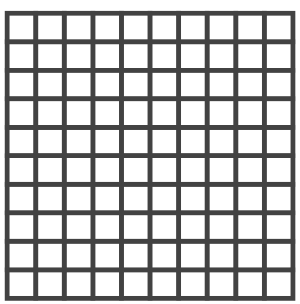
interacting
many-body system



'accidental'
degeneracy

residual effects
select ground state

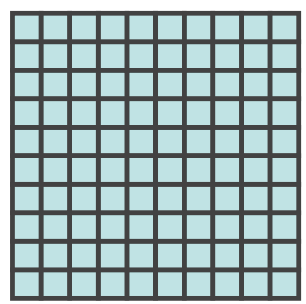
Landau level **degeneracy**



$$2\Phi/\Phi_0$$

orbital states

integer quantum Hall

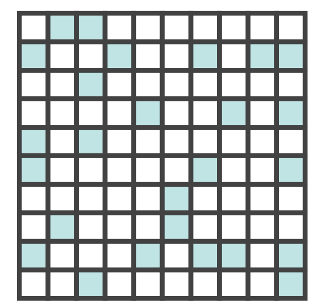


filled level

↓

incompressible liquid

fractional quantum Hall

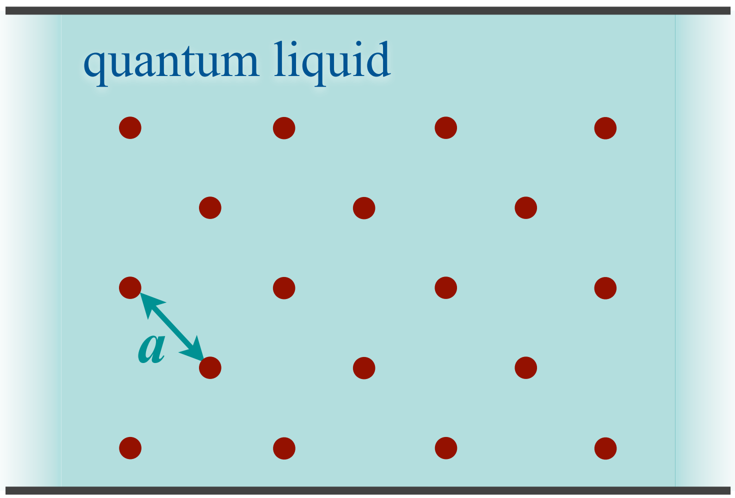


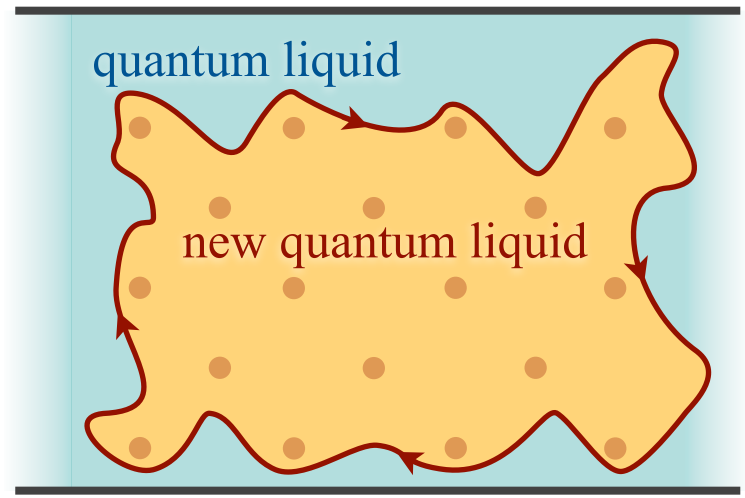
partially filled level

↓ Coulomb repulsion

incompressible liquid

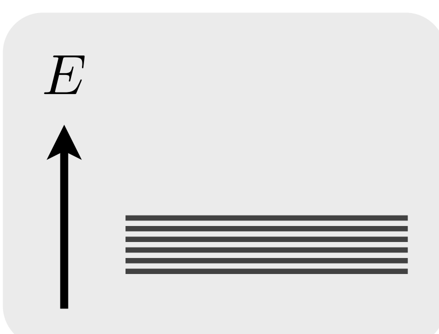
The many-vortex problem



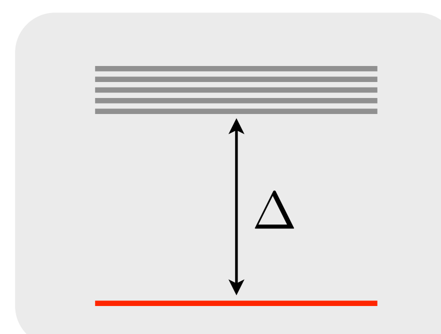


$a \gg \xi_m$

$a \approx \xi_m$



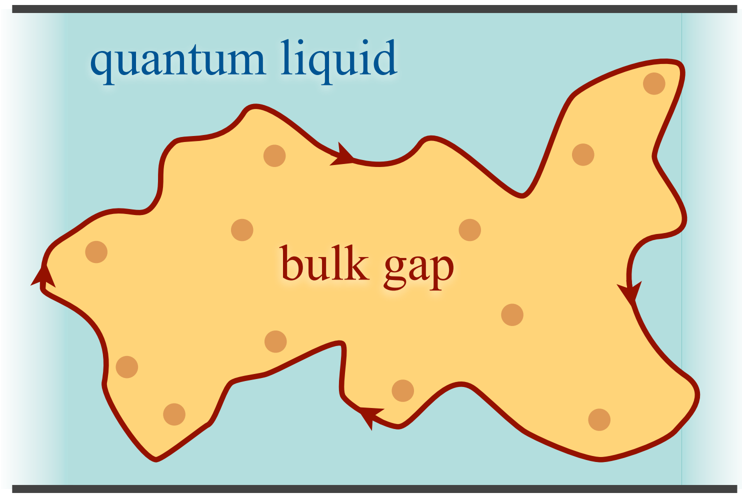
vortex-vortex interactions

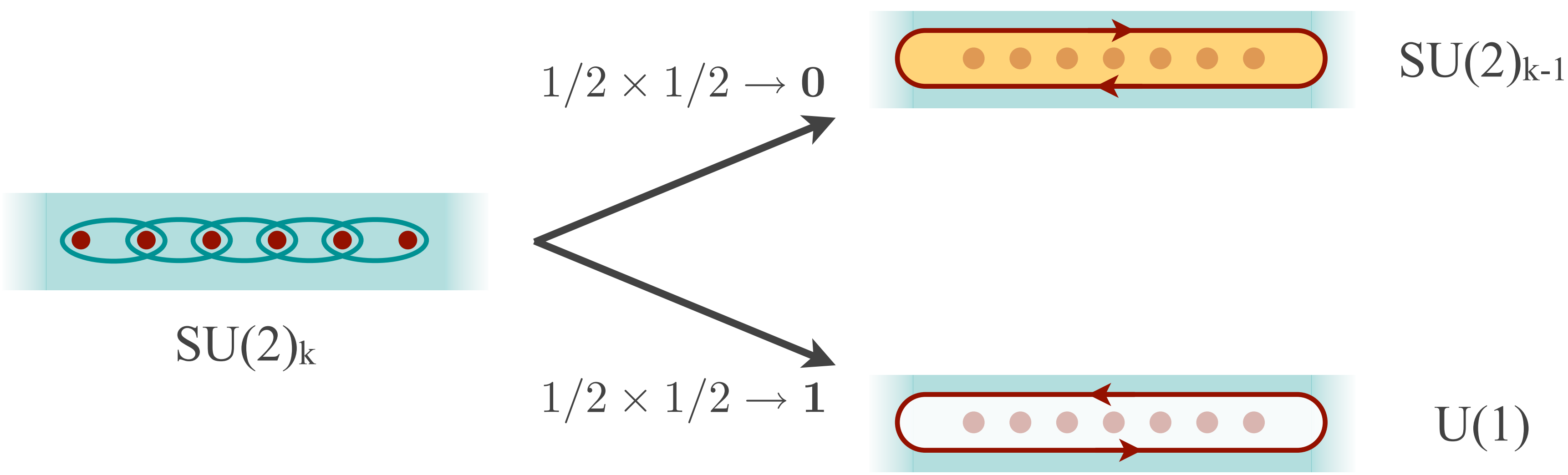


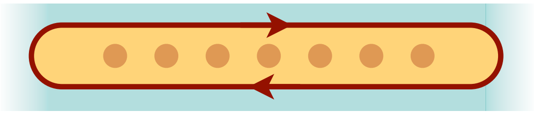
macroscopic degeneracy

unique ground state

The collective state





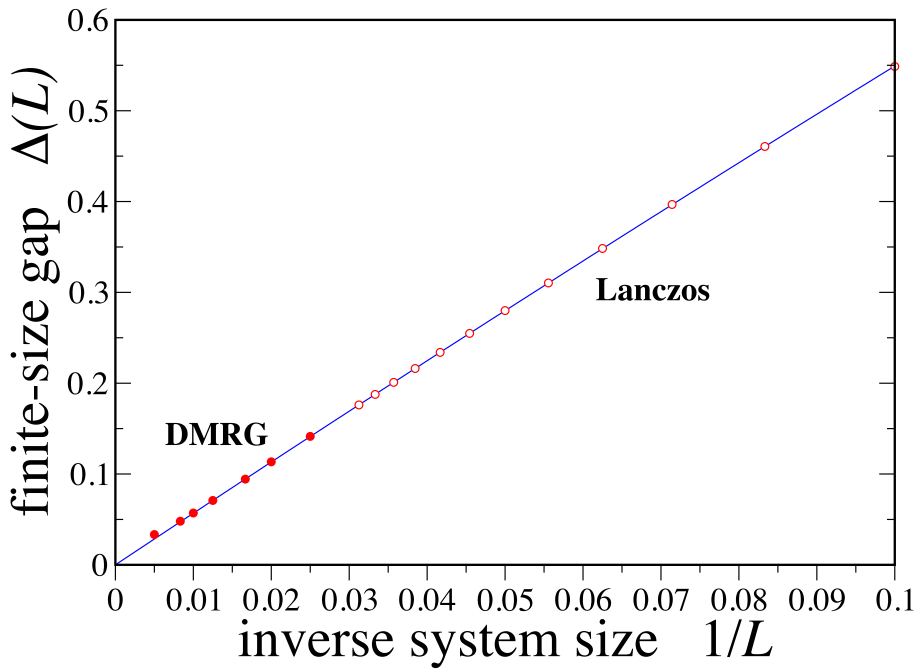


Edge states

Finite-size gap

$$\Delta(L) \propto (1/L)^{z=1}$$

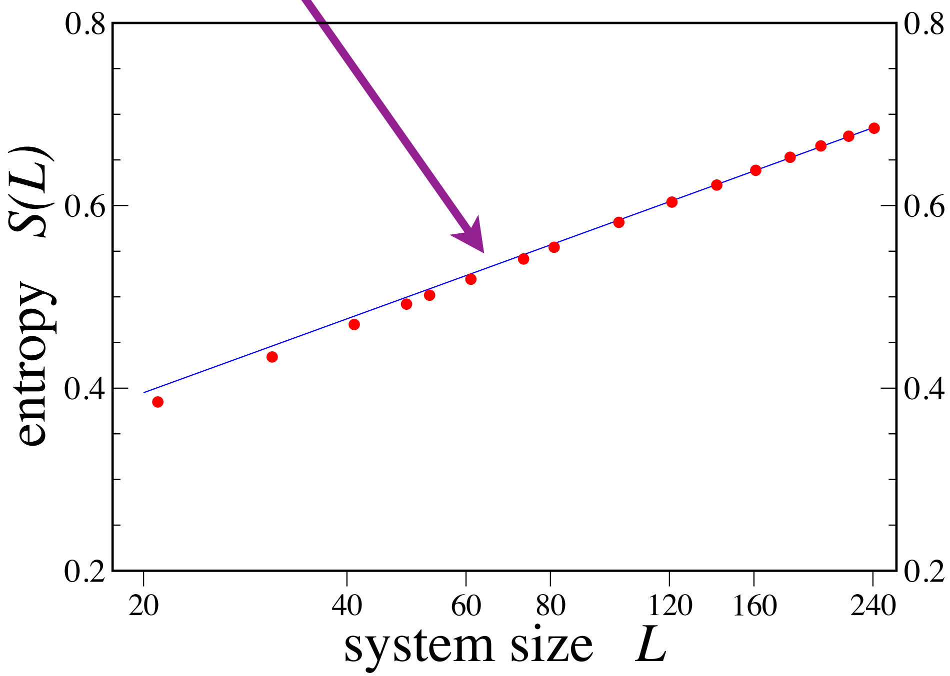
conformal field theory description

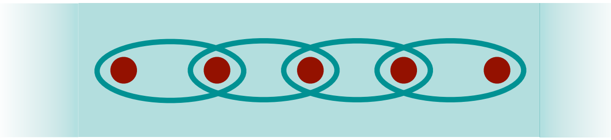


Entanglement entropy

$$S(L) \propto \frac{c}{3} \log L$$

central charge
$c = 7/10$





Gapless theories

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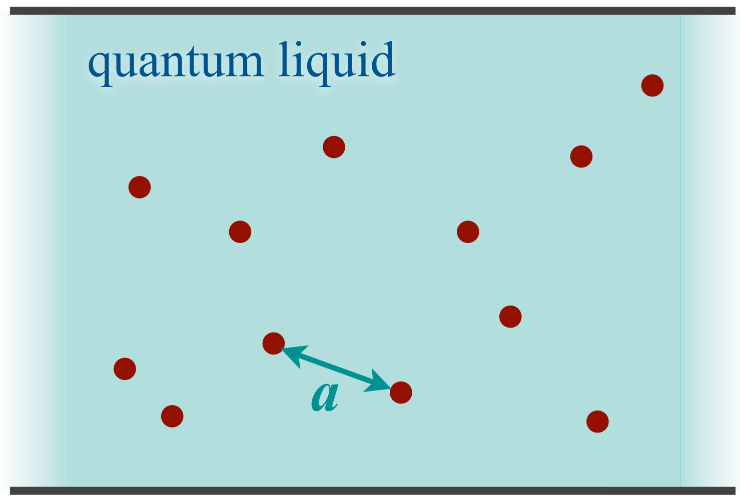
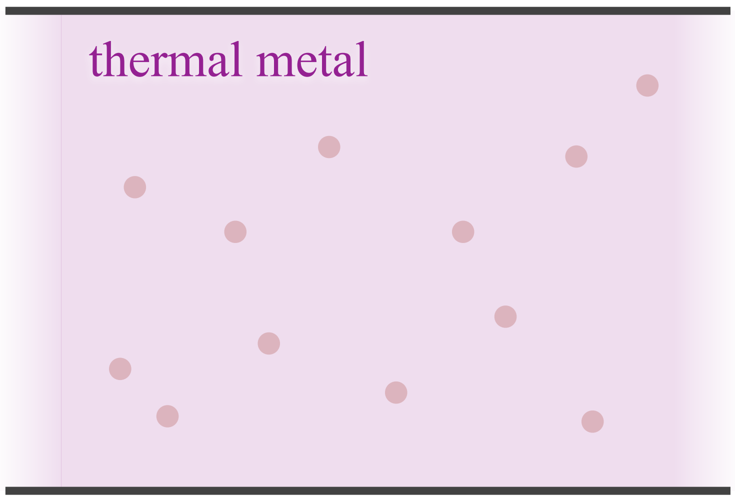
<tr><th>level k</th><th>$1/2 \times 1/2 \rightarrow \mathbf{0}$
'antiferromagnetic'</th><th>$1/2 \times 1/2 \rightarrow \mathbf{1}$
'ferromagnetic'</th></tr>
<tr><td>2</td><td>Ising
c = 1/2</td><td>Ising
c = 1/2</td></tr>
<tr><td>3</td><td>tricritical Ising
c = 7/10</td><td>3-state Potts
c = 4/5</td></tr>
<tr><td>4</td><td rowspan="2">$\frac{SU(2)_{k-1} \times SU(2)_1}{SU(2)_k}$</td><td rowspan="2">$\frac{SU(2)_k}{U(1)}$</td></tr>
<tr><td>5</td></tr>
<tr><td>k</td><td>k-critical Ising
c = 1-6/(k+1)(k+2)</td><td>Z_k-parafermions
c = 2(k-1)/(k+2)</td></tr>
<tr><td>∞</td><td>Heisenberg AFM
c = 1</td><td>Heisenberg FM
c = 2</td></tr>
</table>

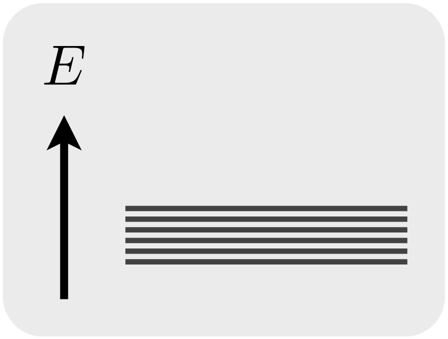
Interactions + disorder

Disorder induced phase transition





$a \gg \xi_m$



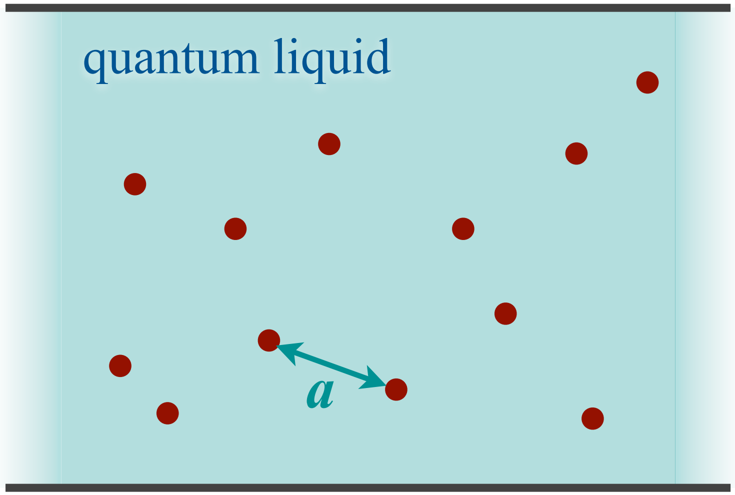
macroscopic degeneracy

disorder +
vortex-vortex
interactions

$a \approx \xi_m$

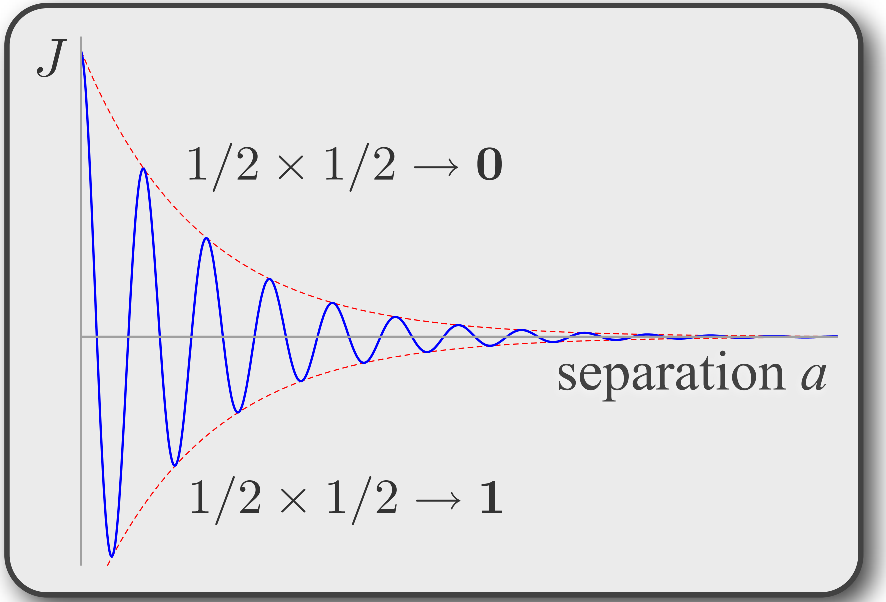
degeneracy is **split**

Interactions and disorder



$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk}$$

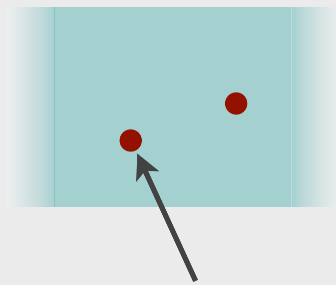
sign disorder
+ strong amplitude modulation



Natural analytical tool:
strong-randomness RG

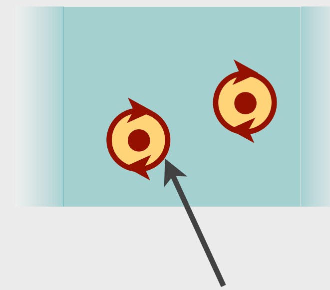
Unfortunately, this does not work.
The system flows *away* from strong disorder under the RG.
No infinite randomness fixed point.

From Ising anyons to Majorana fermions



Ising anyon

$SU(2)_2$
quantum number



Majorana fermion
zero mode

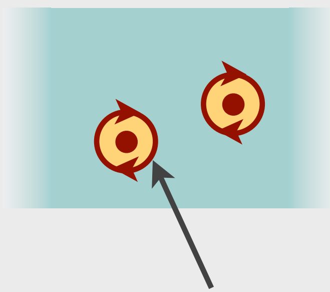
Majorana operator γ_i

$$H = \sum_{\langle jk \rangle} J_{jk} \Pi_{jk} \longrightarrow \mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

interacting Ising anyons
“anyonic Heisenberg model”

free Majorana fermion
hopping model

From Ising anyons to Majorana fermions



Majorana fermion
zero mode

Majorana operator γ_i

Majorana operators

$$\{\gamma_i, \gamma_j\} = \delta_{ij}$$

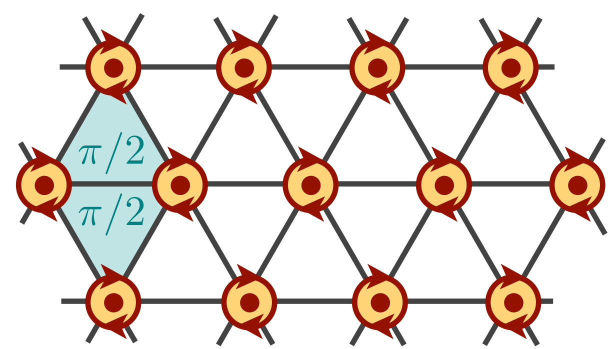
$$\gamma_i^\dagger = \gamma_i$$

$$\gamma_{i1} = (c_i^\dagger + c_i)/2$$

$$\gamma_{i2} = (c_i^\dagger - c_i)/2i$$

$$\mathcal{H} = -\sum_{\langle jk \rangle} i \mathcal{J}_{jk} \gamma_j \gamma_k$$

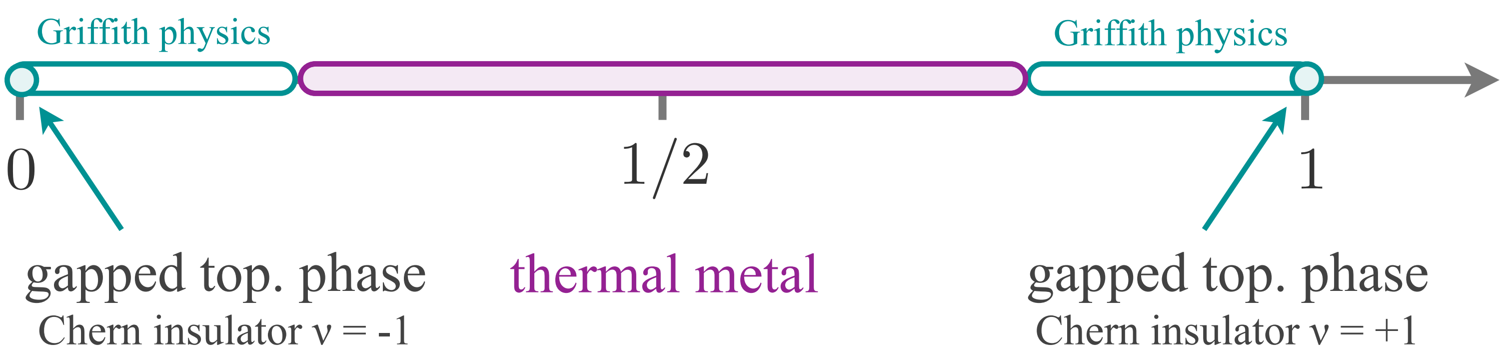
free Majorana fermion
hopping model



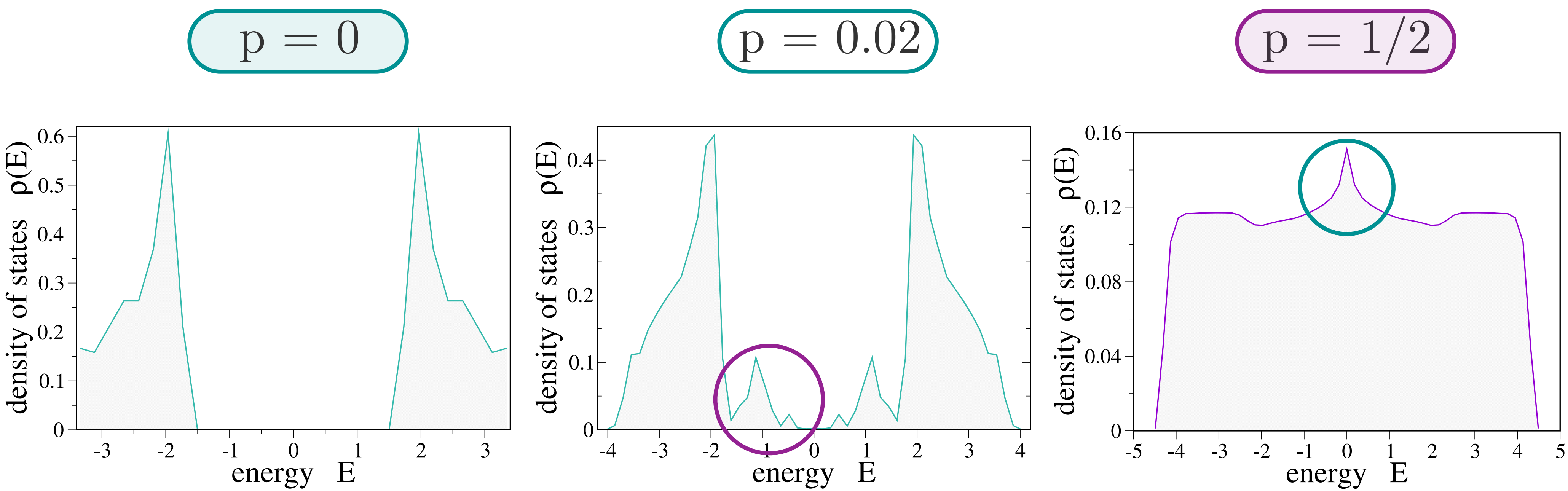
particle-hole symmetry ✓
time-reversal symmetry ✗

symmetry
class D

A disorder-driven metal-insulator transition

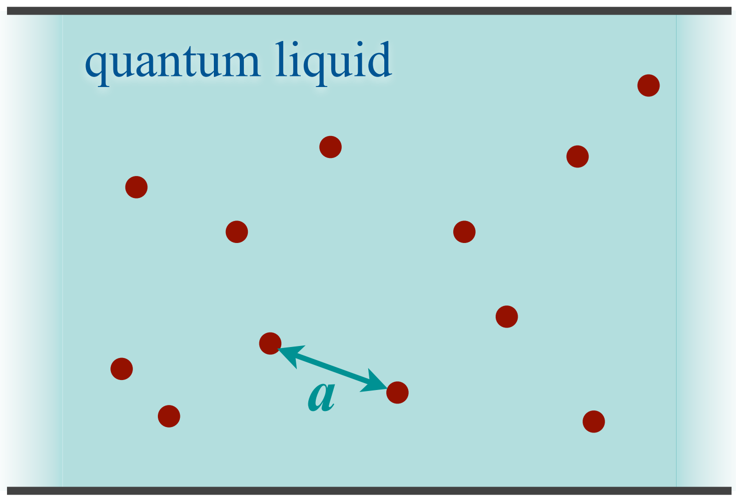


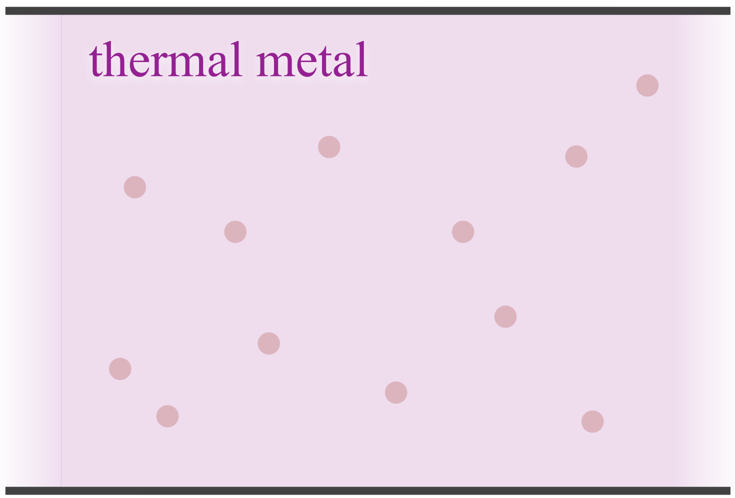
$$J = \begin{cases} +1, & p \\ -1, & 1-p \end{cases}$$



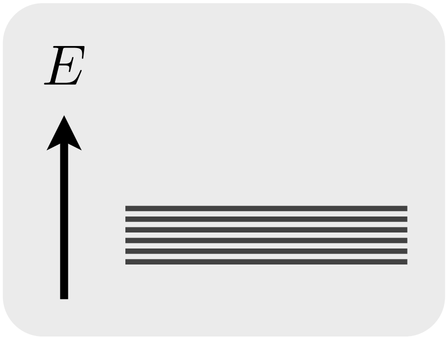
Density of states indicates phase transition.

Disorder induced phase transition





$a \gg \xi_m$



macroscopic degeneracy

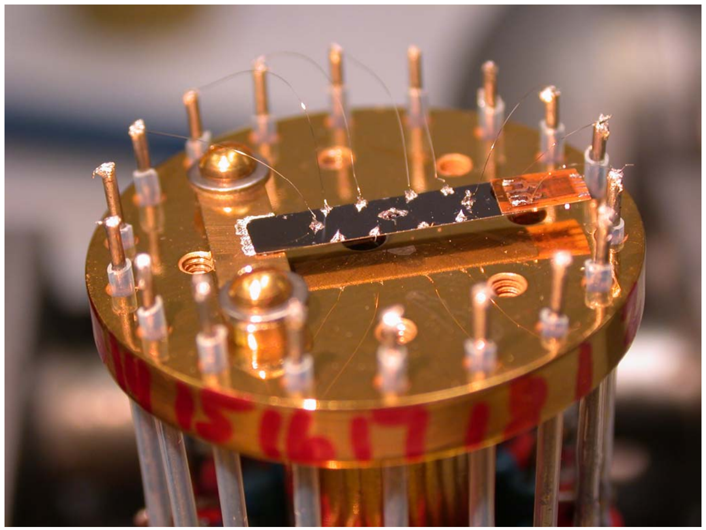
disorder +
vortex-vortex
interactions

$a \approx \xi_m$

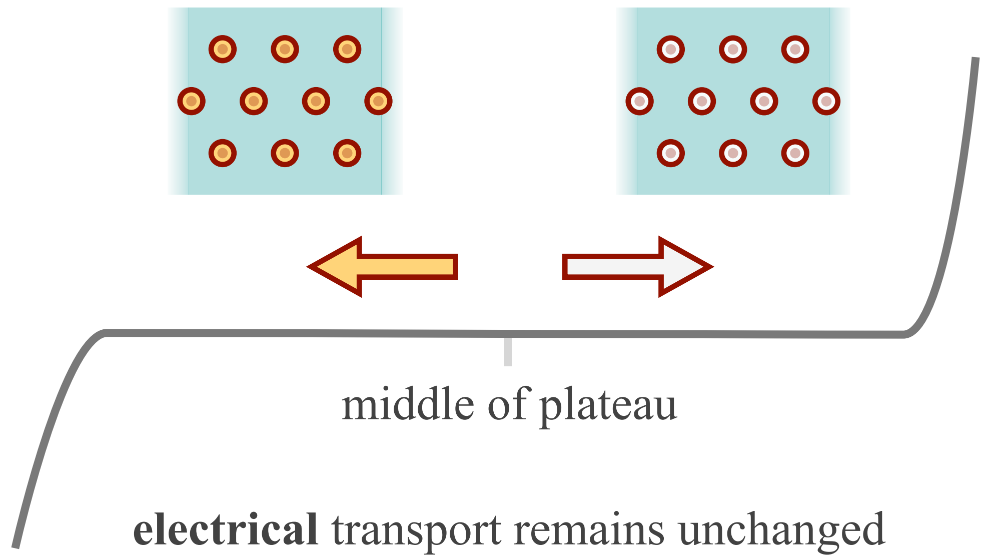
degeneracy is **split**

What does this tell us?

Heat transport



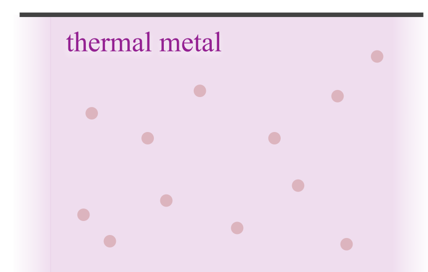
Caltech thermopower experiment

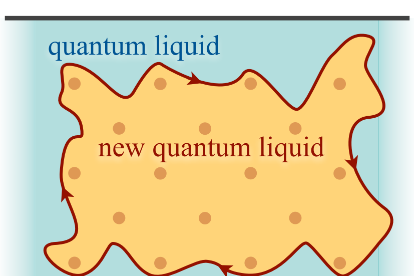


electrical transport remains unchanged

Bulk heat transport
diverges logarithmically
as T → 0.

$$\kappa_{xx}/T \propto \log T$$





Heat transport
along the sample **edges**
changes quantitatively

Collective states – a good thing?

The interaction induced splitting
of the degenerate manifold = qubit states
is yet another obstacle to overcome.

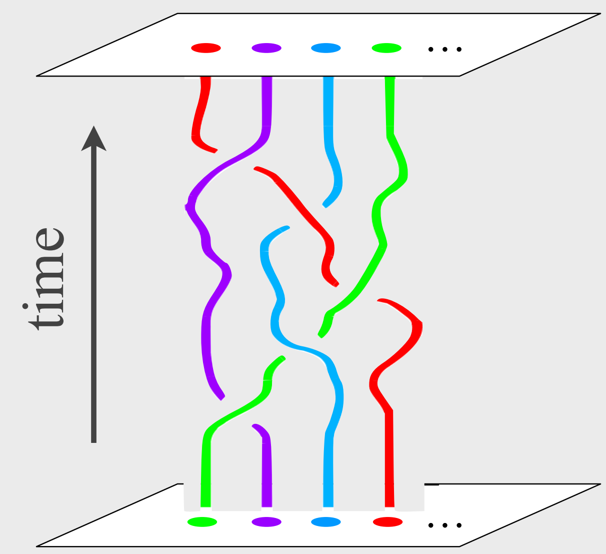
The formation of collective states renders all ideas of manipulating individual anyons inapplicable.

Probably, a topological quantum computer works best at **finite** temperatures.

Topological quantum computing

Degenerate manifold = qubit

Employ **braiding** of non-Abelian vortices to perform computing (unitary transformations).



Summary

- Lord Kelvin was way ahead of his time.
- Topology has re-entered physics in many ways.
- Topological excitations + interactions + disorder can give rise to a plethora of collective phenomena.
  - Topological liquid nucleation
  - Thermal metal
  - Distinct experimental bulk observable (heat transport) in search for Majorana fermions.