Monitored Topology

Measurements, Learning, and Nishimori physics

Simon Trebst
University of Cologne

CRC 183
ENTANGLED STATES OF MATTER

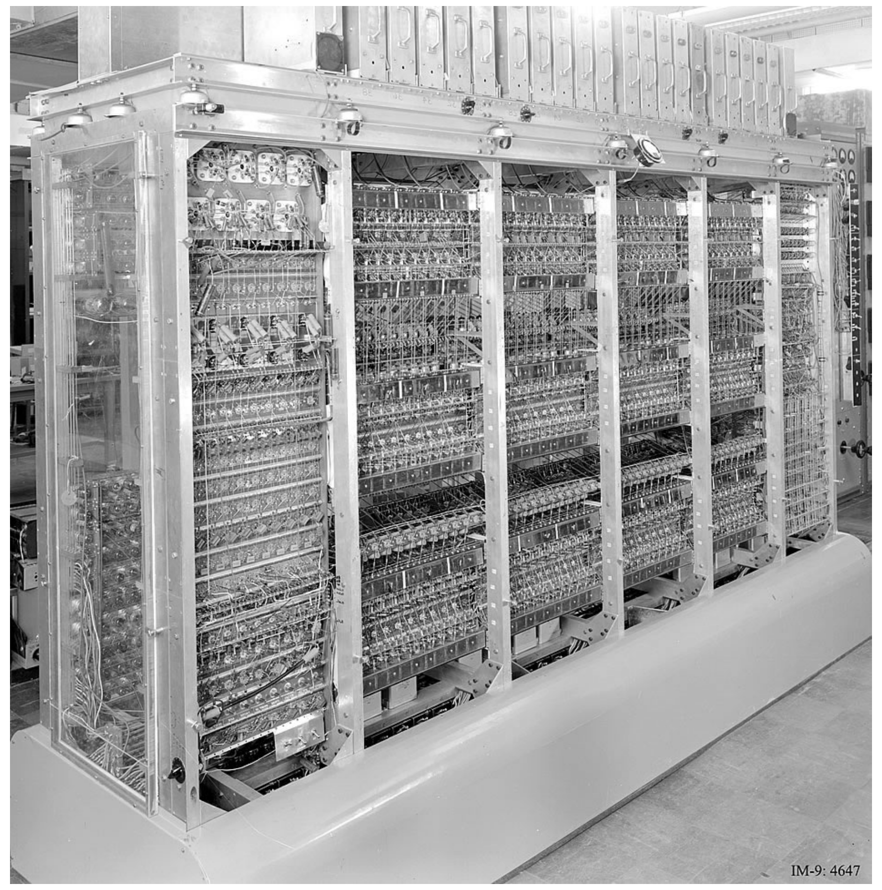
Seminar, TU Munich

Munich, May 2026

ML4Q
MATTER AND LIGHT FOR
QUANTUM COMPUTING

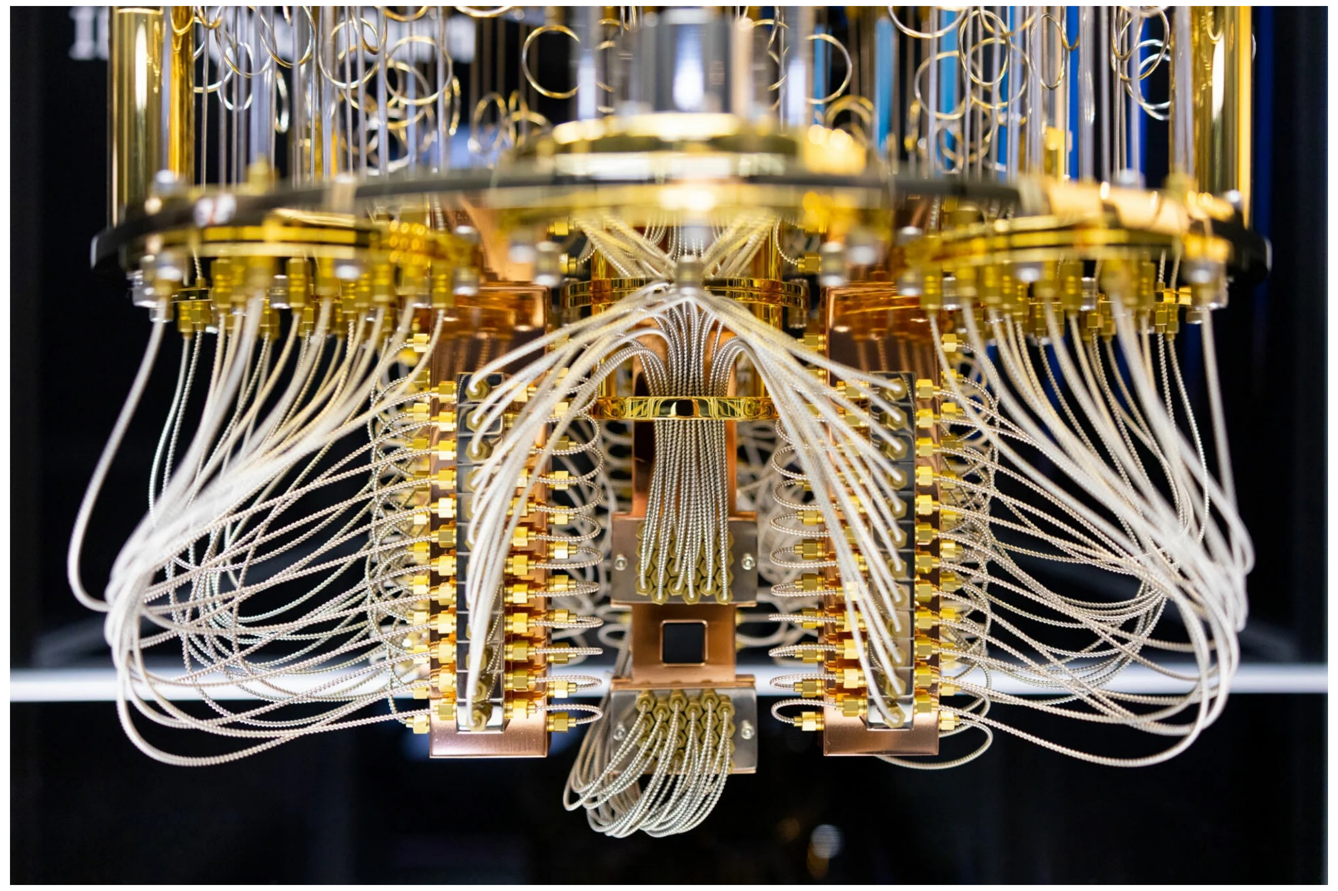
computational physics

Maniac @ Los Alamos



Cray @ ETH Zurich

IBM Quantum (cloud)



(1953) — (1992) — (2019) →

Metropolis algorithm

DMRG
tensor networks

measurement-based
quantum computation

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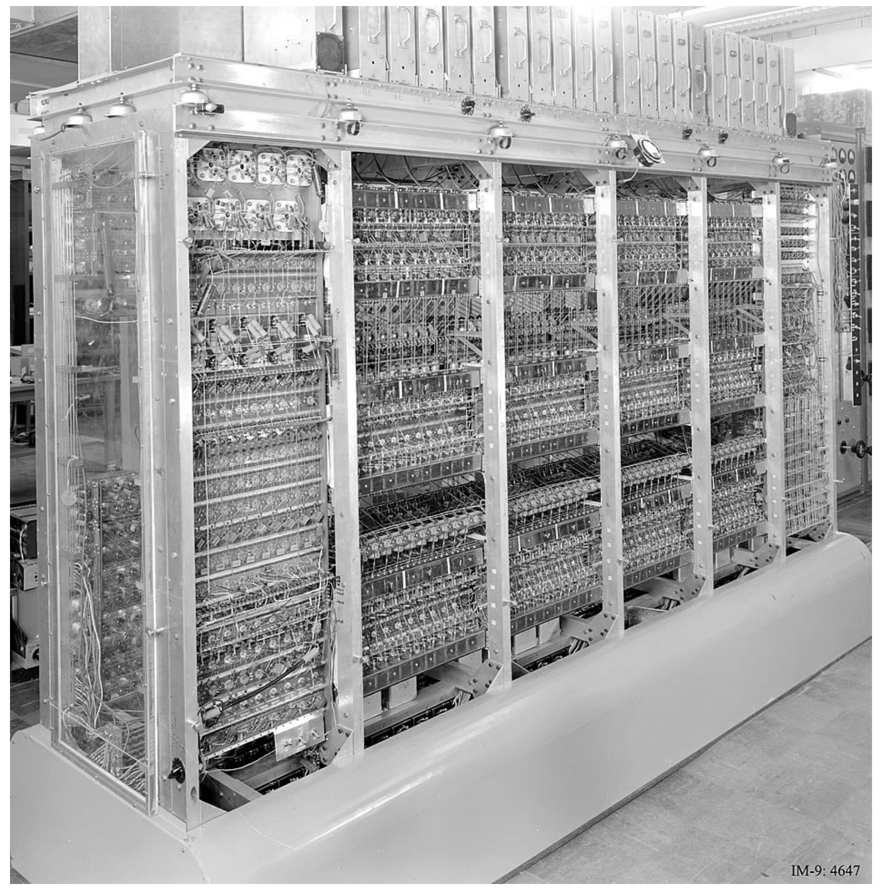
computational physics

Maniac @ Los Alamos



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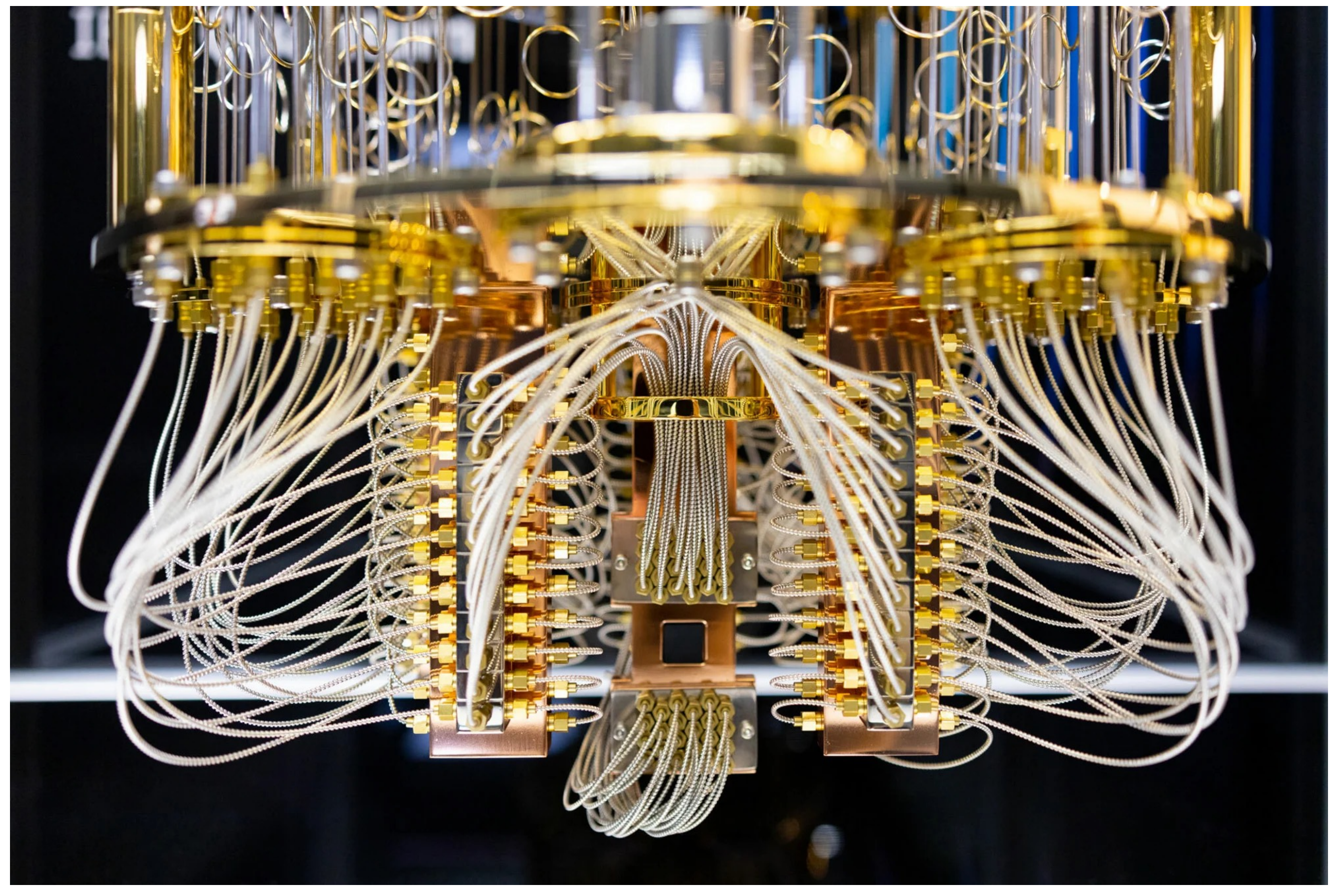

(1953) — (1992) — (2019) →

classical many-body

quantum many-body

$(\text{quantum})^2$ many-body

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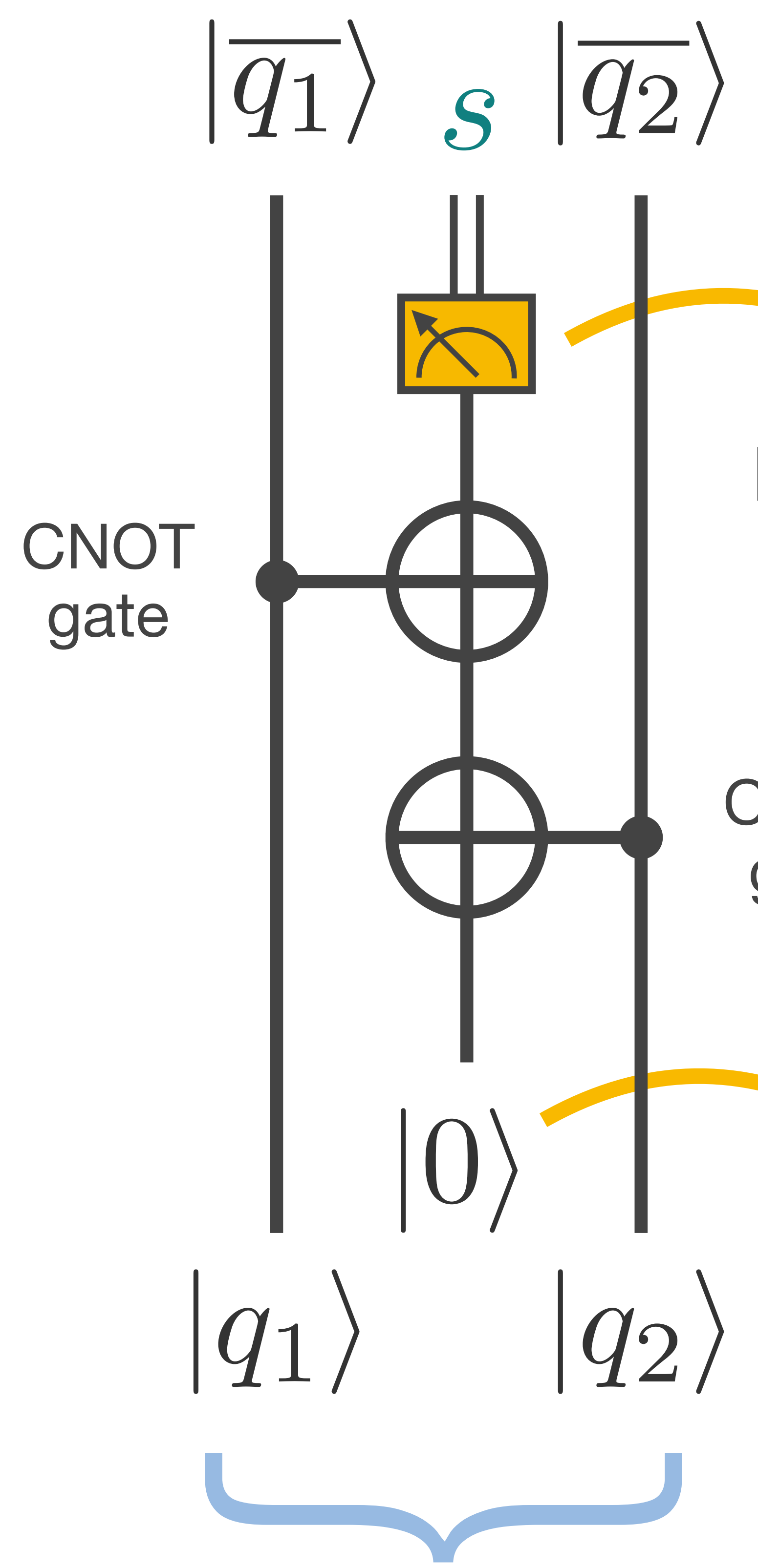
measurement-based quantum computation

quantum circuit for 2-qubit **joint measurement**



measurement

aux. qubit

data qubits

extract information

shape entanglement

condition randomness

quantum error correction

stabilizer & Floquet codes

learning & inference problems

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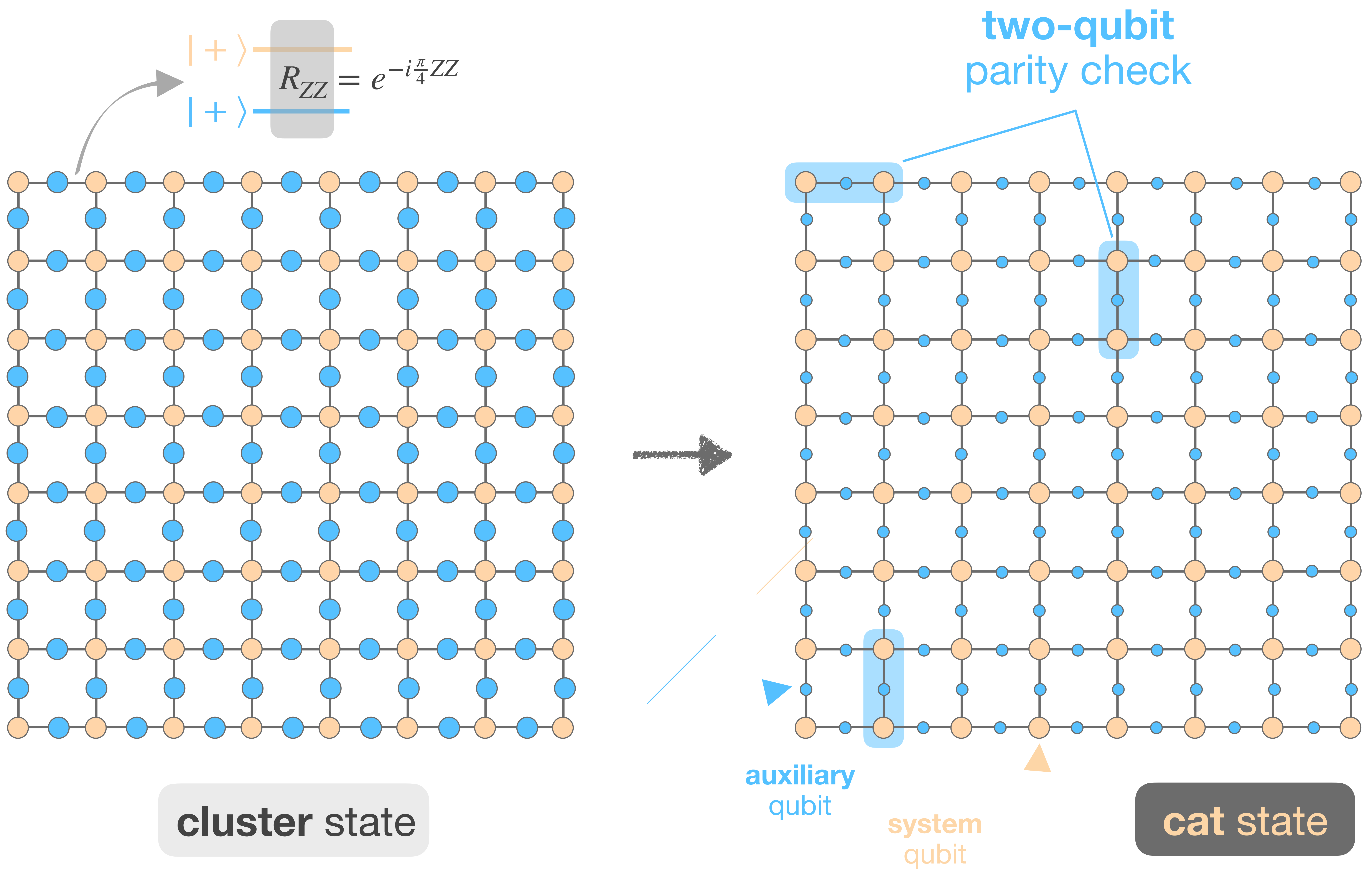
shallow circuits

state preparation

state preparation by measurement



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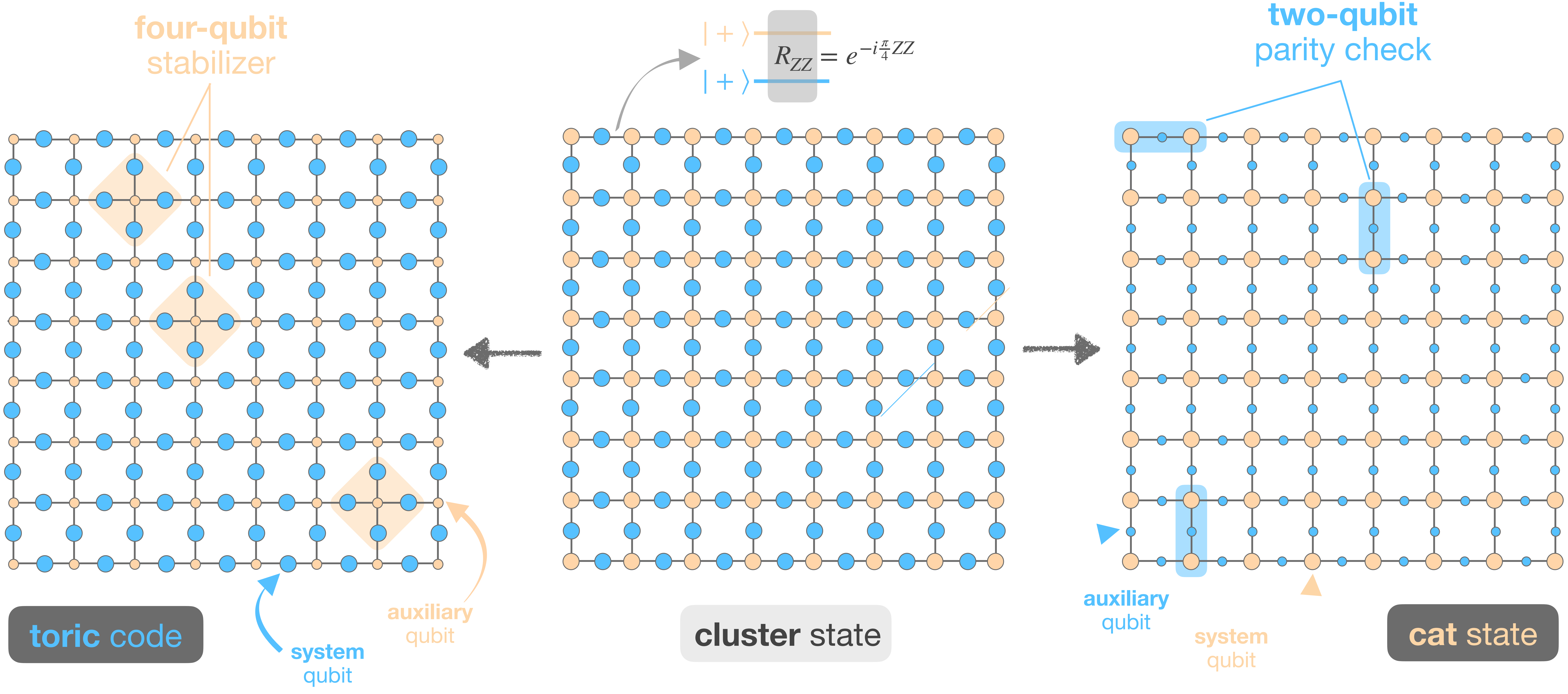
state preparation by measurement



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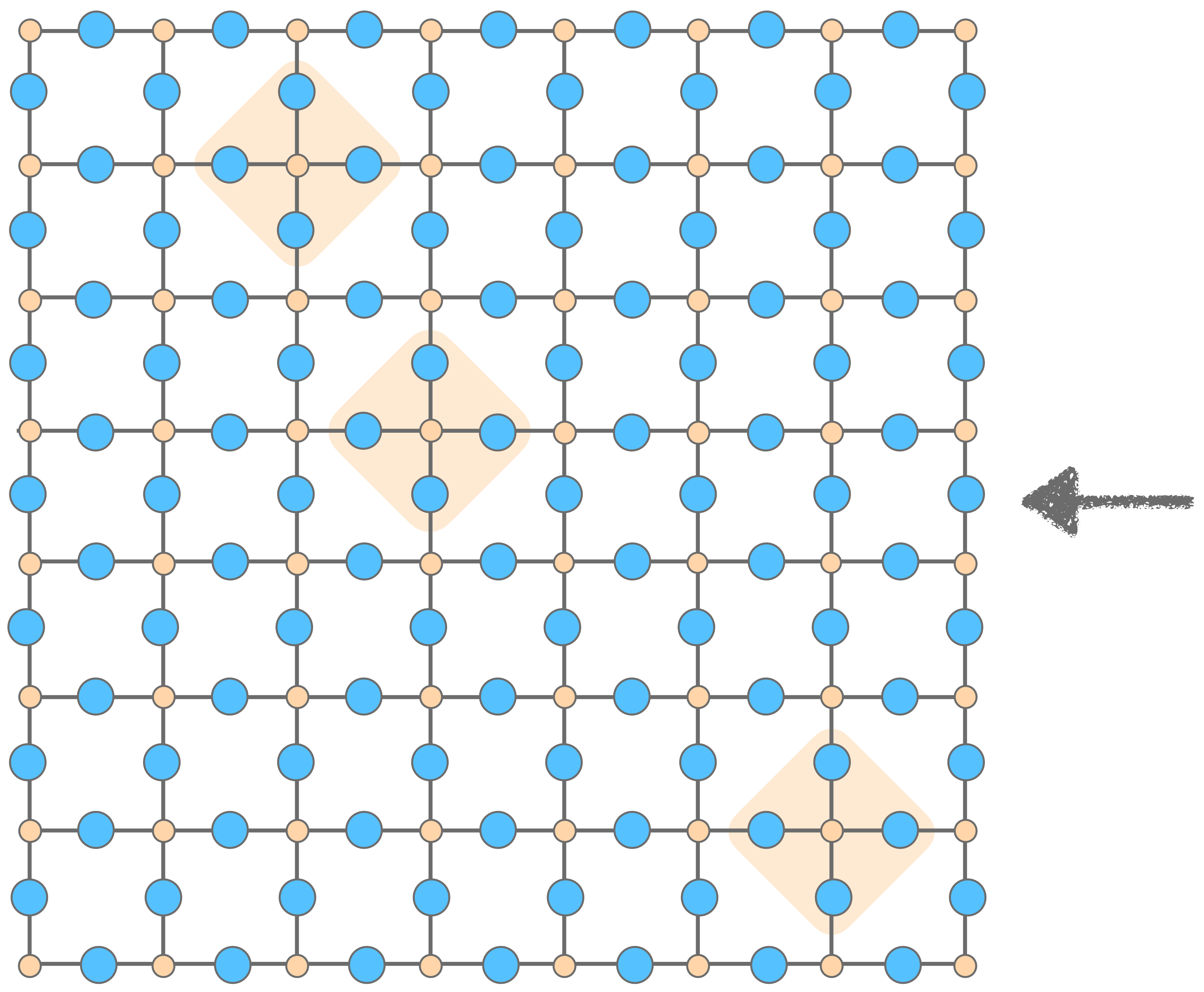
state preparation by measurement

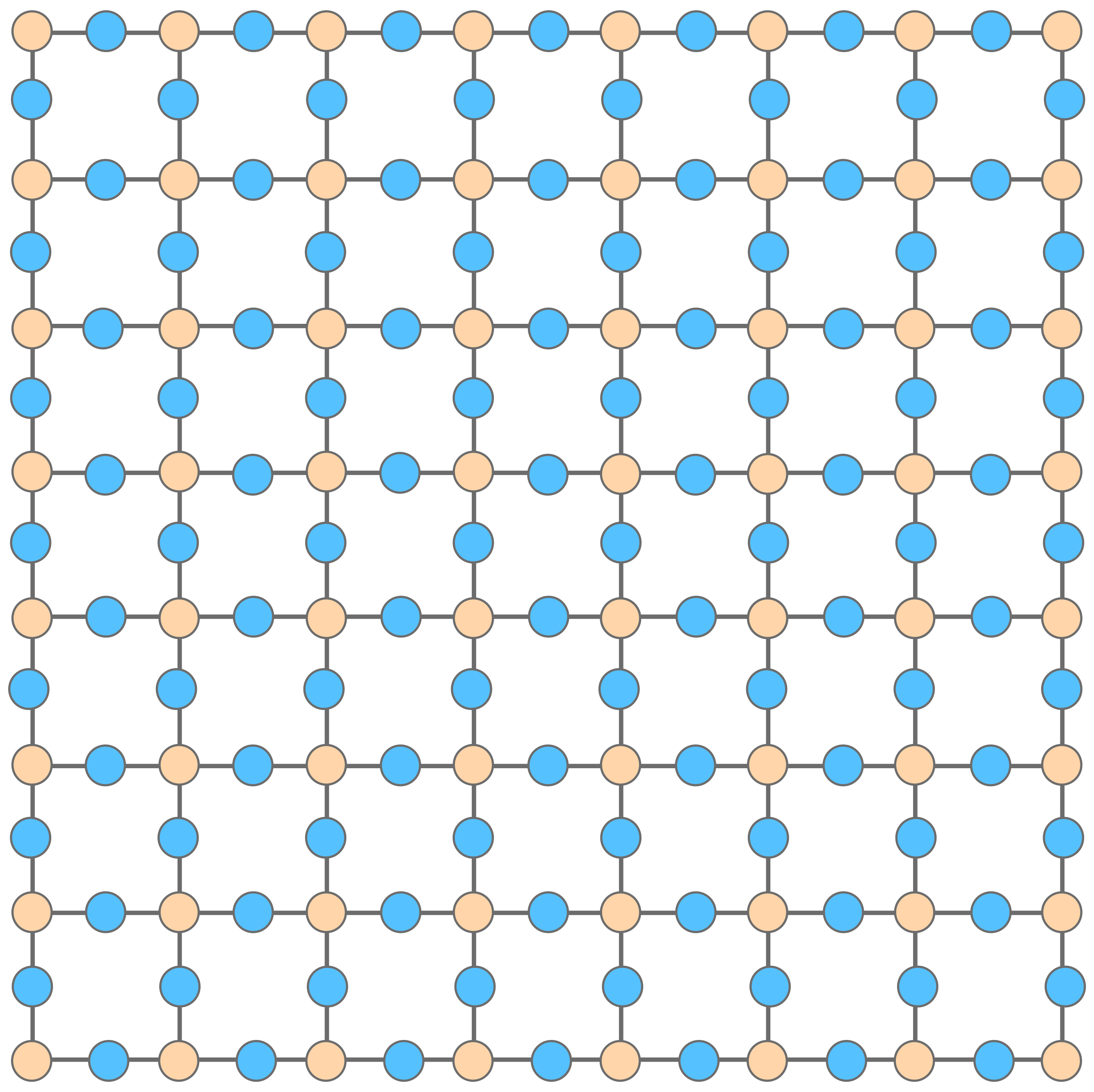
N. Tantivasadakarn, R. Thorngren, A. Vishwanath, and R. Verresen, *Long-Range Entanglement from Measuring Symmetry-Protected Topological Phases*, PRX (2024)



toric code

$Z_2^{(1)}$

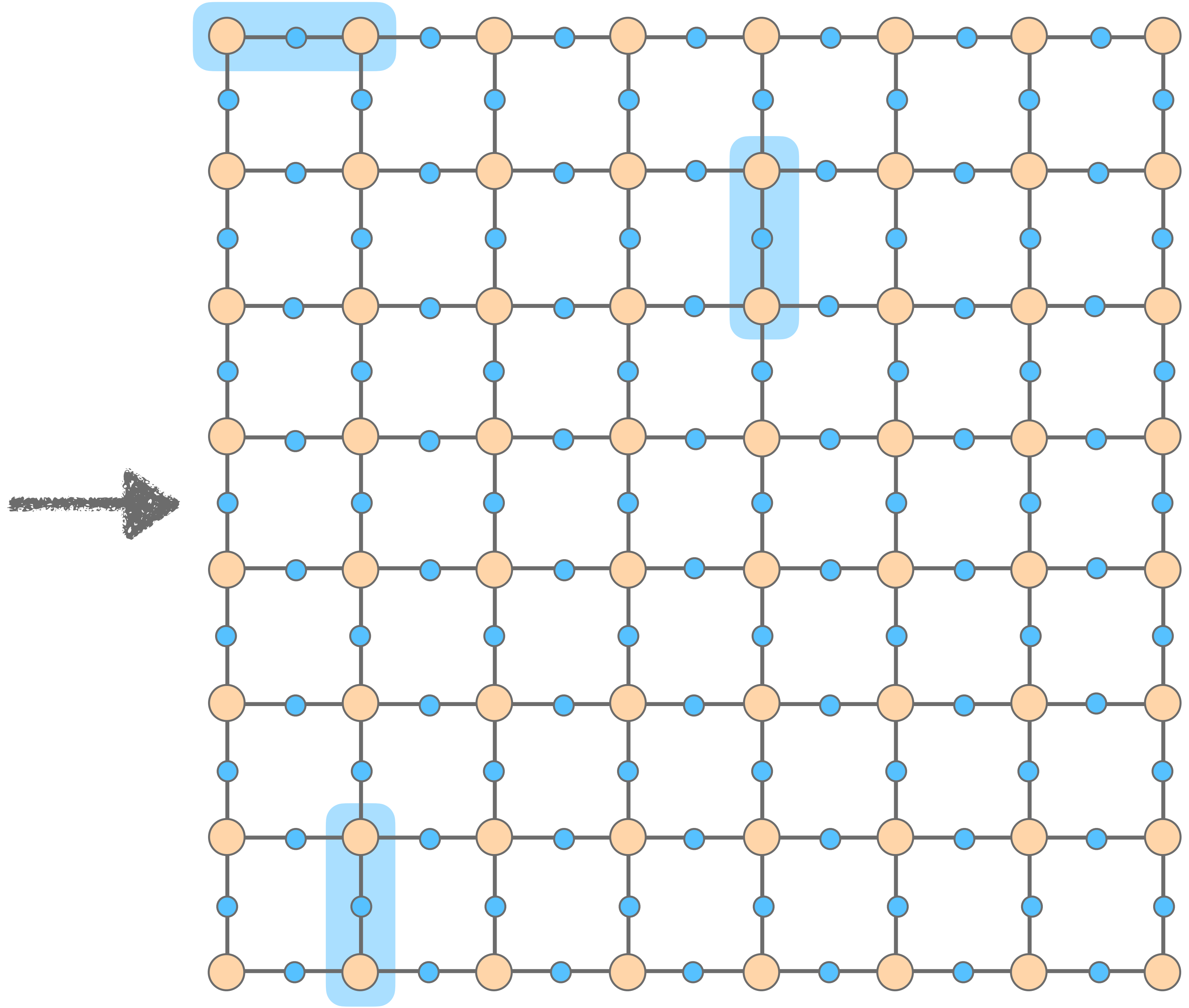
long-range entangled
topological order



cluster state

$Z_2 \times Z_2^{(1)}$

short-range entangled
SPT order



cat state

Z_2

long-range entangled
quantum order

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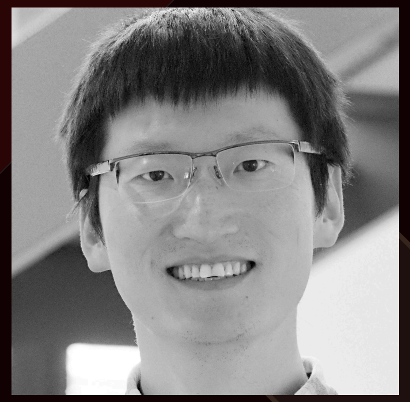
monitored toric codes

arXiv:2512.19786
arXiv:2502.14034
PRX Quantum **5**, 040313 (2024)

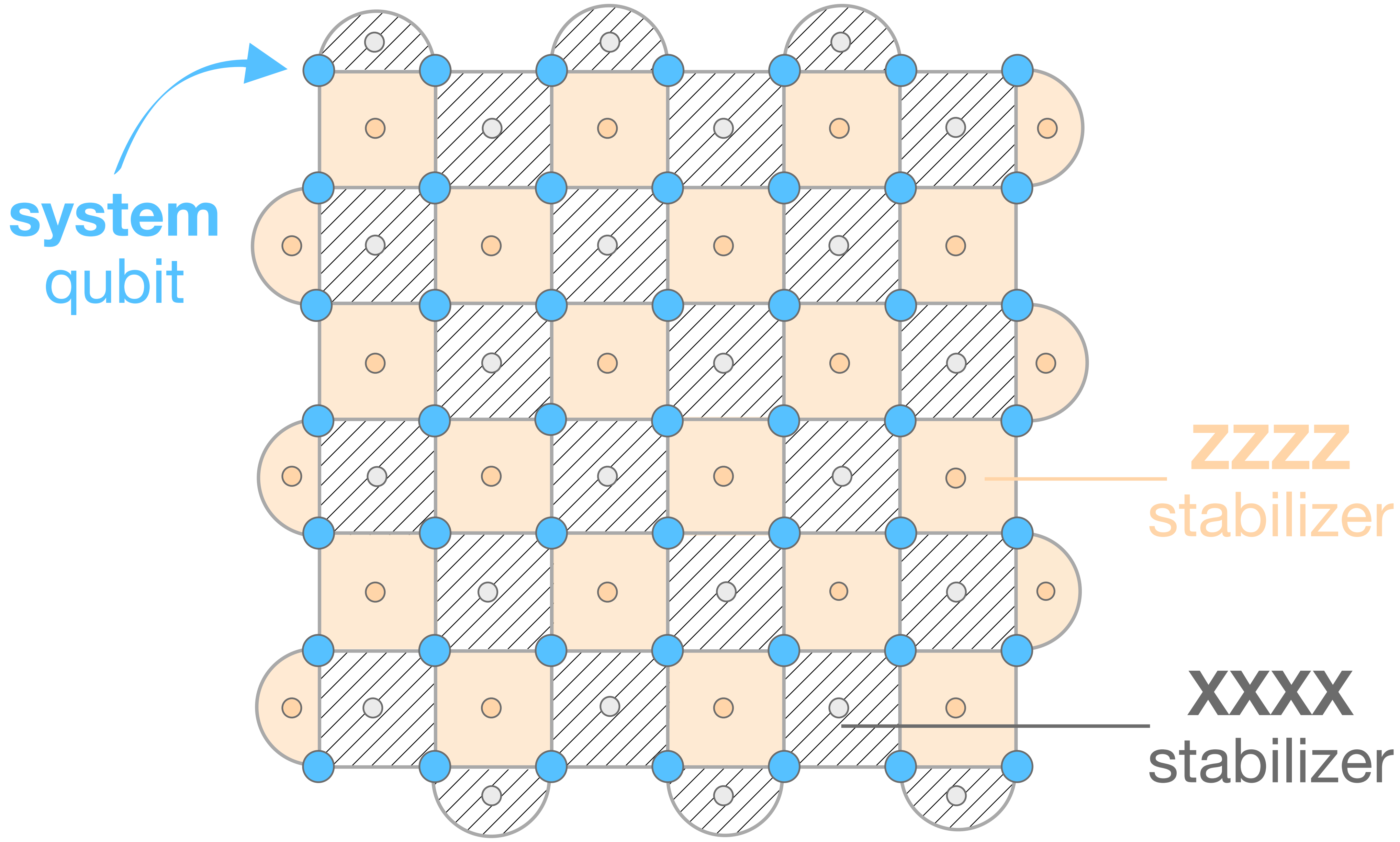
F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu



entanglement by measurement: **surface code**

Kitaev (1997)



d=7 surface code

The toric/surface code
is an ingenious
measurement protocol.

The toric/surface code
is an exactly solvable
Hamiltonian model.

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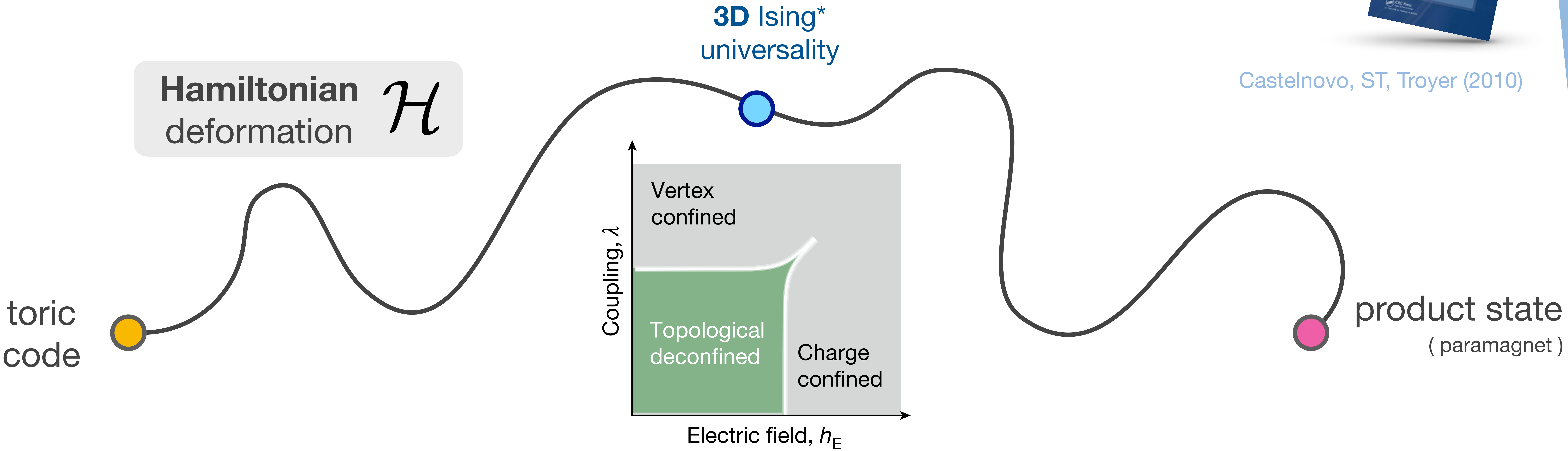
phase transitions & deformations



Castelnovo, ST, Troyer (2010)

Article

Visualizing dynamics of charges and strings in (2+1)D lattice gauge theories

Tyler A. Cochran *et al.*, Nature **642**, 315 (2025)

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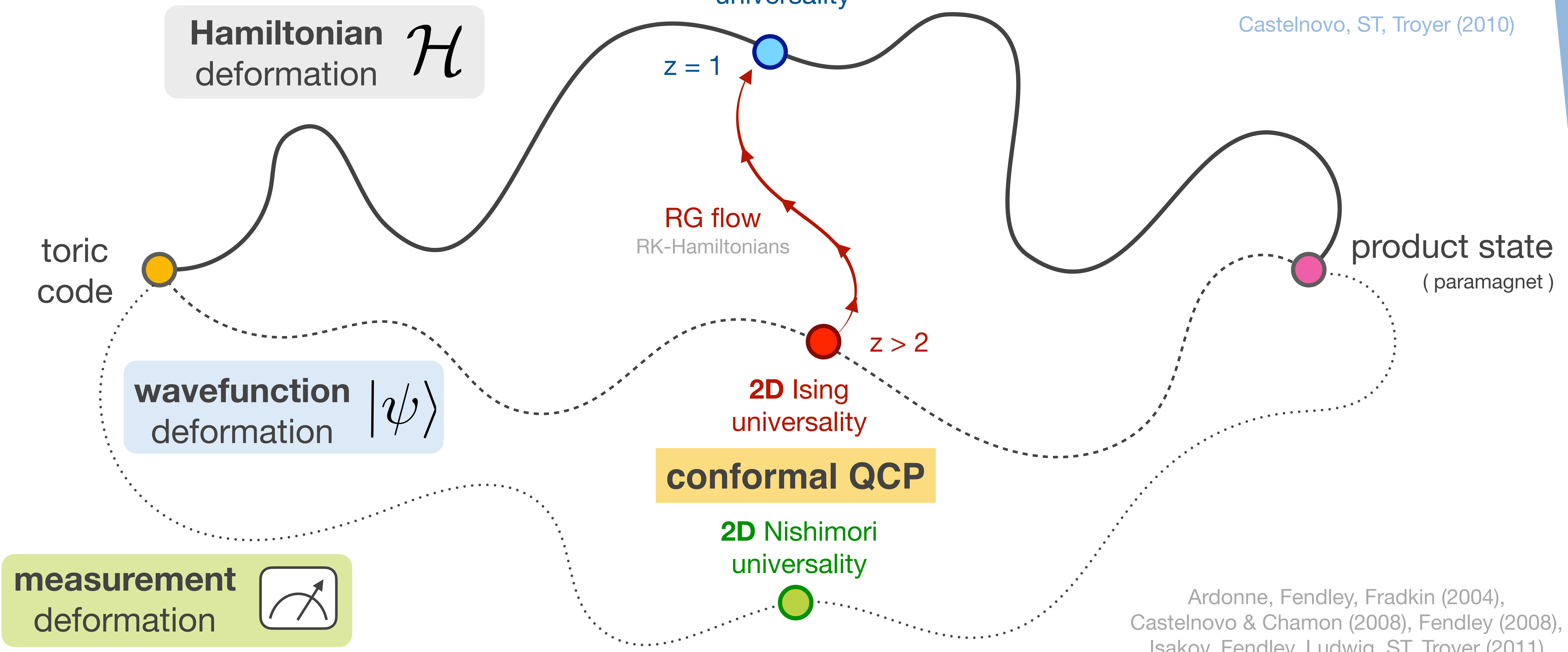
phase transitions & deformations



Castelnovo, ST, Troyer (2010)

Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008),
Isakov, Fendley, Ludwig, ST, Troyer (2011)

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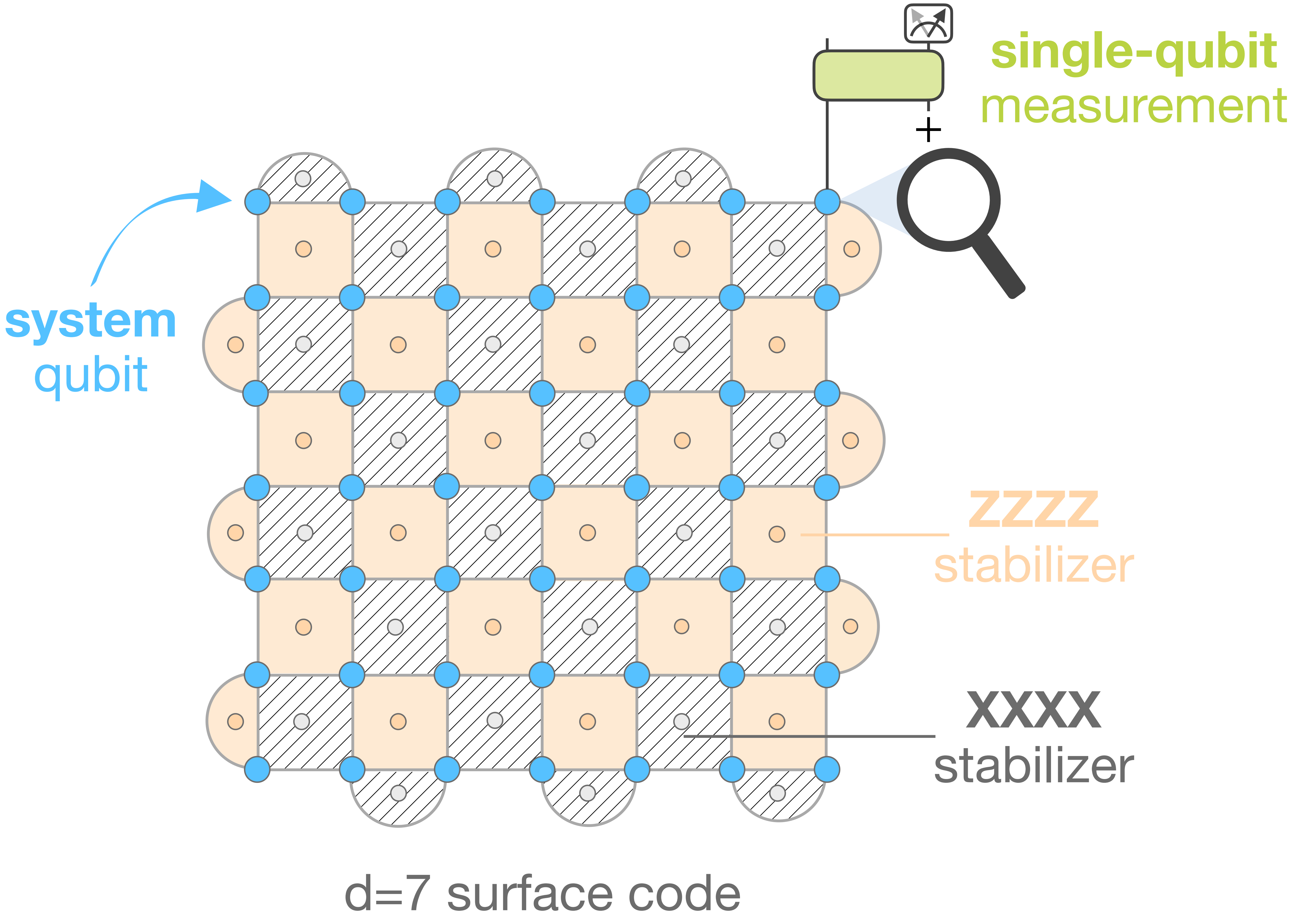
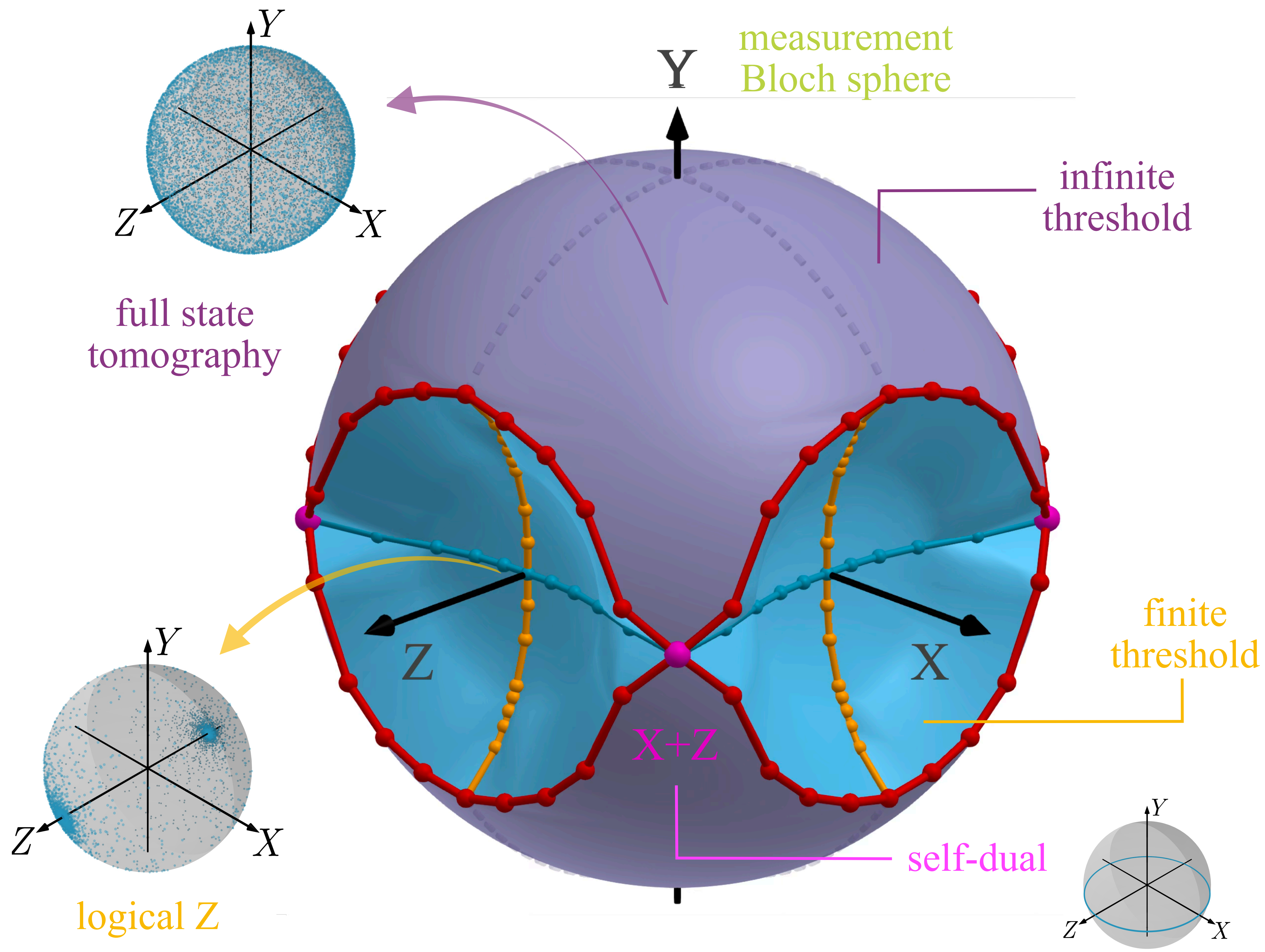
learning by measurement

Infer the logical qubit of a surface code from single-qubits measurements.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786





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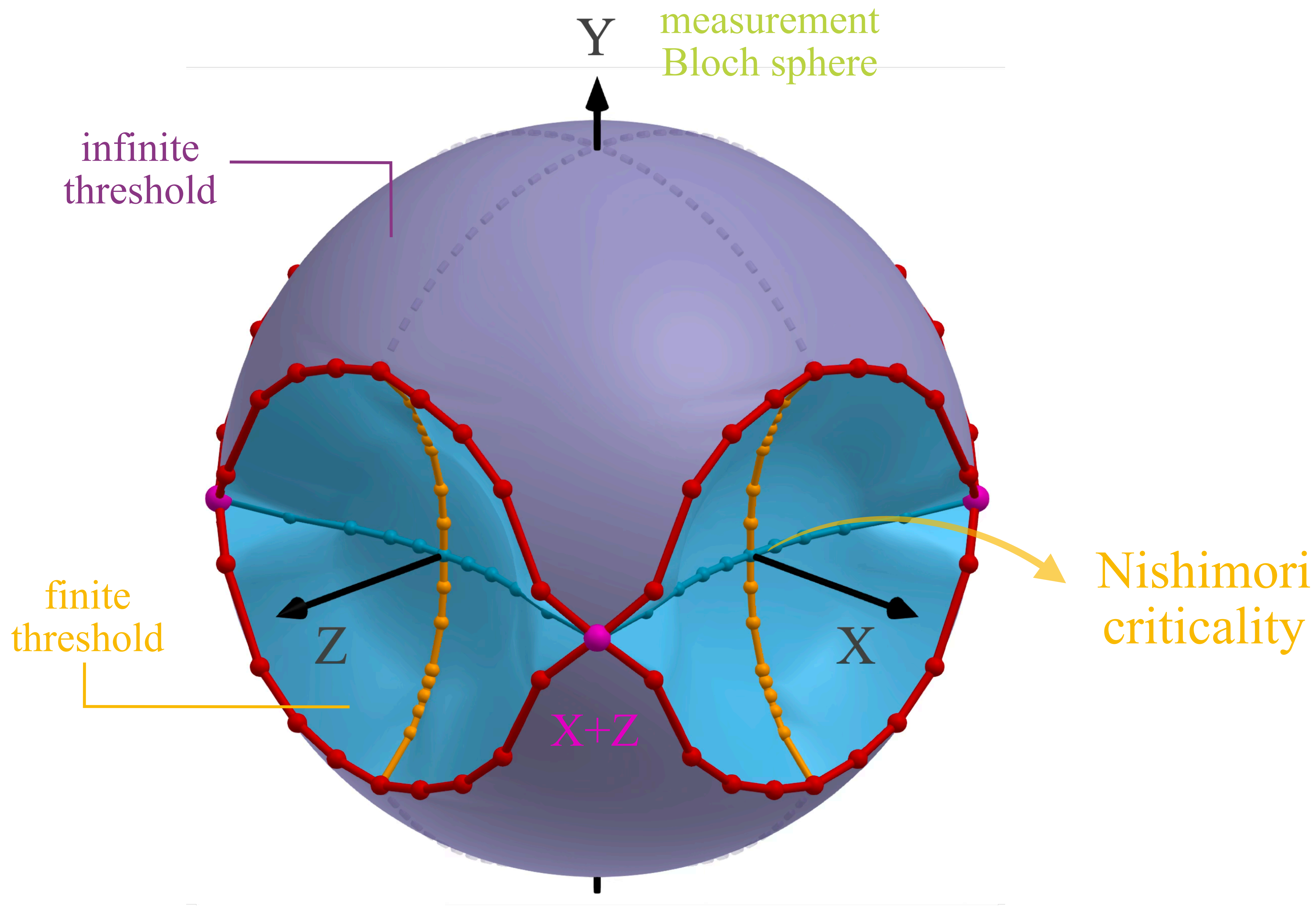
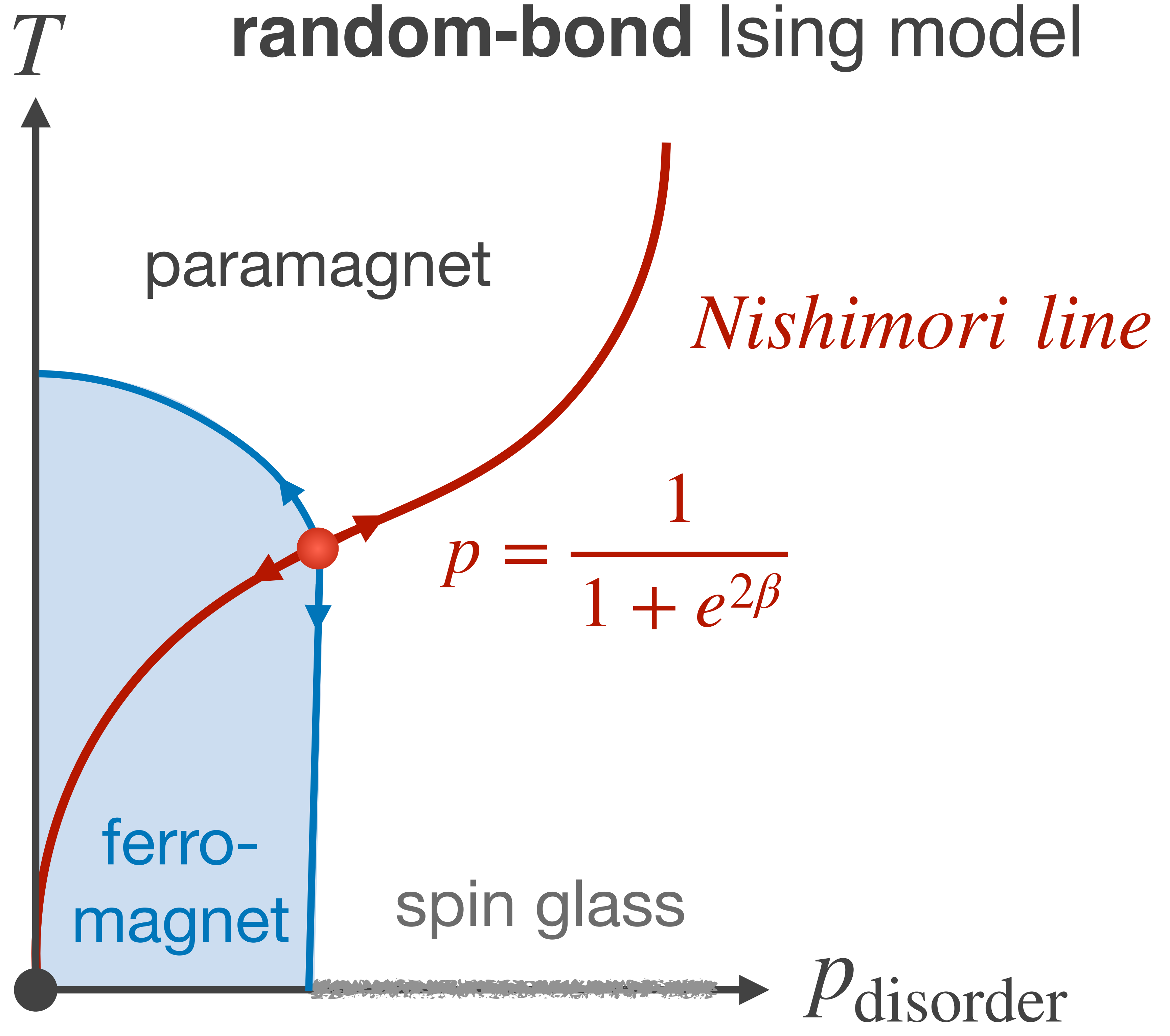
learning by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786





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learning by measurement

The learning thresholds exhibit monitored quantum criticality.

Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786

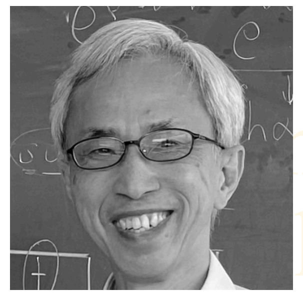
$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

Born's rule

interpret as **classical stat mech model**

$$\tanh \frac{\beta J_\pm}{2} = \pm \tan t$$

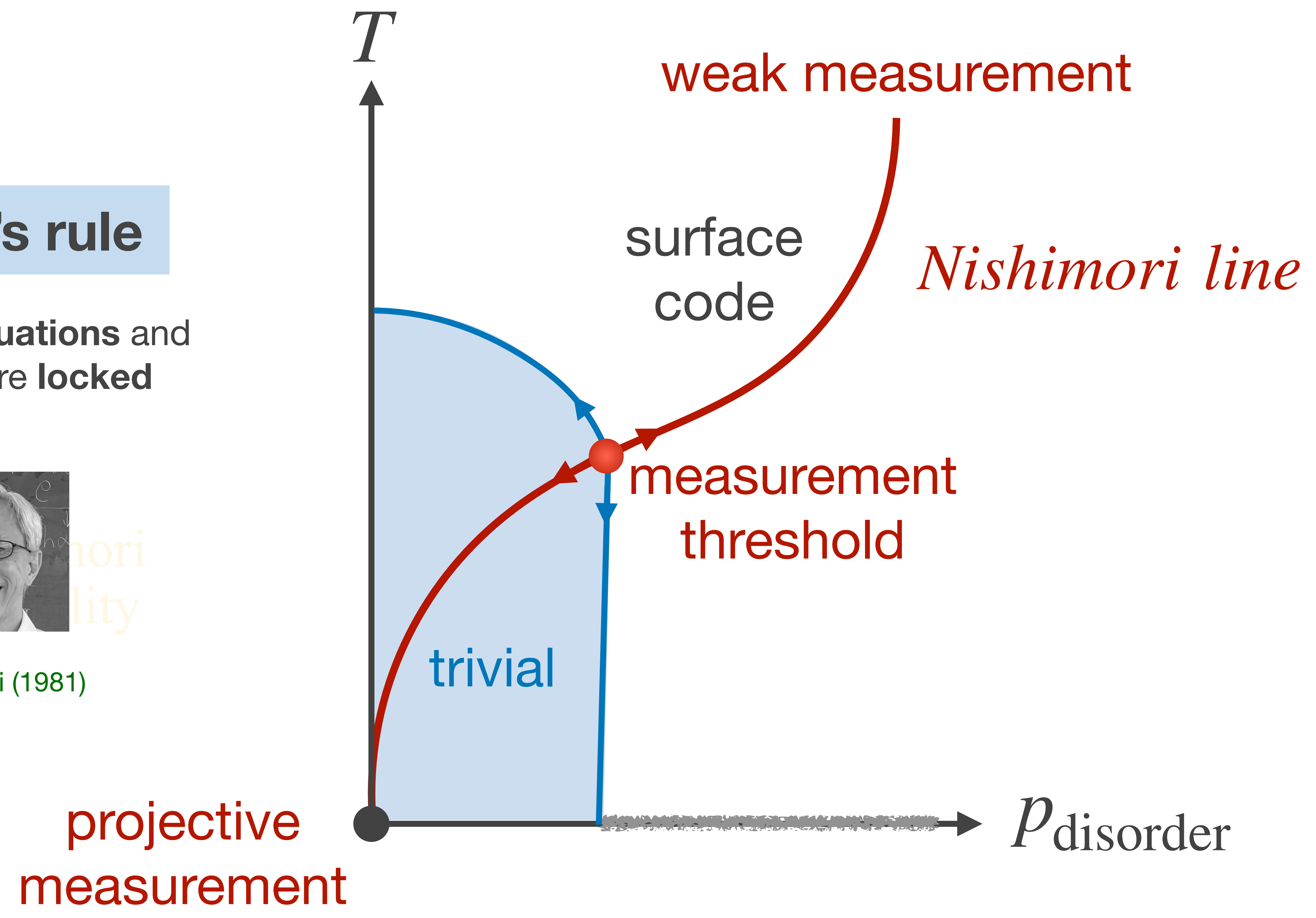
thermal **fluctuations** and **disorder** are **locked**



Nishimori (1981)

$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} J_{s_{ij}} \sigma_i \sigma_j}$$

random bond Ising model



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learning by measurement

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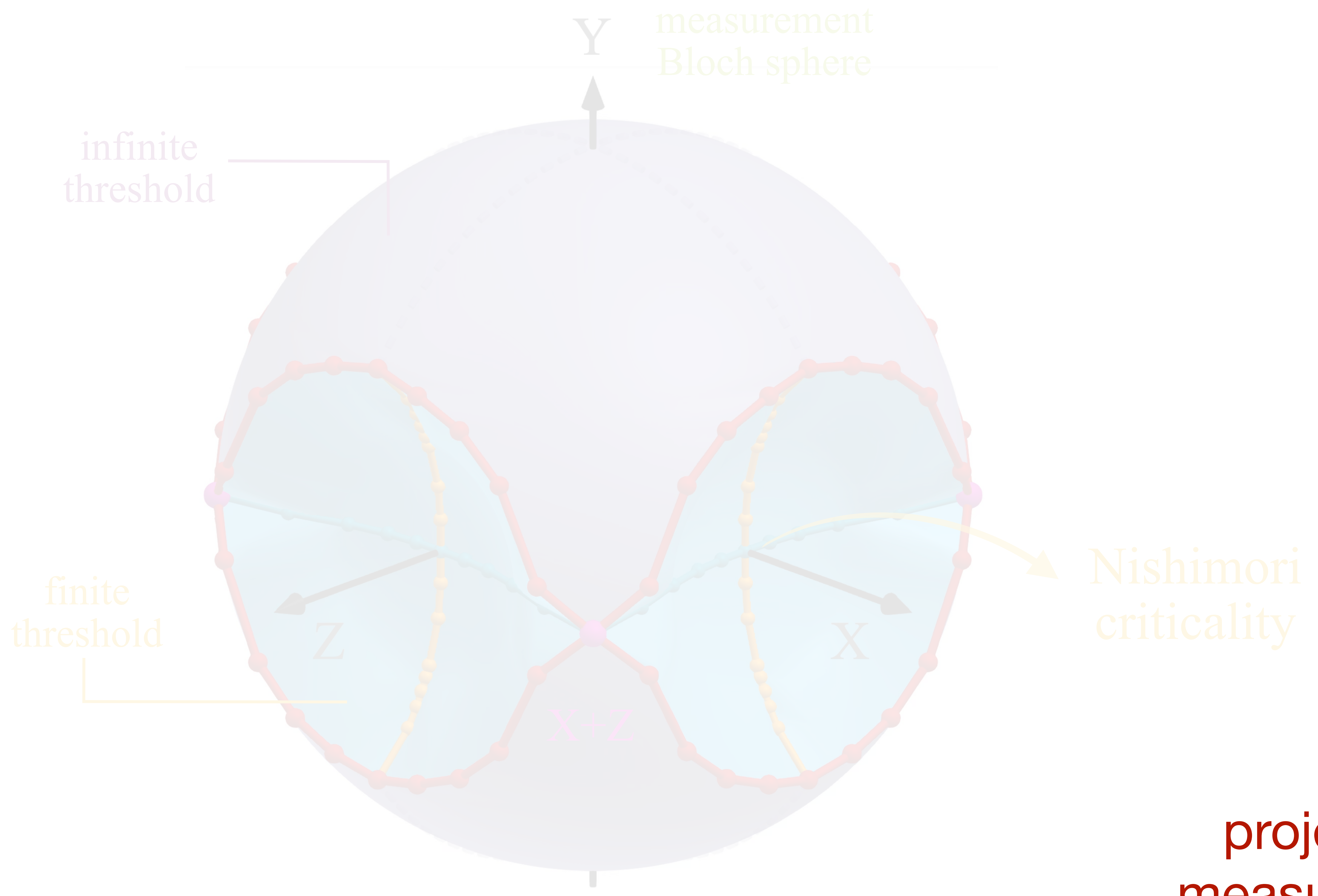
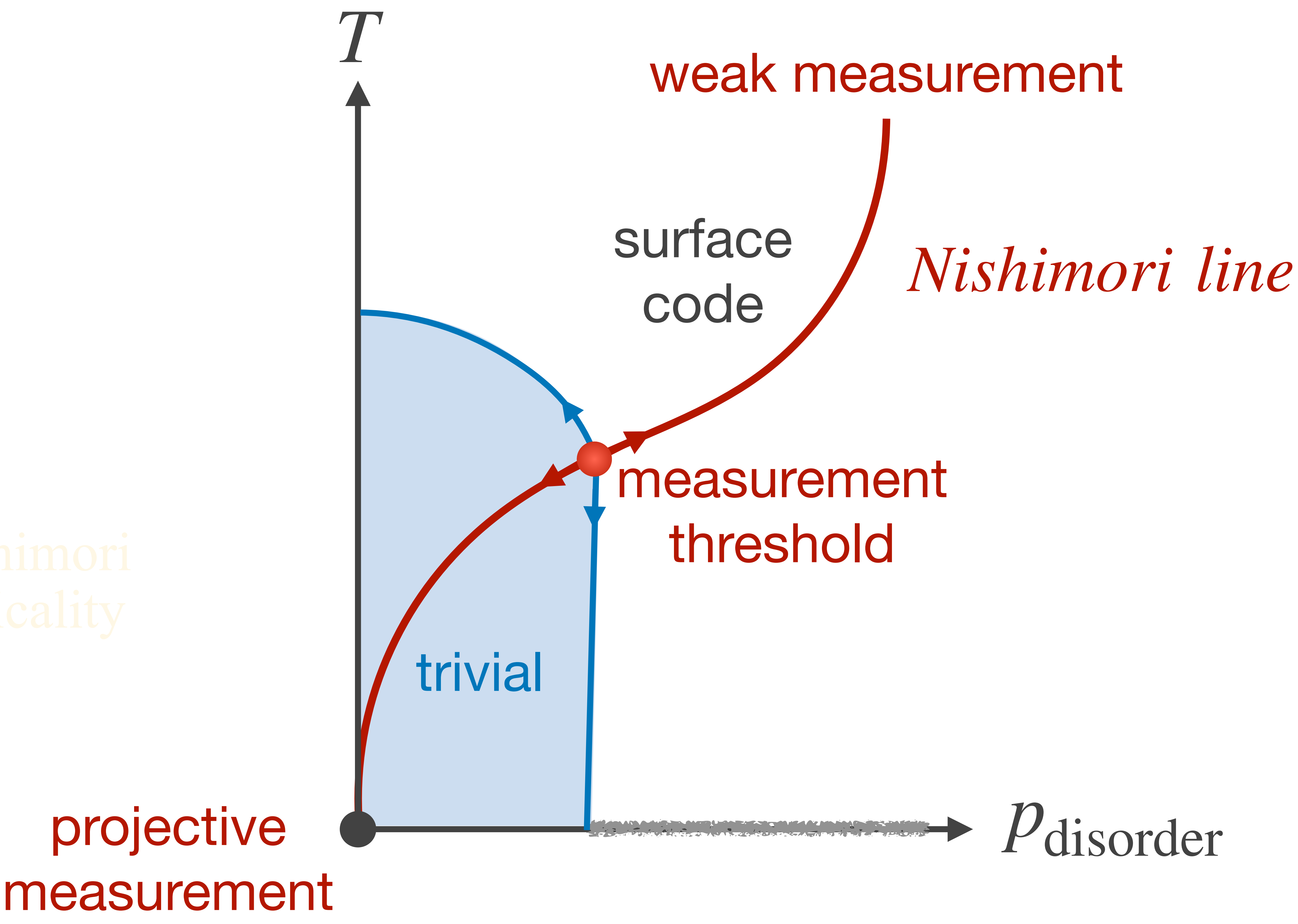
Eckstein, Han, **ST** & Zhu, PRX Quantum 2024, arXiv:2512.19786





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learning by measurement

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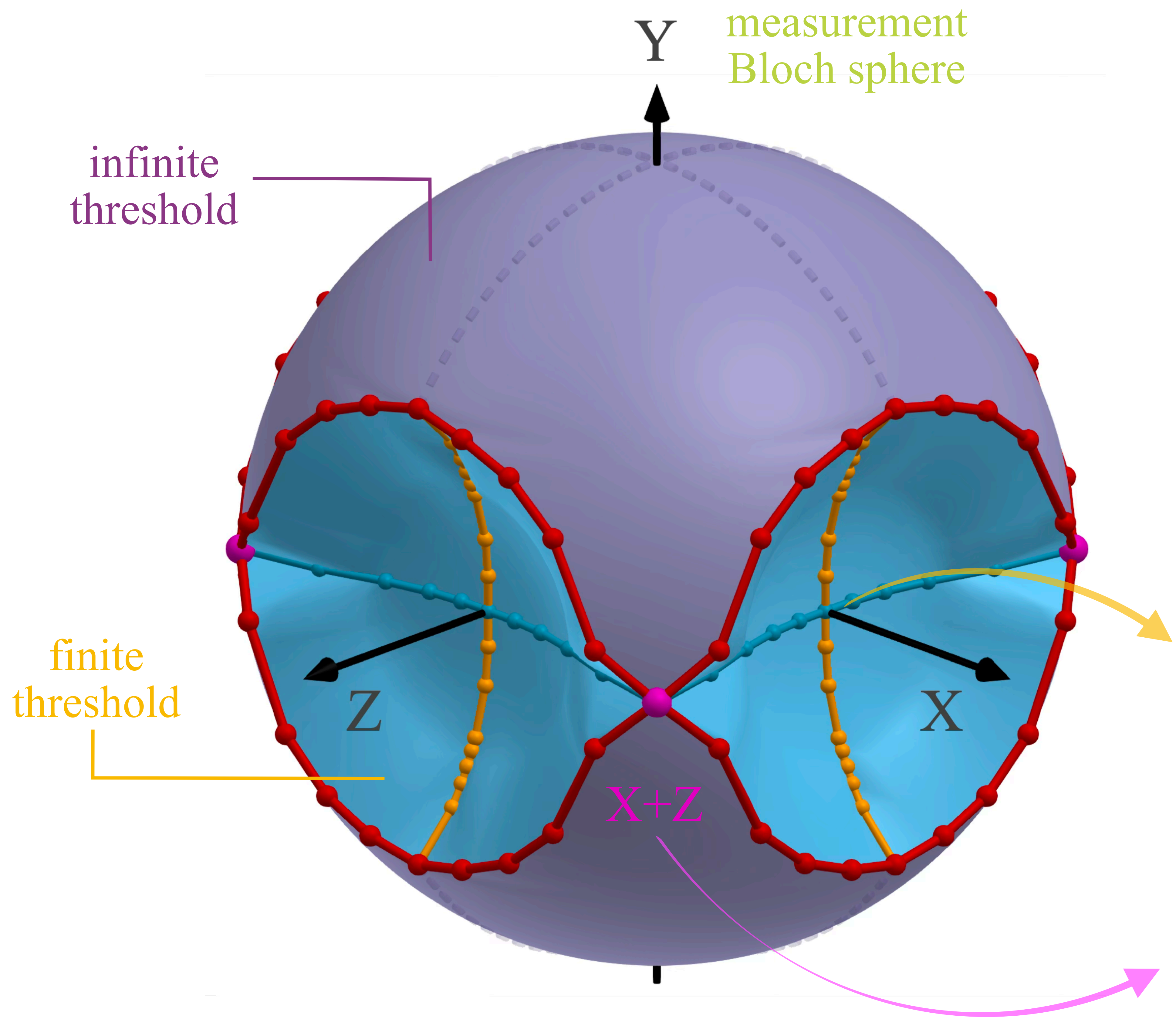
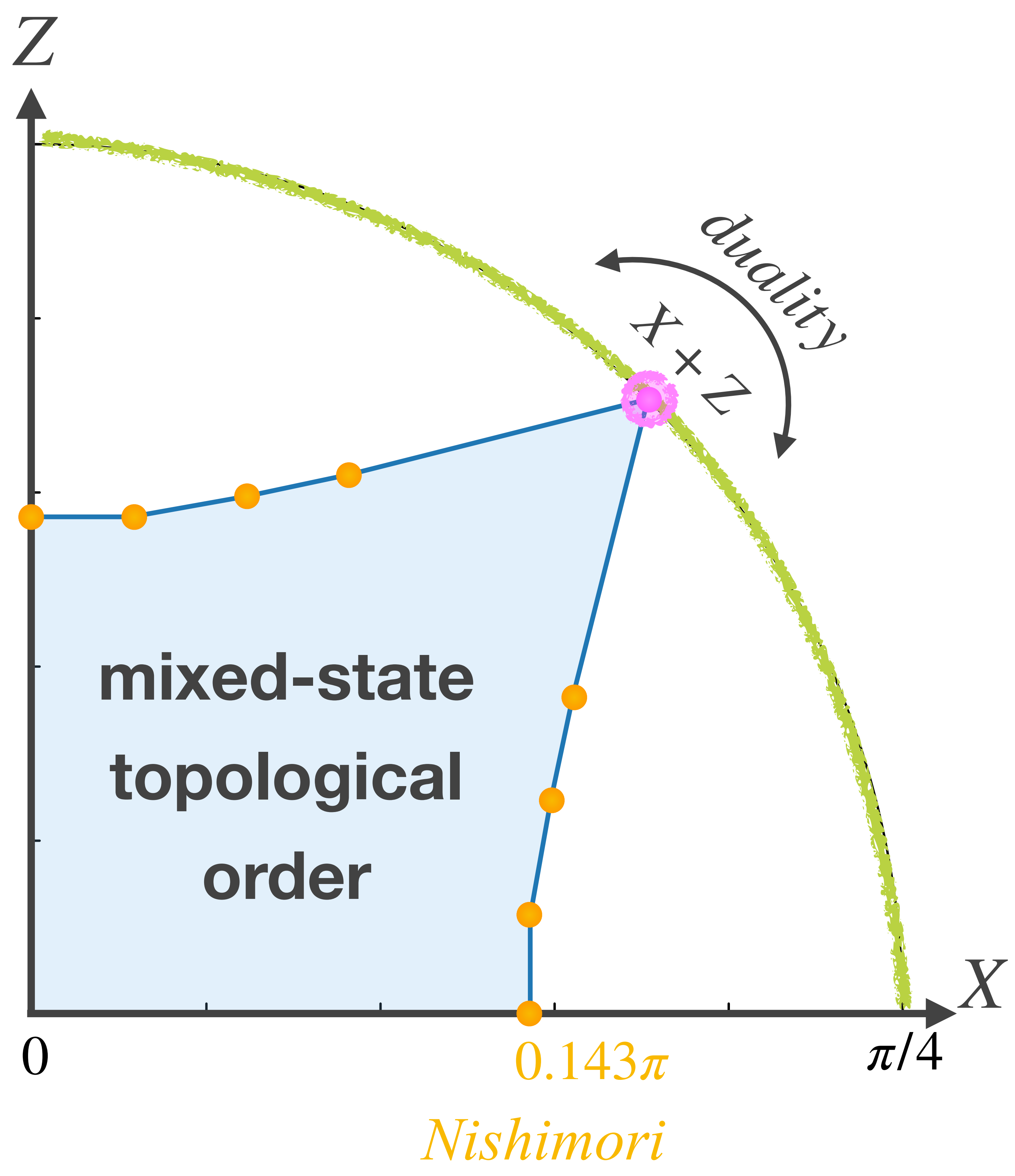


Nishimori criticality

Pütz, Vasseur, Ludwig, **ST** & Zhu, PRX Quantum 2025

self-dual criticality

arXiv:2502.14034



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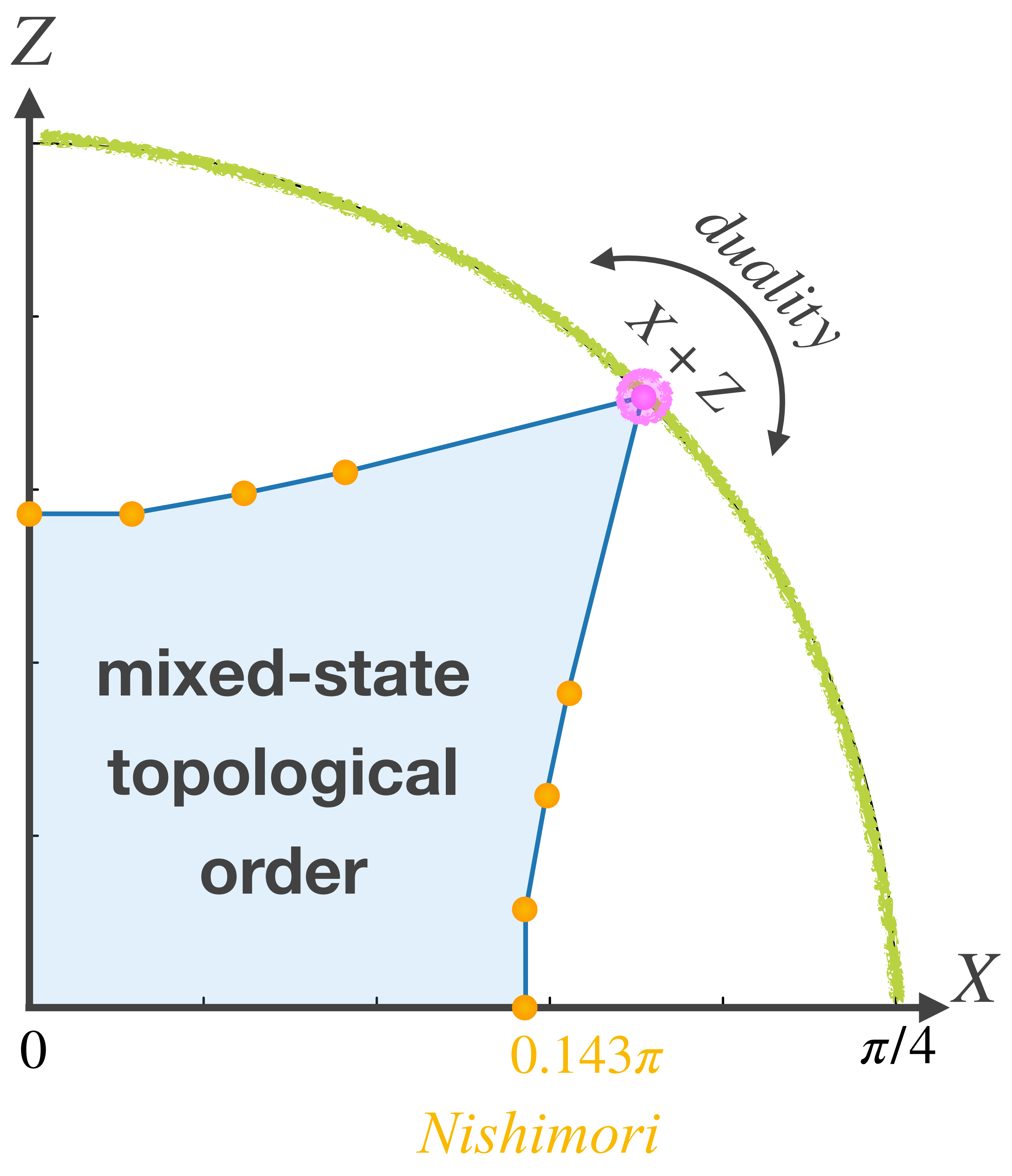
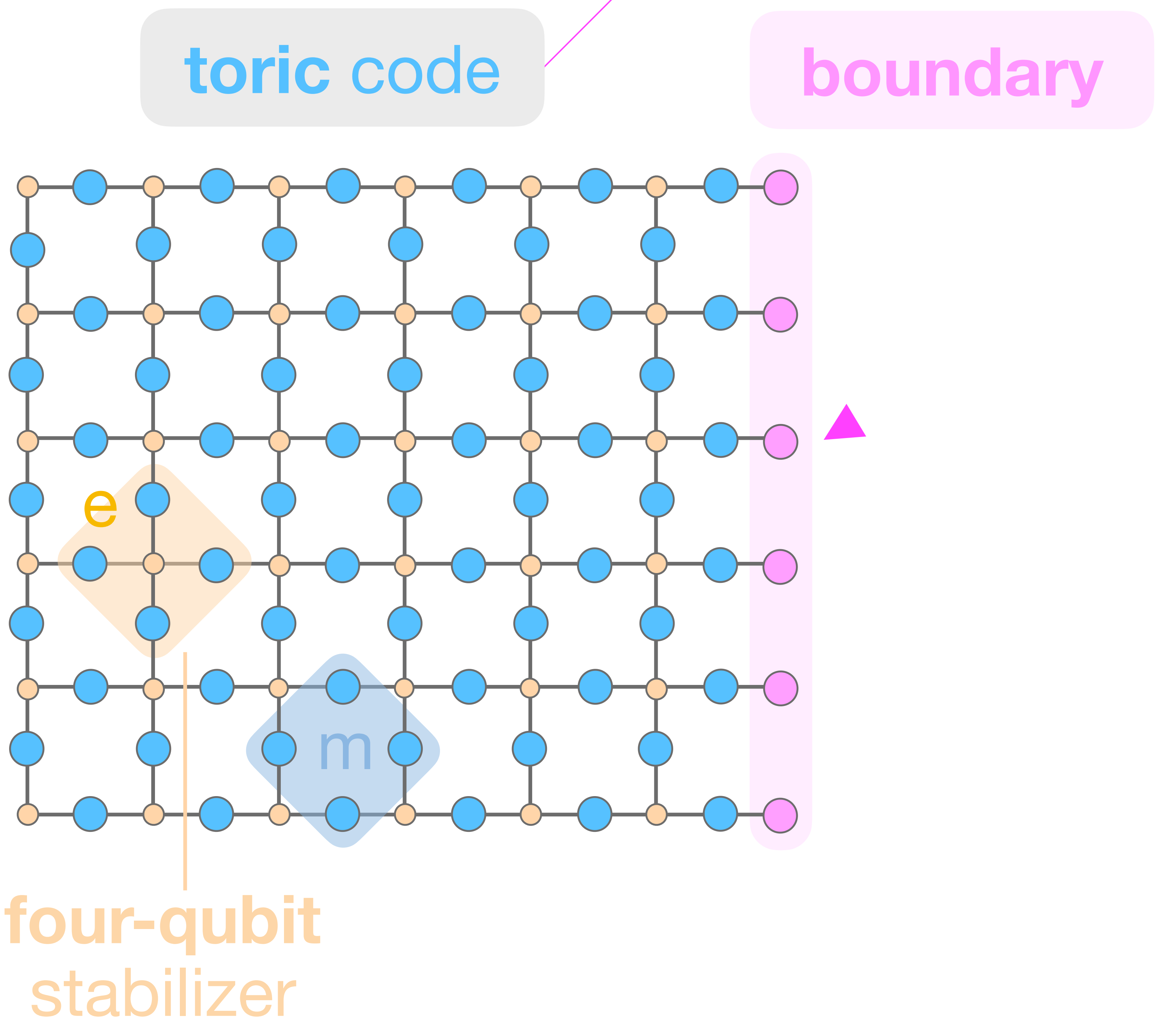
learning by measurement

$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?



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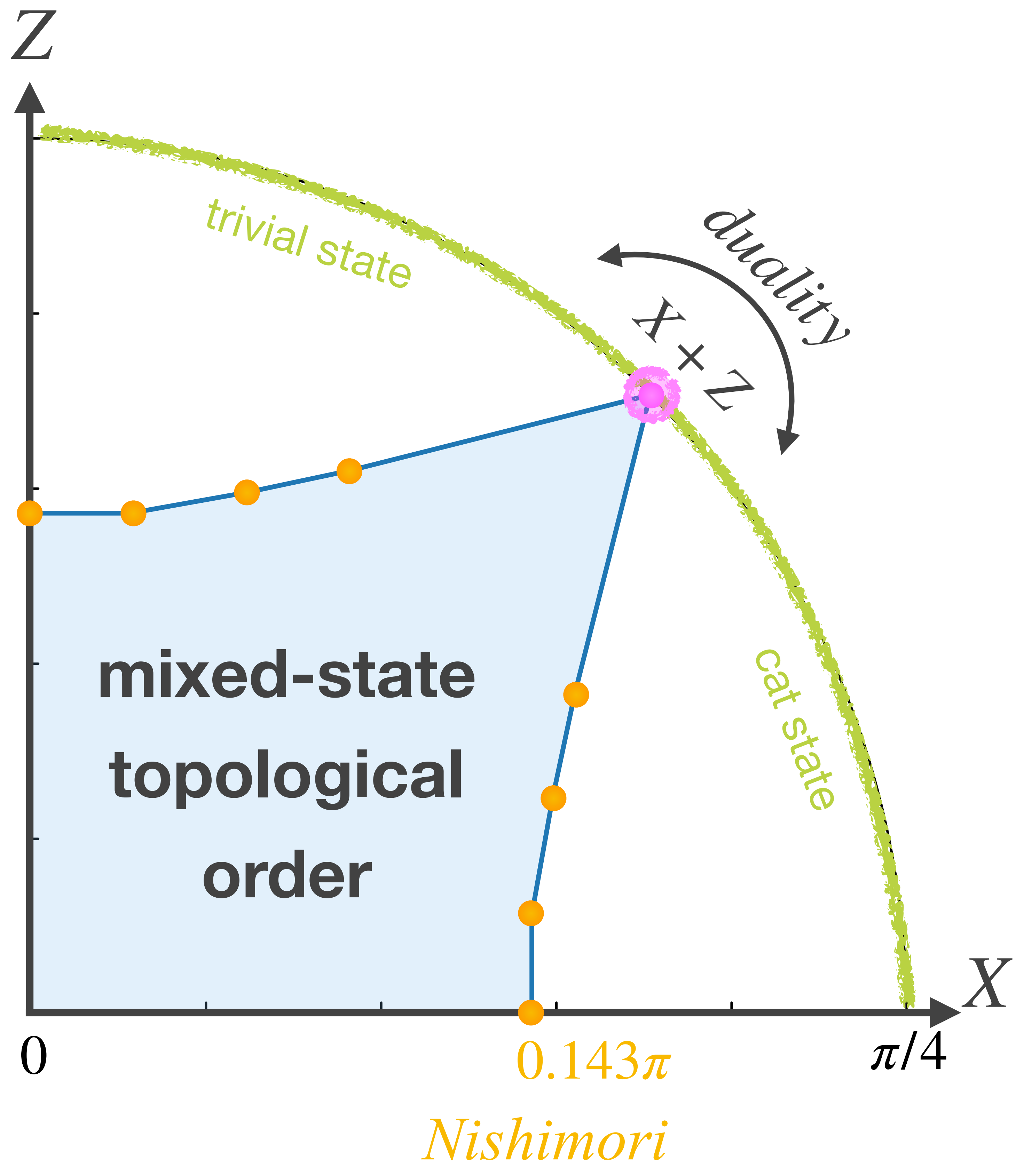
bulk-boundary correspondence

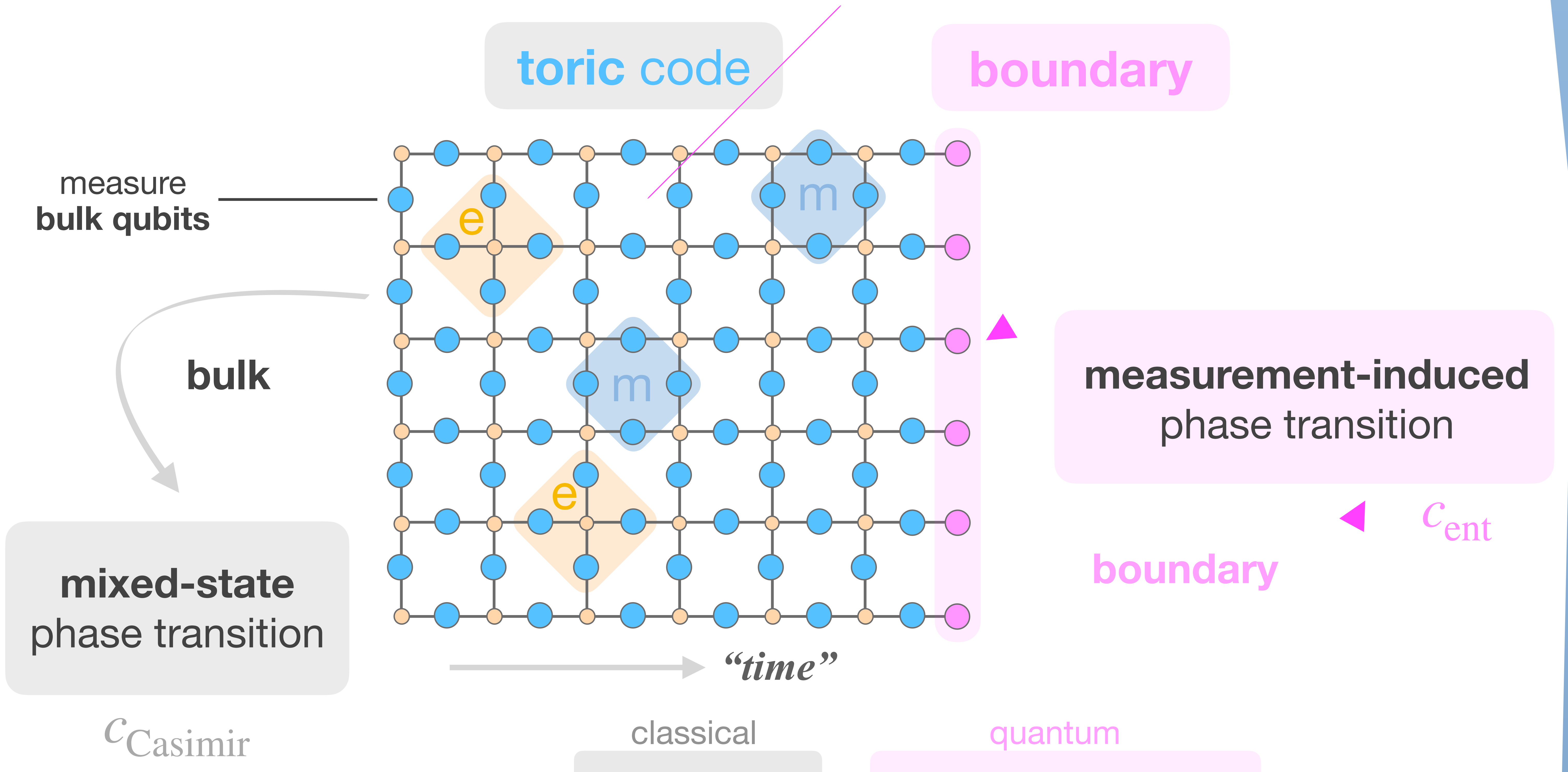
arXiv:2502.14034

$$\Pi_{ij}\left[1 \pm \tanh\frac{\beta}{2}\left(\cos\theta\, Z_{ij} + \sin\theta\, X_{ij}\right)\right]|\psi\rangle$$

measurement **angle**

Why is there a **critical point** when all the bulk qubits **collapse**?





classical quantum

$$\rho = \sum_{em} |em\rangle\langle em|_C \otimes |\psi(em)\rangle\langle\psi(em)|_Q$$

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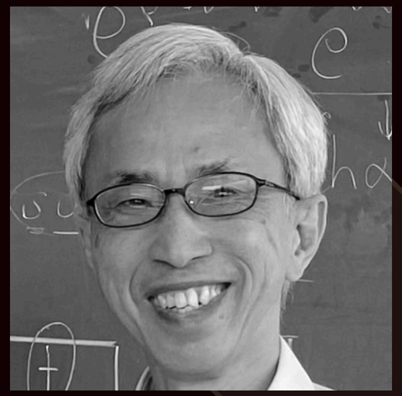
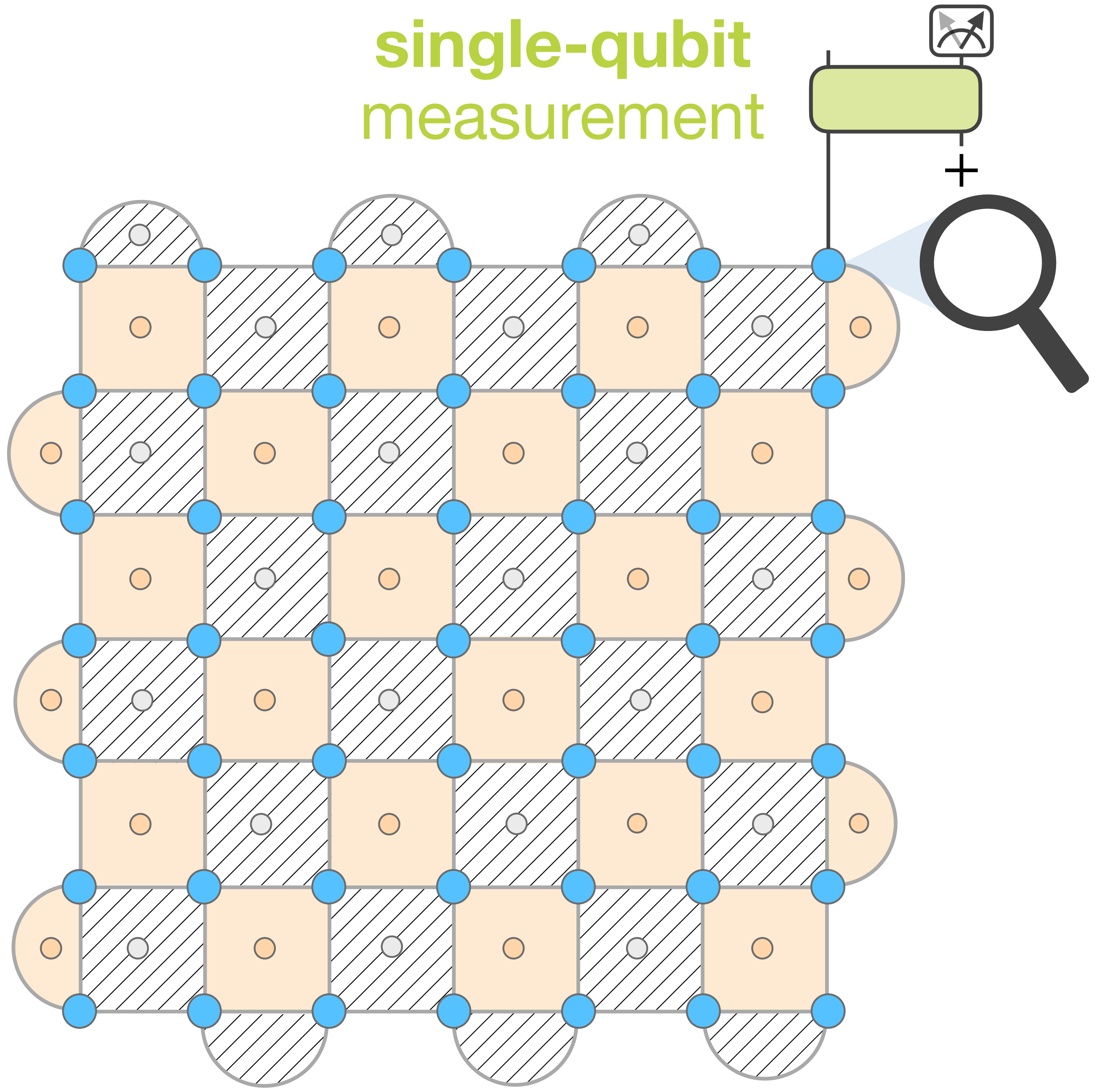
learning *deformed* toric codes

arXiv:2604.06324
PRL **136**, 190402 (2026)

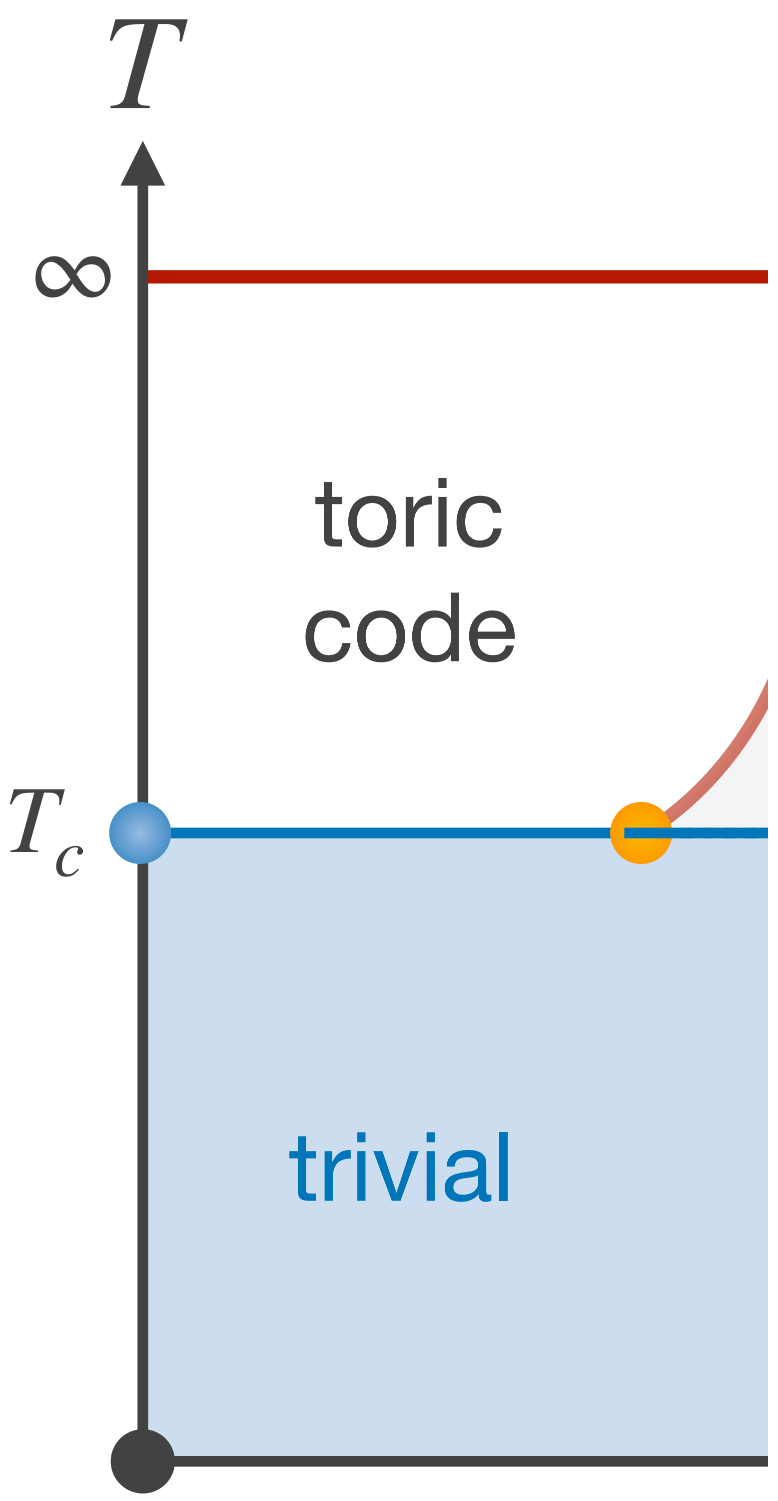
M. Pütz, S. Garratt, H. Nishimori, **R. Patil**, A. Ludwig, G-Y. Zhu

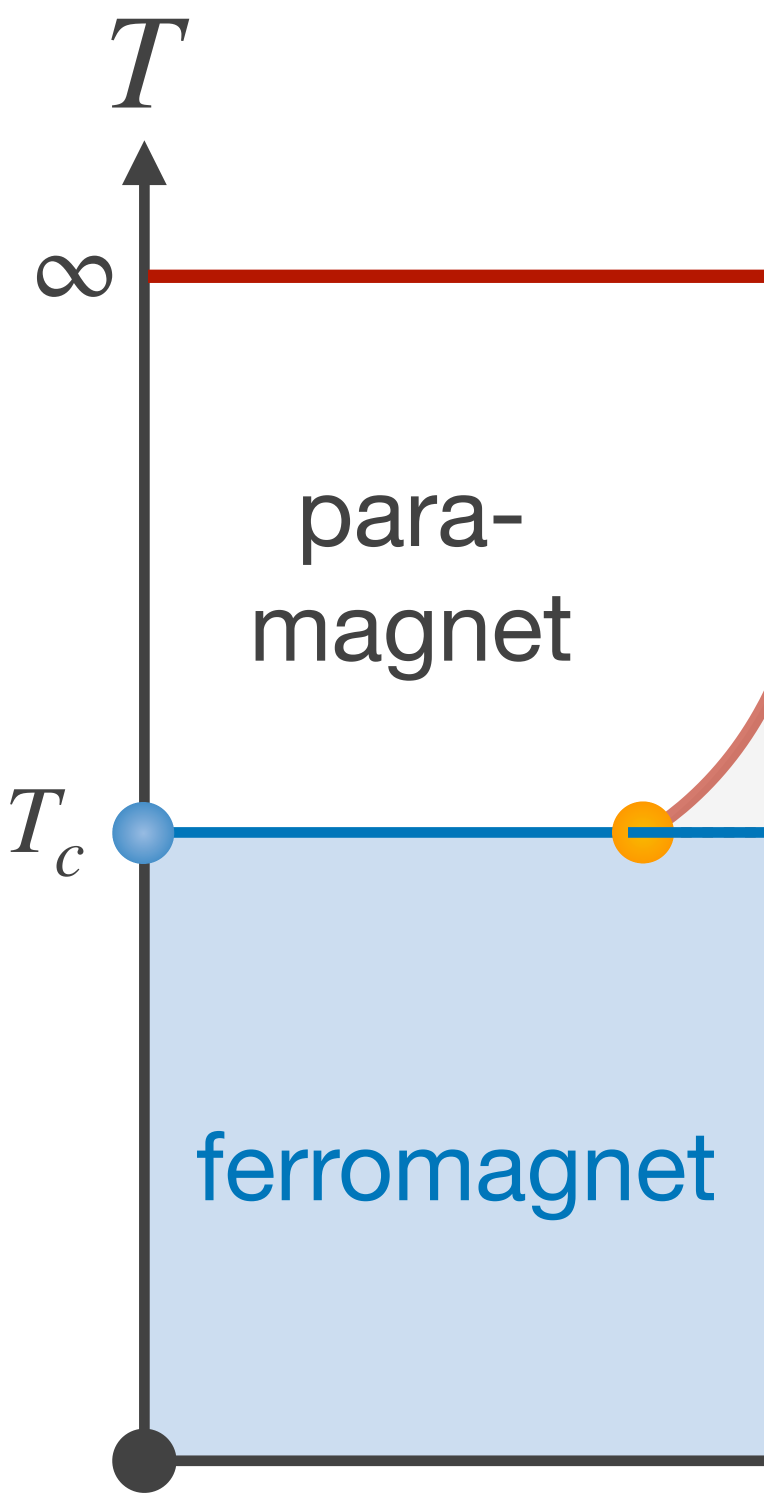
deformed toric code



learning **toric code**



learning **Ising model**



learning
accuracy

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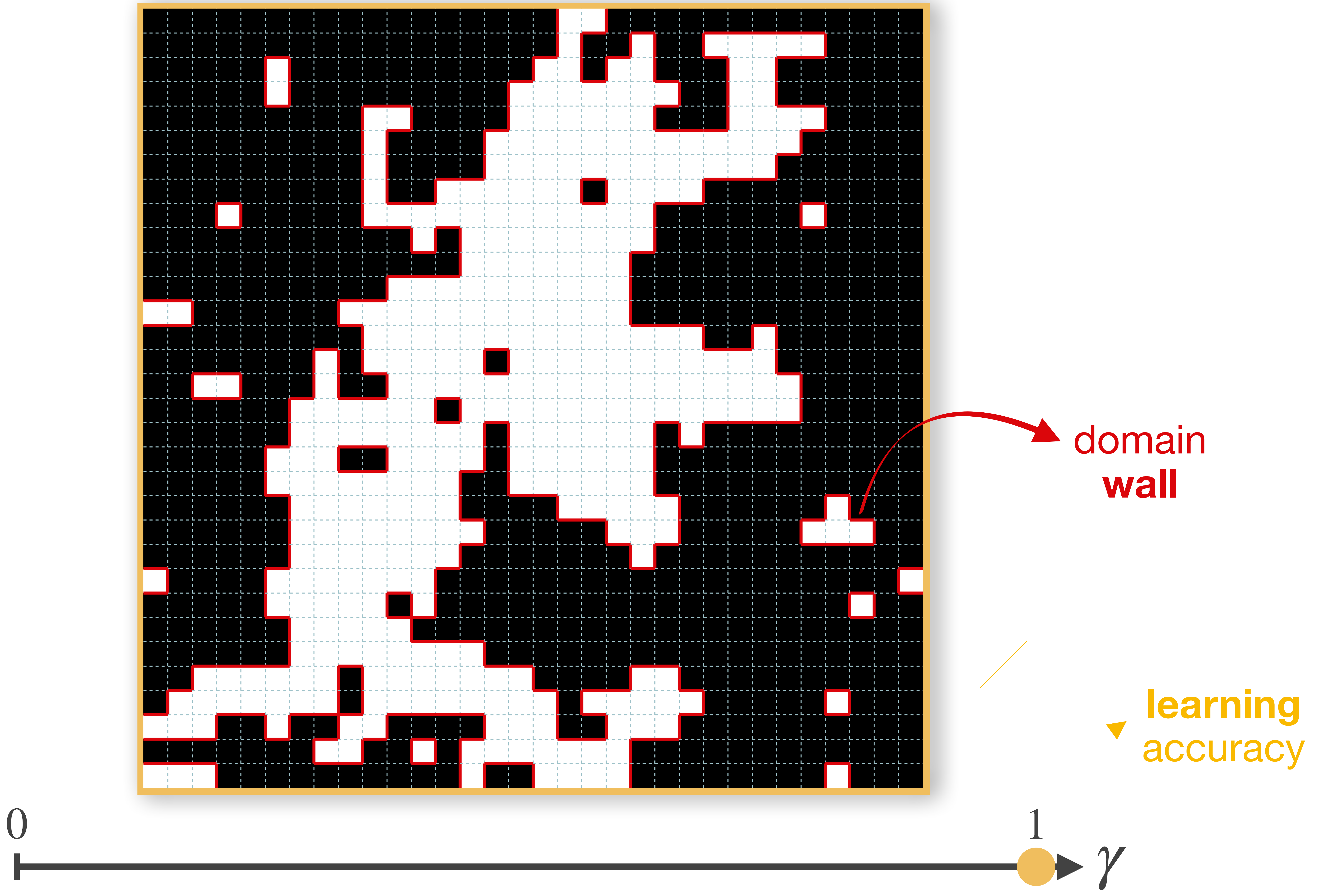
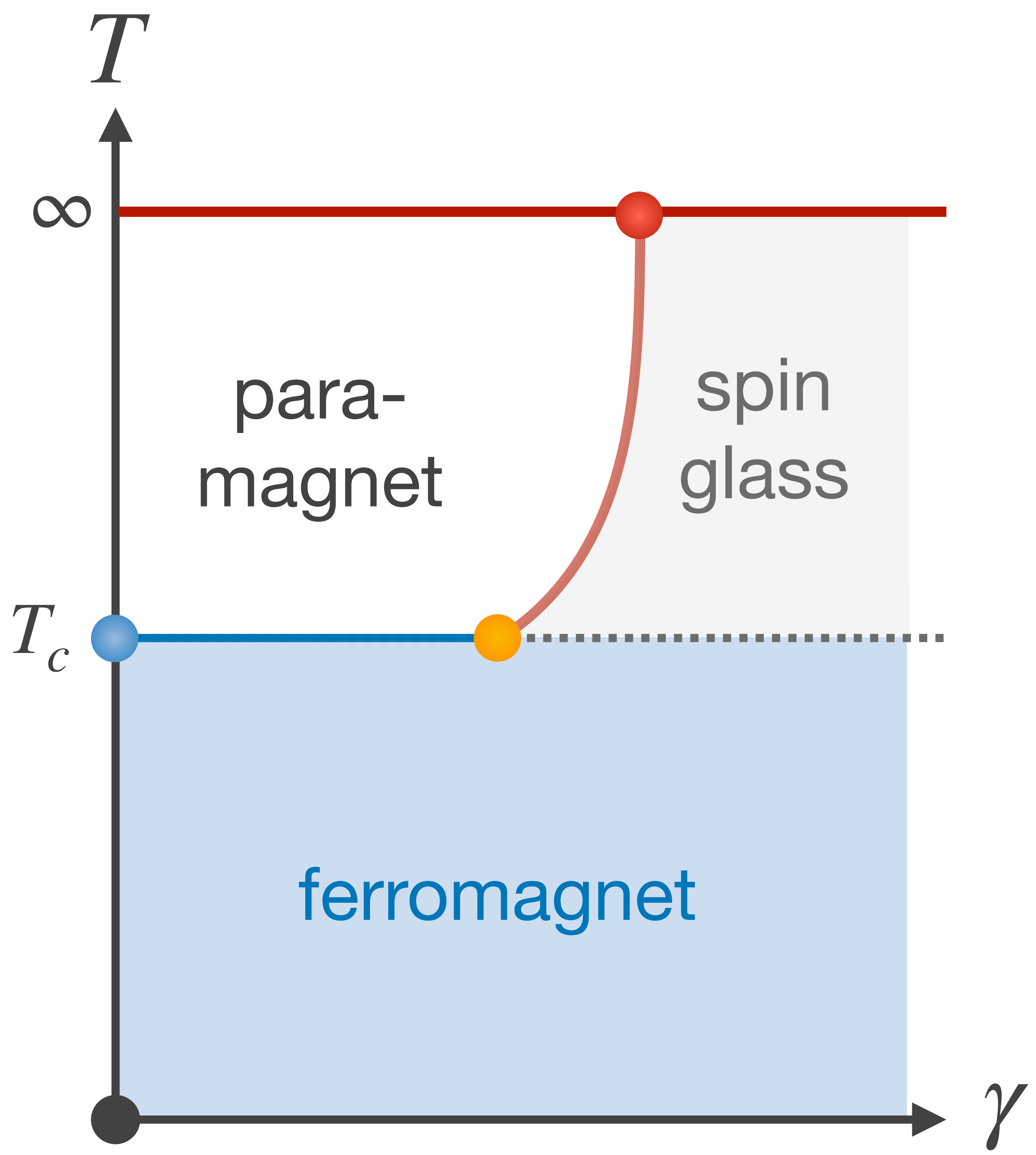
learning Ising model





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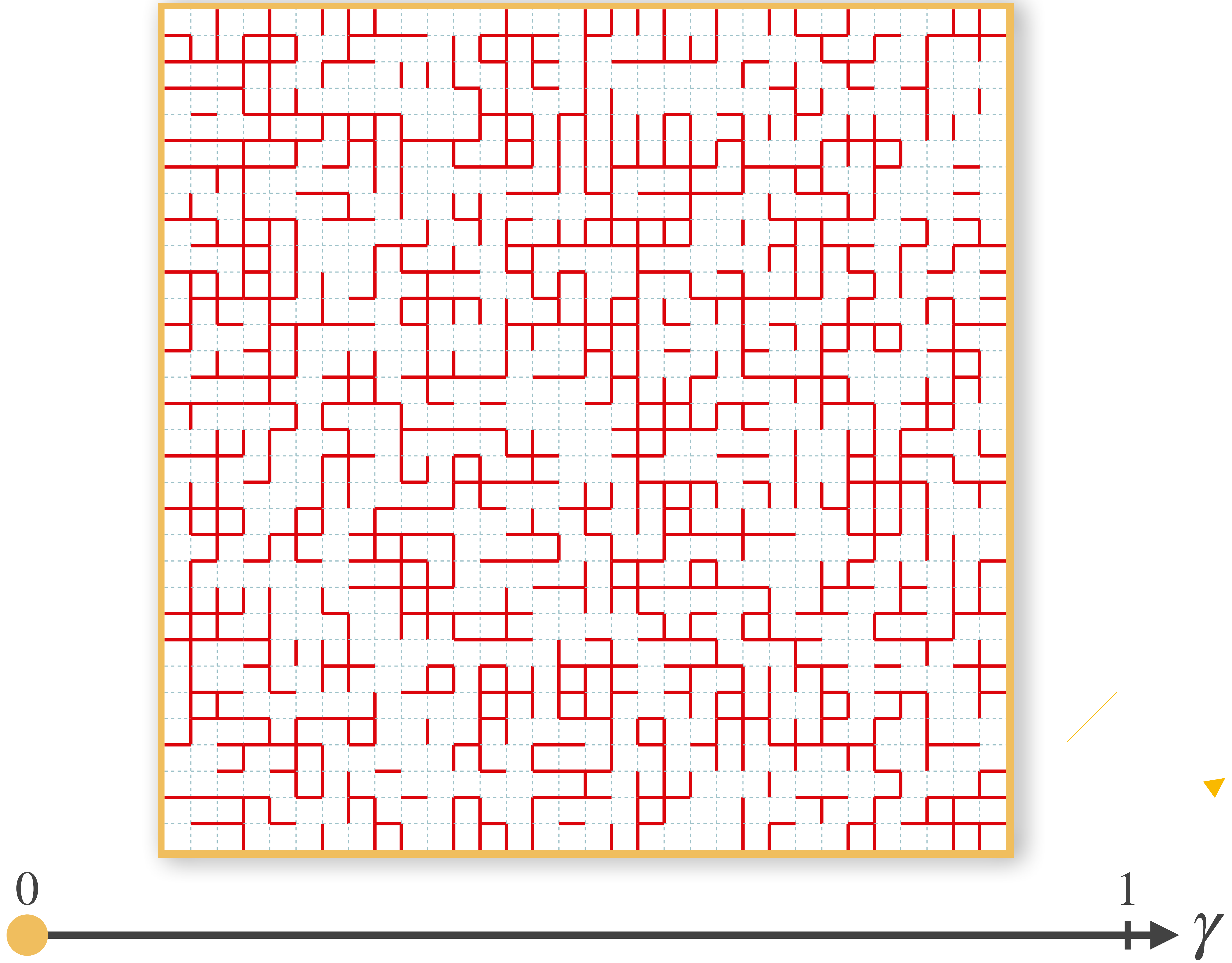
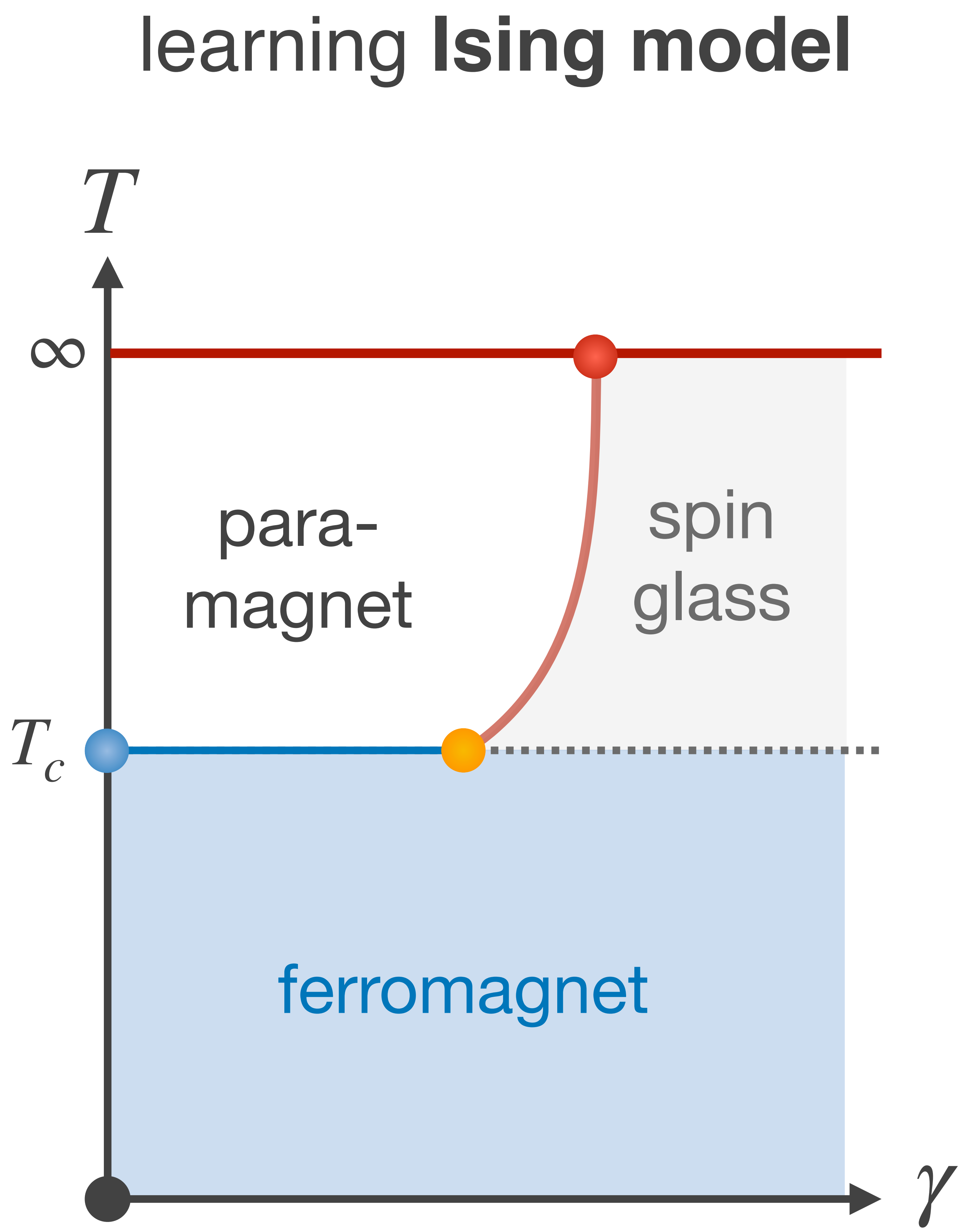
learning Ising model



learning
accuracy



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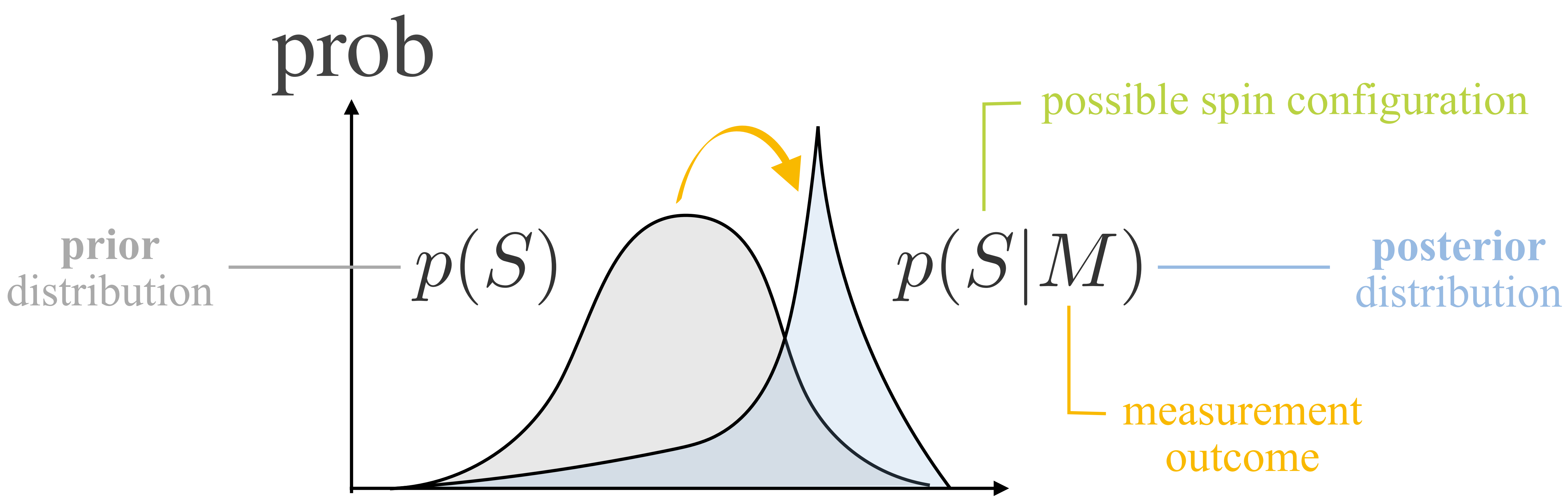
learning Ising model via **Bayesian inference**



conditional distribution

Bayesian inference

$$p(S|M) = \frac{p(S)\,p(M|S)}{p(M)}$$

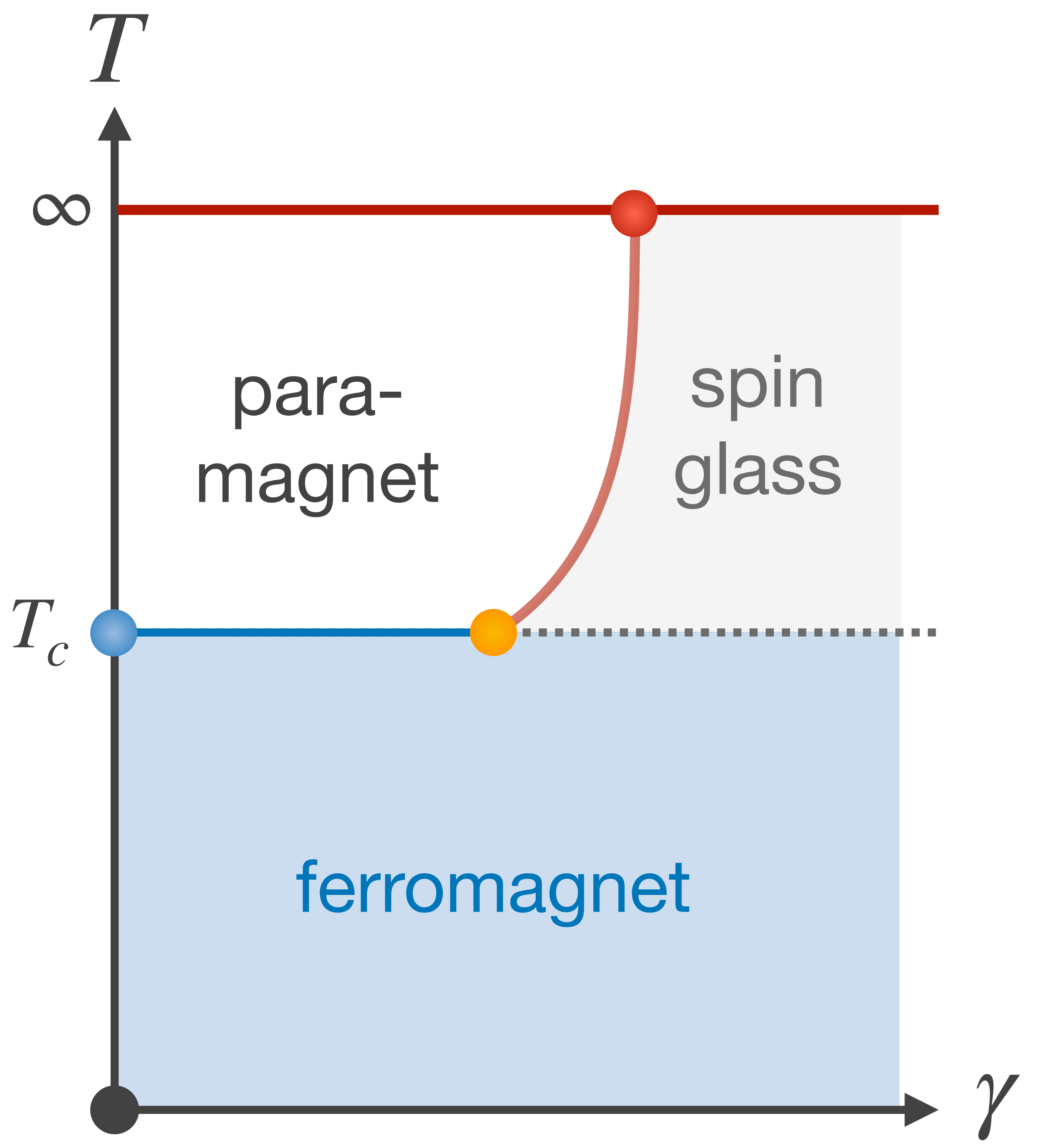
posterior distribution

marginal distribution

Born's rule

$$|\psi\{s\}\rangle$$

learning **Ising model**



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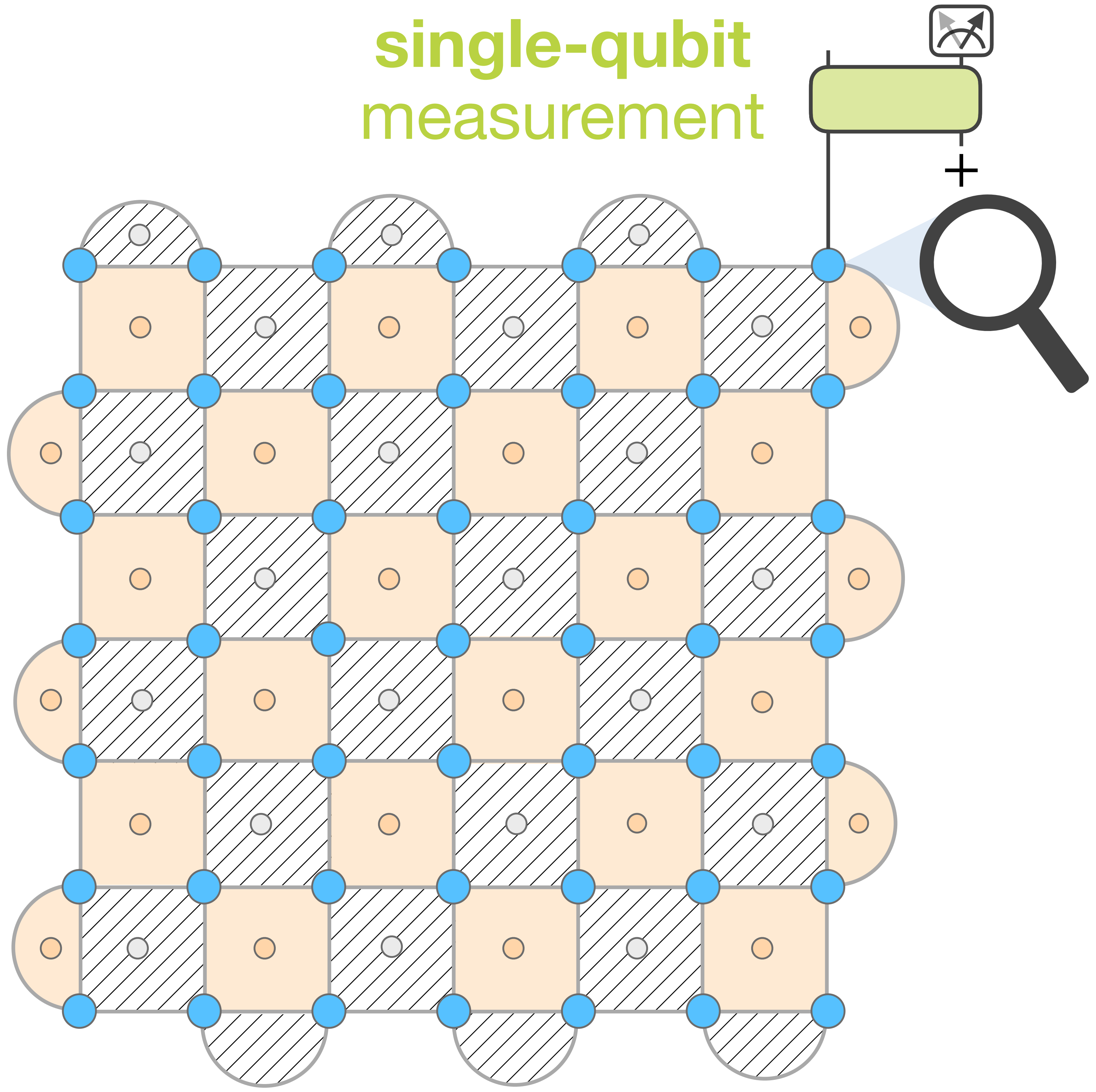
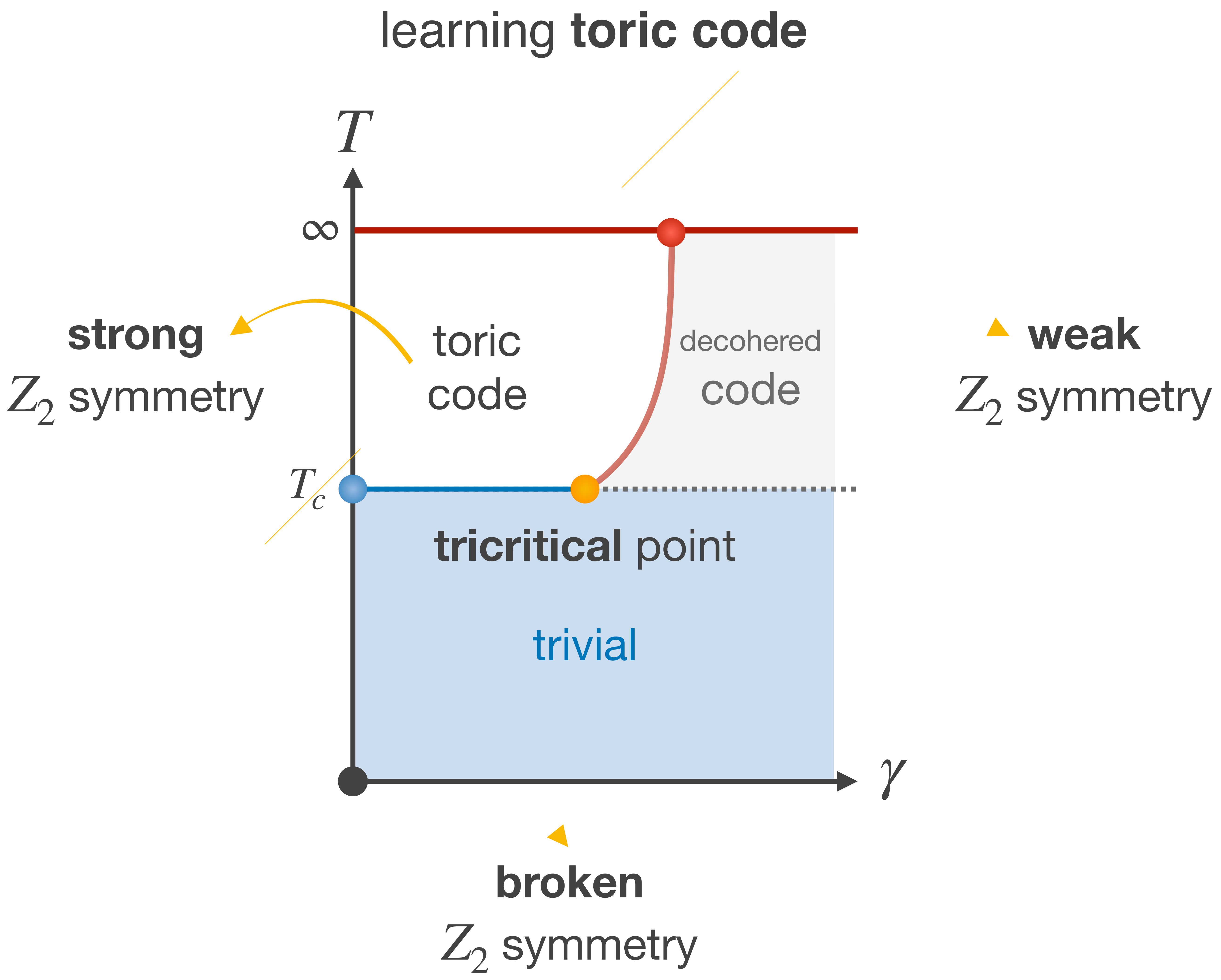
learning deformed toric codes





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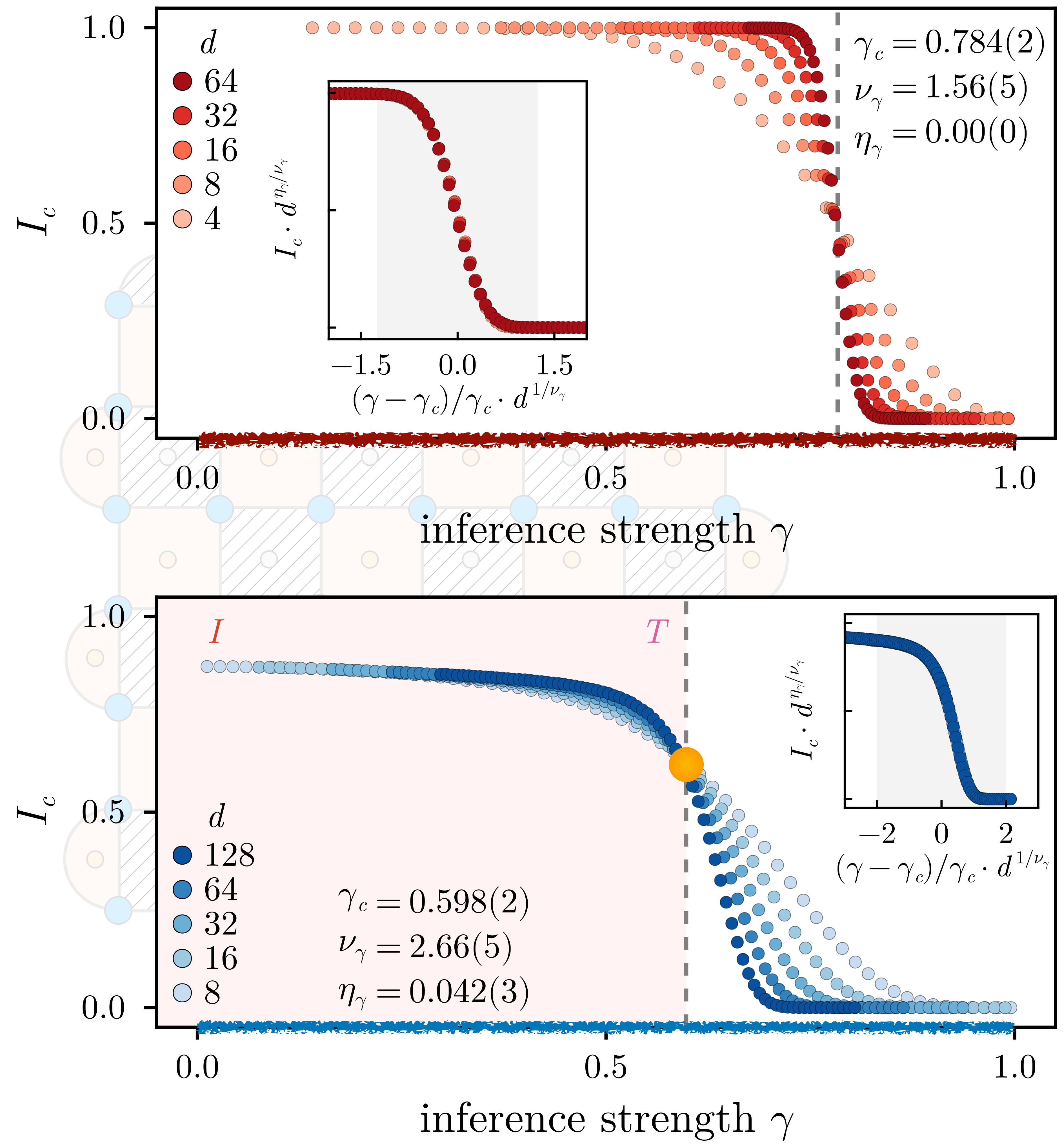
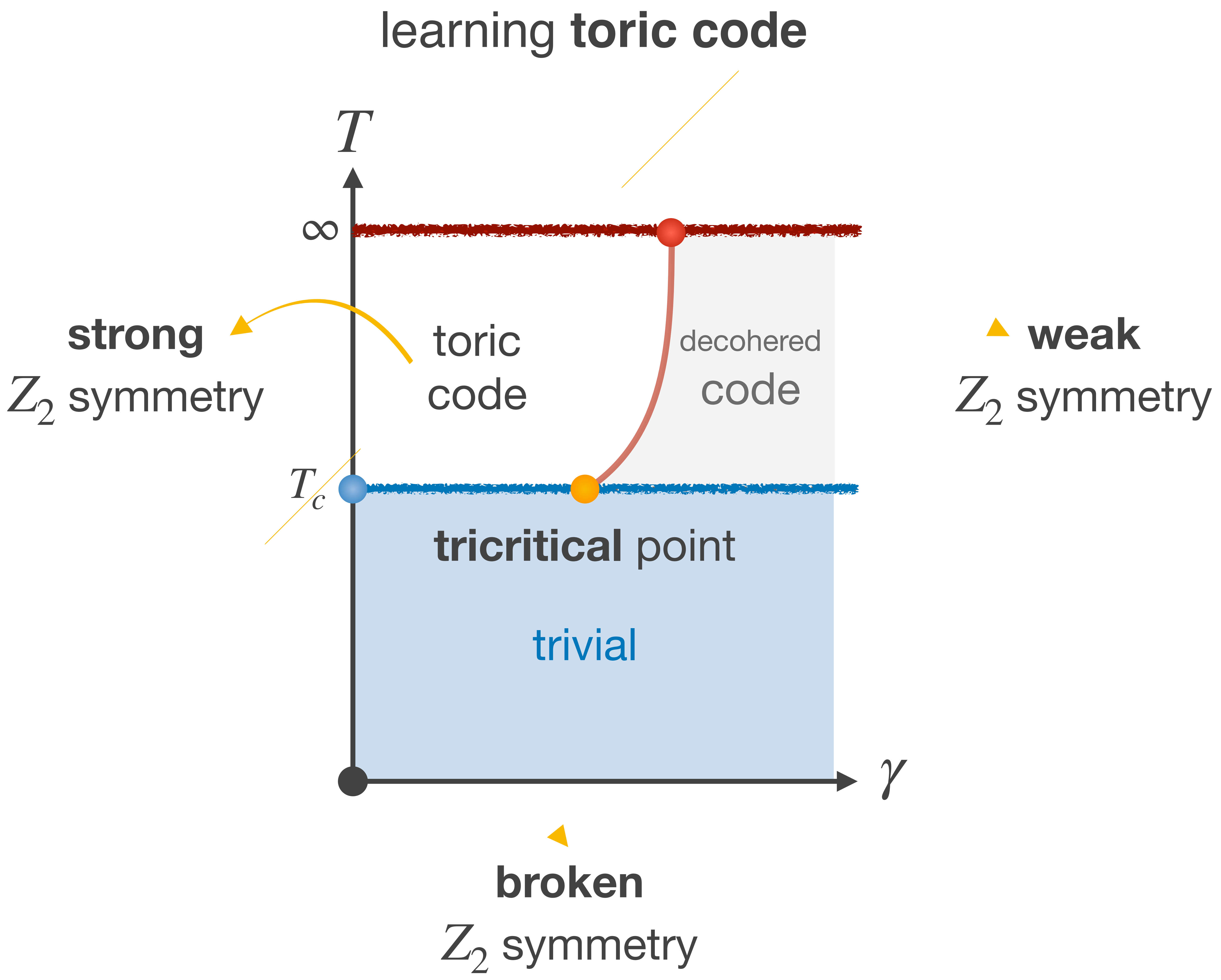
learning deformed toric codes

M. Pütz *et al.*, PRL 136, 190402 (2026)





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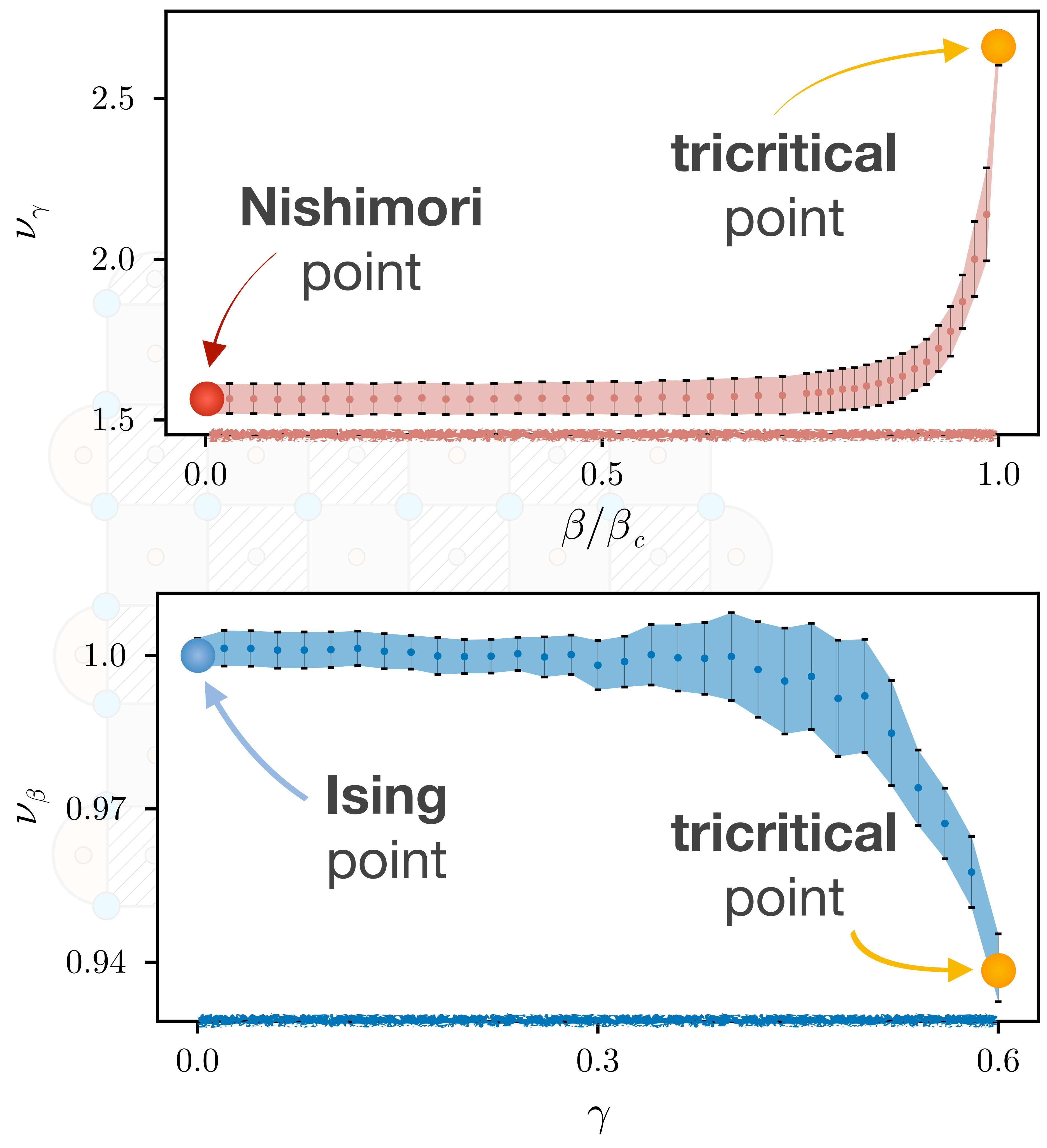

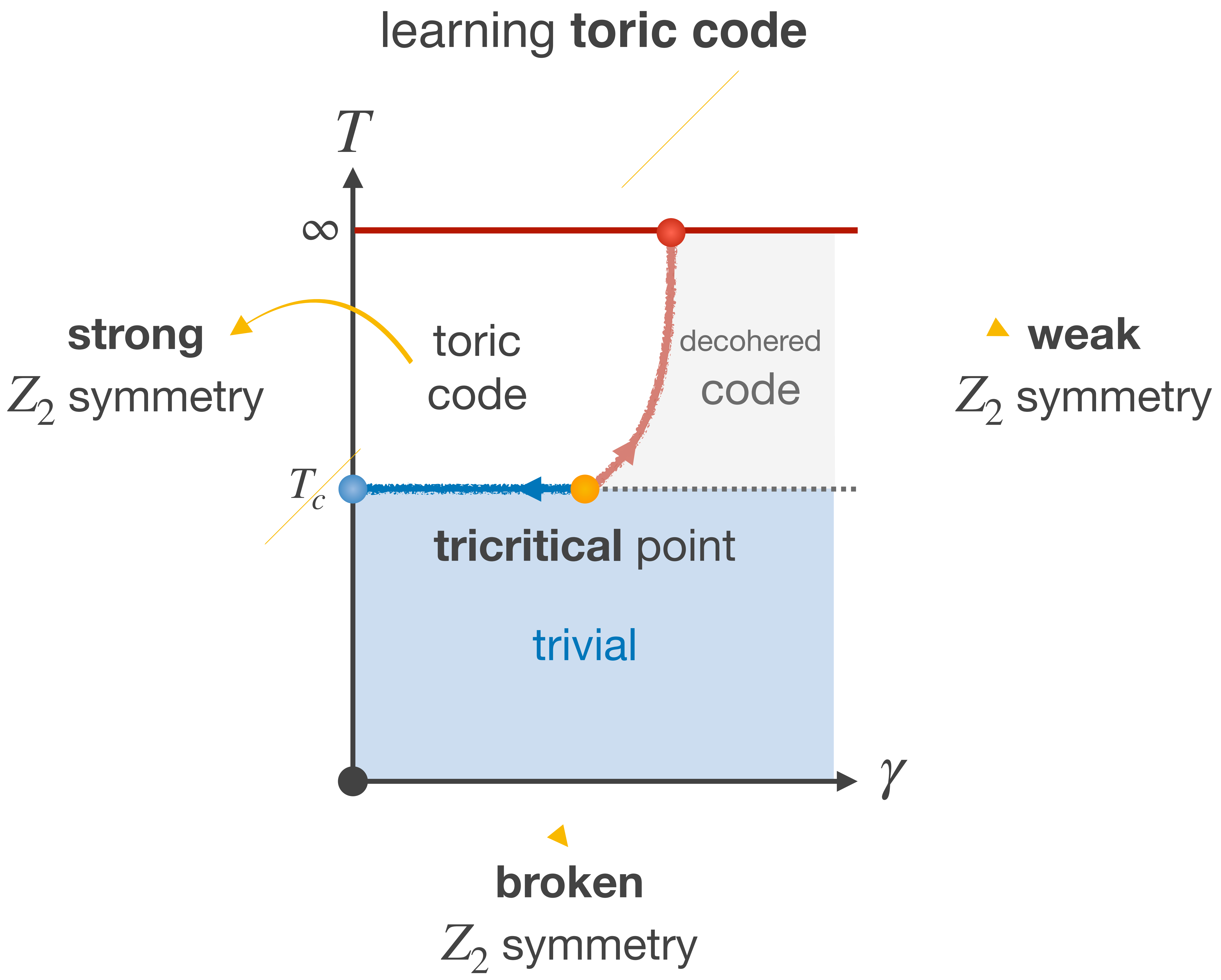

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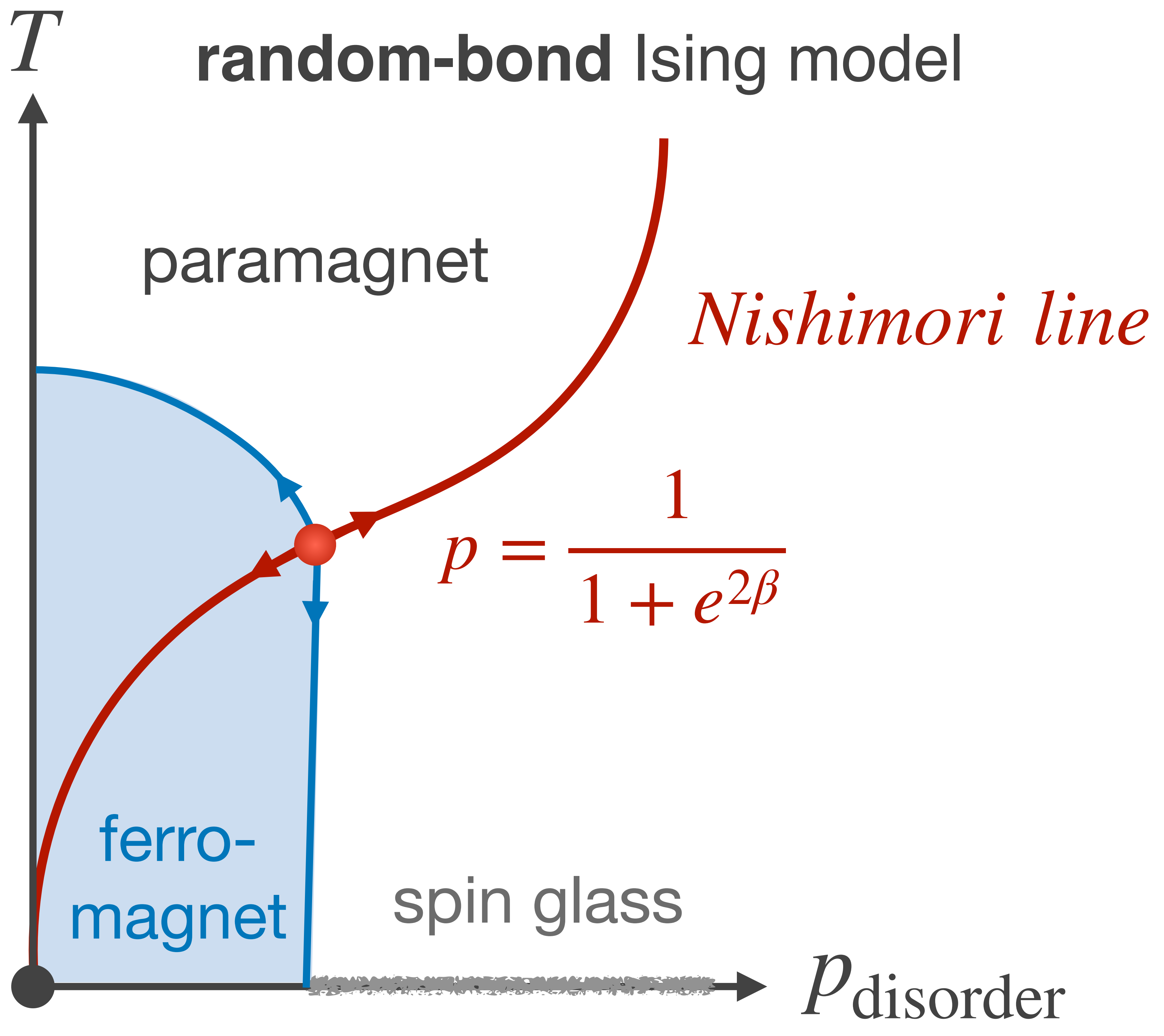
replica theory

arXiv:2604.06324

introduction to **replica theory**

random-bond Ising model



$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \; S_i^z S_j^z$$

Gaussian disorder
mean J_0 and variance Δ^2

We want to calculate **disorder average**

$$\overline{F} = -T\,\overline{\ln Z[J]}$$

disorder average (hard to calculate)

single disorder realization

via the **replica trick**

$$\overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

replicated partition function

replica index

$$\overline{Z^n} = \sum_{\{\sigma^a\}} \prod_{\langle ij \rangle} \left\langle \exp\left(\beta J_{ij} \sum_{a=1}^{n} \sigma_i^{(a)} \sigma_j^{(a)} \right) \right\rangle_J$$

disorder average

Gaussian disorder

$$\exp\left(\beta J_0 \sum_a \sigma_i^{(a)} \sigma_j^{(a)} + \frac{\beta^2 \Delta^2}{2} \left(\sum_a \sigma_i^{(a)} \sigma_j^{(a)} \right)^2 \right)$$

single-replica interaction

replica-replica interaction

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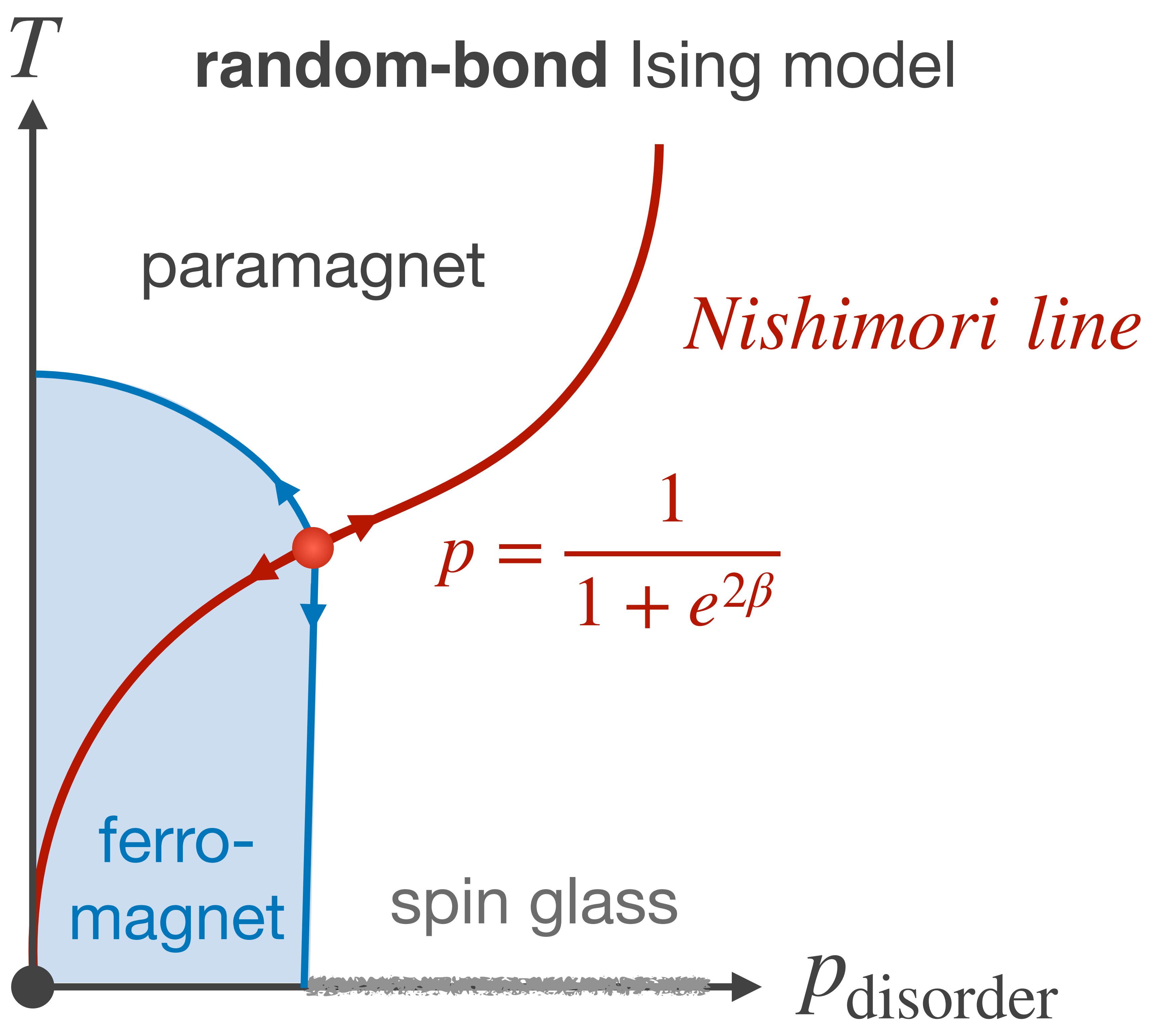
introduction to **replica theory**

random-bond Ising model



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disorder average

becomes an **interacting theory of replicas**

$$\overline{Z^n} = \sum_{\{\sigma^a\}} \exp\left[\underbrace{K_1}_{\beta J_0} \sum_{\langle ij\rangle} \sum_{a} \sigma_i^{(a)} \sigma_j^{(a)} + \underbrace{K_2}_{\beta^2\Delta^2} \sum_{\langle ij\rangle} \sum_{a<b} \sigma_i^{(a)} \sigma_i^{(b)} \sigma_j^{(a)} \sigma_j^{(b)}\right]$$

single-replica interaction

replica-replica interaction

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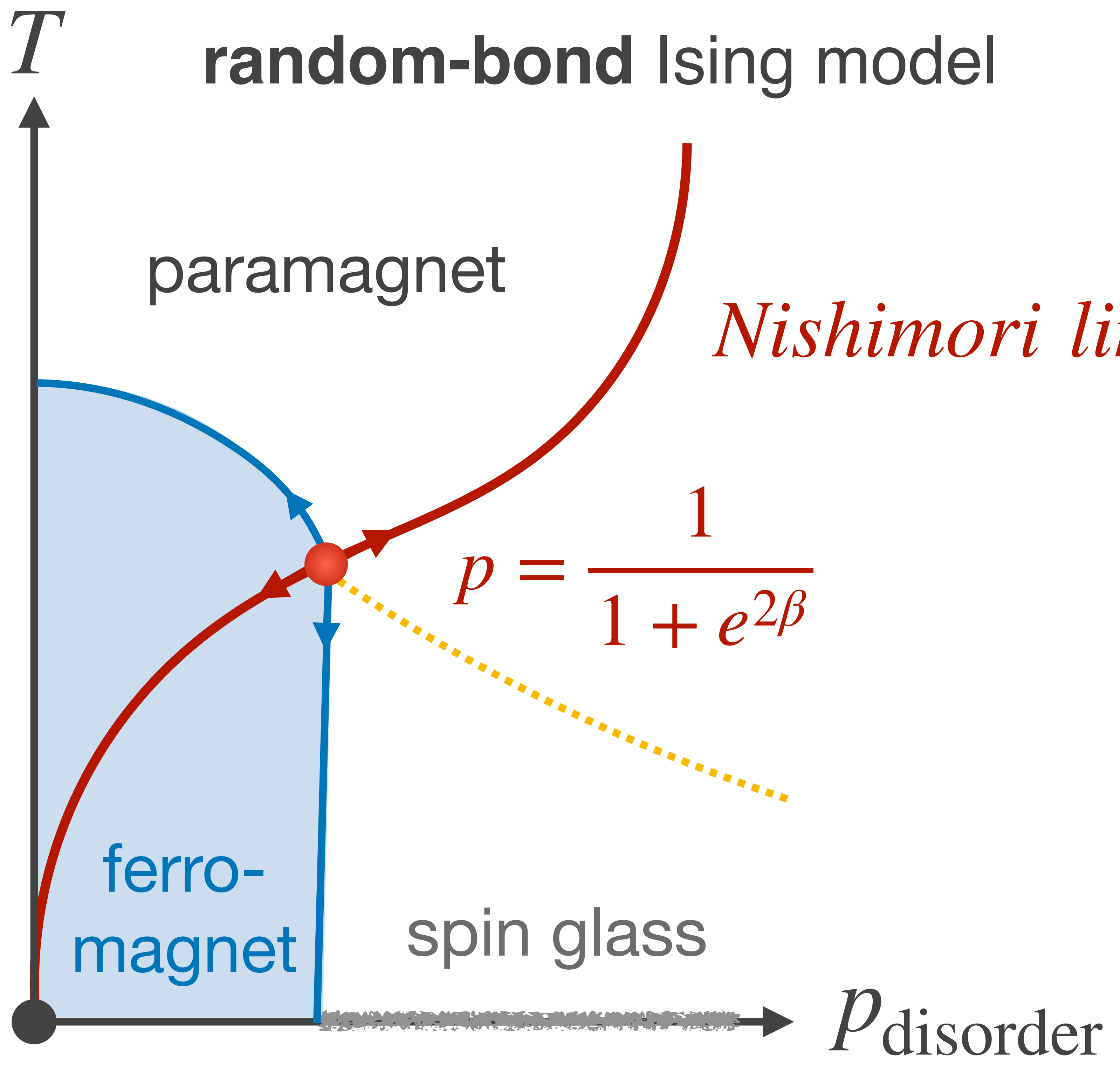
introduction to **replica theory**

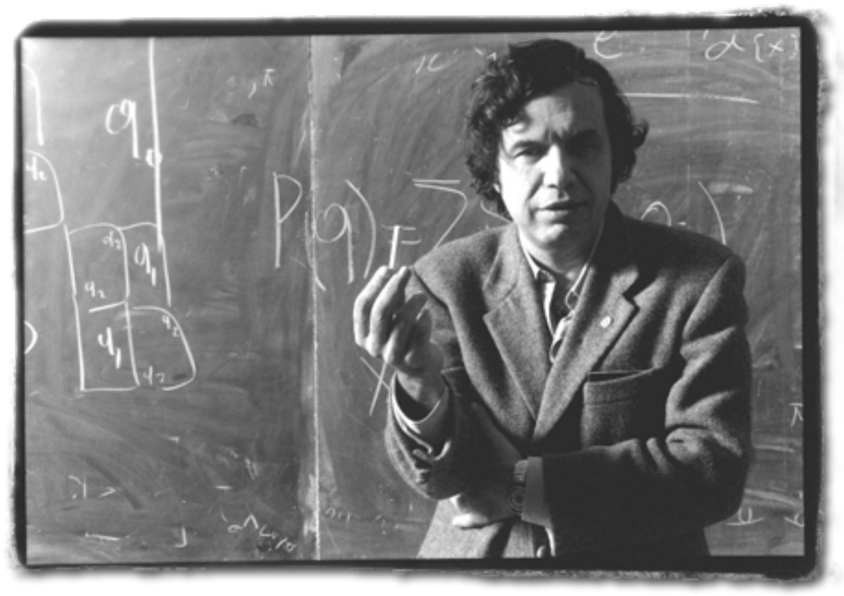
random-bond Ising model



$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \; S_i^z S_j^z$$

Gaussian disorder
mean J_0 and variance Δ^2

Giorgio Parisi



disorder average → theory of **interacting** replicas

$$\overline{Z^n} = \sum_{\{\sigma^a\}} \exp\left[K_1 \sum_{\langle ij \rangle} \sum_{a} \sigma_i^{(a)} \sigma_j^{(a)} + K_2 \sum_{\langle ij \rangle} \sum_{a<b} \sigma_i^{(a)} \sigma_i^{(b)} \sigma_j^{(a)} \sigma_j^{(b)} \right]$$

K_1: βJ_0 — single-replica interaction

K_2: $\beta^2 \Delta^2$ — replica-replica interaction

▸ **attractive** interactions

Edwards-Anderson order parameter

$$q_{\text{EA}} = \overline{\langle \sigma_i \rangle^2} = \lim_{n \to 0} \left\langle \sigma_i^{(a)} \sigma_i^{(b)} \right\rangle_{\text{rep}}$$

$a \neq b$

◂ replica-replica **locking**

▸ **replica symmetry** breaking

Nobel 2021

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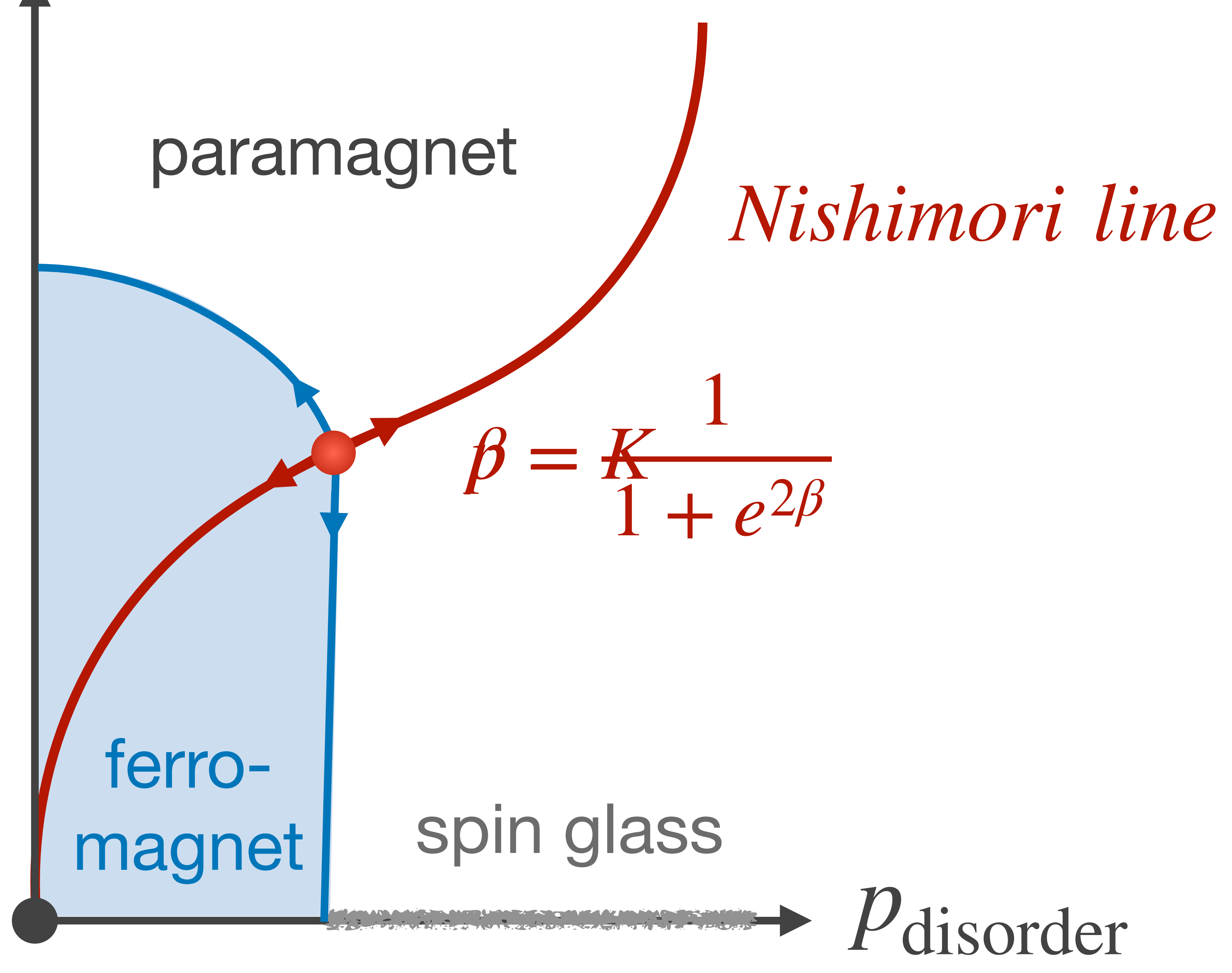
Nishimori line & replica symmetry

random-bond Ising model



$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij}\, S_i^z S_j^z$$

binary disorder

$P(J) \propto e^{KJ}$

$$P(J_{ij} = +1) = p \qquad K = \frac{1}{2}\ln\left(\frac{1-p}{p}\right)$$
$$P(J_{ij} = -1) = 1-p$$

start from the **replicated partition function**

$$\overline{Z^n} = \sum_{\{\sigma^{(a)}\}} \prod_{\langle ij \rangle} \overline{\exp\left(\beta J_{ij} \sum_{a=1}^{n} \sigma_i^{(a)} \sigma_j^{(a)}\right)}$$

bond average

$$\overline{e^{\beta J S}} = (1-p)\, e^{\beta S} + p\, e^{-\beta S}$$

$\beta = K$

and restrict to **Nishimori condition**

$$\overline{e^{\beta J S}}\Big|_{\beta=K} = \frac{e^{K(S+1)} + e^{-K(S+1)}}{2\cosh K} = \frac{\cosh(K(S+1))}{\cosh K}$$

gives **partition function on the Nishimori line**

$$\overline{Z^n}\Big|_{\beta=K} = (\cosh K)^{-N_b} \sum_{\{\sigma^{(a)}\}} \prod_{\langle ij \rangle} \cosh\left(K\left(\textstyle\sum_a \sigma_i^{(a)} \sigma_j^{(a)} + 1\right)\right)$$

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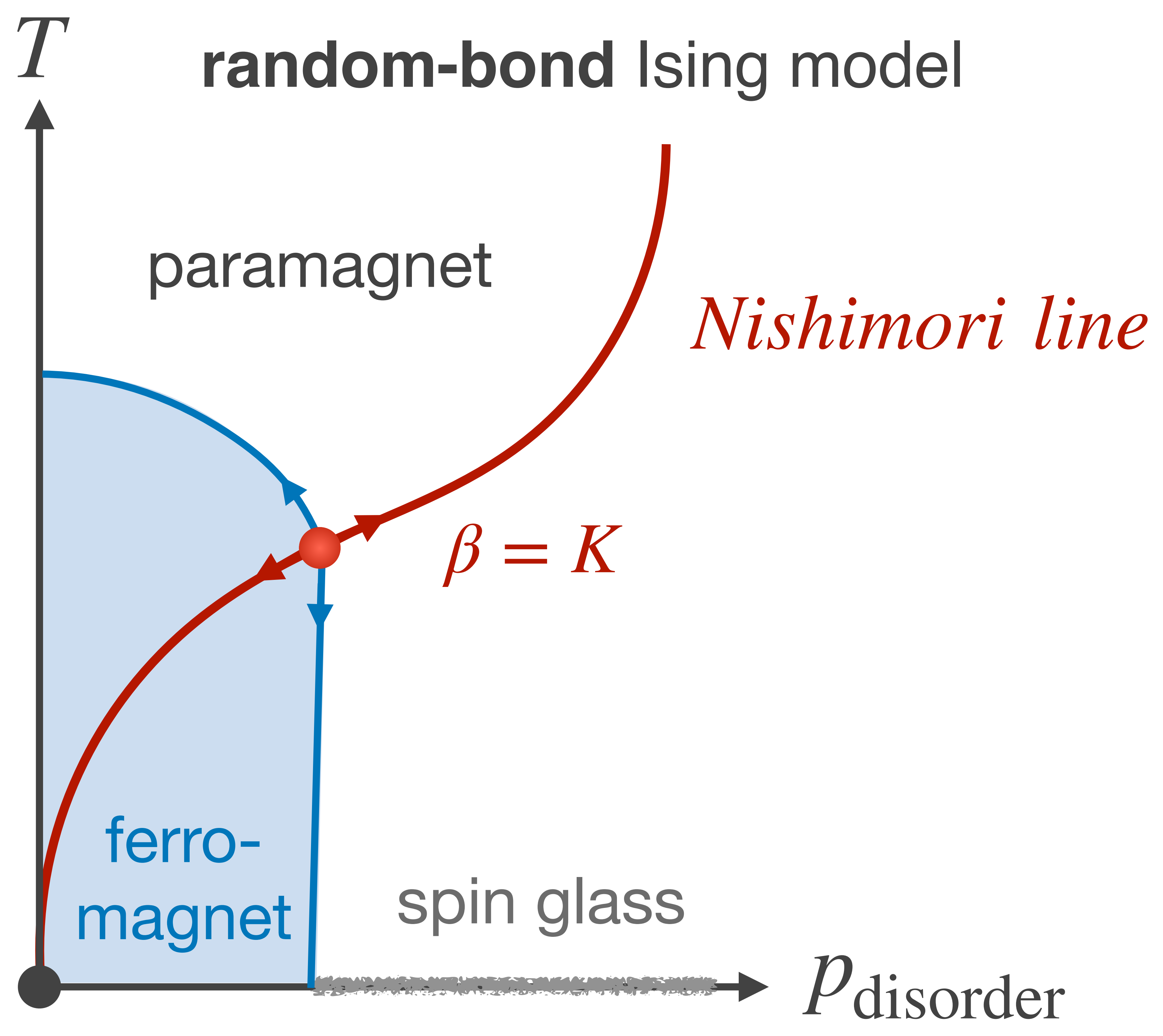
Nishimori line & replica symmetry

random-bond Ising model



$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \, S_i^z S_j^z$$

binary disorder $P(J) \propto e^{KJ}$

$$K = \frac{1}{2} \ln \left(\frac{1-p}{p} \right)$$

replicated partition function on the **Nishimori line**

$$\overline{Z^n}\Big|_{\beta=K} = (\cosh K)^{-N_b} \sum_{\{\sigma^{(a)}\}} \prod_{\langle ij \rangle} \cosh\Big(K \big(\textstyle\sum_a \sigma_i^{(a)} \sigma_j^{(a)} + 1 \big) \Big)$$

perform a **gauge change** $\sigma_i^{(a)} \to s_i \sigma_i^{(a)}$

$s_i \in \{\pm 1\}$ for each site

$$\overline{Z^n}\Big|_{\beta=K} = (\cosh K)^{-N_b} \sum_{\{\sigma^{(a)}\}} \prod_{\langle ij \rangle} \cosh\Big(K \big(\textstyle\sum_a \sigma_i^{(a)} \sigma_j^{(a)} + s_i s_j \big) \Big)$$

promote to **(n+1)-th replica**

$$\overline{Z^n}\Big|_{\beta=K} = \frac{(\cosh K)^{-N_b}}{2^N} \sum_{\{\sigma^{(a)}\}}^{n+1} \prod_{\langle ij \rangle} \cosh\Big(K \big(\textstyle\sum_a \sigma_i^{(a)} \sigma_j^{(a)} \big) \Big)$$

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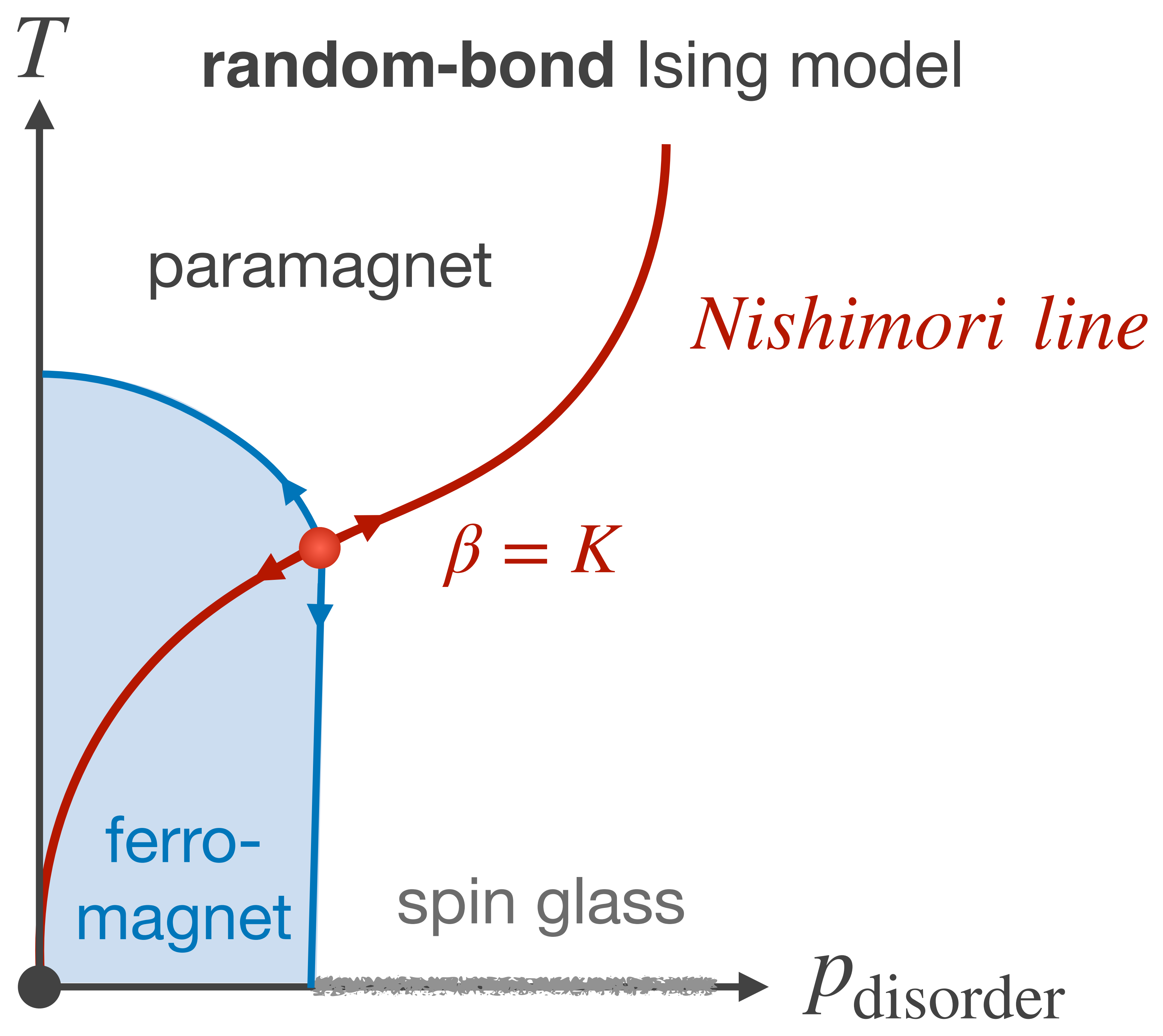
Nishimori line & **replica symmetry**

random-bond Ising model



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Nishimori line exhibits **enlarged symmetry**

- **replica permutation** symmetry

  $$S_n \to S_{n+1}$$

- local **gauge invariance**

  $$\text{global } Z_2 \to \text{local } Z_2$$

quenched disorder $= n \to 0$ replica limit
Nishimori line $= n \to 1$ replica limit

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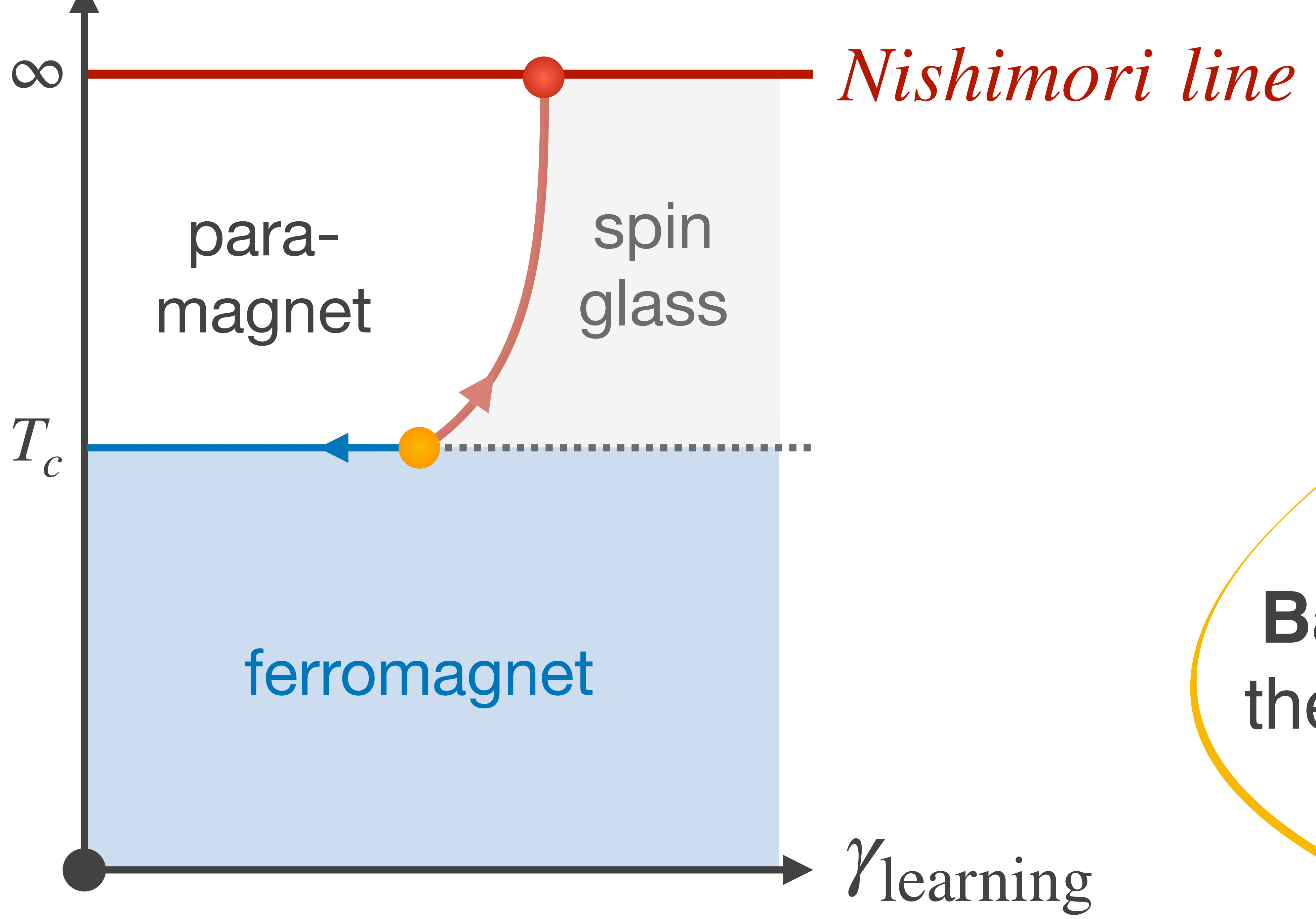
replica theory for the **learning model**

learning Ising model



$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

measurement $M_x = O_x + \eta_x$ — Gaussian disorder

mean 0 and variance Δ^2

conditional probability of measurement outcome

$$p(M|S) = \exp\left(-\frac{1}{2\Delta^2} \sum_x (O_x - M_x)^2 + \ln d \right)$$

given spin configuration; measurement; normalization $d = \sqrt{\frac{1}{(2\pi\Delta^2)^{N_{\text{meas}}}}}$

measurement outcome; variance; observable

Bayes' theorem

$$H_{\text{meas}}[S, M] = \mathcal{H} + \frac{1}{2\Delta^2} \sum_x (O_x - M_x)^2 - \ln d$$

$$p(S|M) = \frac{p(S)\, p(M|S)}{p(M)} = \frac{1}{Z[M]} \exp(-H_{\text{meas}}[S, M])$$

posterior distribution; **marginal** distribution; statistical ensemble

$$p(M) = \frac{Z[M]}{Z}$$

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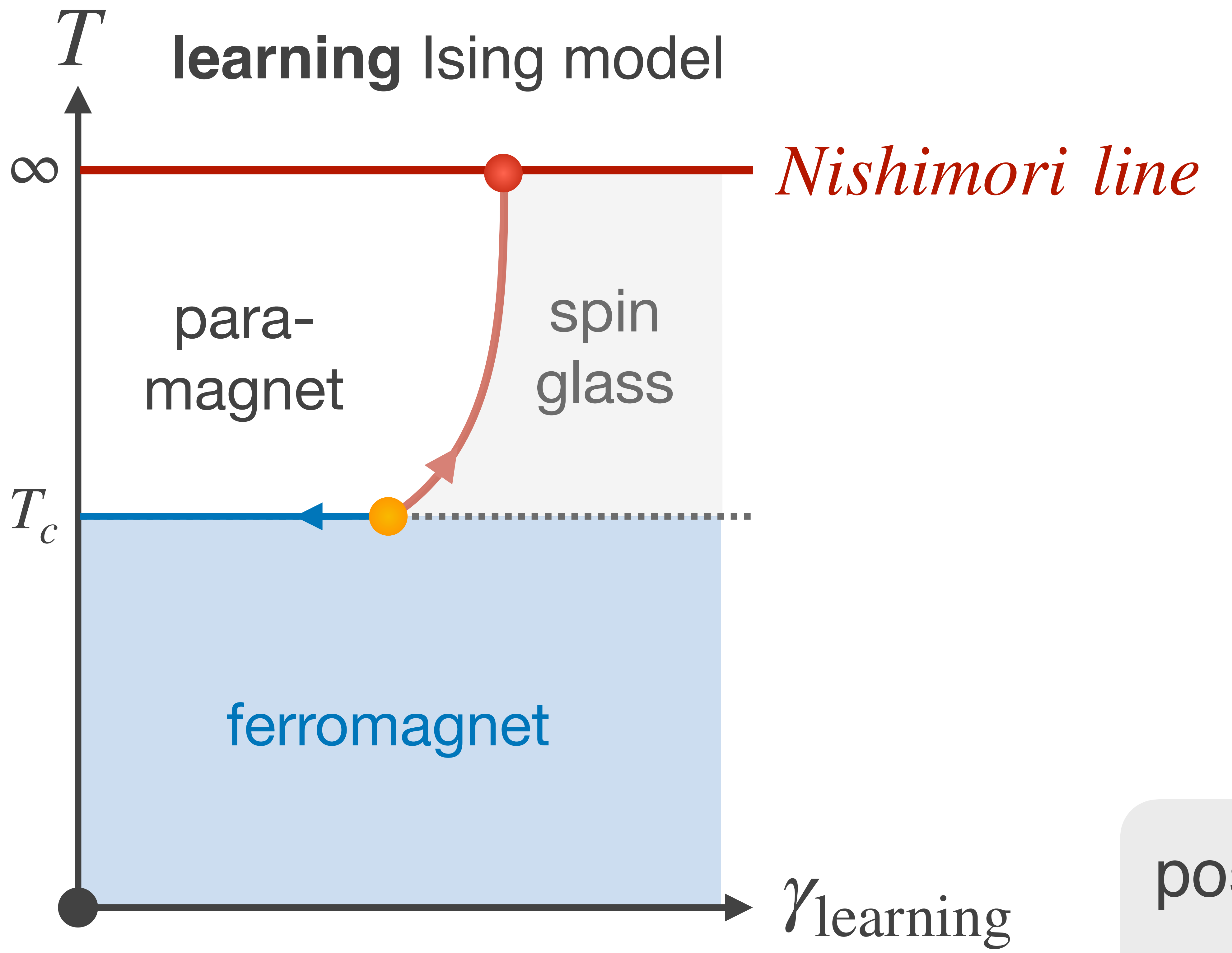
replica theory for the **learning model**

learning Ising model



$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

measurement $M_x = O_x + \eta_x$ — Gaussian disorder

mean 0 and variance Δ^2

replica trick for posterior ensemble

$$p_K(S^1, S^2, \ldots, S^K, M)$$

K replicas

conditioned on the same measurement outcome

$$p_K(S^1, S^2, \ldots, S^K, M) = p(M) \prod_{\alpha=1}^{K} p(S^\alpha | M)$$

$$= \frac{Z[M]}{Z} \cdot \frac{1}{Z[M]^K} \exp\left(-\sum_{\alpha=1}^{K} H_{\text{meas}}[S^\alpha, M] \right)$$

marginal distribution

replicated statistical ensemble

posterior ensemble captured by **n → 1 replica limit**

$$= \lim_{n \to 1} \frac{Z[M]^{n-K}}{Z} \exp\left(-\sum_{\alpha=1}^{K} H_{\text{meas}}[S^\alpha, M] \right)$$

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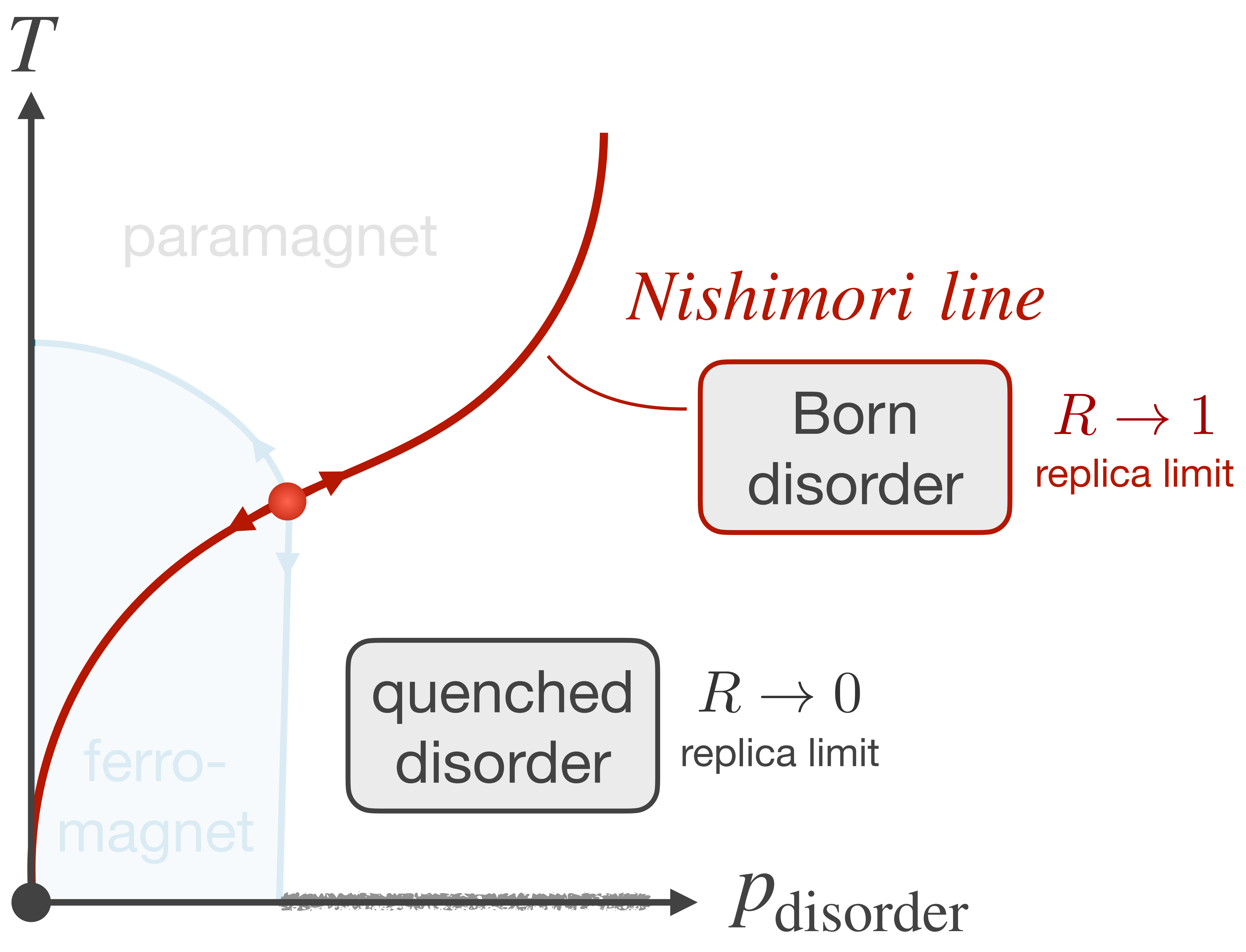
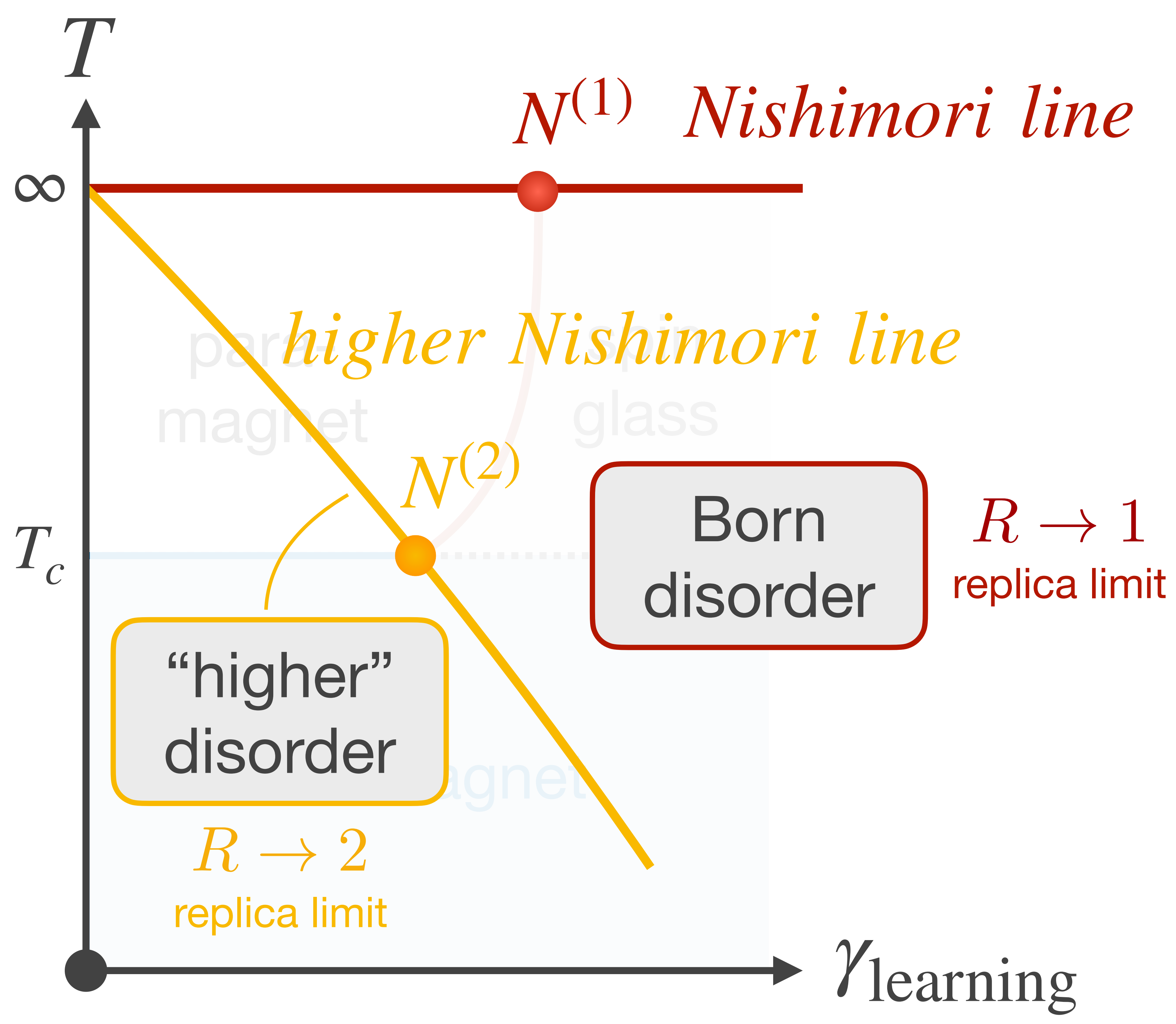
conditional randomness by measurement

random-bond Ising model $R \to 0$



$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij}\, S_i^z S_j^z$$

learning Ising model $R \to 1$



$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

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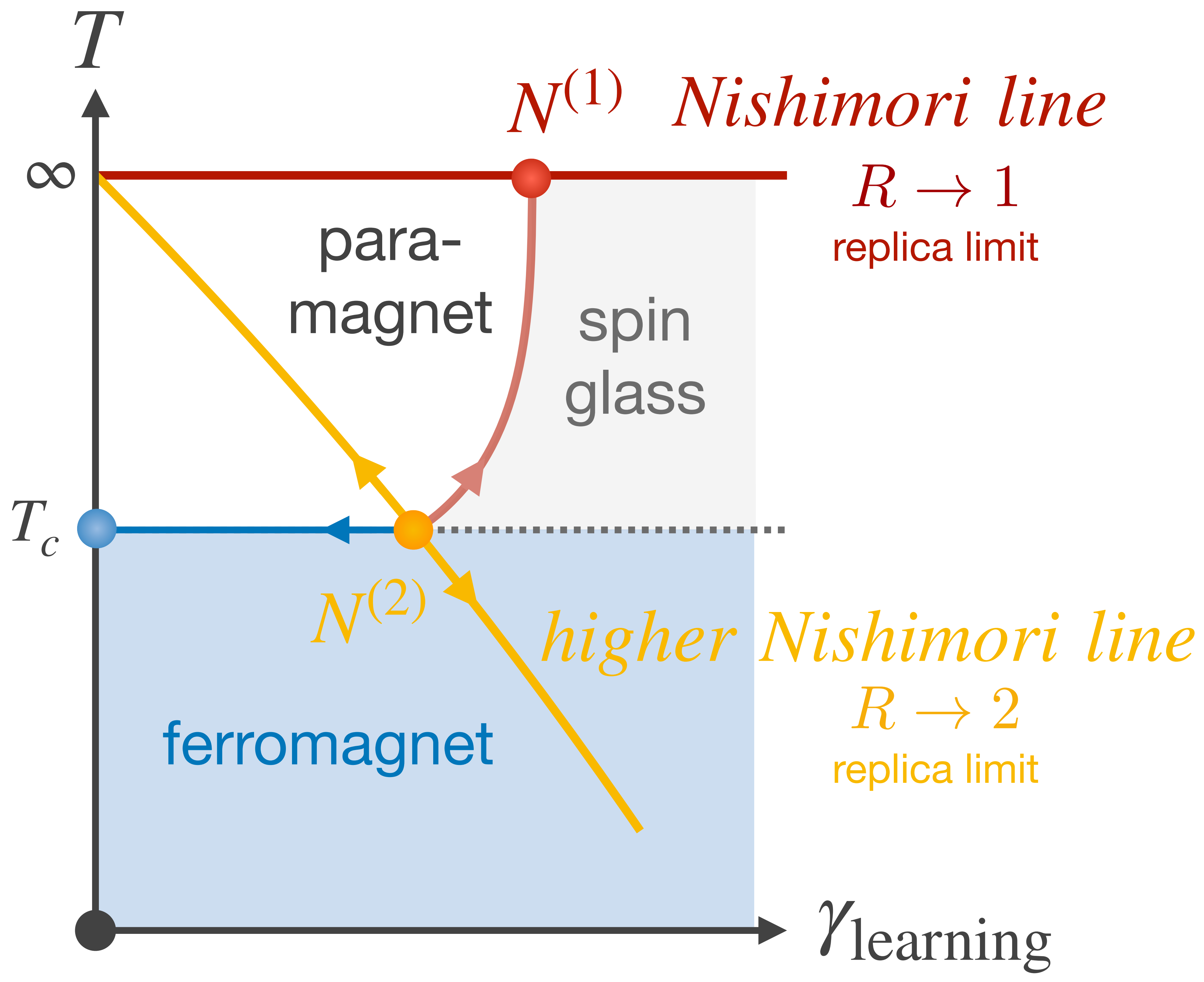
higher Nishimori line

arXiv:2604.06324

higher Nishimori line

R. Patil *et al.*, arXiv:2604.06324

learning Ising model $R \to 1$



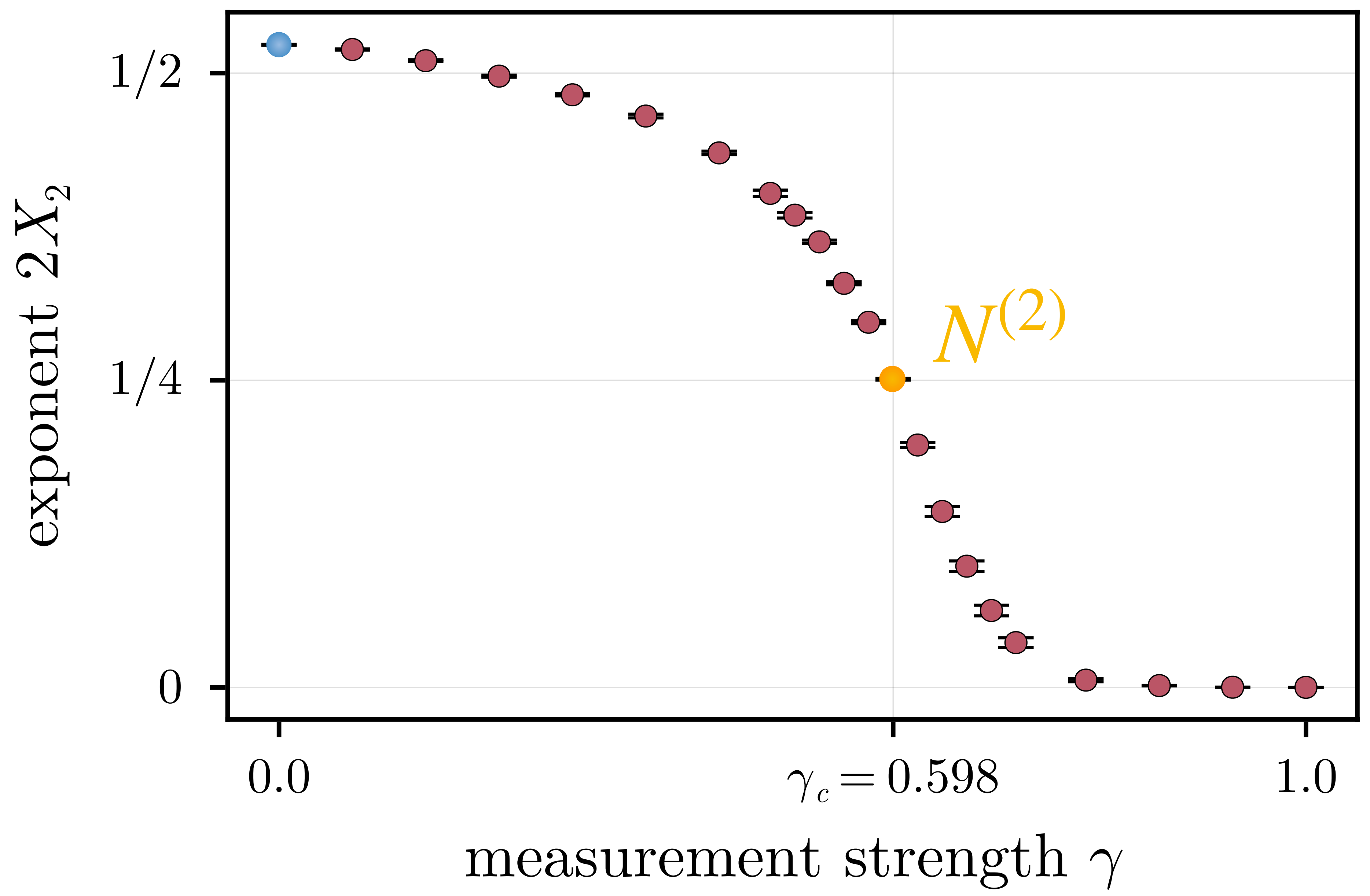
$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

power-law exponent of EA correlator

$$\overline{\langle \sigma_i \sigma_j \rangle^2} \sim \frac{1}{|i-j|^{1/4}}$$

replica-replica **locking**

two-point spin correlator at Ising point



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higher Nishimori line

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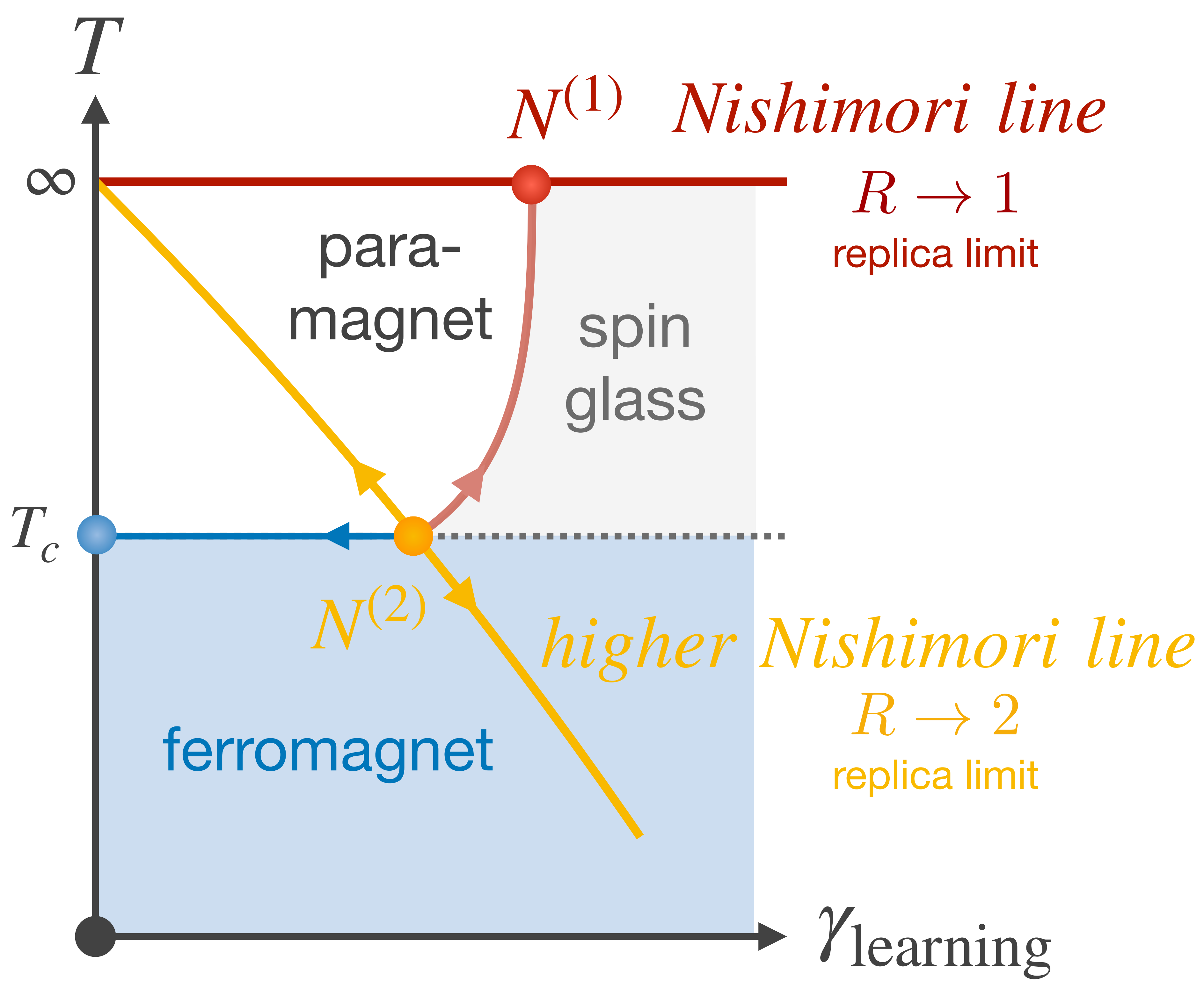
learning Ising model $R \to 1$



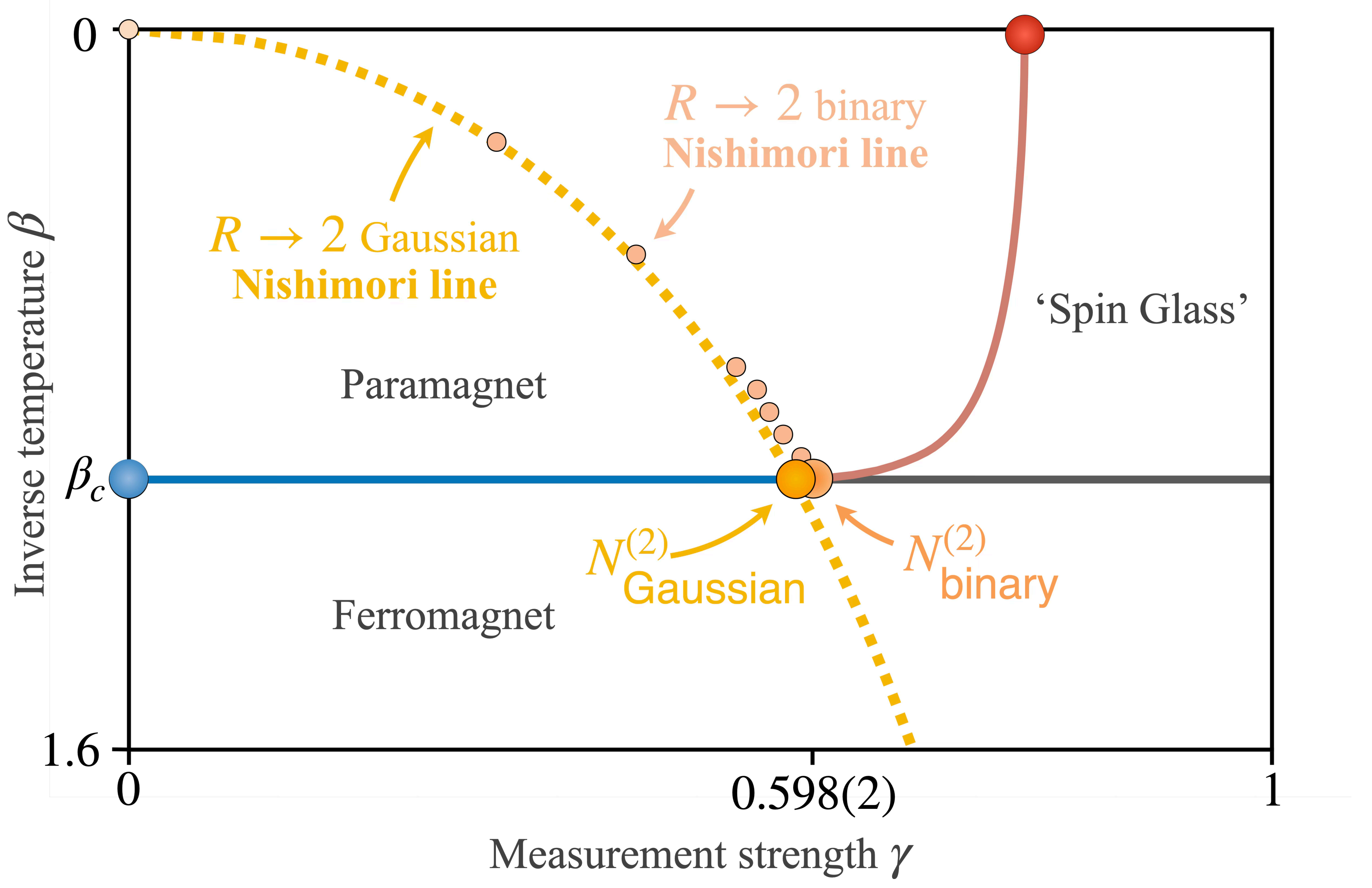
$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

generalized **Nishimori condition**

$$\overline{\langle \sigma_i \sigma_j \rangle^2} \simeq \overline{\langle \sigma_i \sigma_j \rangle}$$

replica-replica **locking**

two-point spin correlator at Ising point



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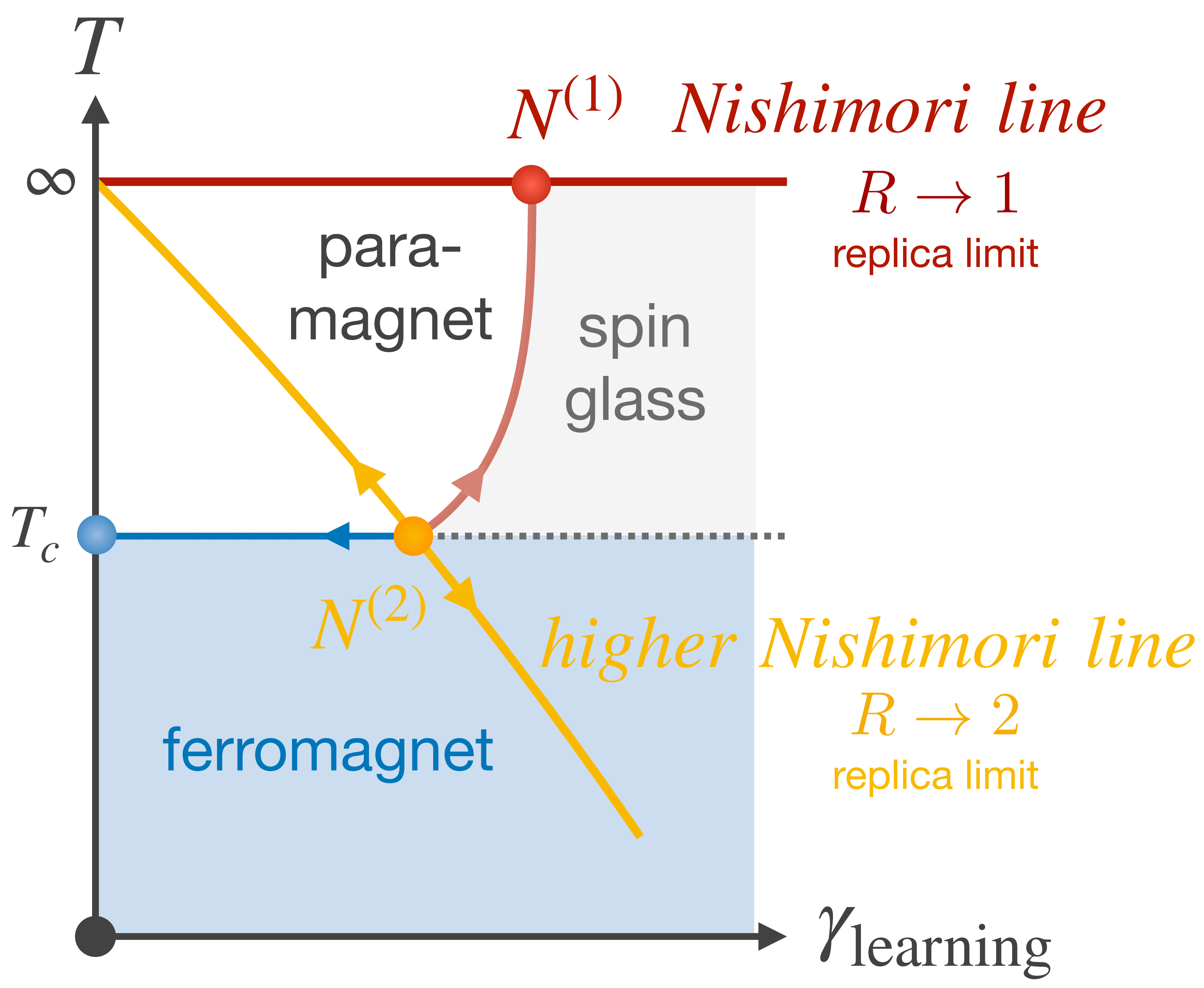
generalized **RG flows** and c-**theorem**

R. Patil *et al.*, arXiv:2604.06324

learning Ising model $R \to 1$

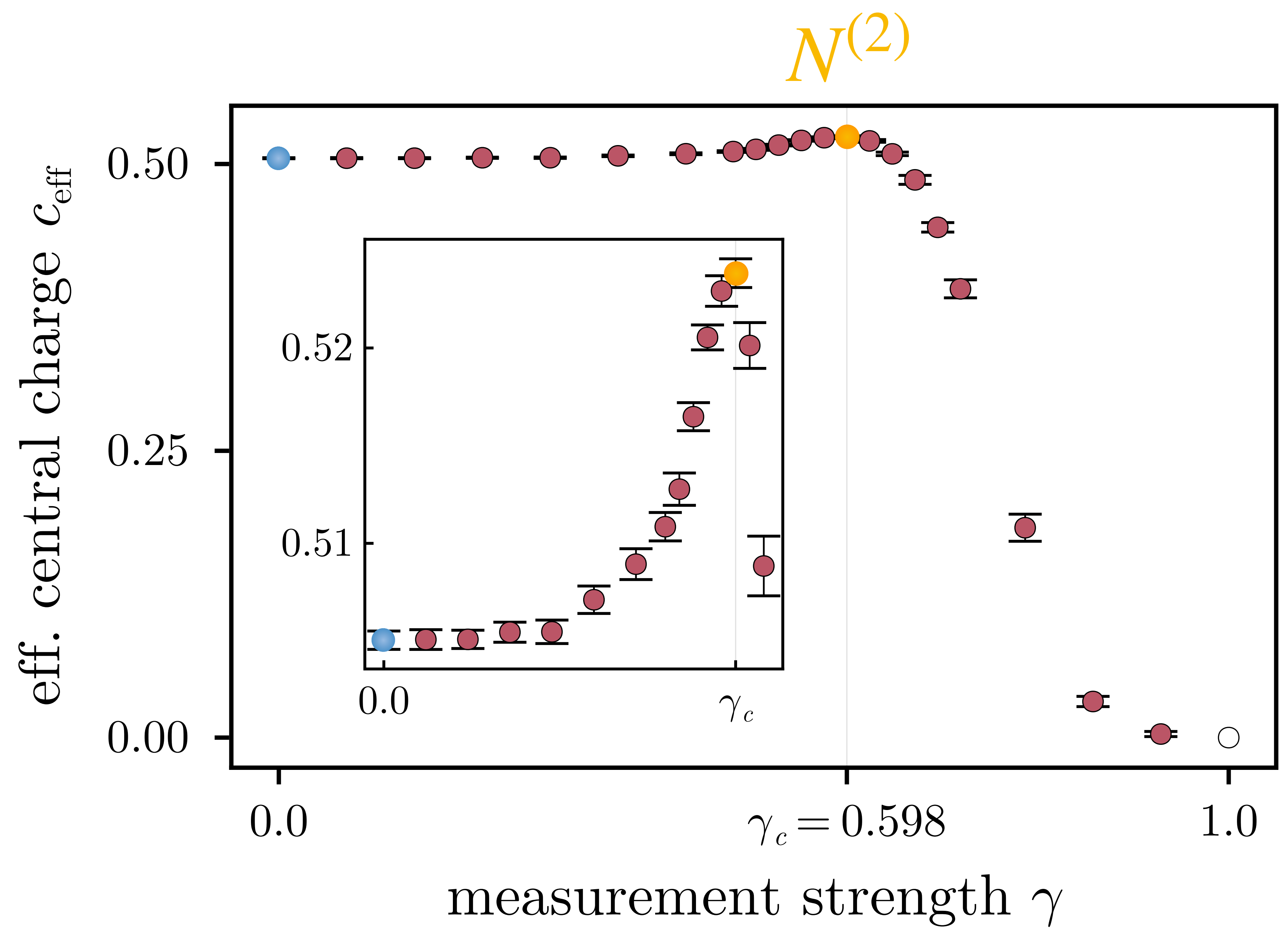


$$\mathcal{H} = -\sum_{\langle i,j \rangle} S_i^z S_j^z$$

RG flow of **Casimir central charge**

$$c_{\text{eff}} > c_{\text{Ising}} = 1/2$$

R. Patil & A. Ludwig, arXiv:2507.07959



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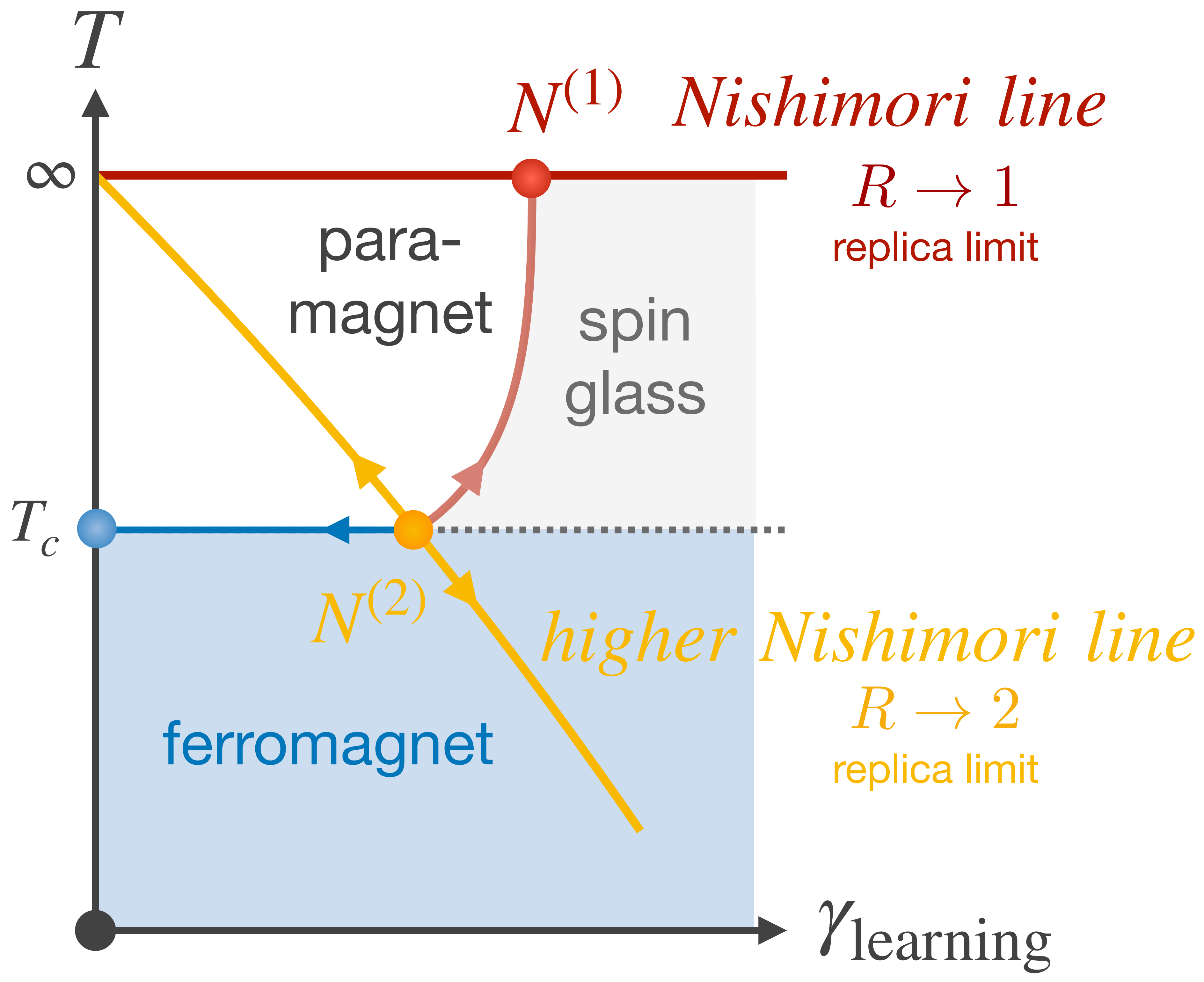
generalized **RG flows** and c-**theorem**

R. Patil *et al.*, arXiv:2604.06324

learning Ising model $R \to 1$



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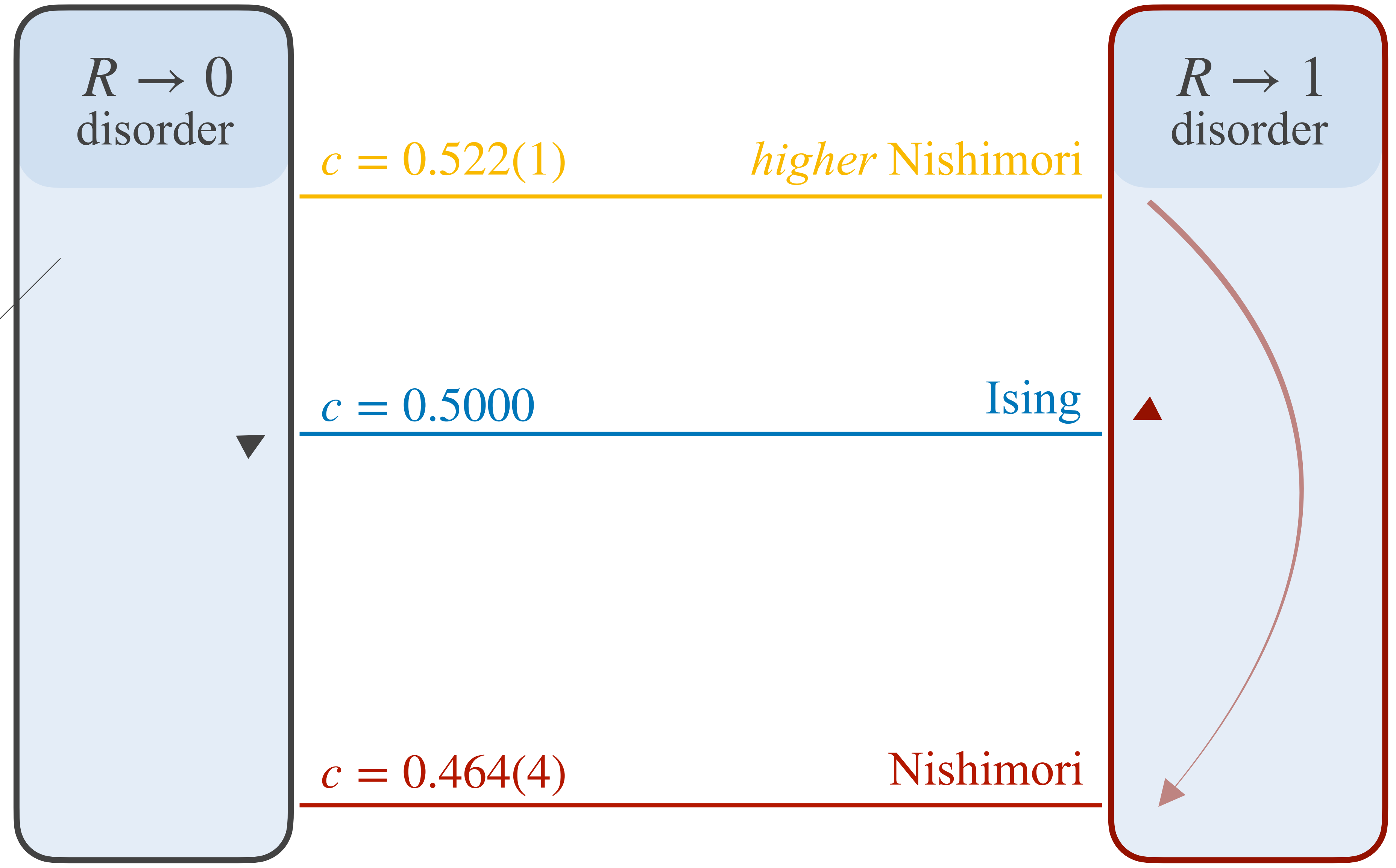

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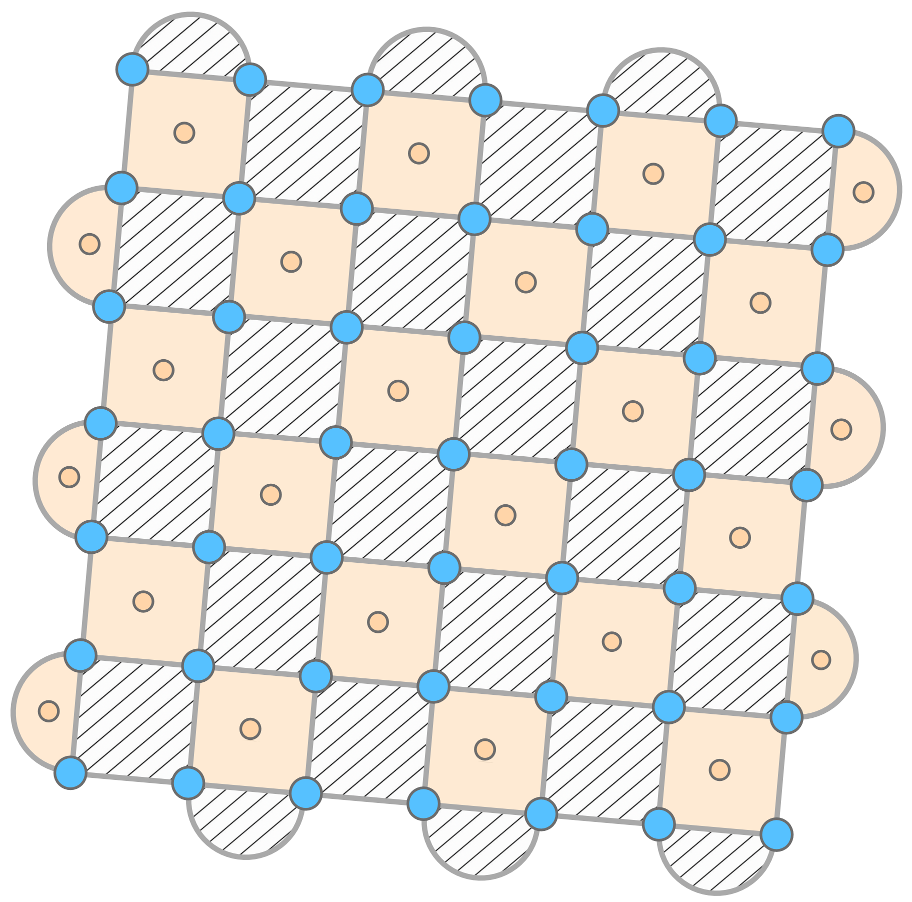
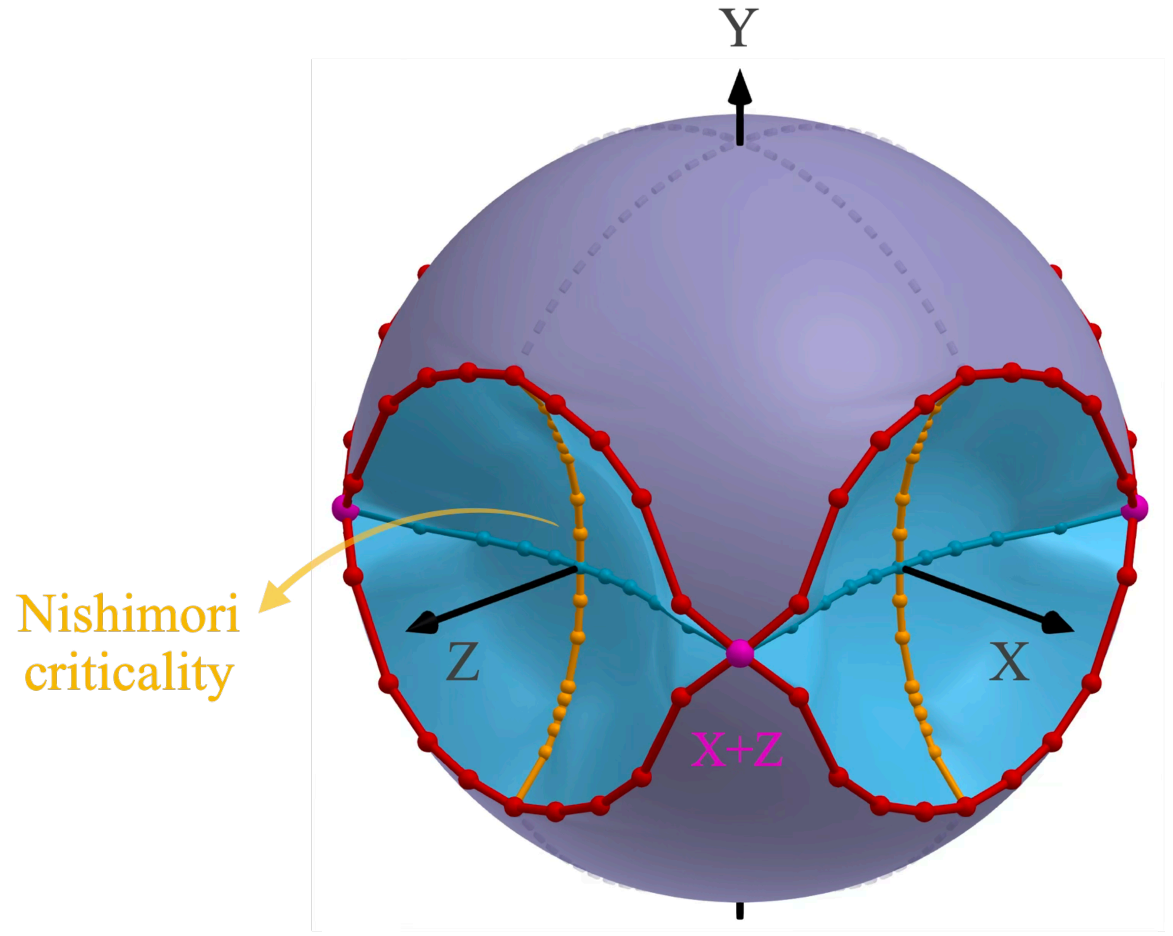
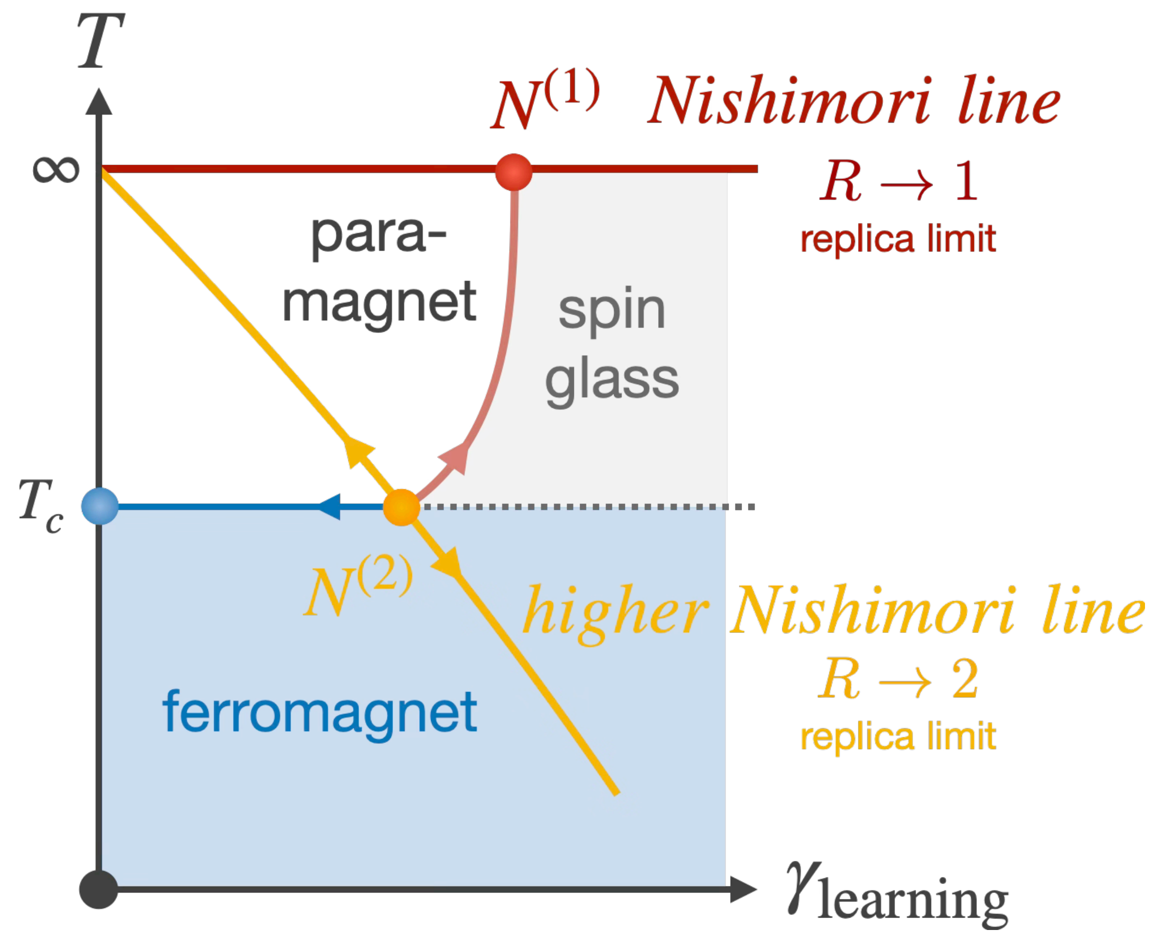
closing remarks

summary

- **measurements** are a powerful tool to create entangled states
  - **shallow** circuit constructions for cat & toric code states
  - trade-off in form of **auxiliary qubits** and **decoding**
- **monitoring** topology
  - **stability** of protocols & thresholds
  - **criticality** & **Nishimori** physics
- **learning** topology
  - **conditioned** disorder & **replica** theory
  - new **universal behavior** & **higher Nishimori** physics







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Thanks!