Quantum Computational Physics

Nishimori physics, quantum measurements, and monitored criticality

CRC183 ENTANGLED STATES OF MATTER

Simon Trebst
University of Cologne

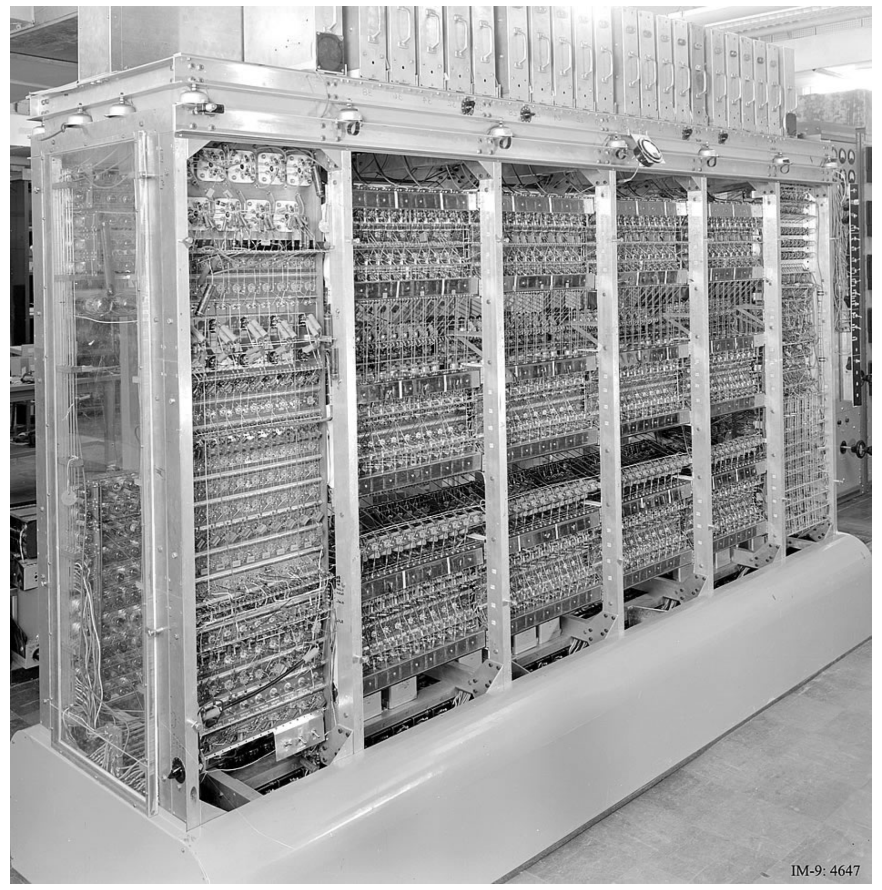
ML4Q MATTER AND LIGHT FOR QUANTUM COMPUTING

SQAI-NCTS Workshop on Quantum Technologies and Machine Learning

National Taiwan University, Taipei, August 2025

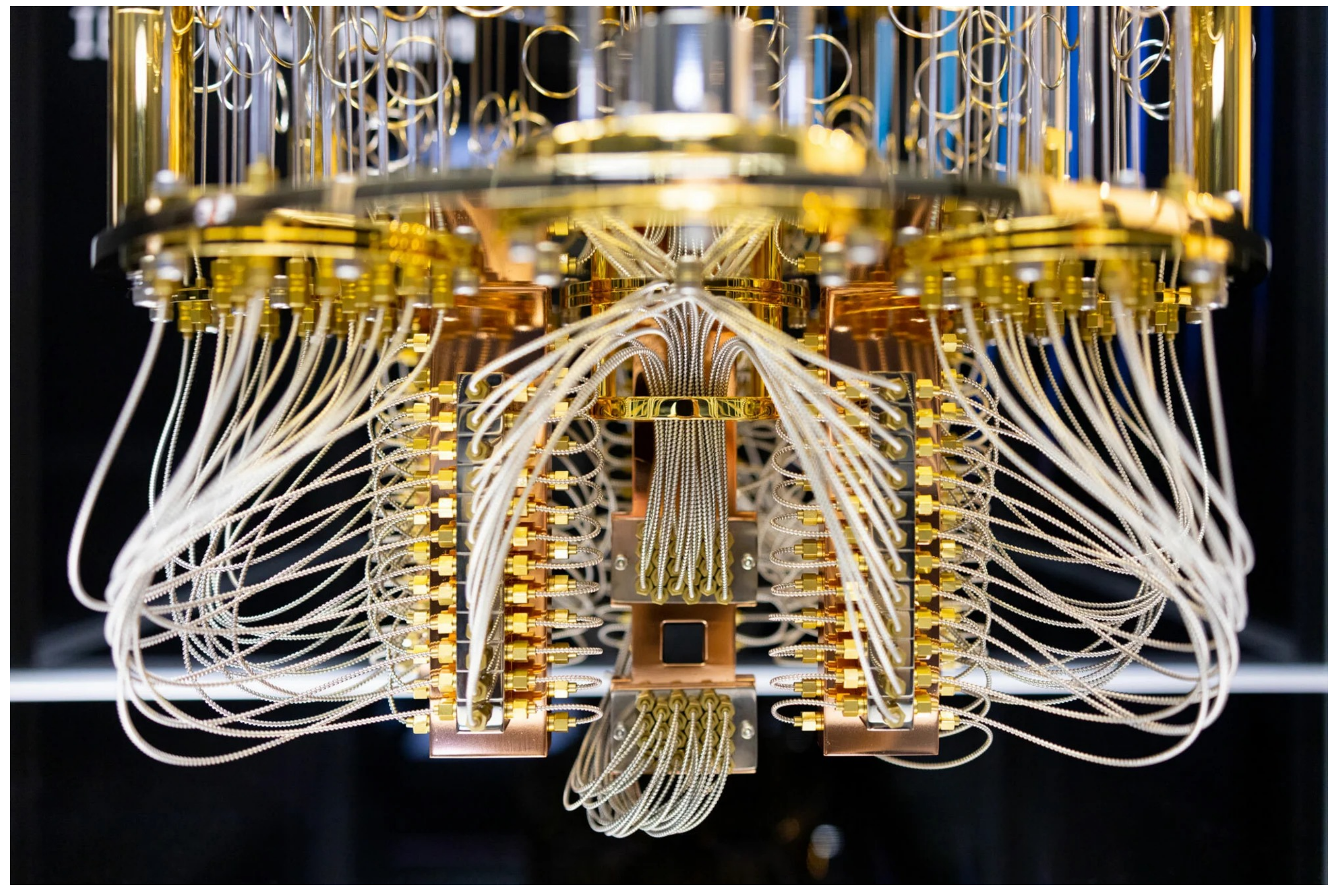
computational physics

Maniac @ Los Alamos



Cray @ ETH Zurich

IBM Quantum (cloud)



(1953) — (1992) — (2019) →

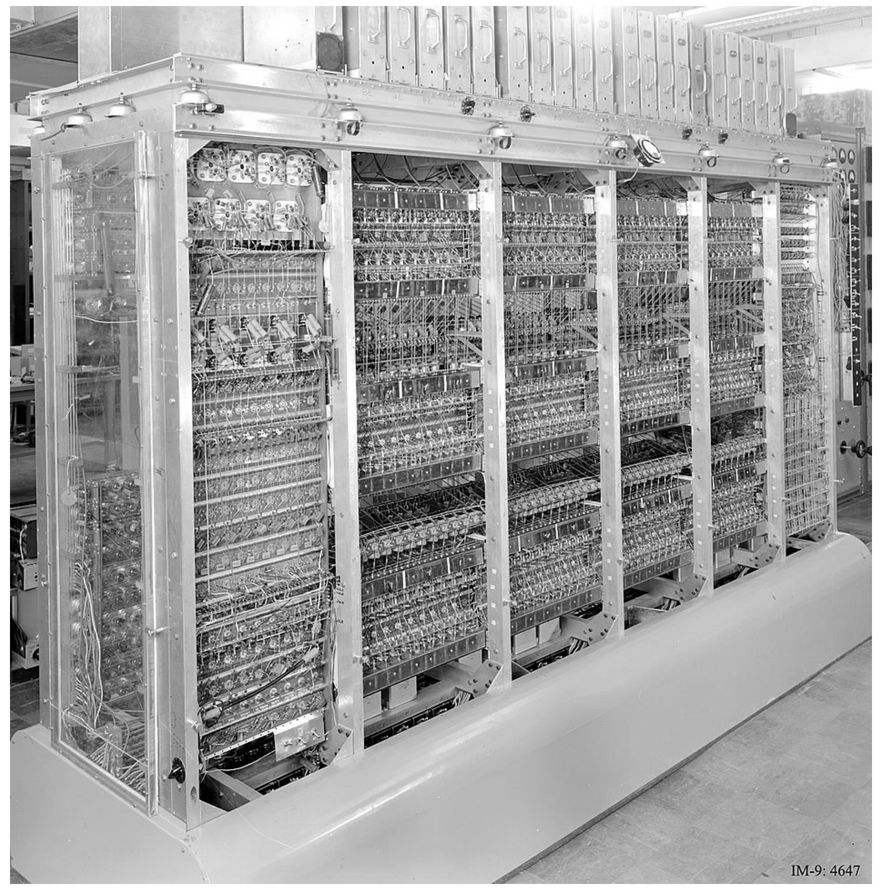
classical many-body

quantum many-body

$(\text{quantum})^2$ many-body

classical many-body physics

Maniac @ Los Alamos



— (1953) —

classical many-body

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, and Augusta H. Teller,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

and

Edward Teller,* *Department of Physics, University of Chicago, Chicago, Illinois*

(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations [of] state for substances consisting of interacting individual molecules is described. The method consists of modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphe[re] system have been obtained on the Los Alamos MANIAC and are presented here. These results are compar[ed] to the free volume equation of state and to a four-term virial coefficient expansion.

I. INTRODUCTION

THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric.

II. THE GENERAL METHOD FOR AN … POTENTIAL BETWEEN THE PAR…

In order to reduce the problem to a fe… numerical work, we can, of course, consid… number of particles. This number N may… several hundred. Our system consists of a… taining N particles. In order to minimiz… effects we suppose the complete substance…

Monte Carlo sampling nevertheless became one of the most widely used algorithms for its ability to sample from arbitrary distributions.

GUEST EDITORS' INTRODUCTION

In putting together this issue of *Computing in Science & Engineering*, we knew three things: it would be difficult to list just 10 algorithms; it would be fun to assemble the authors and read their papers; and, whatever we came up with in the end, it would be controversial. We tried to assemble the 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century.

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

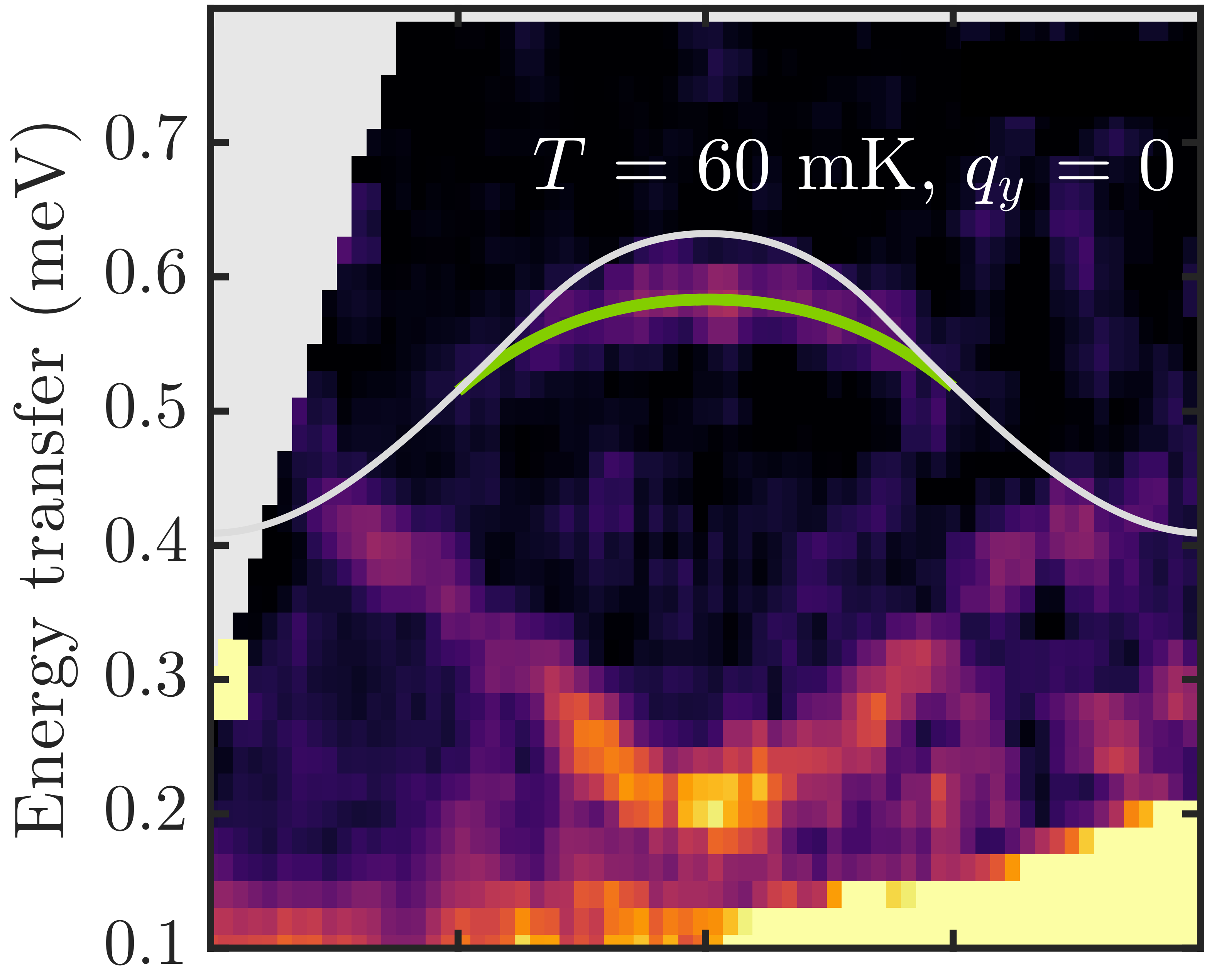
quantum many-body physics

Cray @ ETH Zurich

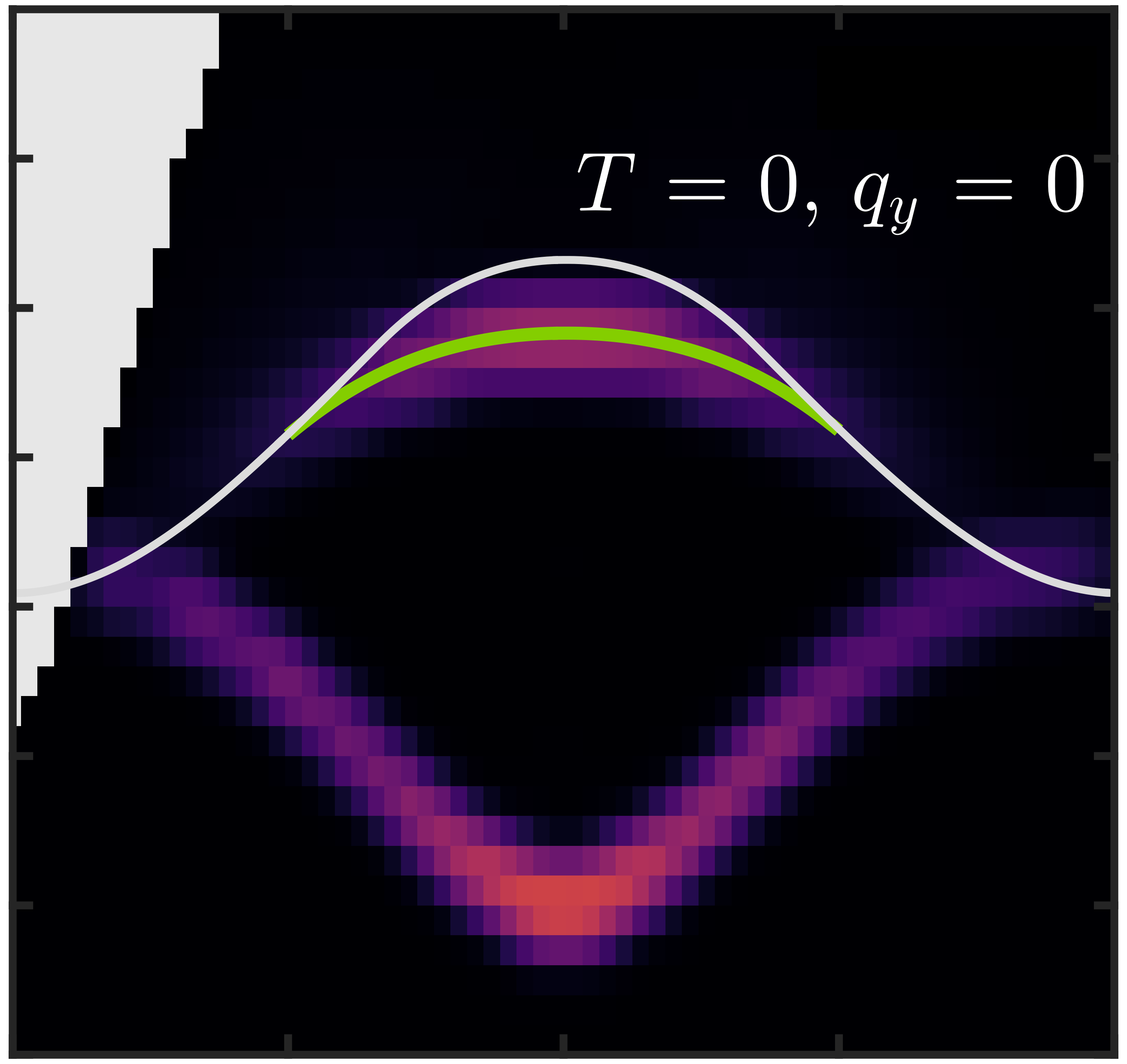
— (1992) —

quantum many-body

Collective phenomena in quantum systems become accessible to **quantitative numerical simulations**, such as superconductivity, Bose-Einstein condensation & topological materials.



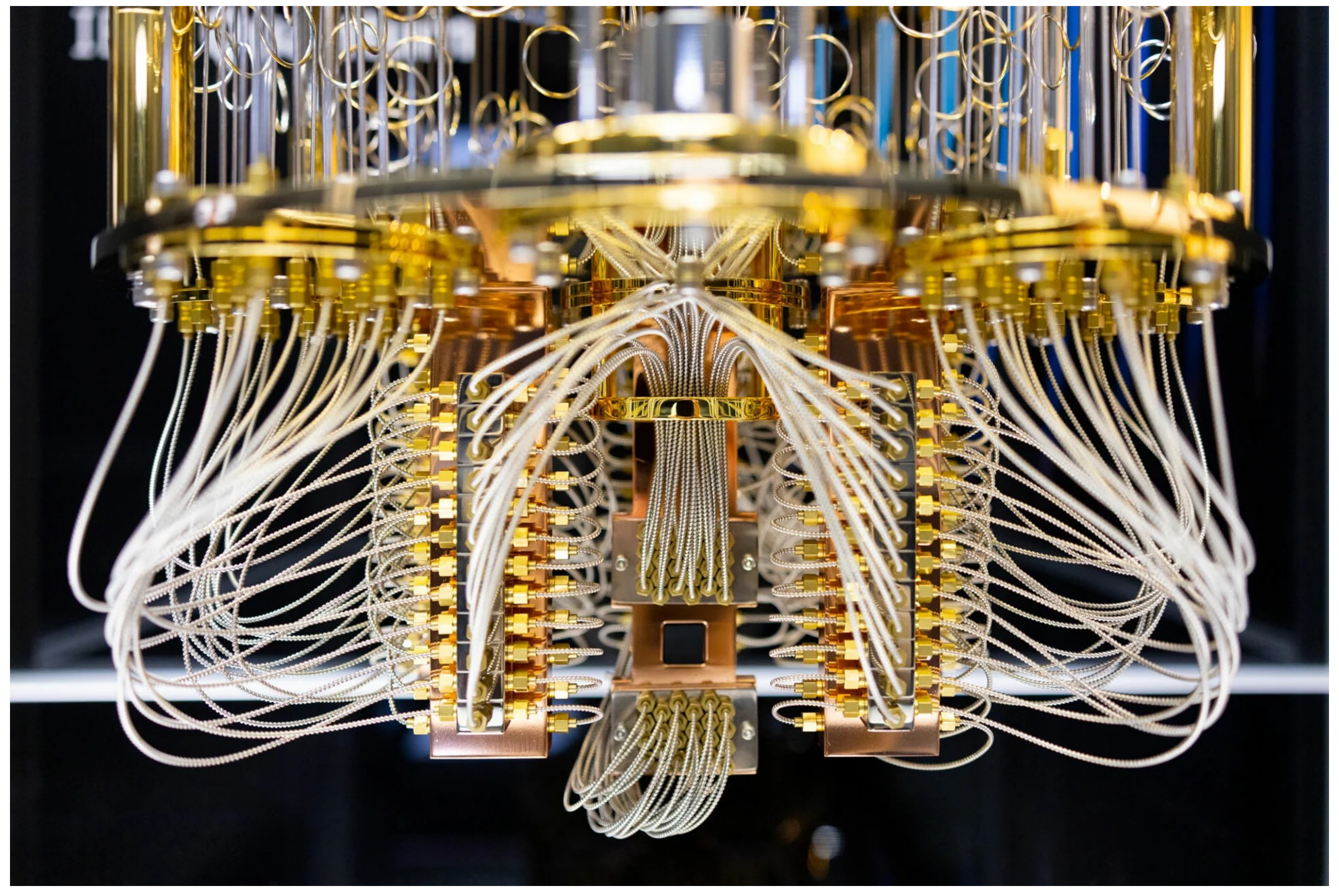
neutron scattering on quantum material



density matrix renormalization group (DMRG) numerical simulation

(quantum)^2 many-body

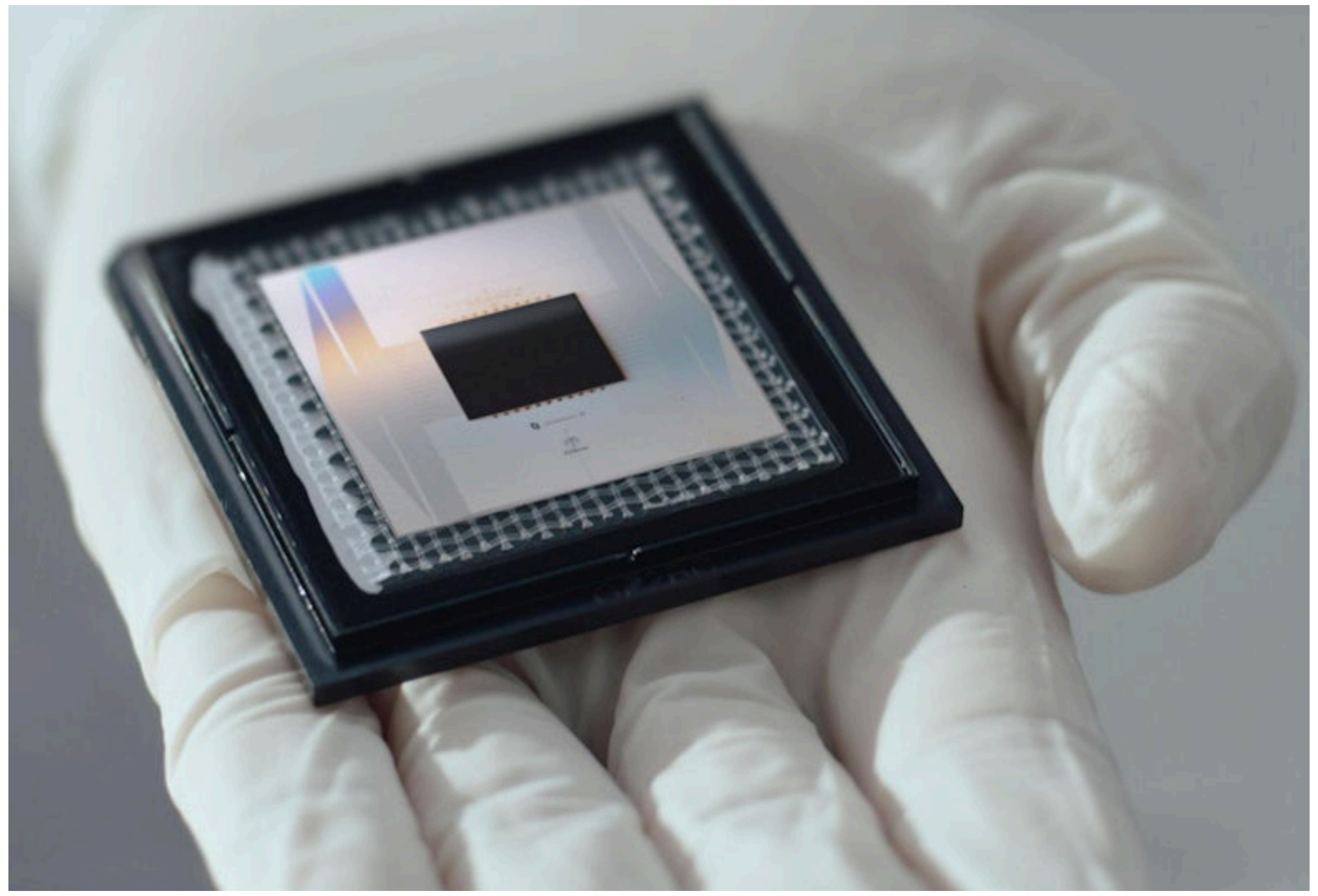
IBM Quantum (cloud)



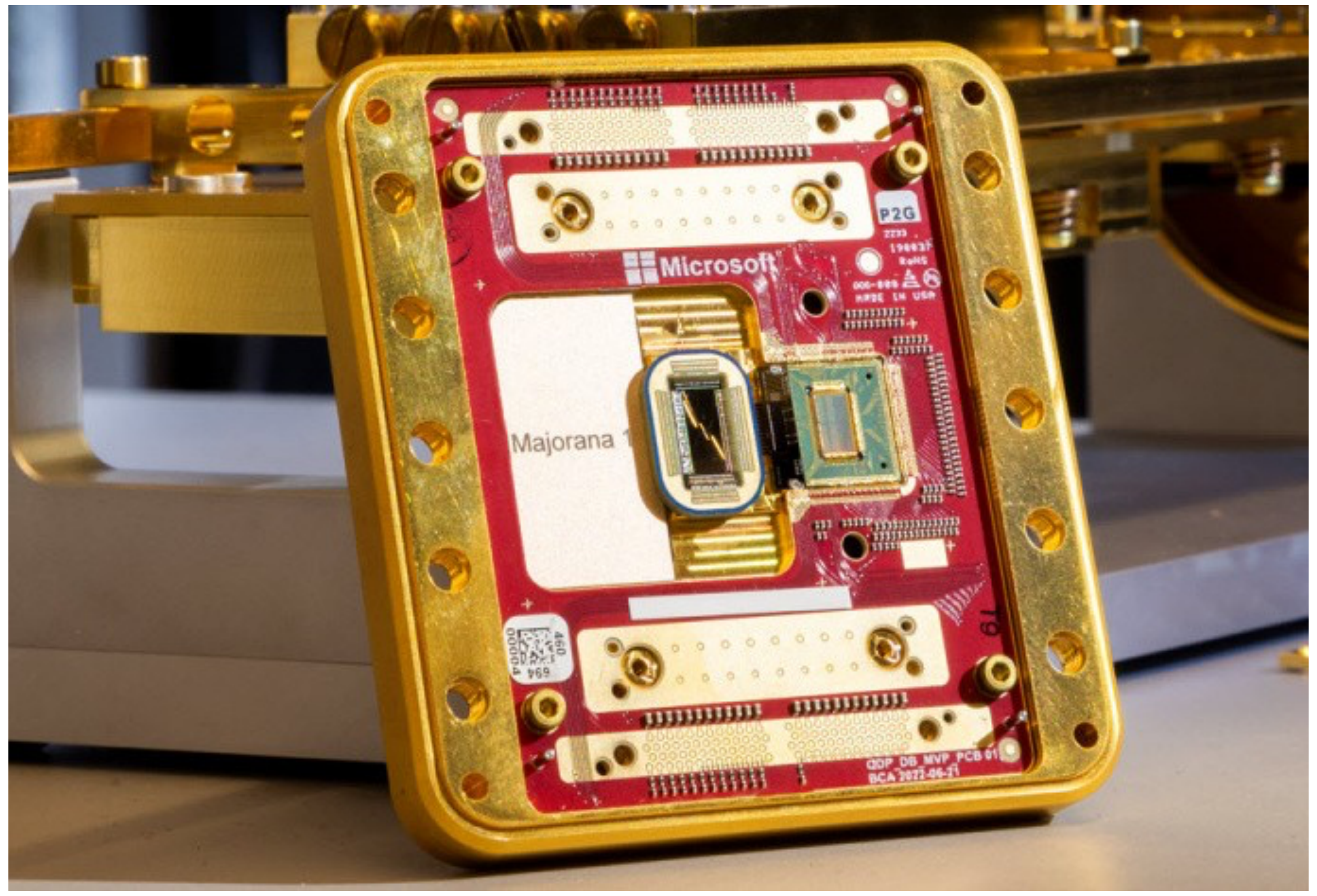
(2019)

(quantum)^2
many-body

Quantum processors are emerging as a new experimental platform to simulate **“quantum on quantum”**, i.e. the non-linear **dynamics** of quantum systems with unprecedented control.



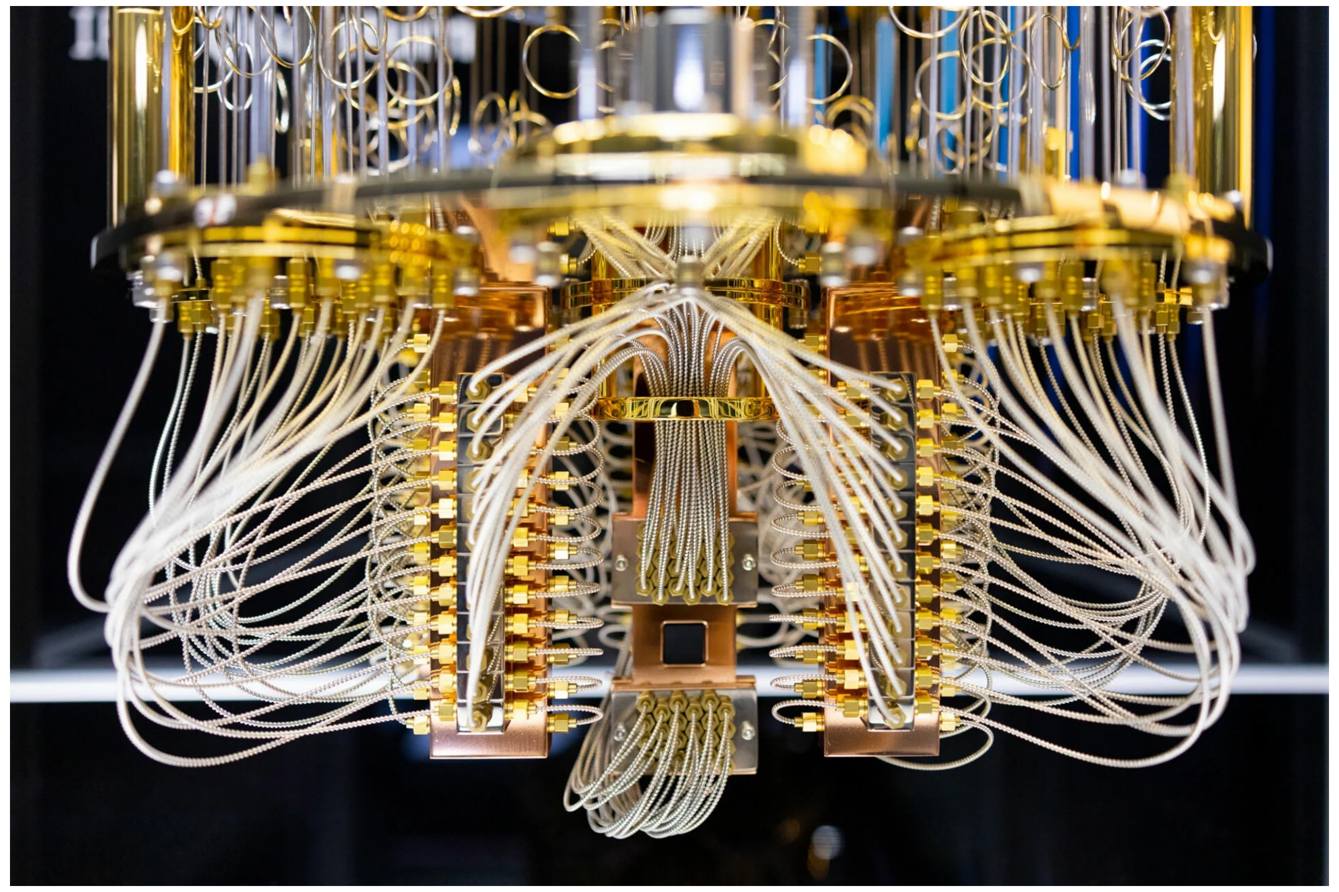
Google’s Willow chip
(December 2024)



Microsoft’s Majorana 1 chip
(February 2025)

(quantum)^2 many-body

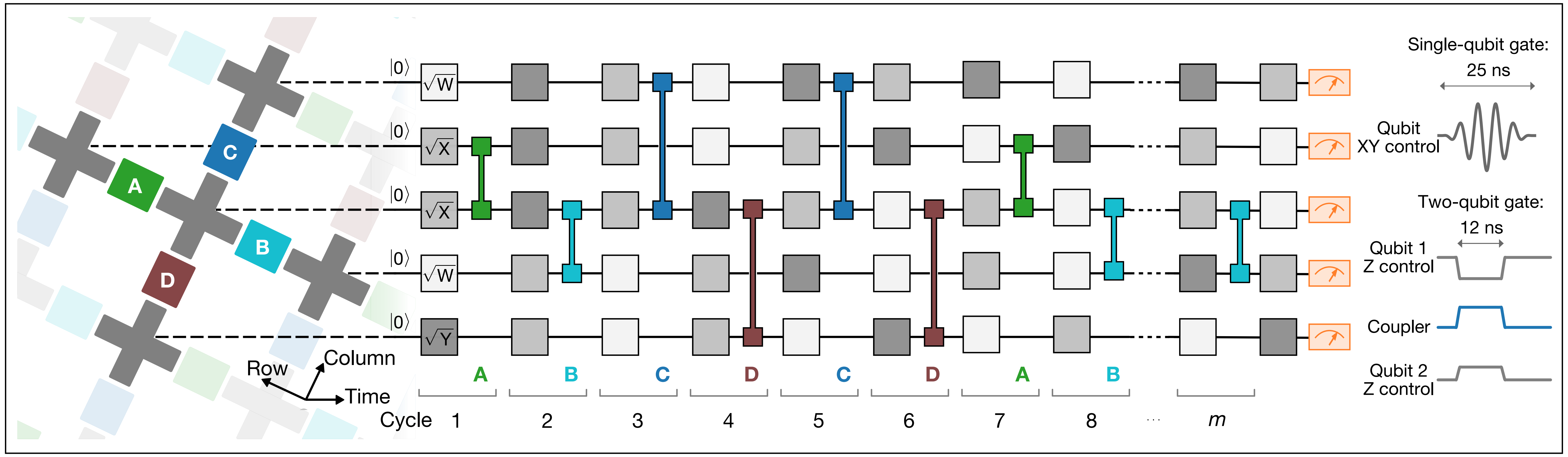
IBM Quantum (cloud)



(2019)

(quantum)^2
many-body

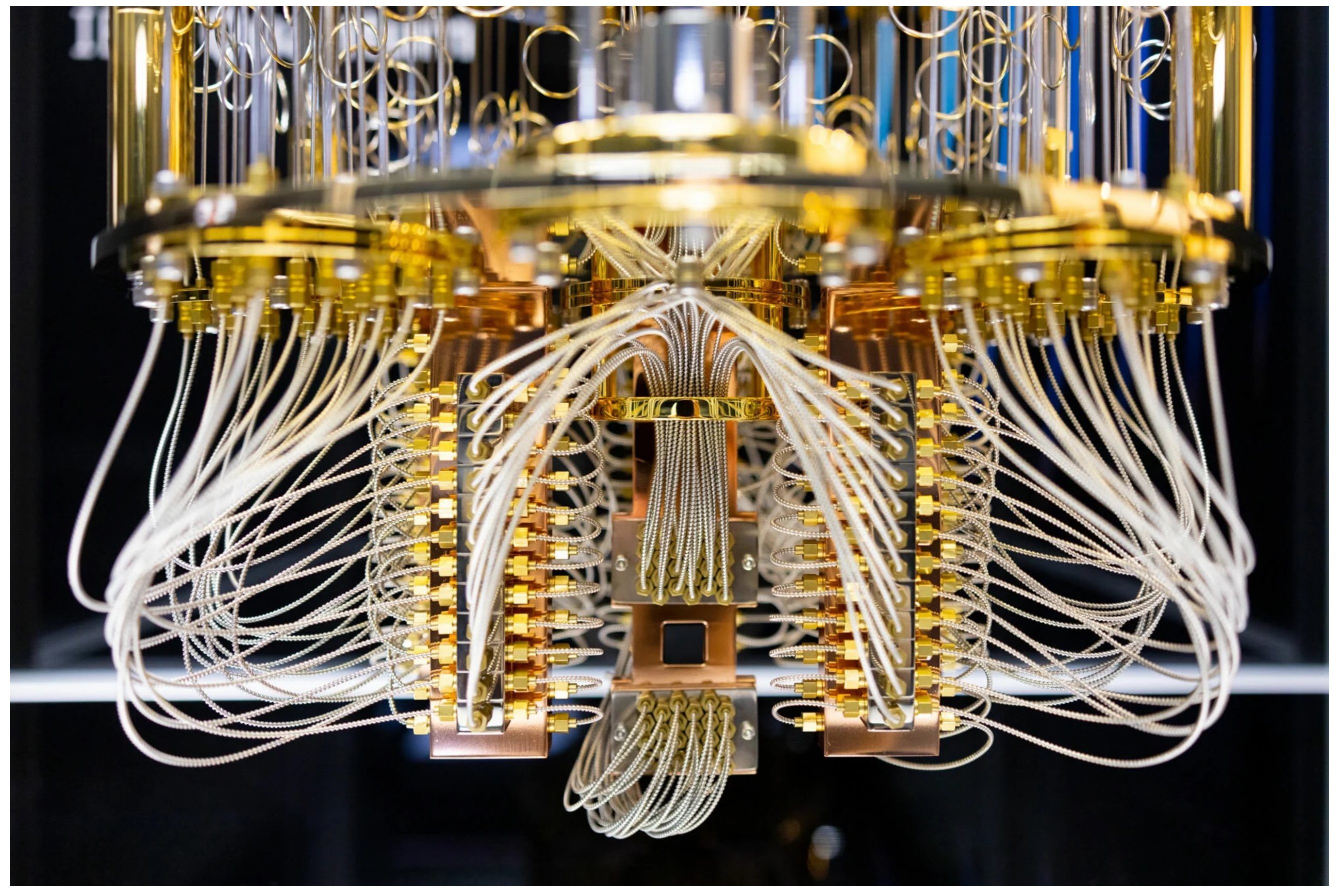
Quantum processors are emerging as a new experimental platform to simulate **"quantum on quantum"**, i.e. the non-linear **dynamics** of quantum systems with unprecedented control.



quantum circuits are the "assembler-level" of quantum computing

(quantum)^2 many-body

IBM Quantum (cloud)



Quantum processors are emerging as a new experimental platform to simulate "**quantum on quantum**", i.e. the quantum **dynamics** of quantum systems with unprecedented control.

Article

Quantum supremacy using a programmable superconducting processor

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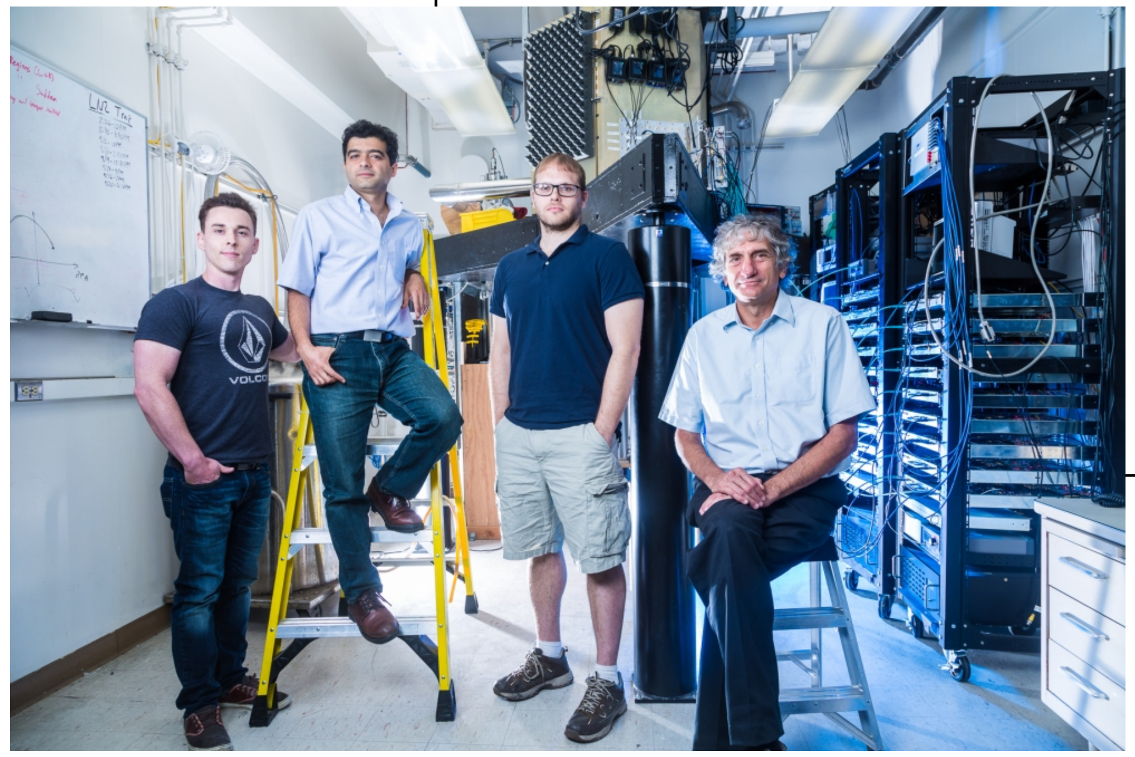
Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1], Rupak Biswas[3], Sergio Boixo[1], Fernando G. S. L. Brandao[1,4], David A. Buell[1], Brian Burkett[1], Yu Chen[1], Zijun Chen[1], Ben Chiaro[5], Roberto Collins[1], William Courtney[1], Andrew Dunsworth[1], Edward Farhi[1], Brooks Foxen[1,5], Austin Fowler[1], Craig Gidney[1], Marissa Giustina[1], Rob Graff[1], Keith Guerin[1], Steve Habegger[1], Matthew P. Harrigan[1], Michael J. Hartmann[1,6], Alan Ho[1], Markus Hoffmann[1], Trent Huang[1], Travis S. Humble[7], Sergei V. Isakov[1], Evan Jeffrey[1], Zhang Jiang[1], Dvir Kafri[1], Kostyantyn Kechedzhi[1], Julian Kelly[1], Paul V. Klimov[1], Sergey Knysh[1], Alexander Korotkov[1,8], Fedor Kostritsa[1], David Landhuis[1], Mike Lindmark[1], Erik Lucero[1], Dmitry Lyakh[9], Salvatore Mandrà[3,10], Jarrod R. McClean[1], Matthew McEwen[5], Anthony Megrant[1], Xiao Mi[1], Kristel Michielsen[11,12], Masoud Mohseni[1], Josh Mutus[1], Ofer Naaman[1], Matthew Neeley[1], Charles Neill[1], Murphy Yuezhen Niu[1], Eric Ostby[1], Andre Petukhov[1], John C. Platt[1], Chris Quintana[1], Eleanor G. Rieffel[3], Pedram Roushan[1], Nicholas C. Rubin[1], Daniel Sank[1], Kevin J. Satzinger[1], Vadim Smelyanskiy[1], Kevin J. Sung[1,13], Matthew D. Trevithick[1], Amit Vainsencher[1], Benjamin Villalonga[1,14], Theodore White[1], Z. Jamie Yao[1], Ping Yeh[1], Adam Zalcman[1], Hartmut Neven[1] & John M. Martinis[1,5]*

— (2019) —

(quantum)^2
many-body

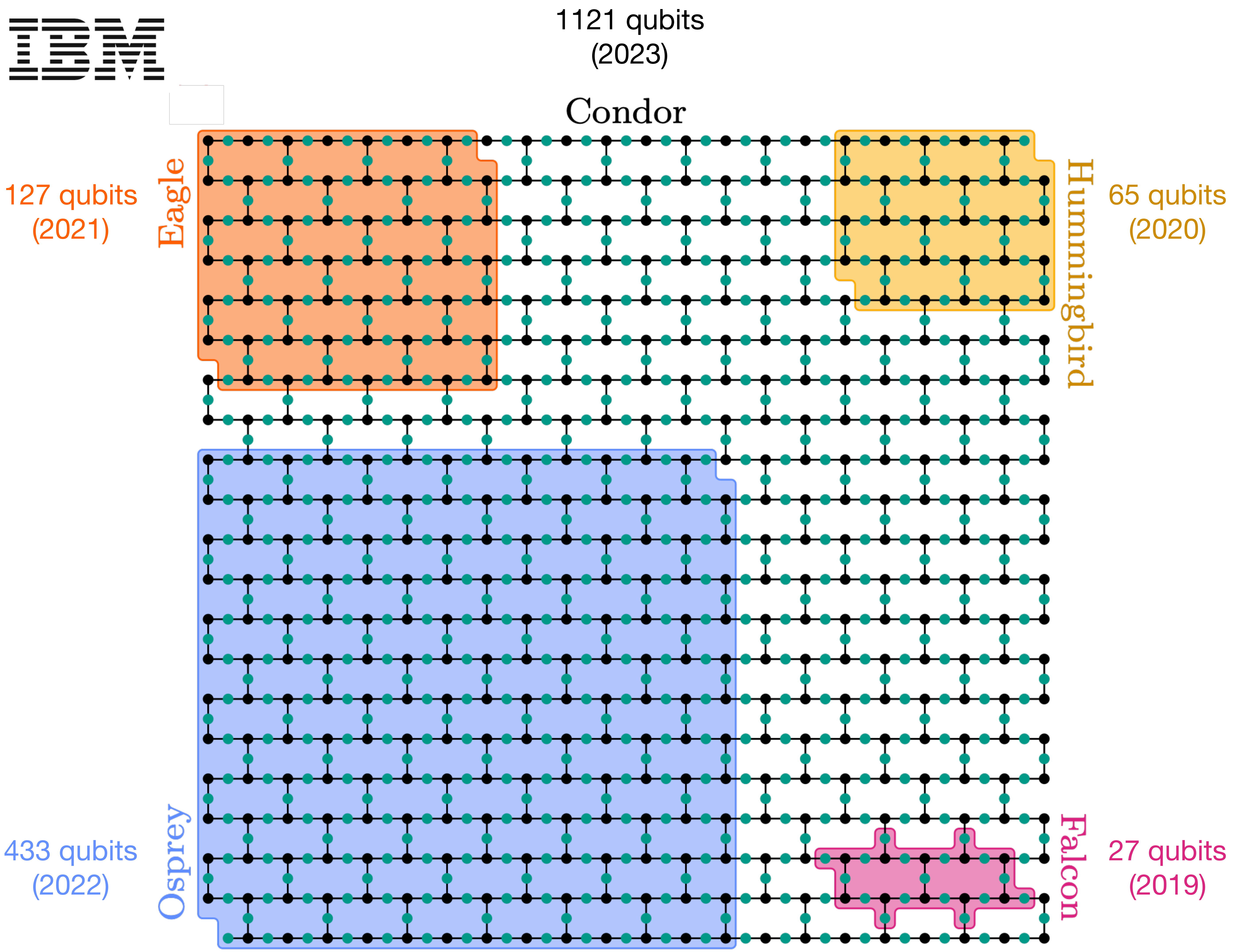


John Martinis (2019)

The promise of quantum computers is that **certain computational tasks** might be executed **exponentially faster** on a quantum processor than on a classical processor.

quantum hardware & simulations

IBM



(2019)

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Quantum supremacy using a programmable superconducting processor

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Nature **574**, 505–510 (2019) | Cite this article

(2021)

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER, Y.-J LIU, A. SMITH, C. KNAPP, [...], AND P. ROUSHAN +93 authors Authors Info & Affiliations

SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237-1241 • DOI: 10.1126/science.abi8378

(2023)

Article | Open access | Published: 11 May 2023

Non-Abelian braiding of graph vertices in a superconducting processor

Google Quantum AI and Collaborators

Nature **618**, 264–269 (2023) | Cite th

(2025)

nature physics

Article https://doi.org/10.1038/s41567-024-02696-6

Nishimori transition across the error threshold for constant-depth quantum circuits

Received: 13 October 2023
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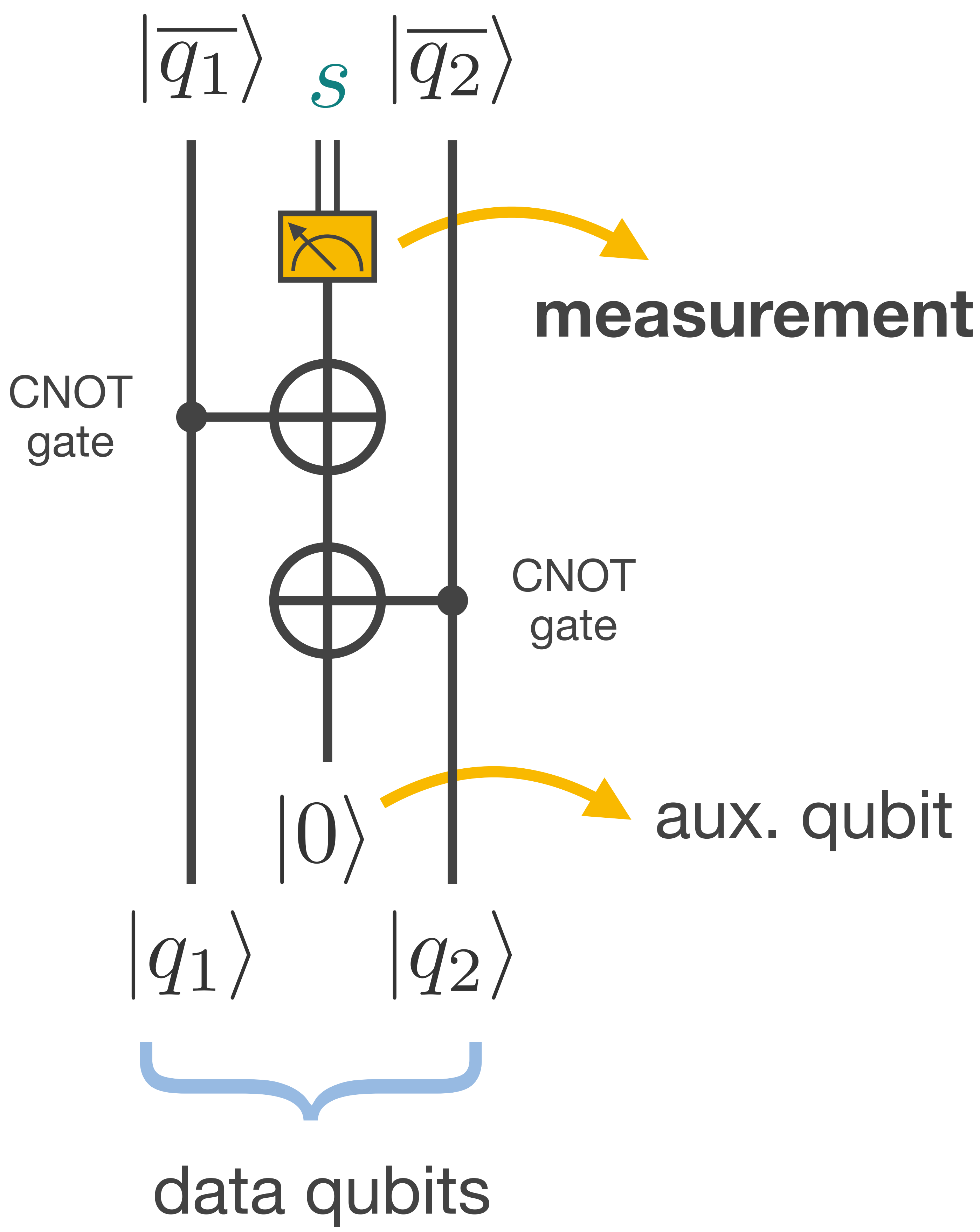
Published online: 16 December 2024

Edward H. Chen[1,2], Guo-Yi Zhu[3], Ruben Verresen[4], Alireza Seif[5], Elisa Bäumer[6], David Layden[1,5], Nathanan Tantivasadakarn[4,7], Guanyu Zhu[5], Sarah Sheldon[5], Ashvin Vishwanath[4], Simon Trebst[3] & Abhinav Kandala[5]

quantum measurements

joint measurements & entanglement

quantum circuit for 2-qubit
joint measurement



extract information

$|q_1\rangle|q_2\rangle$	$|\overline{q_1}\rangle|\overline{q_2}\rangle$	s
$|0\,0\rangle$	$|0\,0\rangle$	0
$|0\,1\rangle$	$|0\,1\rangle$	1
$|1\,0\rangle$	$|1\,0\rangle$	1
$|1\,1\rangle$	$|1\,1\rangle$	0

syndrome measurement

ZZ **parity check**

shape entanglement

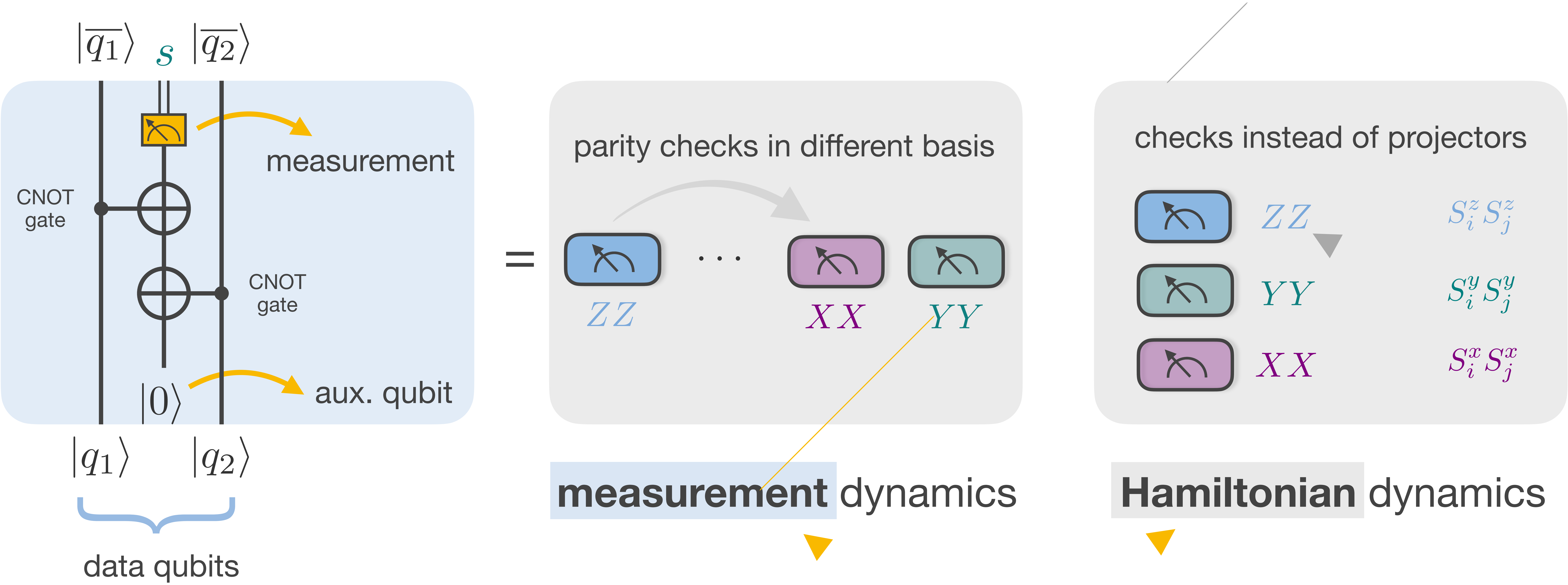
product state

$|q_1\rangle|q_2\rangle$	$|\overline{q_1}\rangle|\overline{q_2}\rangle$	s
$|++\rangle$	p = 1/2 $\left(|00\rangle + |11\rangle\right)/\sqrt{2}$	0
	p = 1/2 $\left(|01\rangle + |10\rangle\right)/\sqrt{2}$	1

Bell state

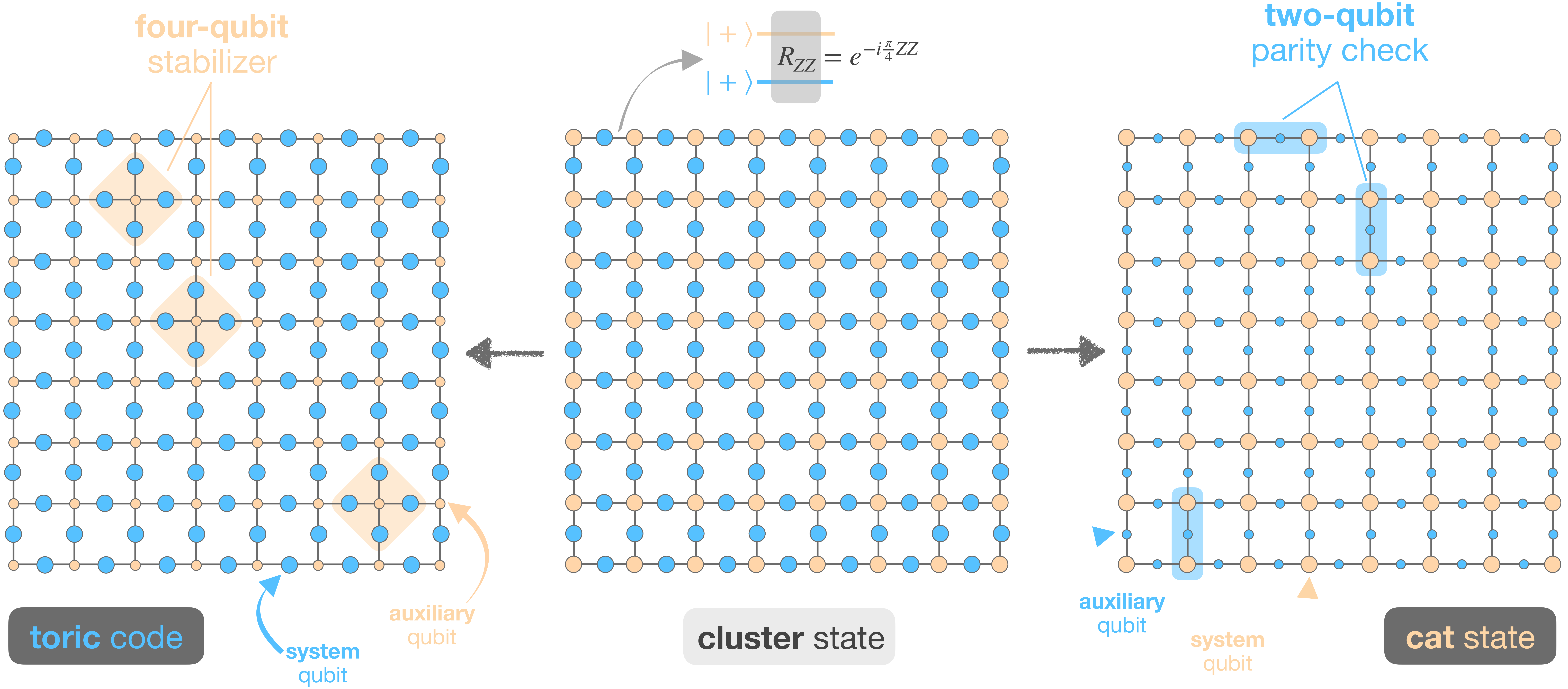
$$|++\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

joint measurements & entanglement



new toolbox for quantum many-body physics

joint measurements & entanglement



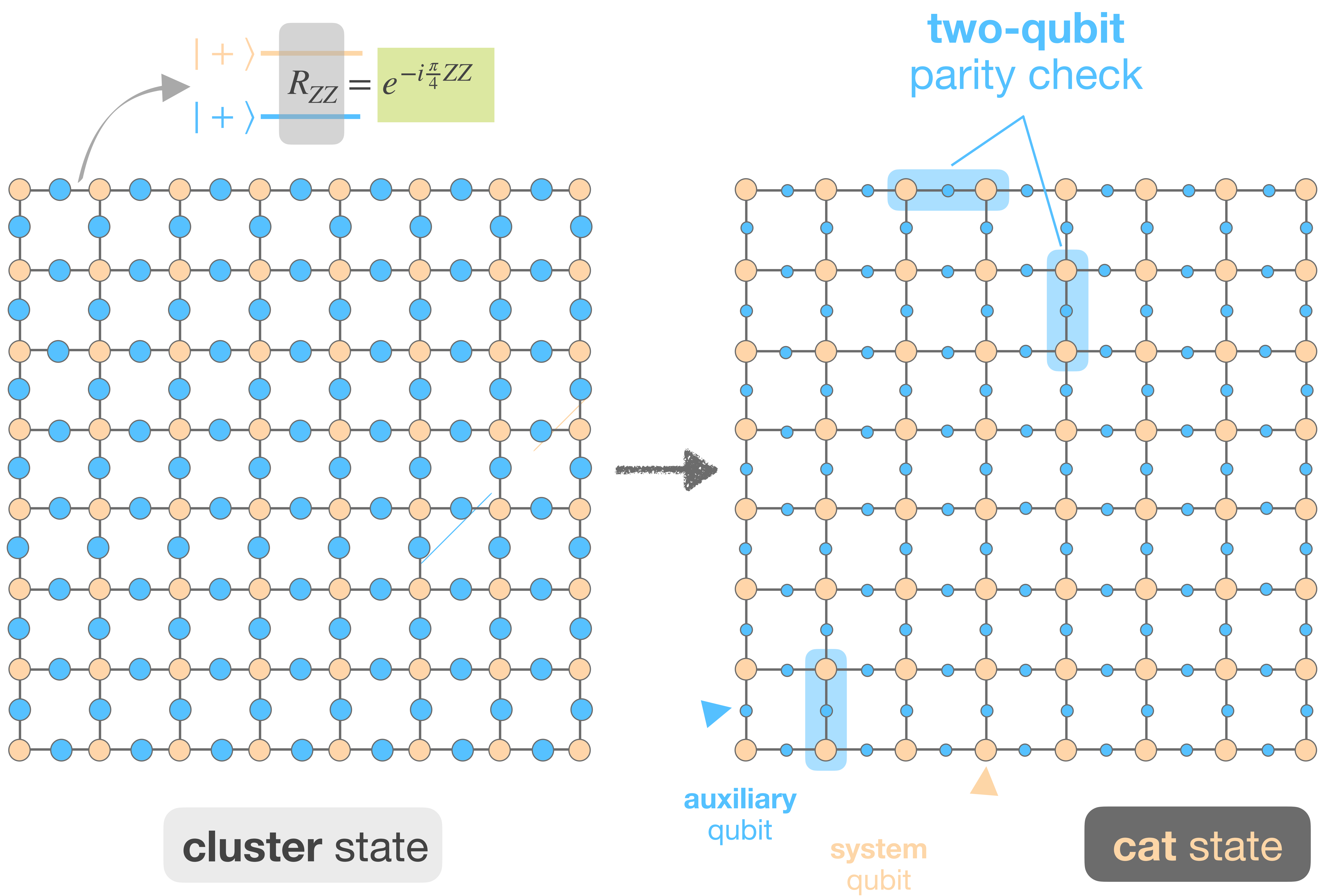
measurement-induced phase transitions

What happens if our **resource** — the cluster state — is **imperfectly prepared**?

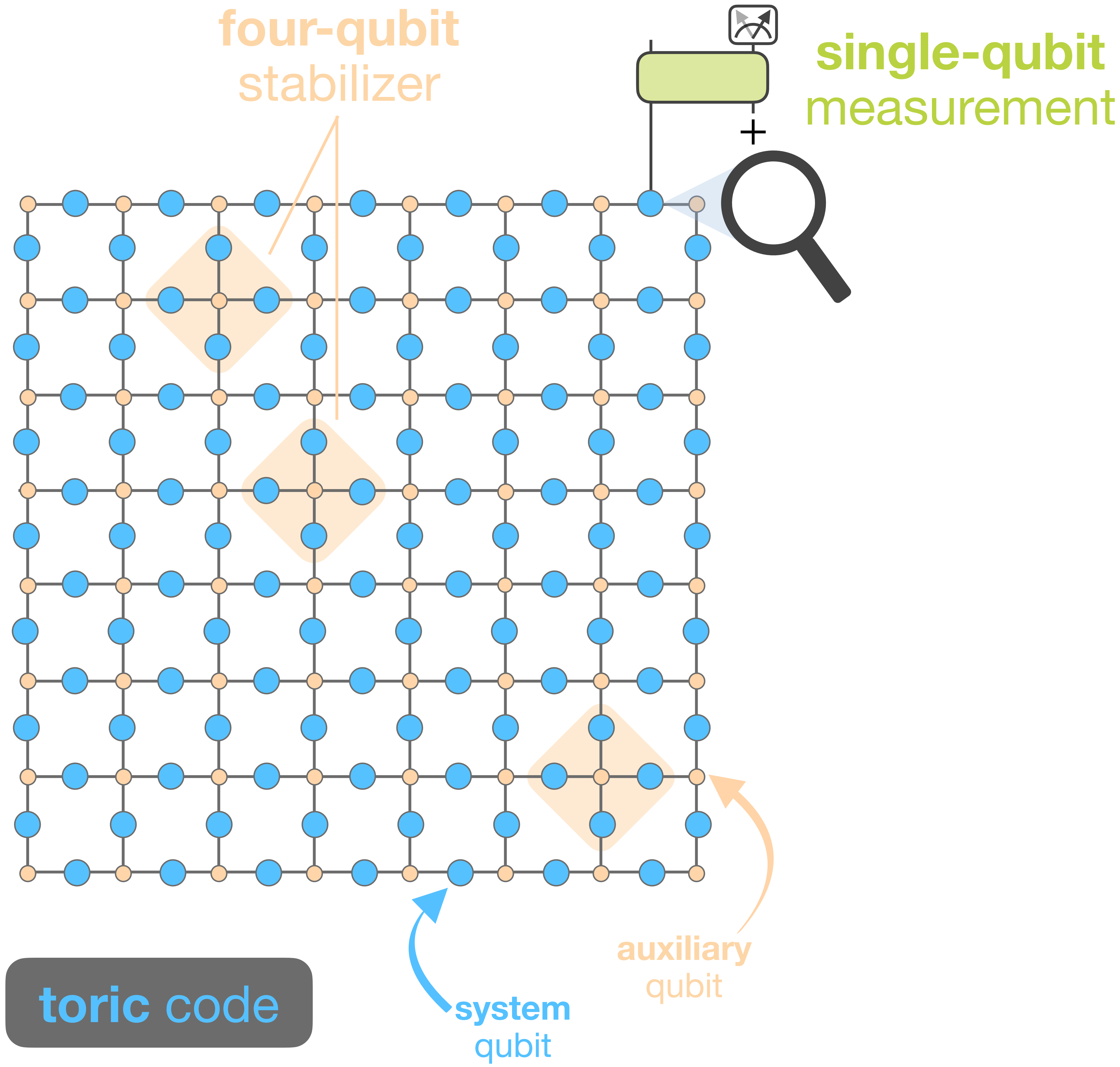
$$0 \leq t \leq \frac{\pi}{4}$$

Is the prepared **cat state stable** to this coherent deformation? Is there a **threshold**?

Nishimori transition



measurement-induced phase transitions



What happens if, for the toric code, we weakly **monitor all system qubits**?

$$0 \leq t \leq \frac{\pi}{4}$$

Is the monitored **toric code stable** to this coherent deformation? Is there a **threshold**?

Nishimori transition

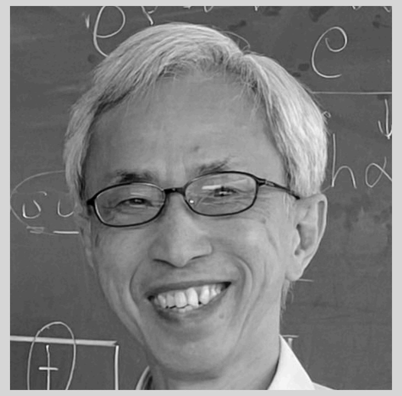
Nishimori physics

Phys. Rev. Lett. 131, 200201 (2023)

G-Y. Zhu, N. Tantivasadakarn, A. Vishwanath, R. Verresen



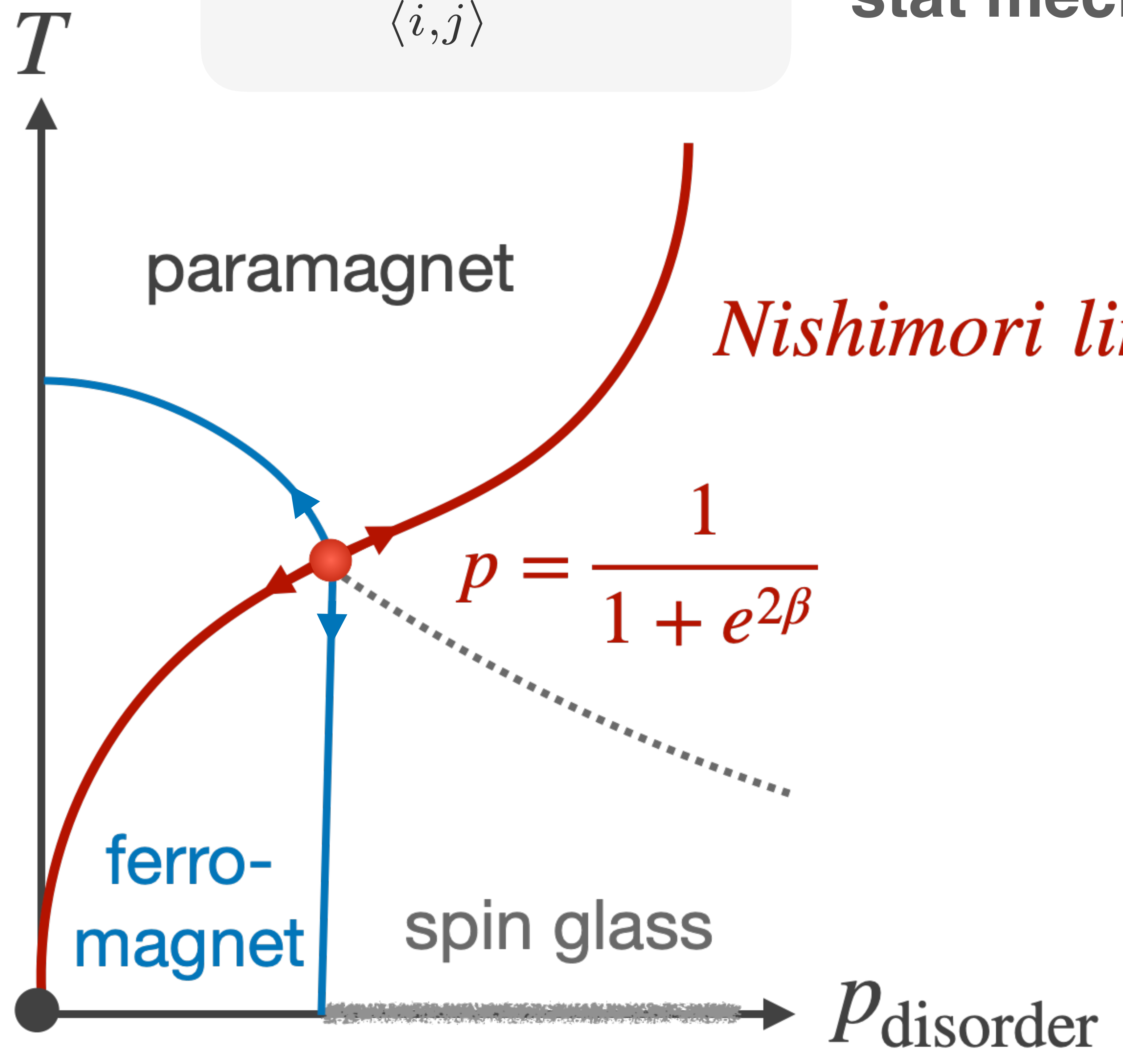

Nishimori physics



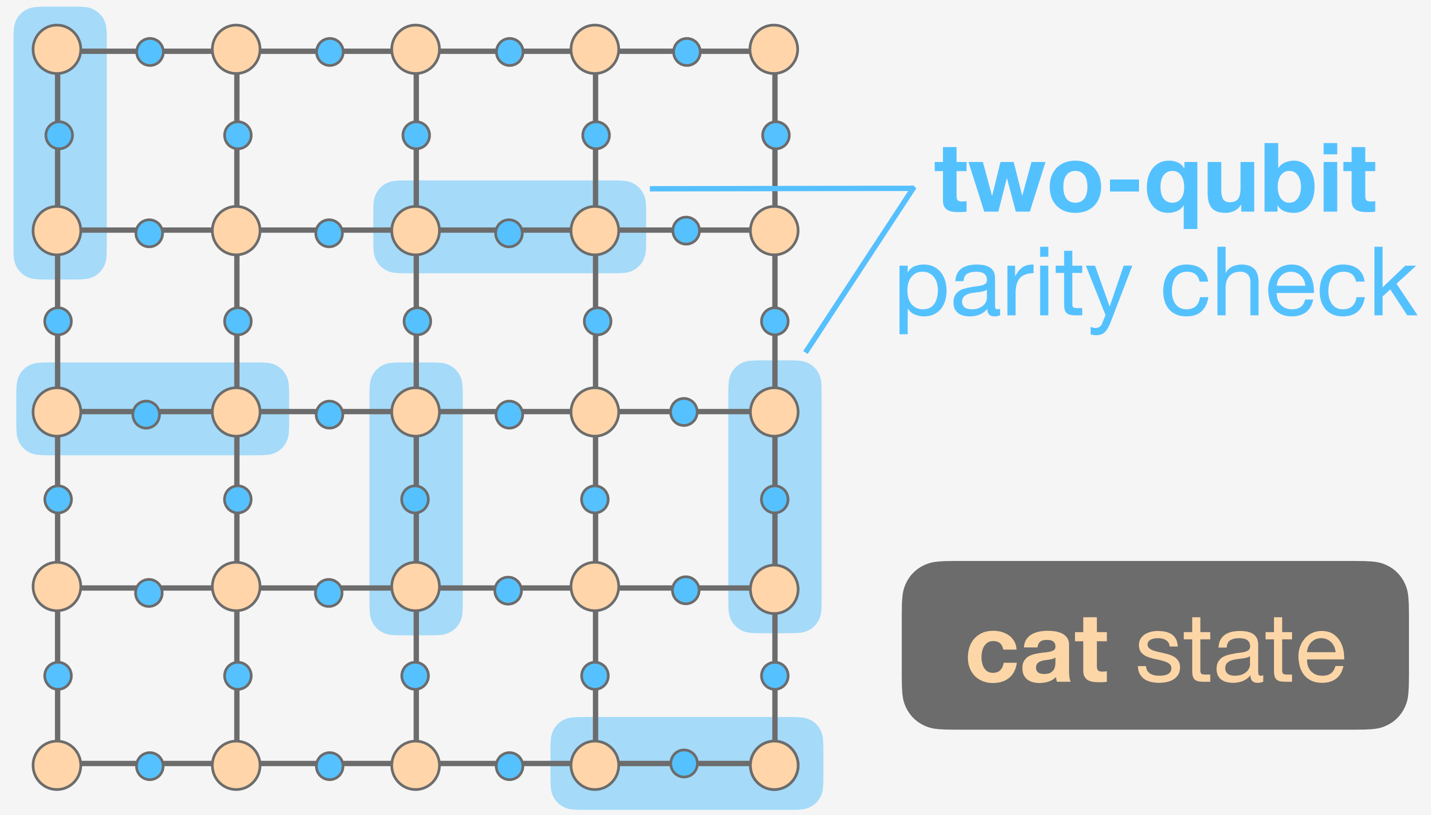
Nishimori (1981)

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \; S_i^z S_j^z$$

classical
stat mech



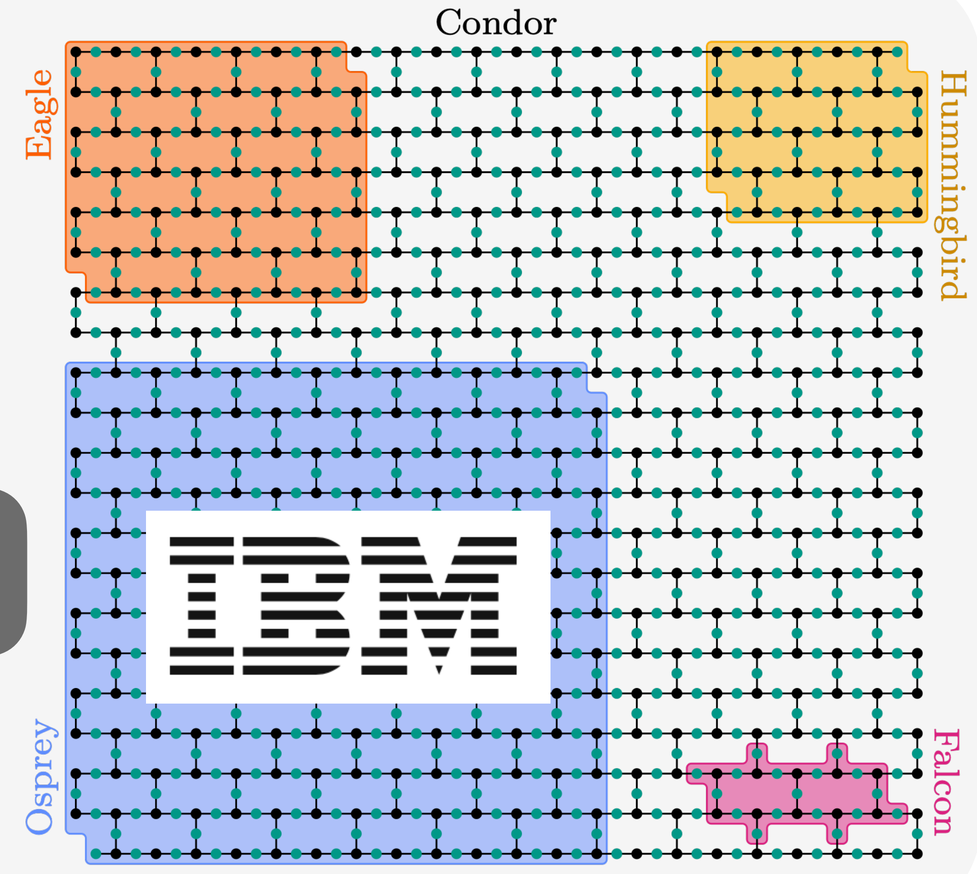
monitored
circuits



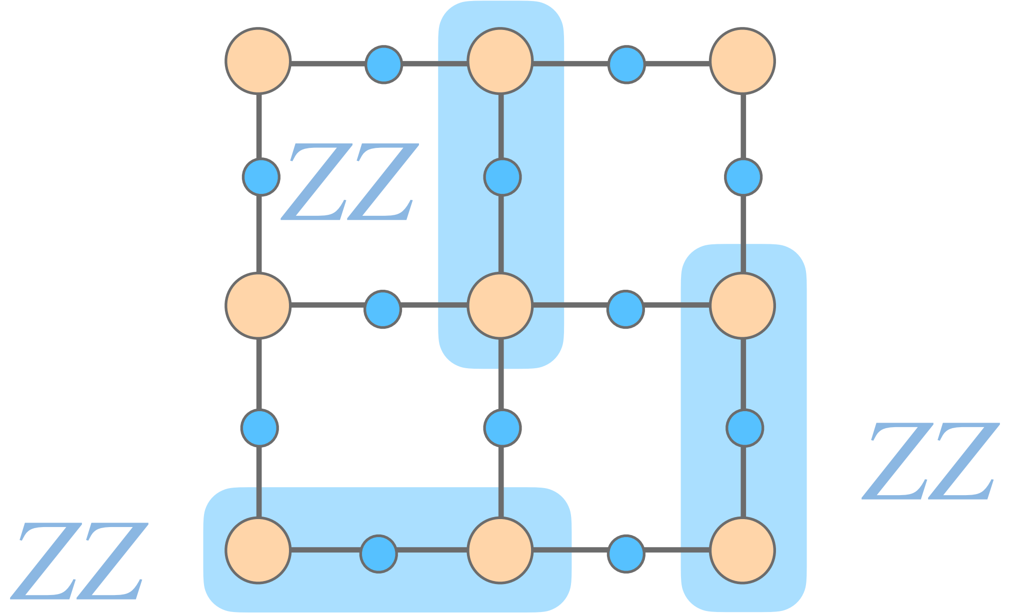
NISQ
devices

127-qubit
device

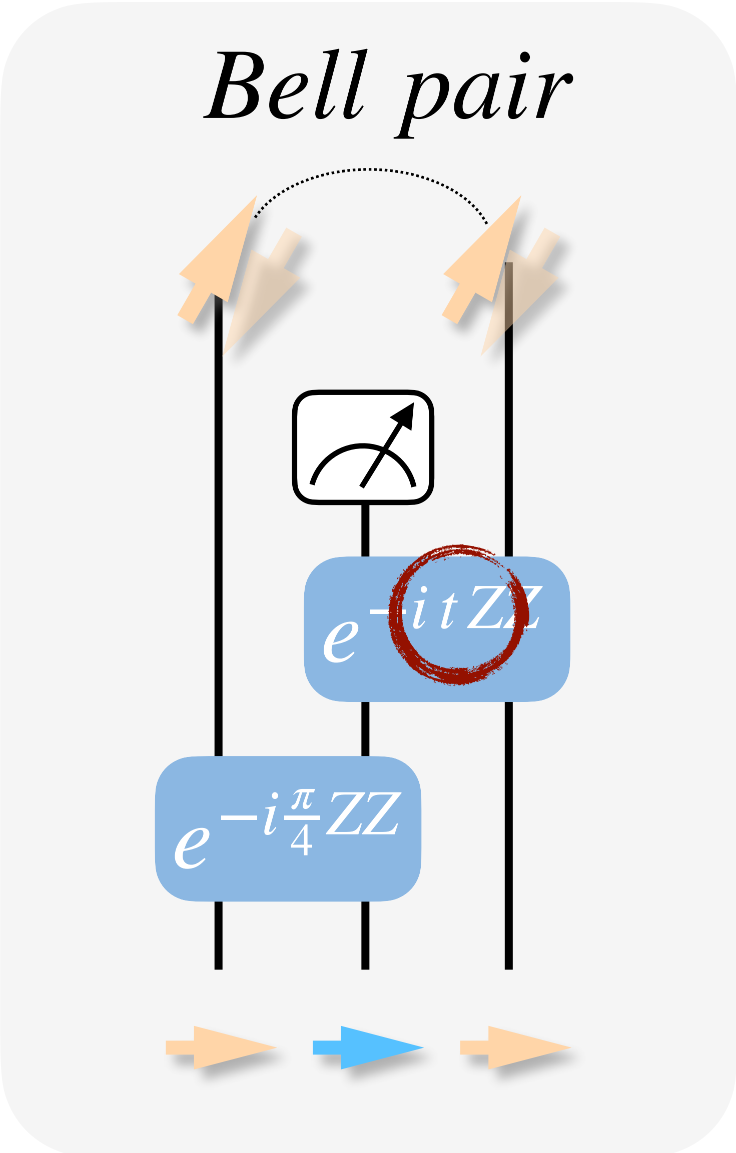
Sherbrooke



Nishimori's cat



Nishimori's cat



$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

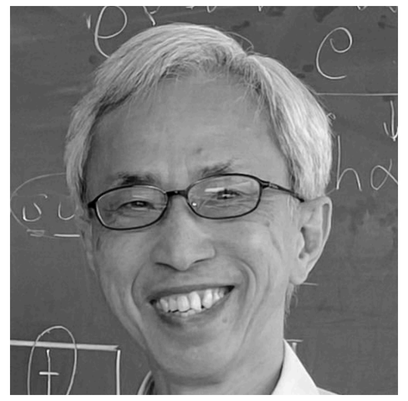
Born's rule

interpret as **classical stat mech model**

$$\tanh \frac{\beta J_\pm}{2} = \pm \tan t$$

thermal **fluctuations** and **disorder** are **locked**

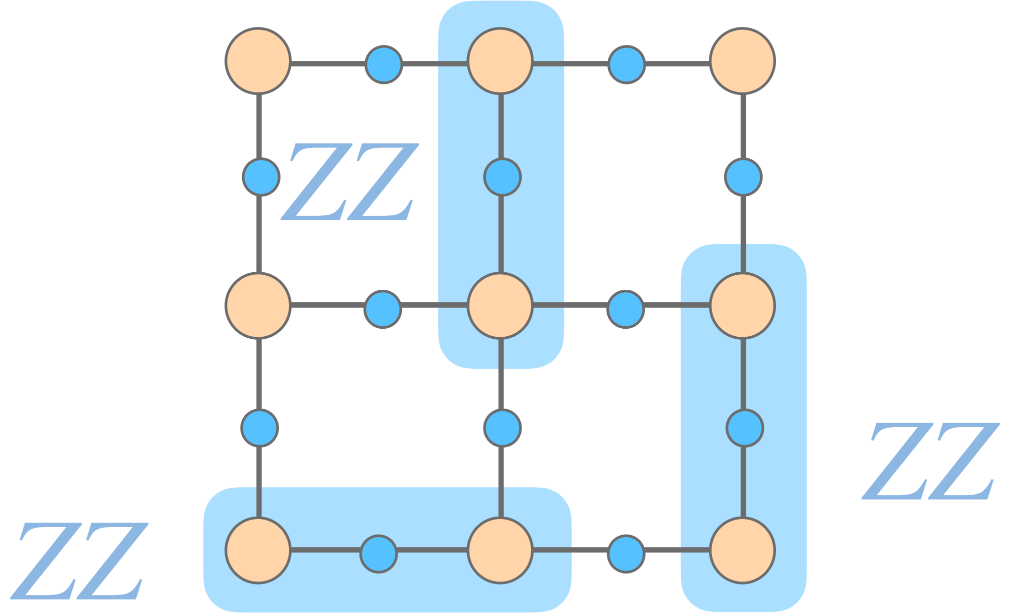
$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} J_{s_{ij}} \sigma_i \sigma_j}$$



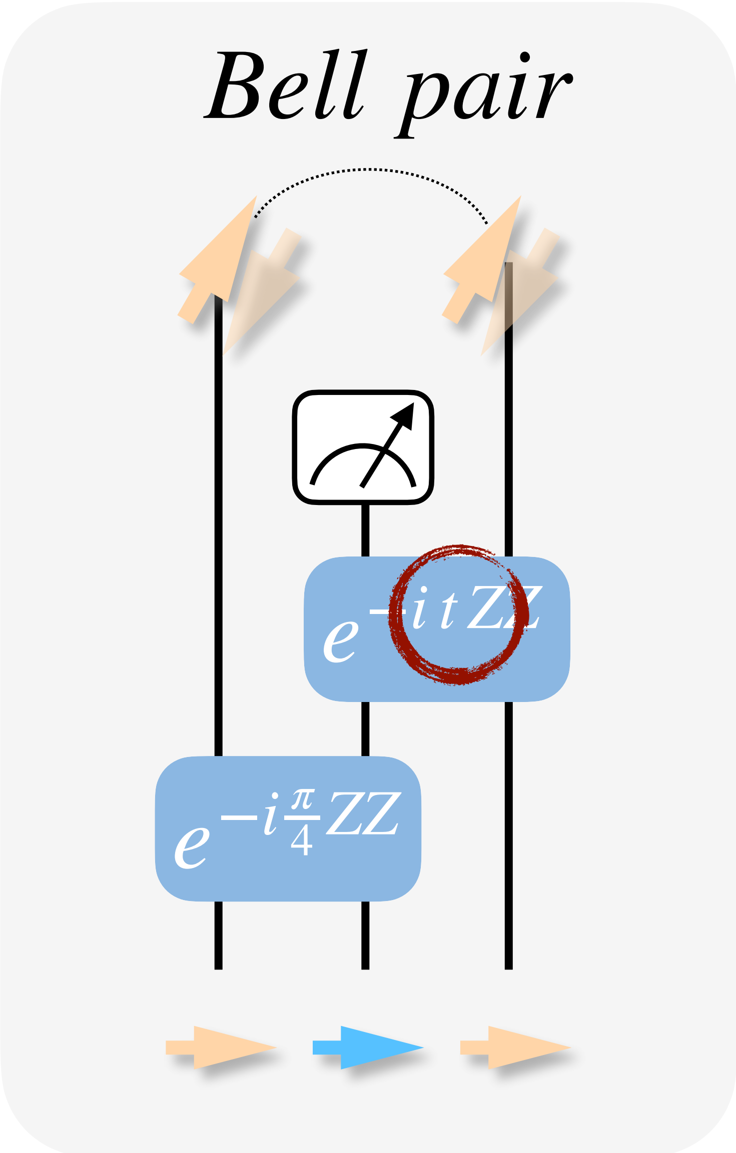
Nishimori (1981)

random bond Ising model

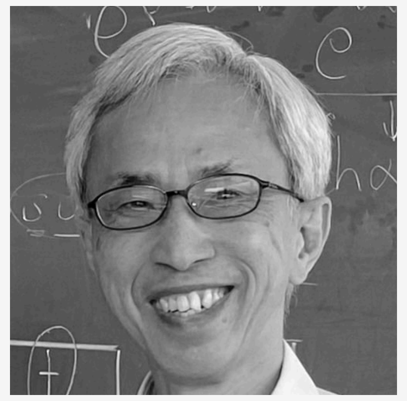
Nishimori's cat



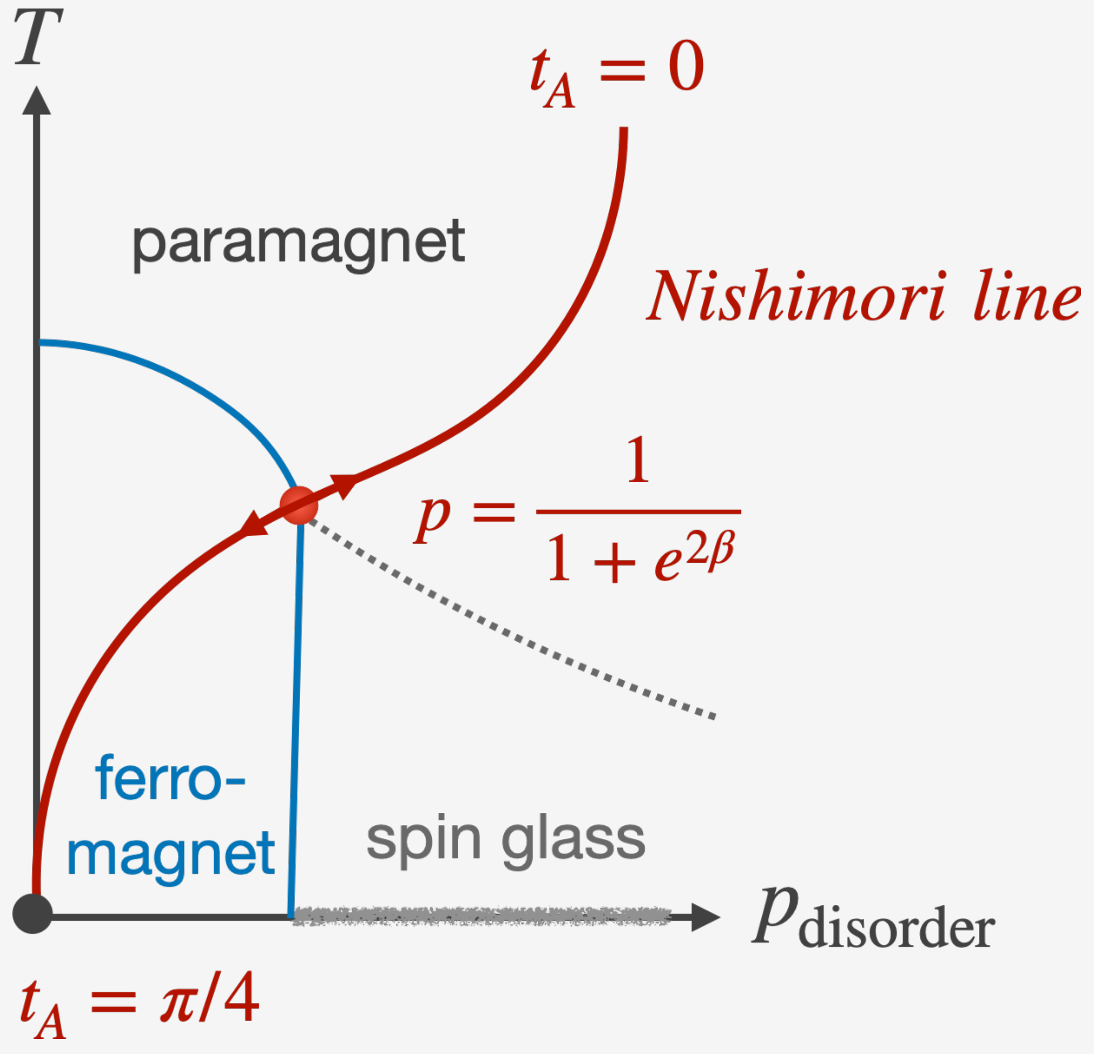
Nishimori's cat

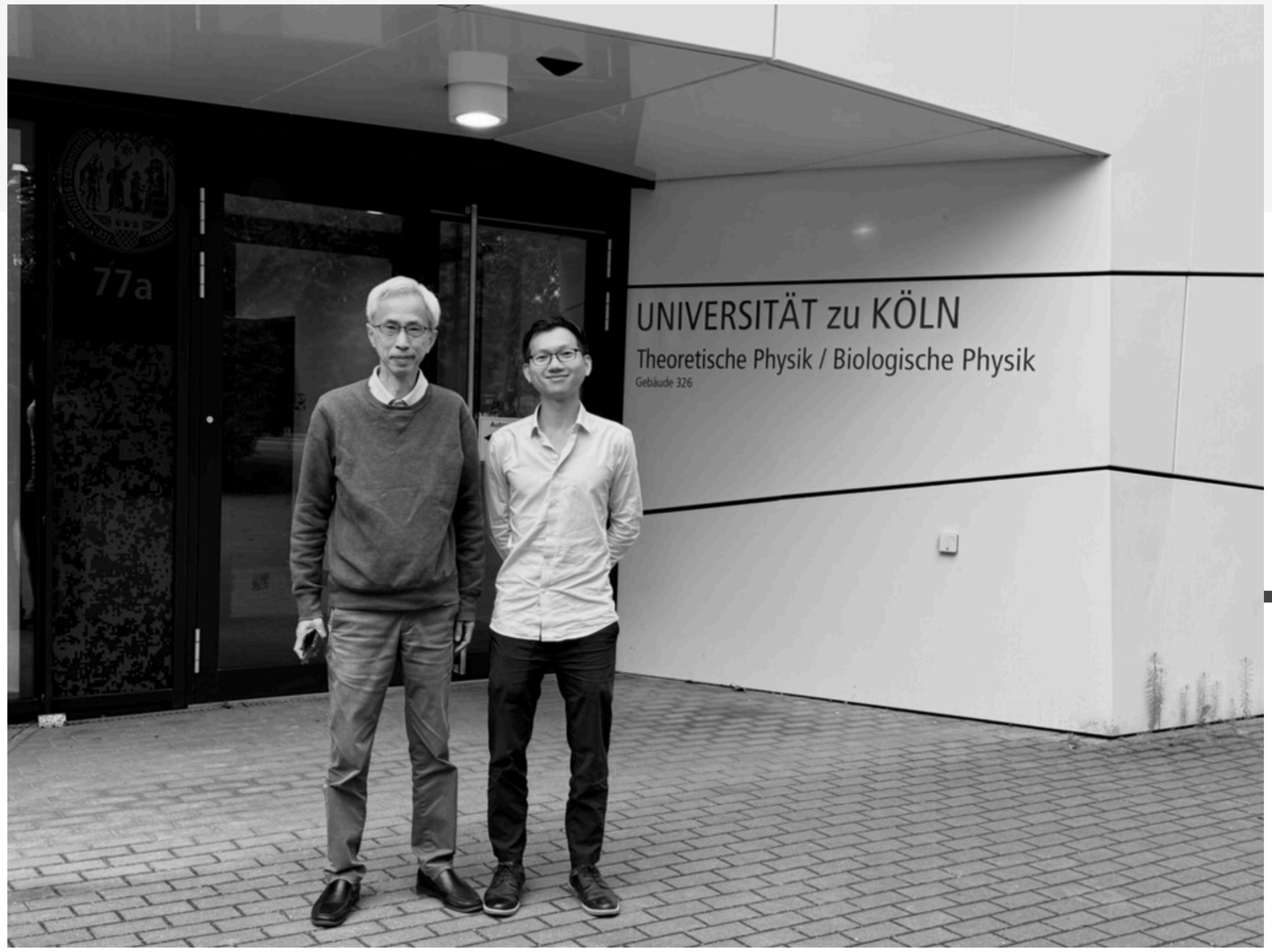


thermal fluctuations and disorder are **locked**

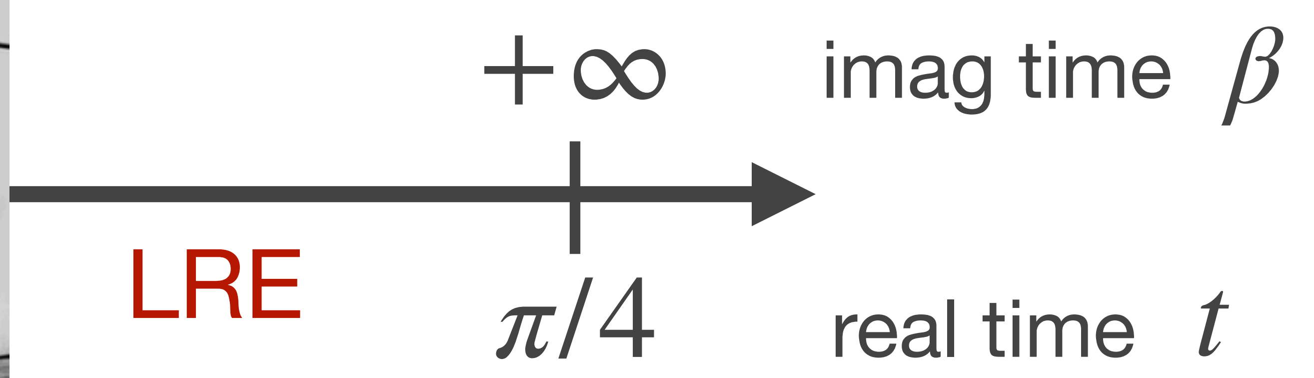


Nishimori (1981)

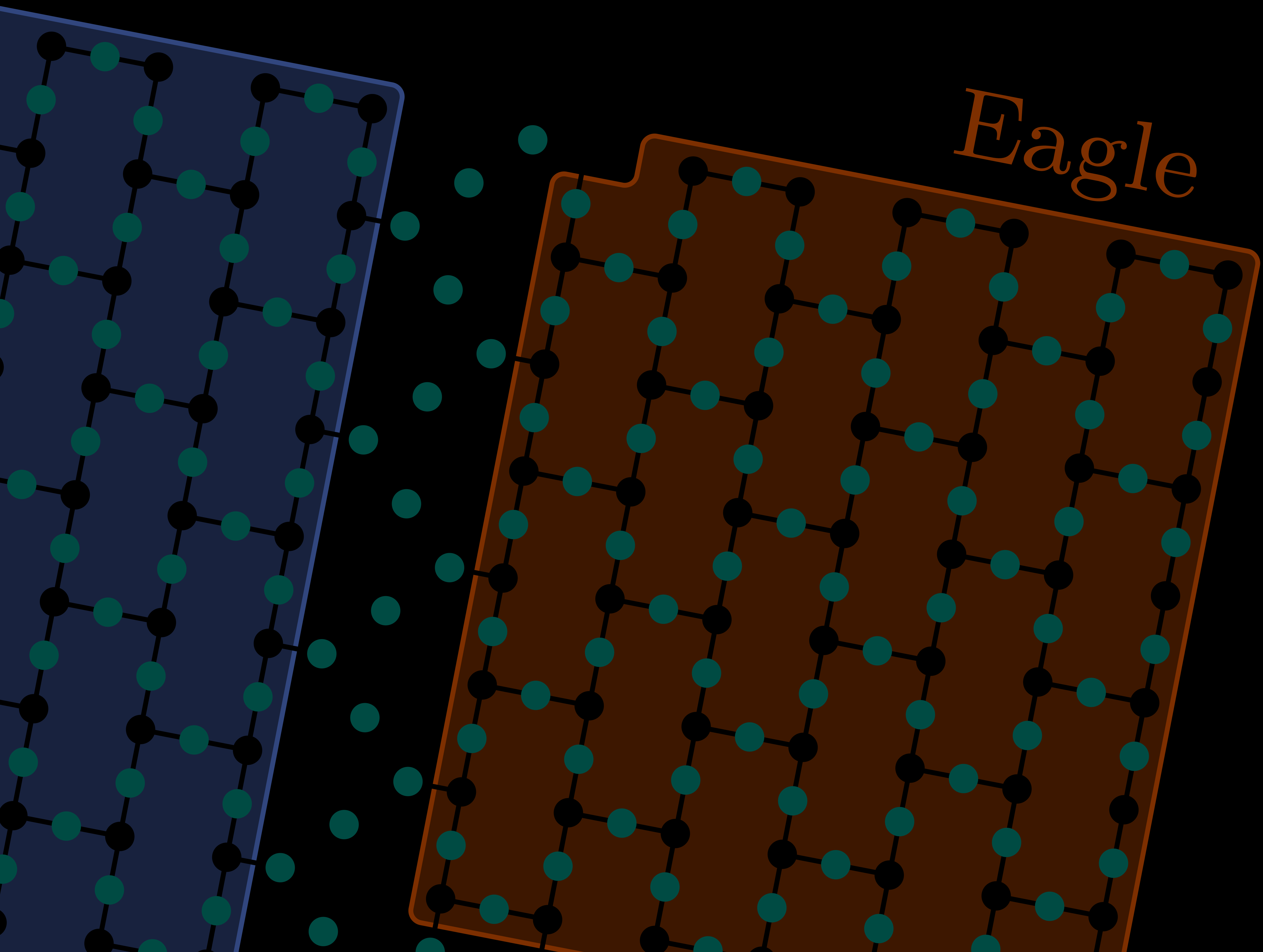




"low temperature"



strong
measurement

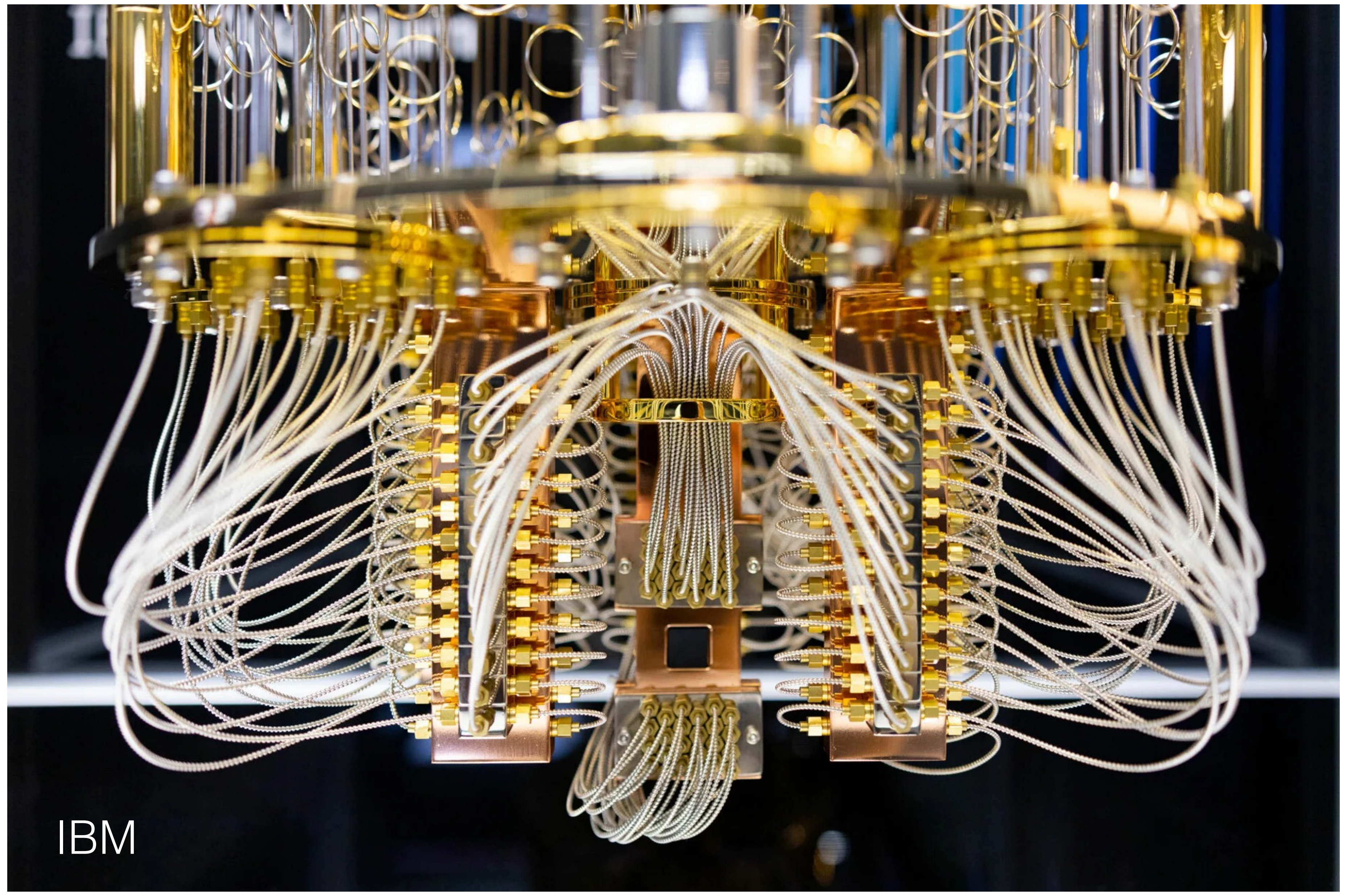


experiment

Nature Physics 21, 161 (2025)

E. Chen, G-Y. Zhu, R. Verresen, A. Seif, E. Bäumer, D. Layden, N. Tantivasadakarn, G. Zhu, S. Sheldon, A. Vishwanath, A. Kandala

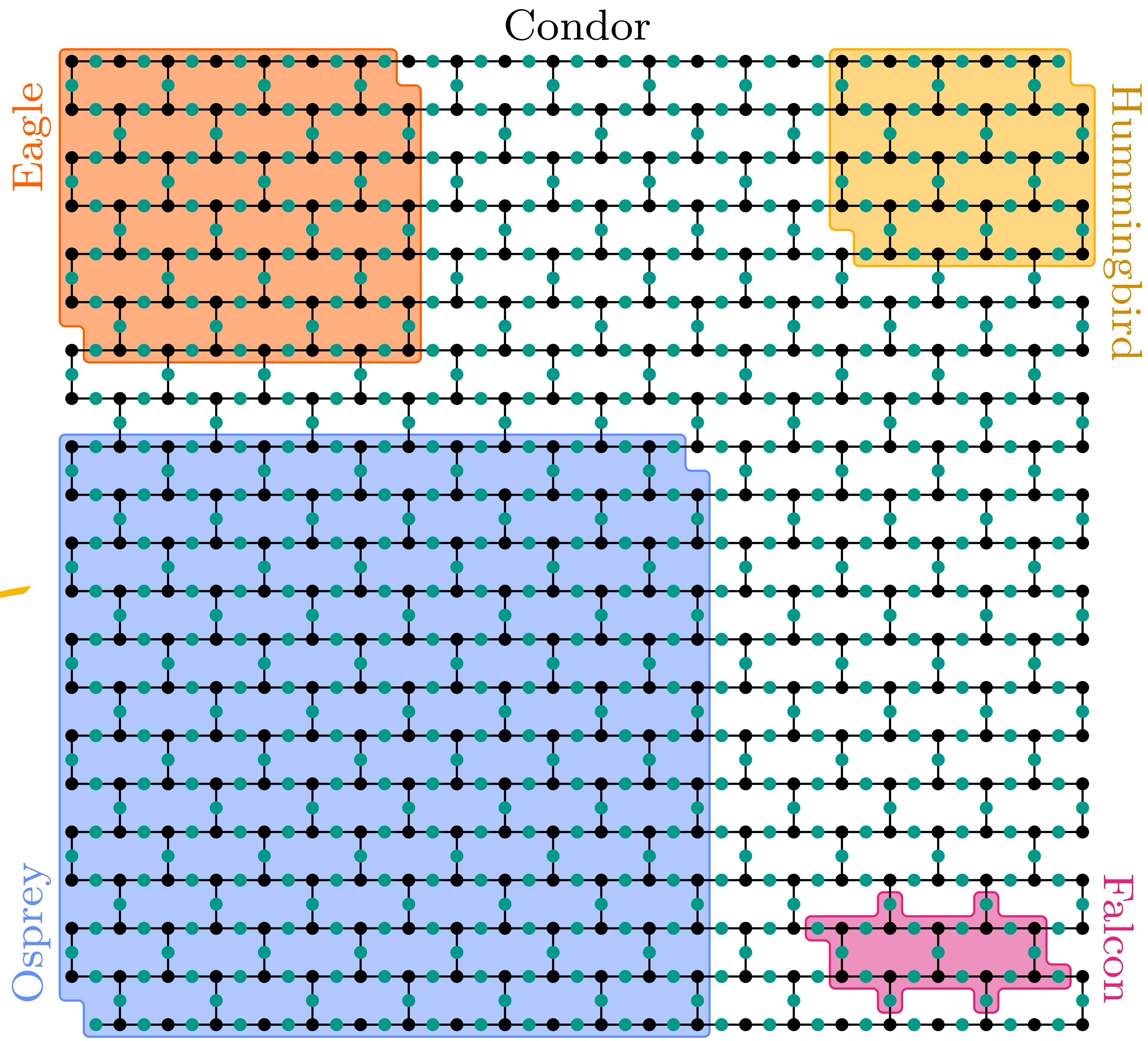
IBM quantum cloud devices



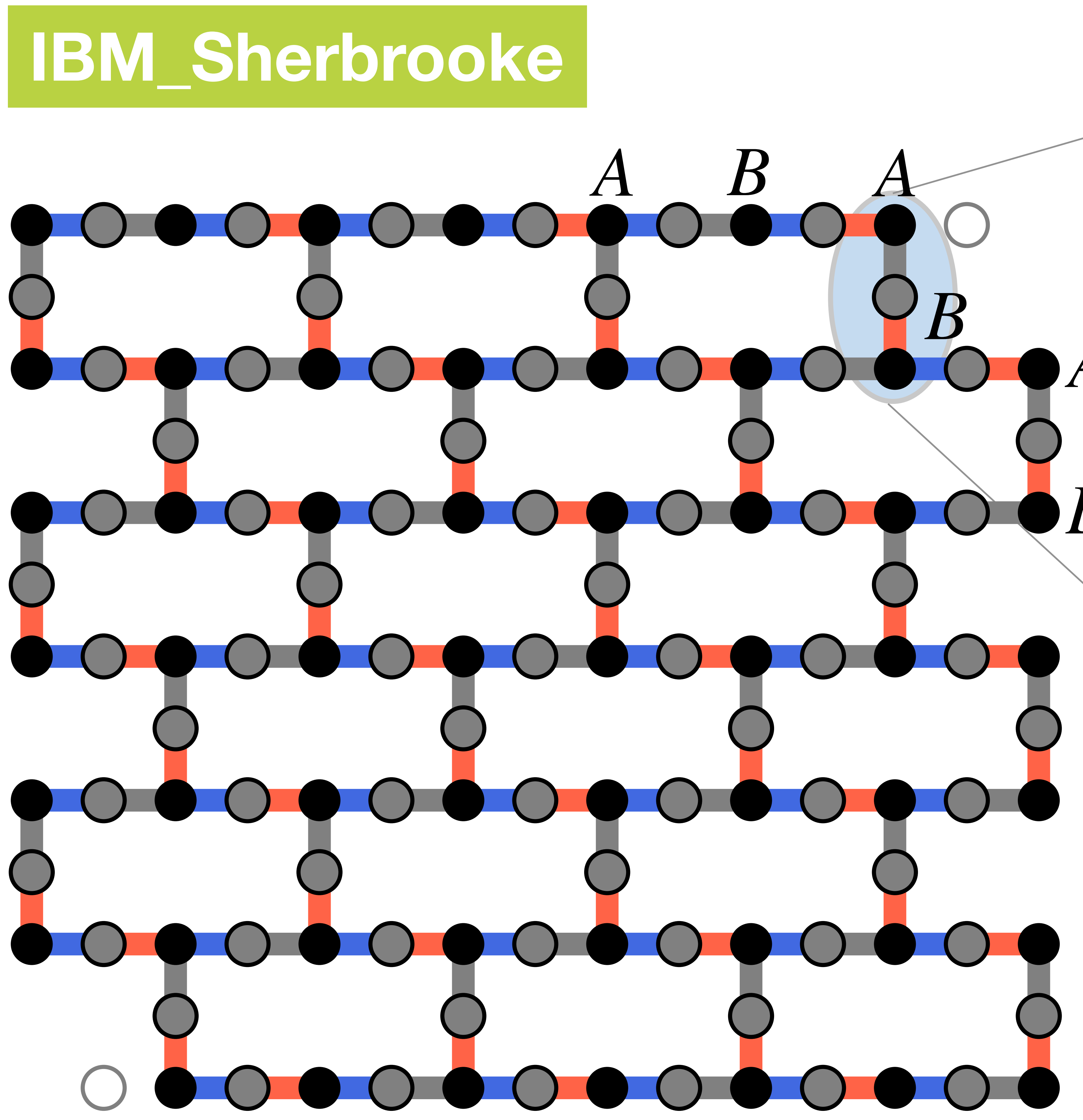
NISQ devices built on transmon qubits

noisy intermediate scale quantum devices

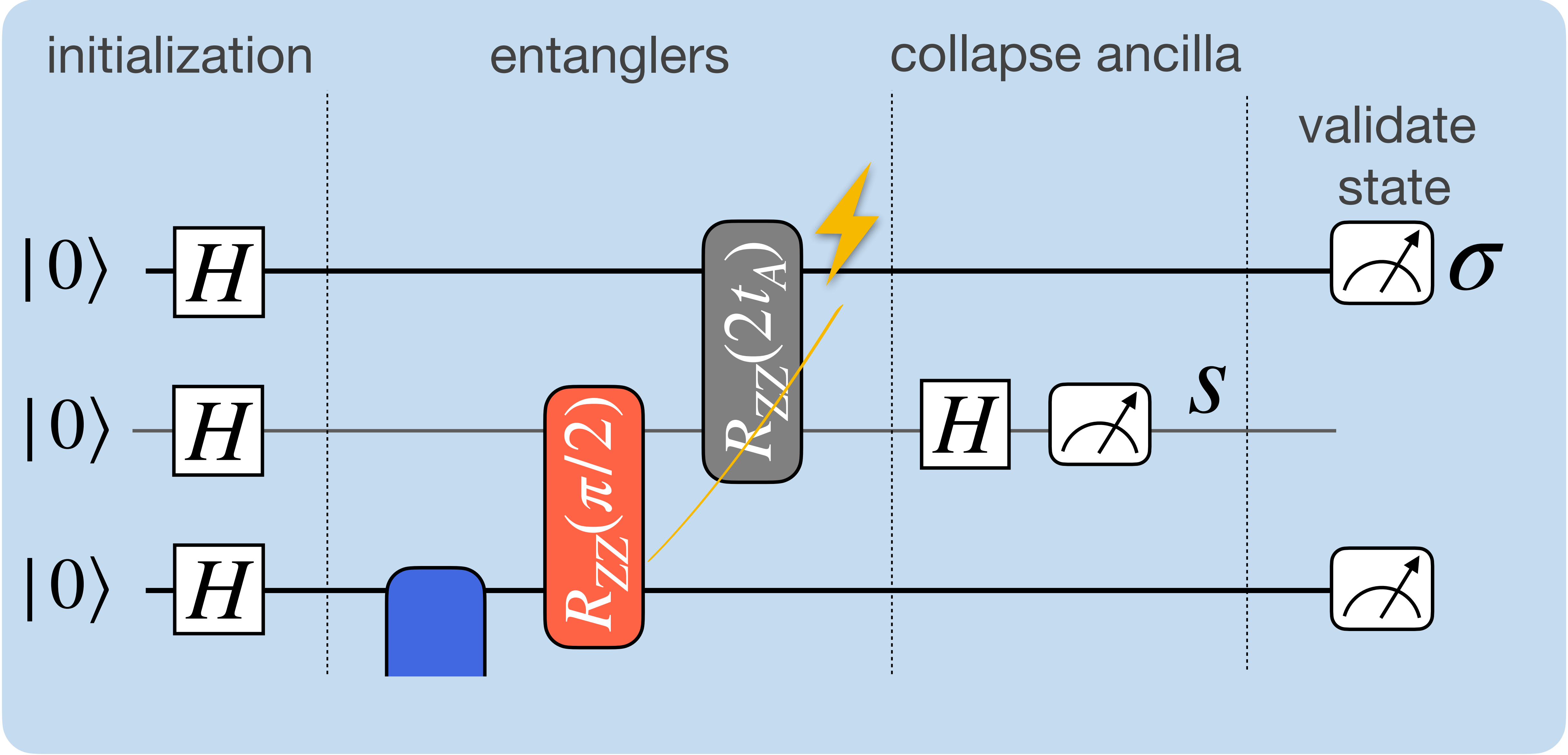
heavy-hexagon geometry + **Ising evolution gates**

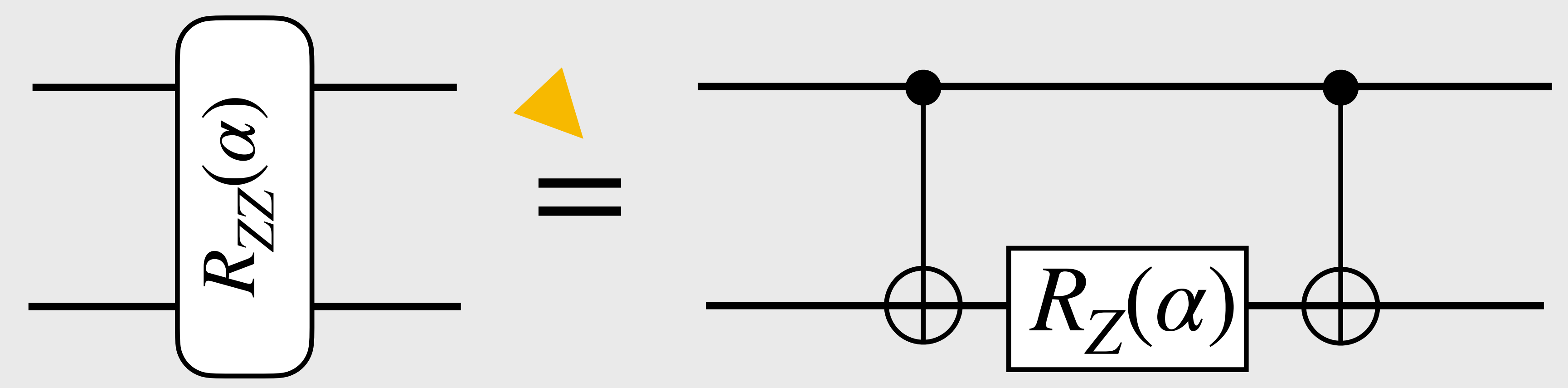


protocol on IBM heavy-hexagon lattice



- **system** qubit: site of honeycomb lattice
- **auxiliary** qubit: bond of honeycomb lattice

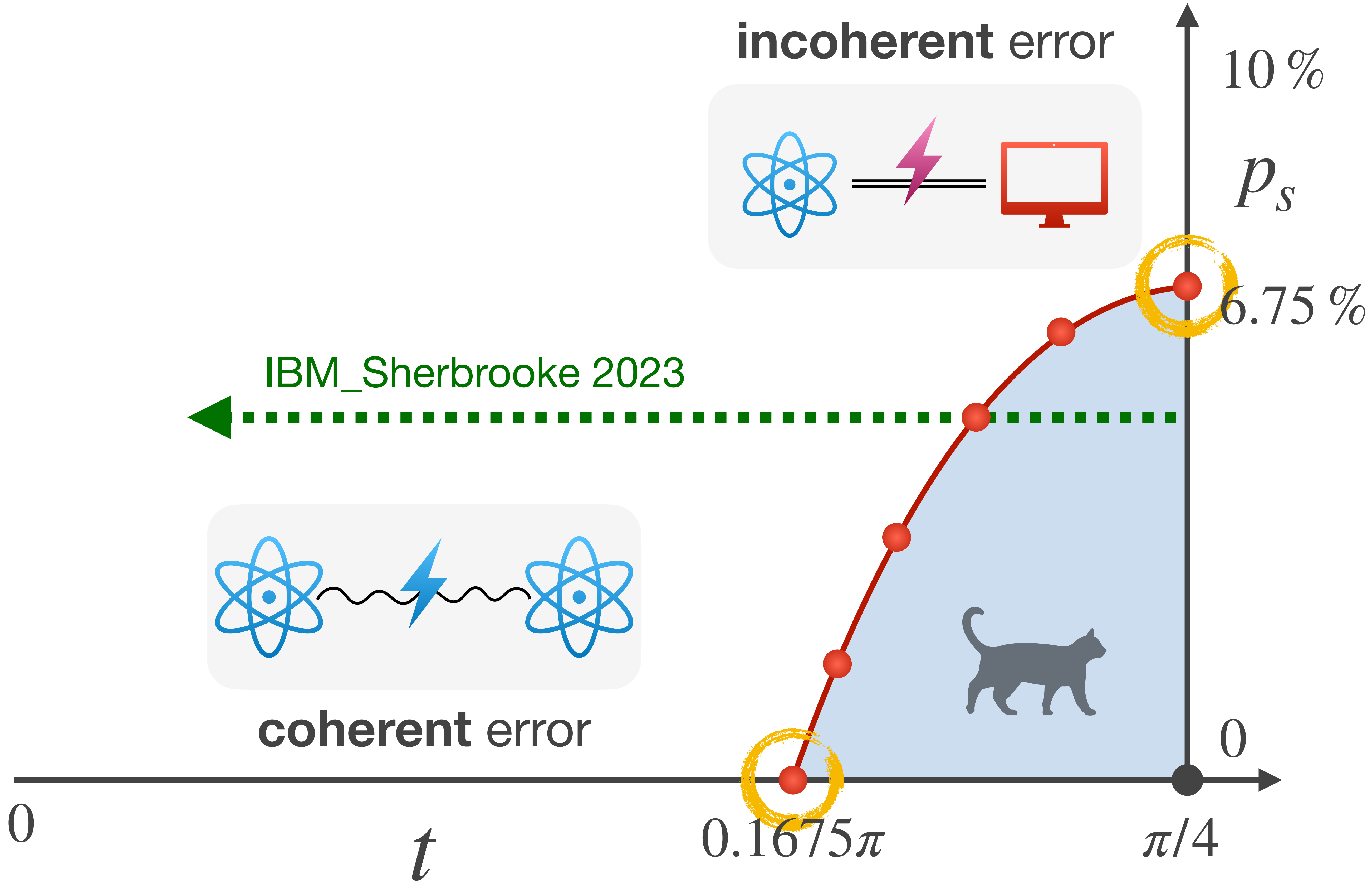


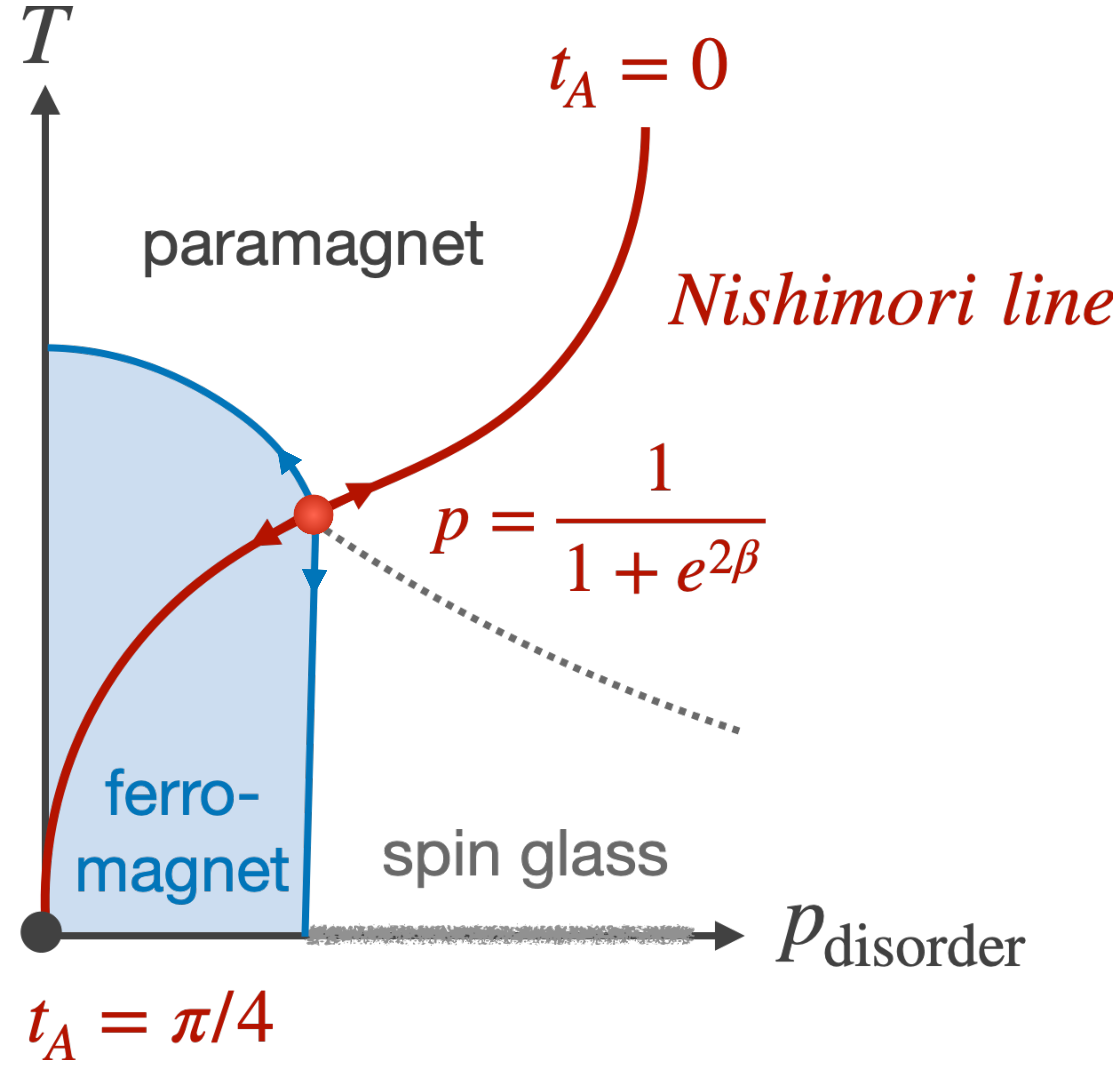


- $\alpha < \pi/4$ injects **tunable coherent error**
- generically a **non-Clifford** gate

incoherent & coherent errors

Dennis, Kitaev, Landahl, Preskill 2002

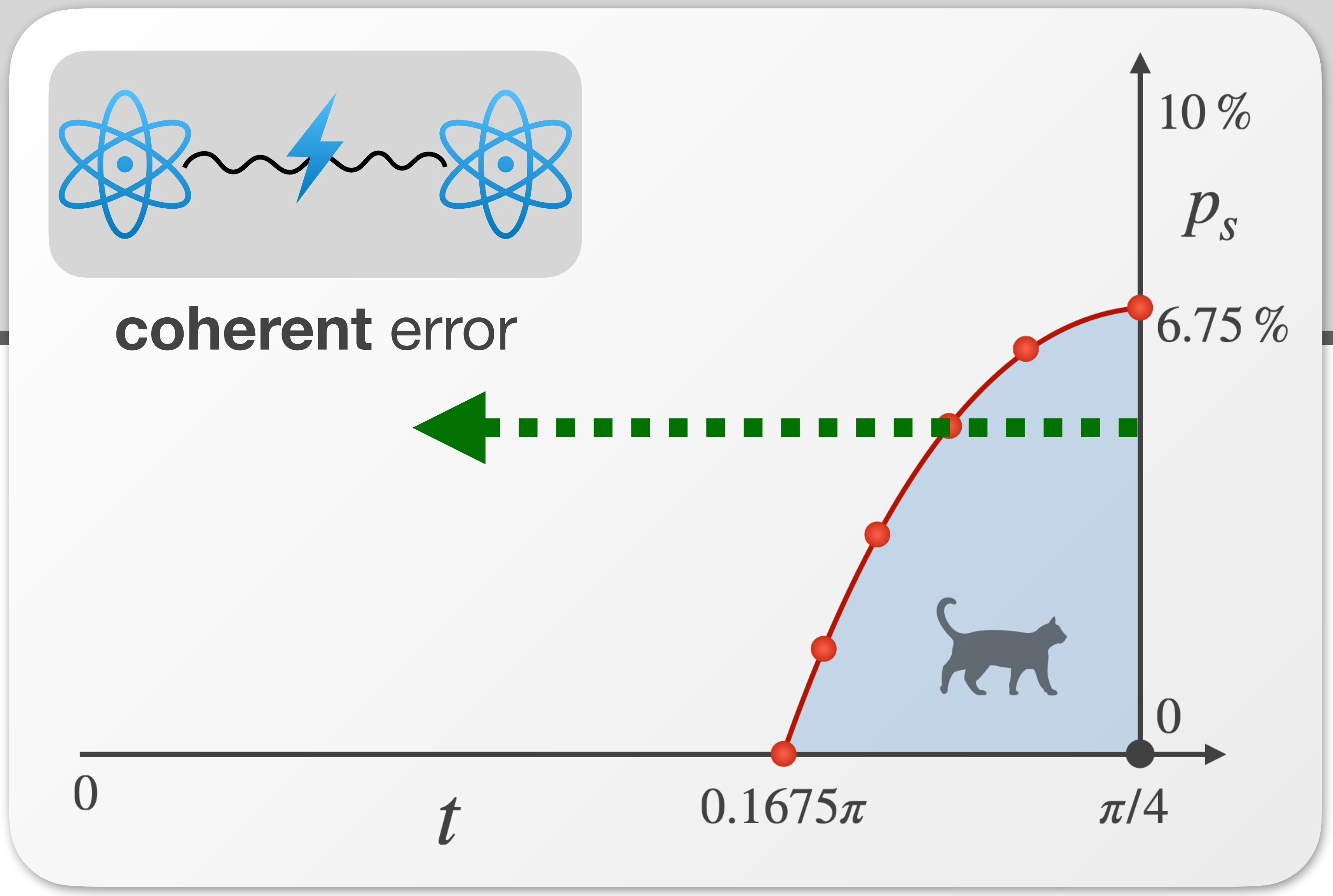


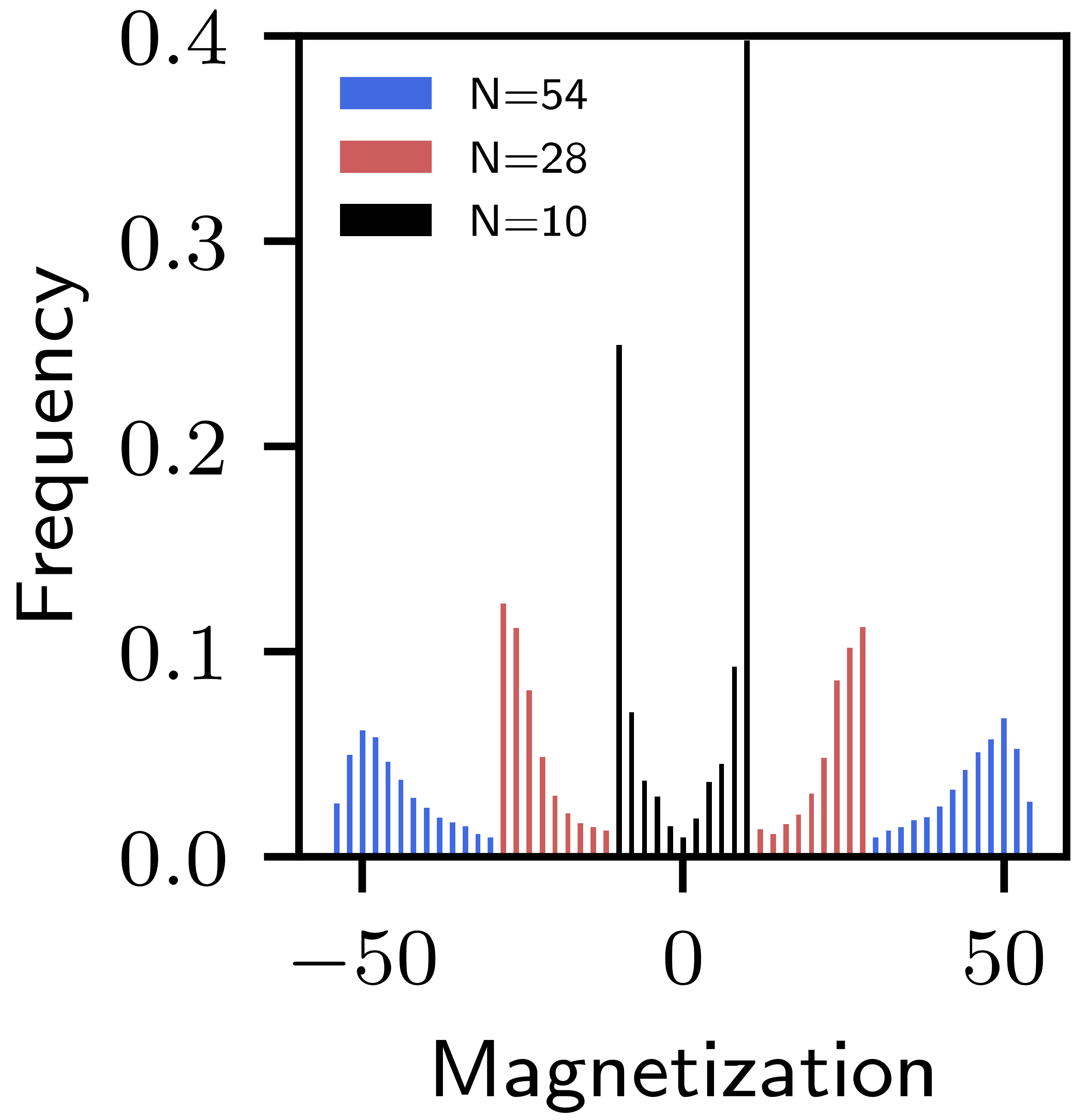


entire phase diagram is mapped to **Nishimori line**

$$\tilde{p} = \frac{1-(1-2p_s)\sin(2t_A)}{2}$$

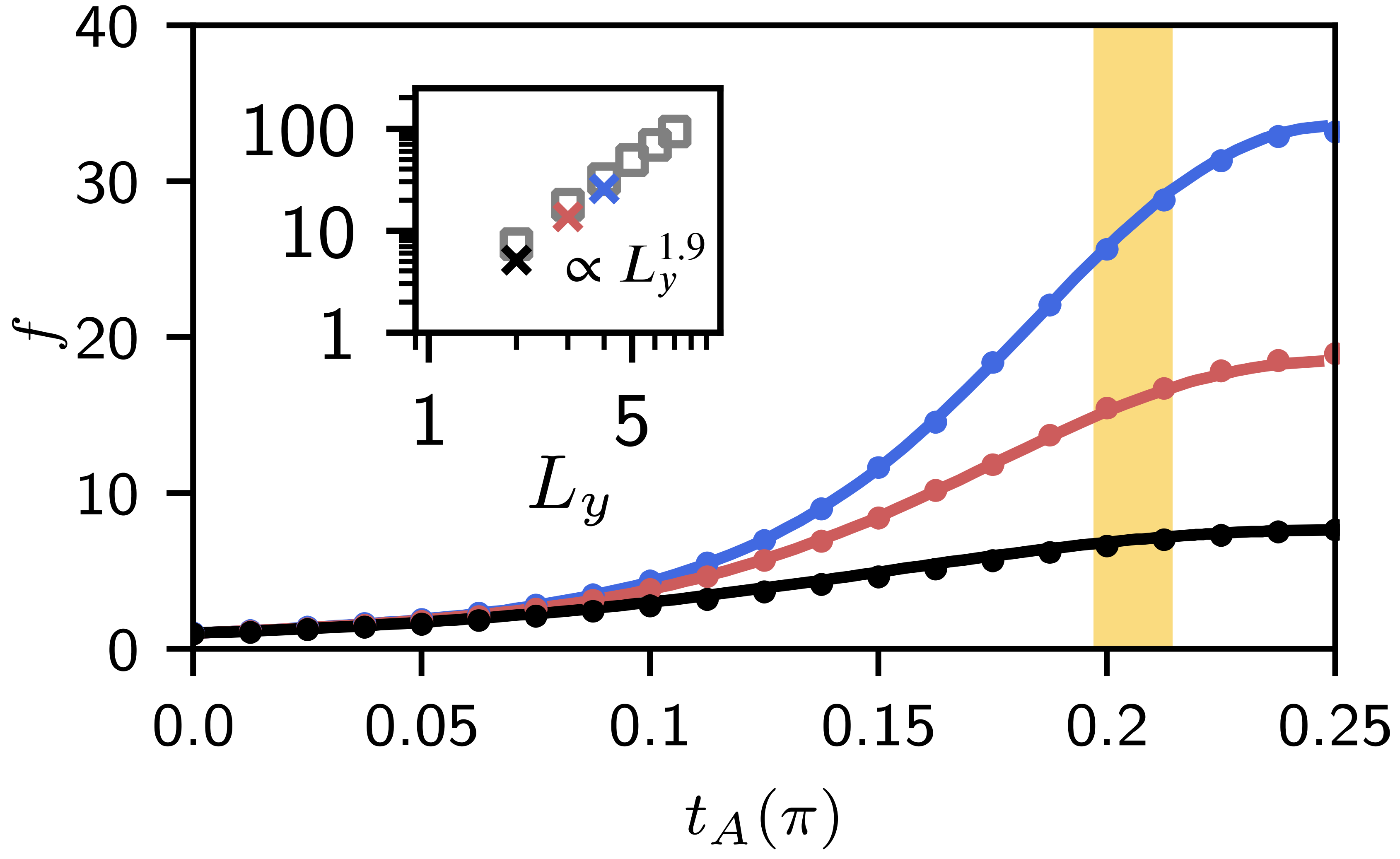
coherent error transition





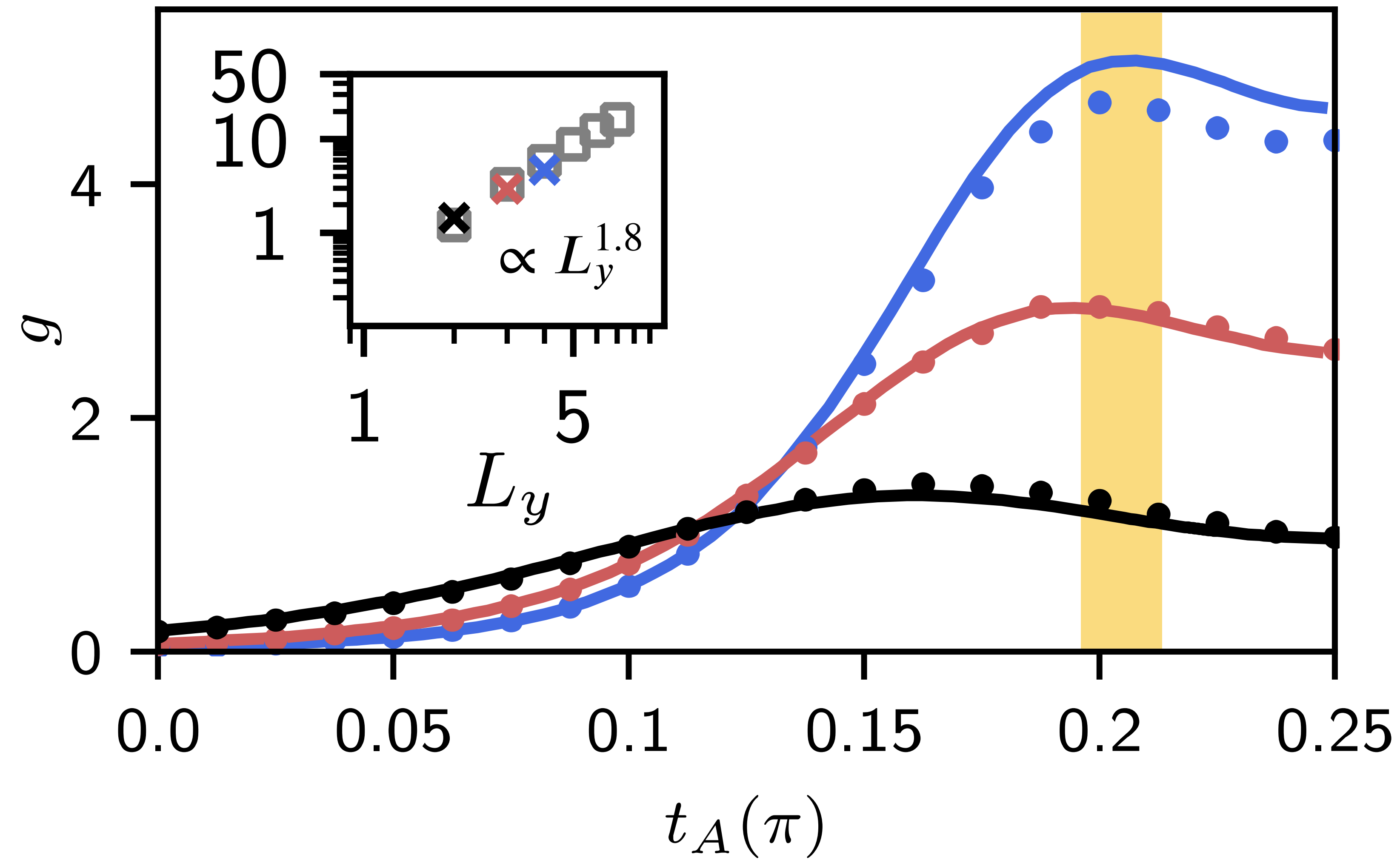
$$f = \frac{1}{N}\left(\langle M^2\rangle - \langle M\rangle^2\right)$$

classical **correlations**

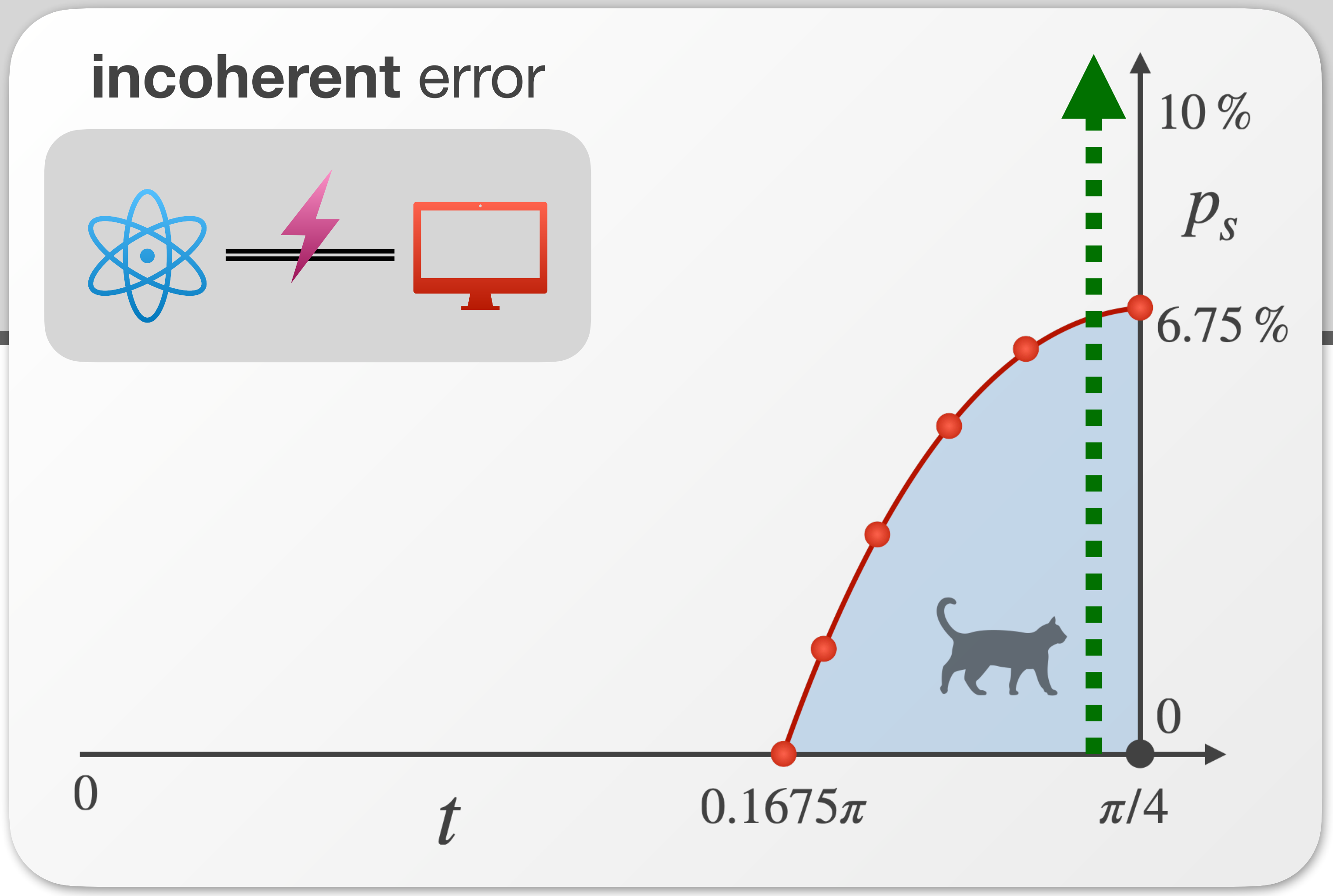


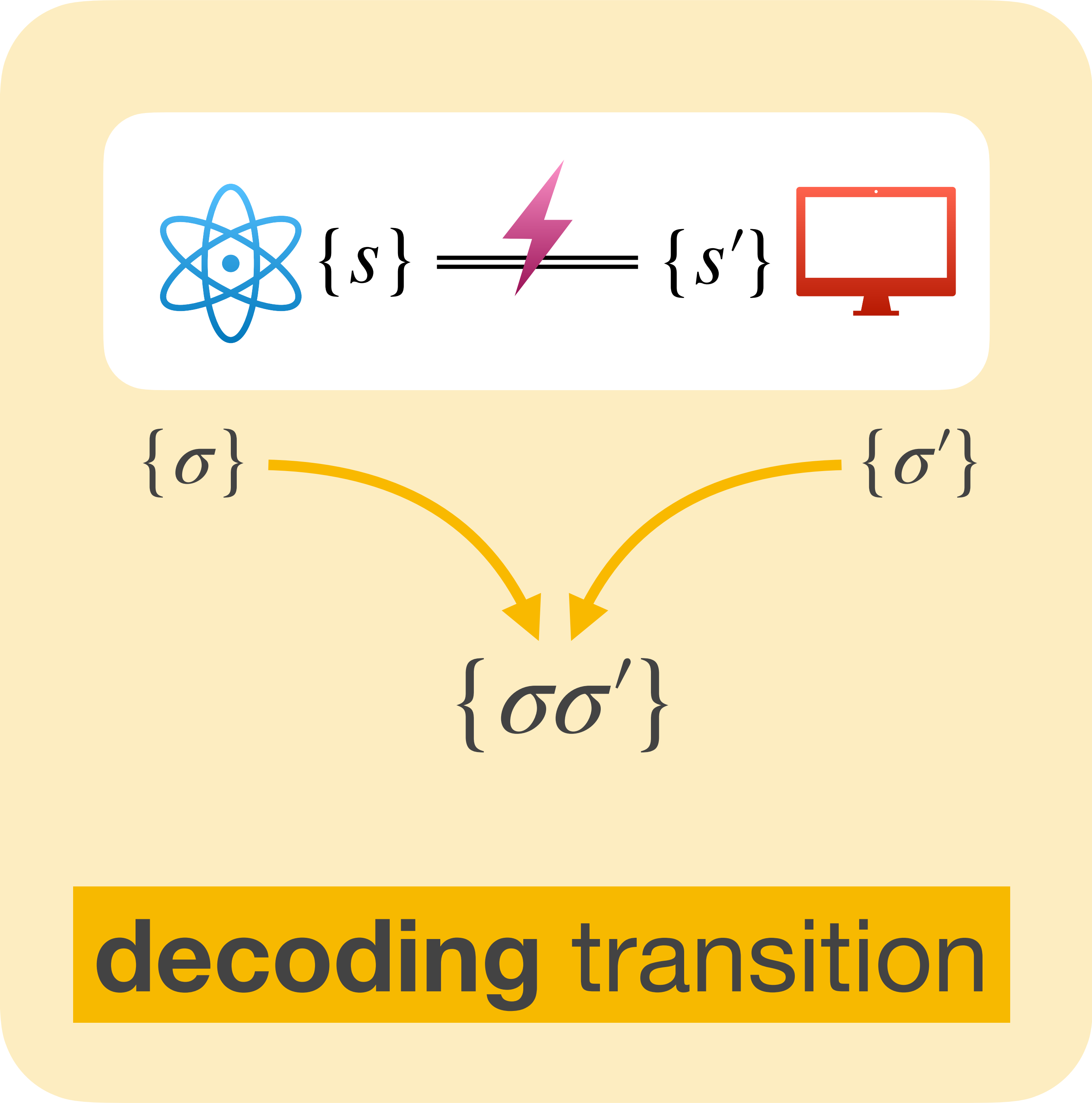
normalized **variance**

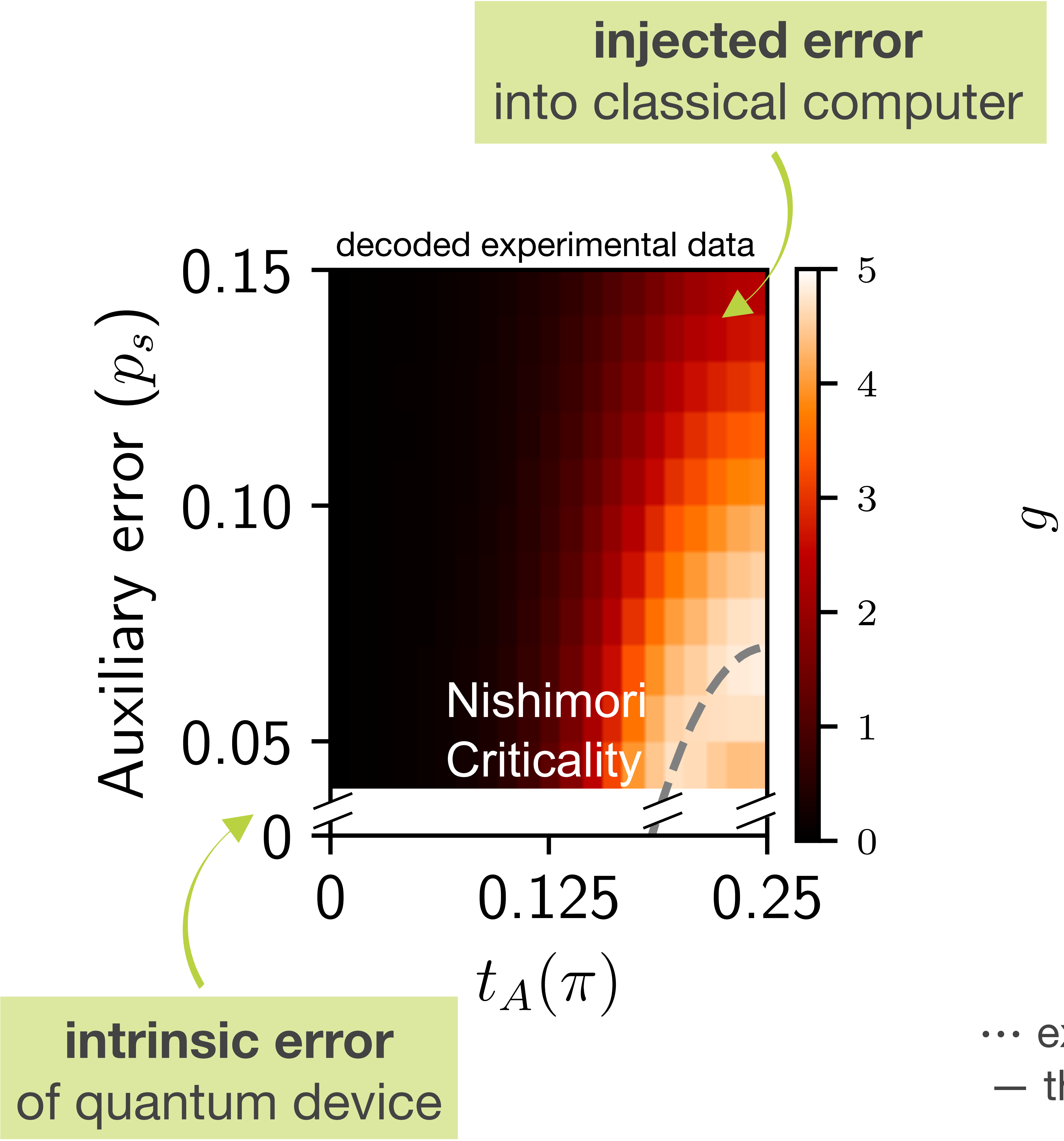
$$g = \frac{1}{N^3}\left(\langle M^4\rangle - \langle M^2\rangle^2\right)$$

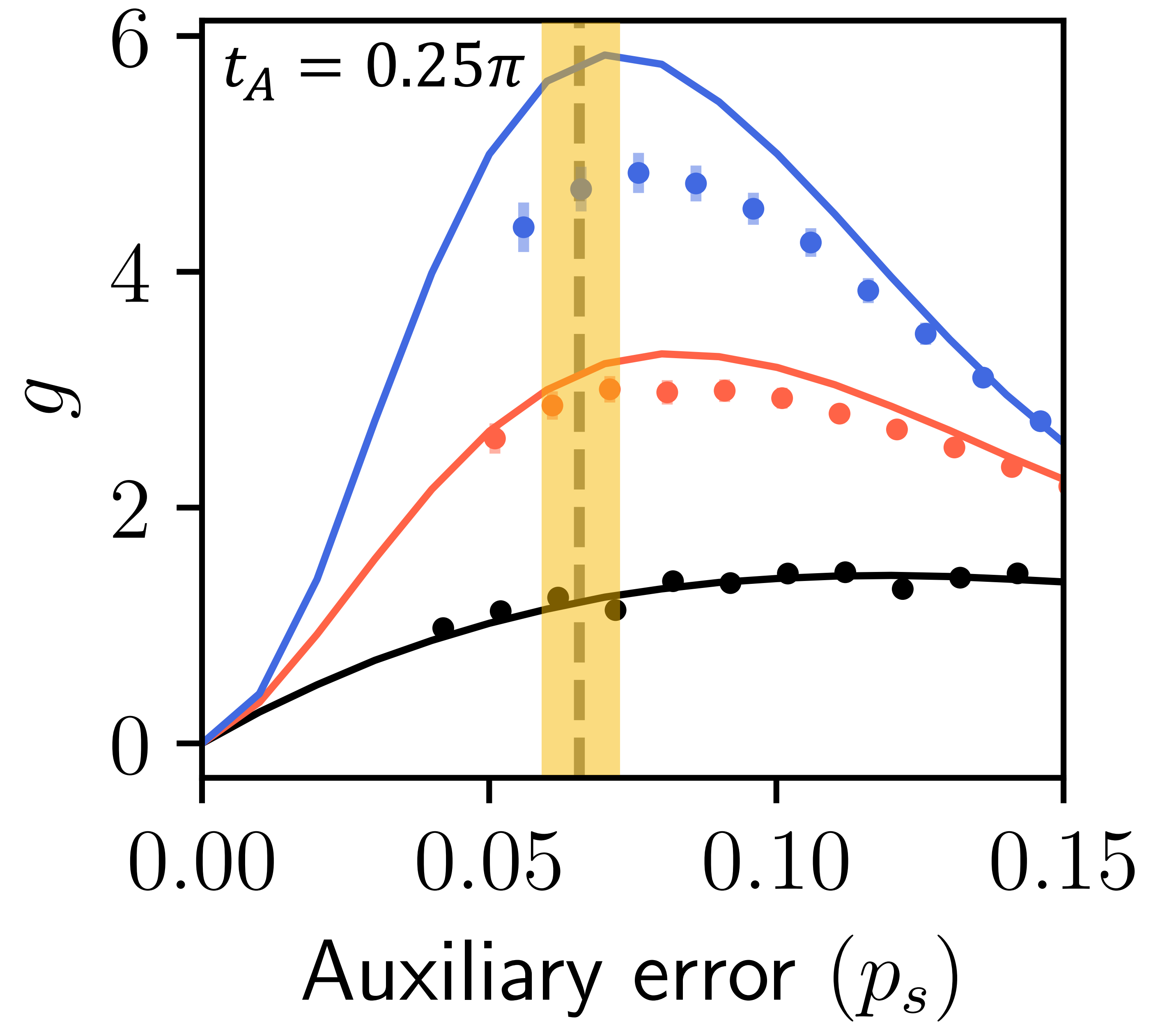


incoherent error transition

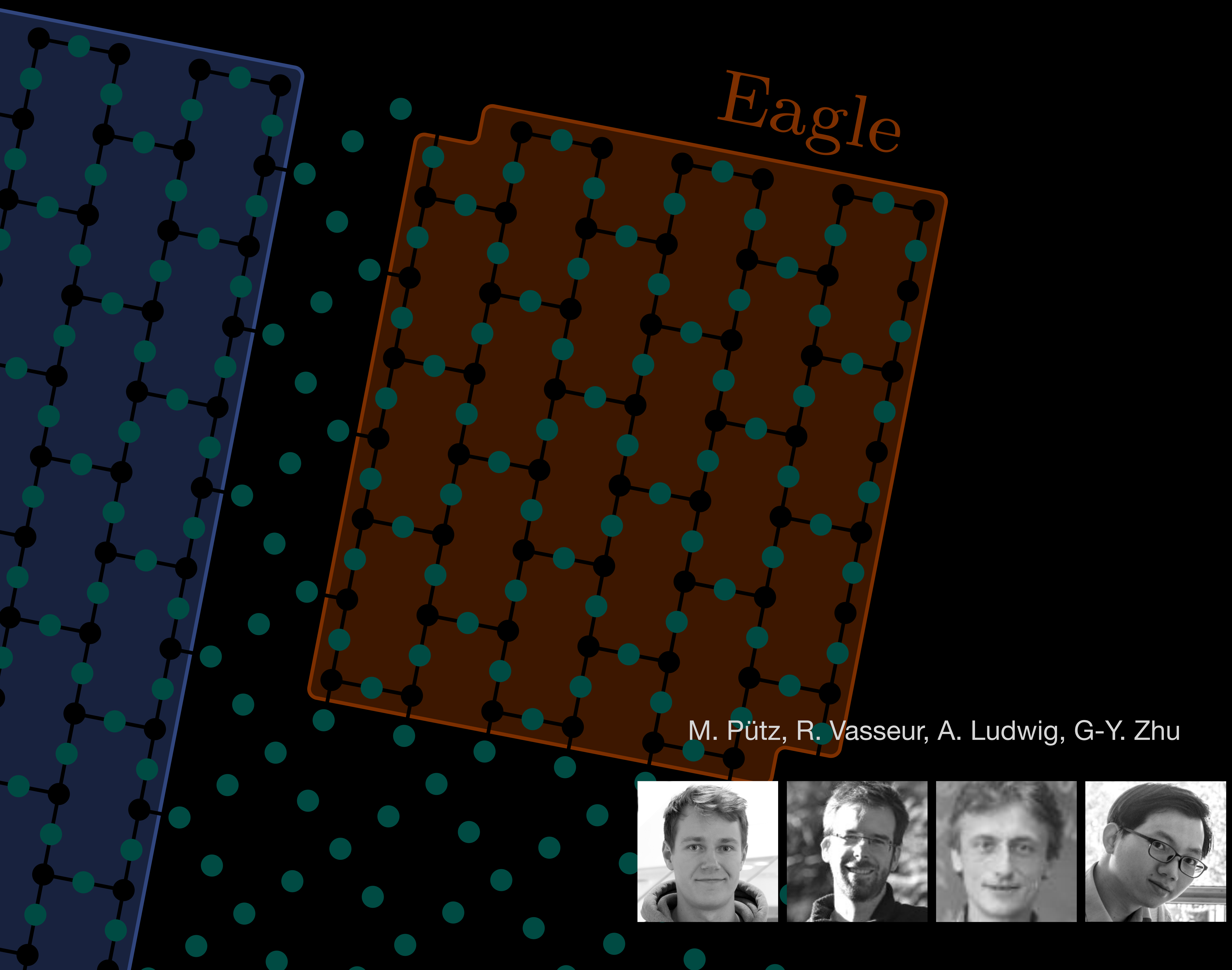








··· experimental data on IBM Sherbrooke
— theoretical benchmark with noise



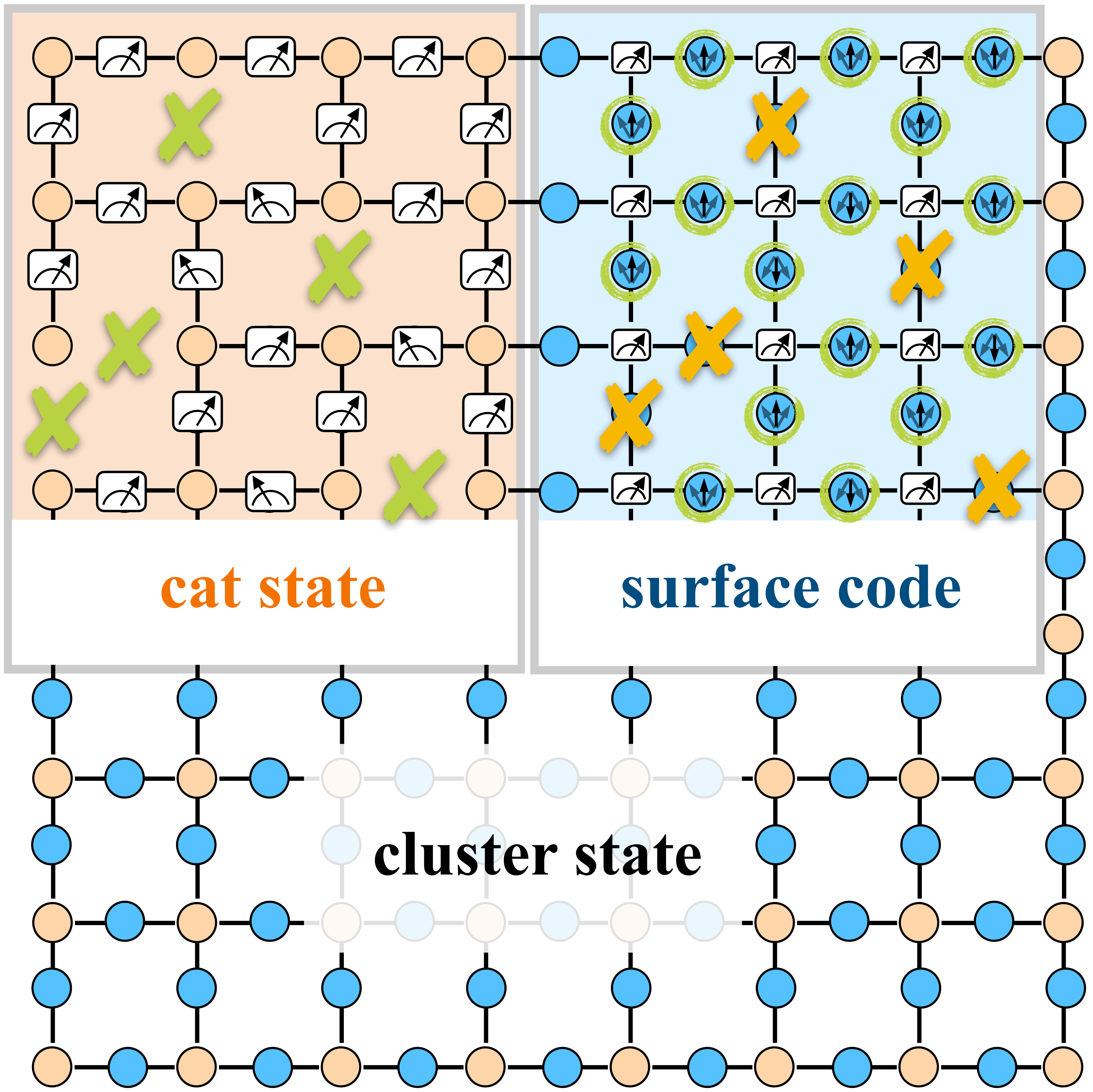
qubit loss

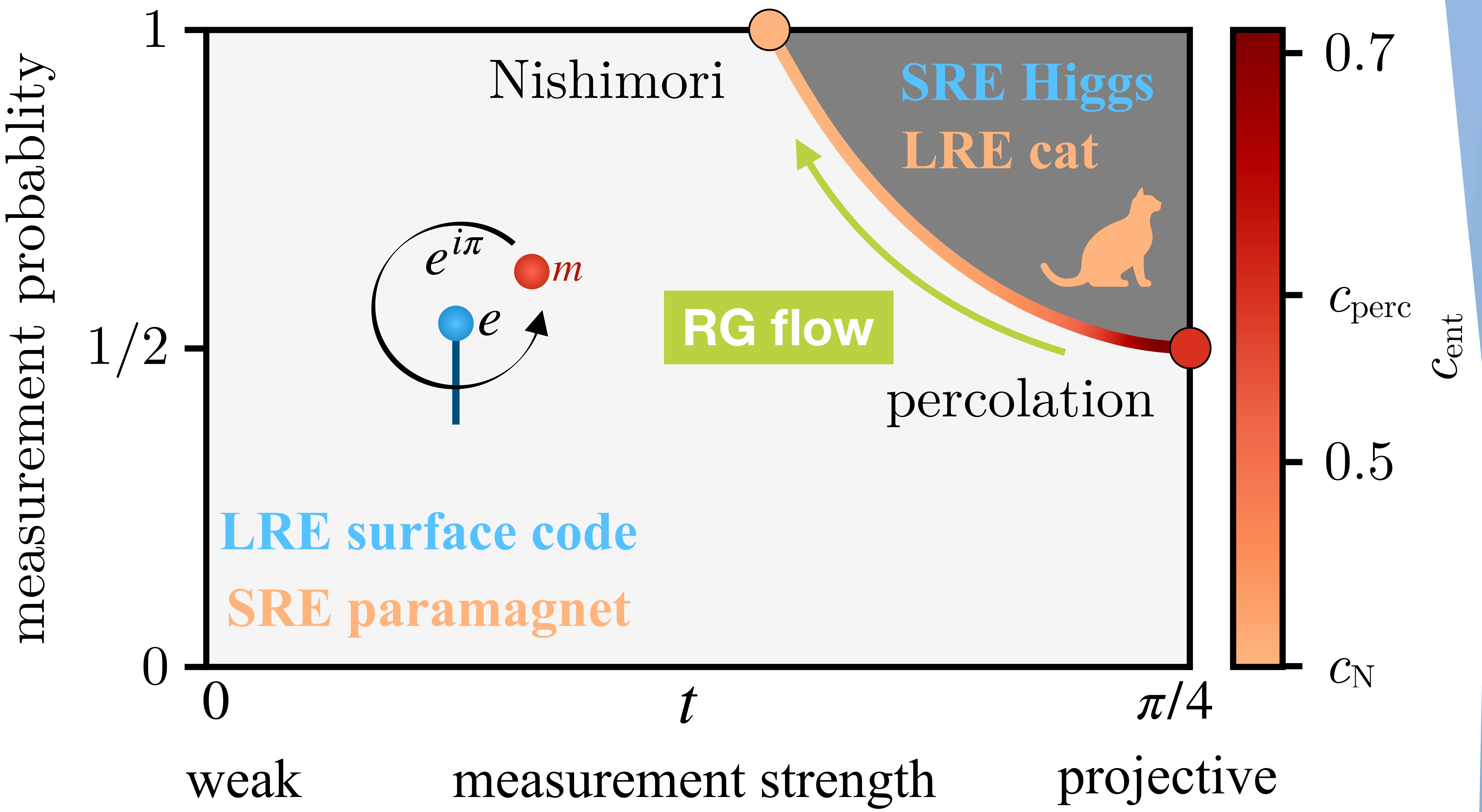
arXiv:2505.22720

M. Pütz, R. Vasseur, A. Ludwig, G-Y. Zhu

Nishimori / percolation & **RG flows**

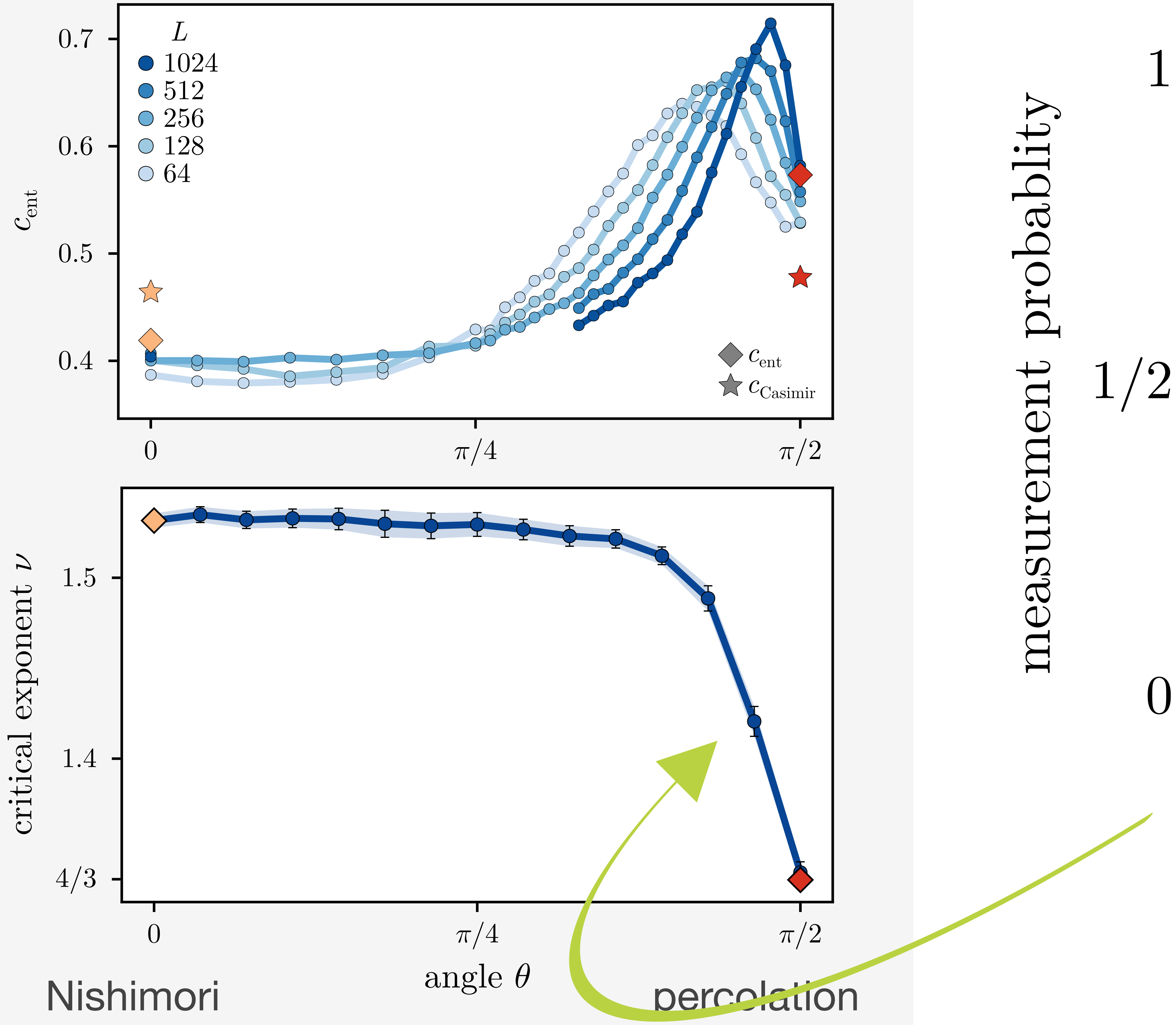
qubit loss

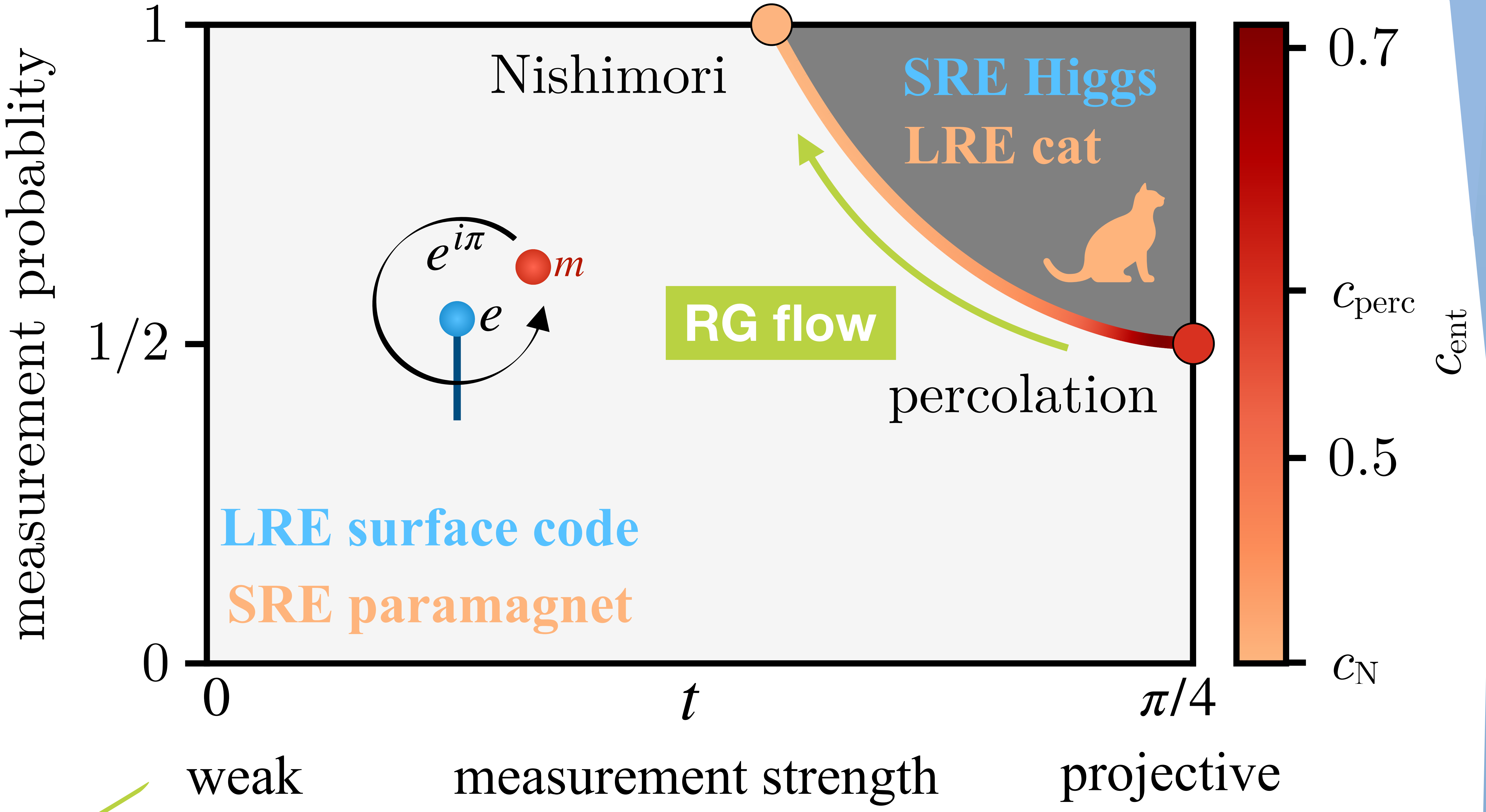




RG flow from percolation
to Nishimori criticality

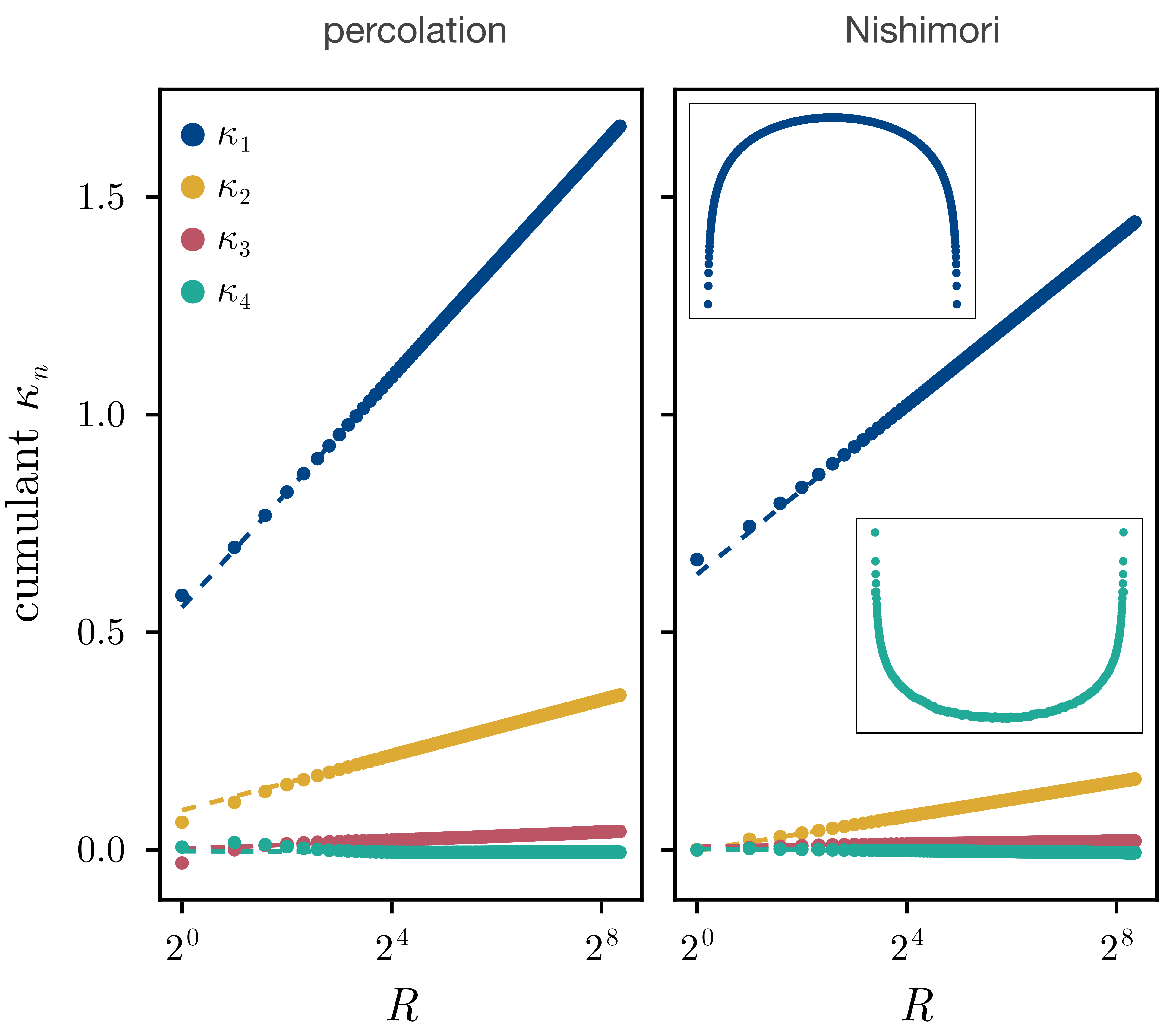
Nishimori / percolation & **RG flows**



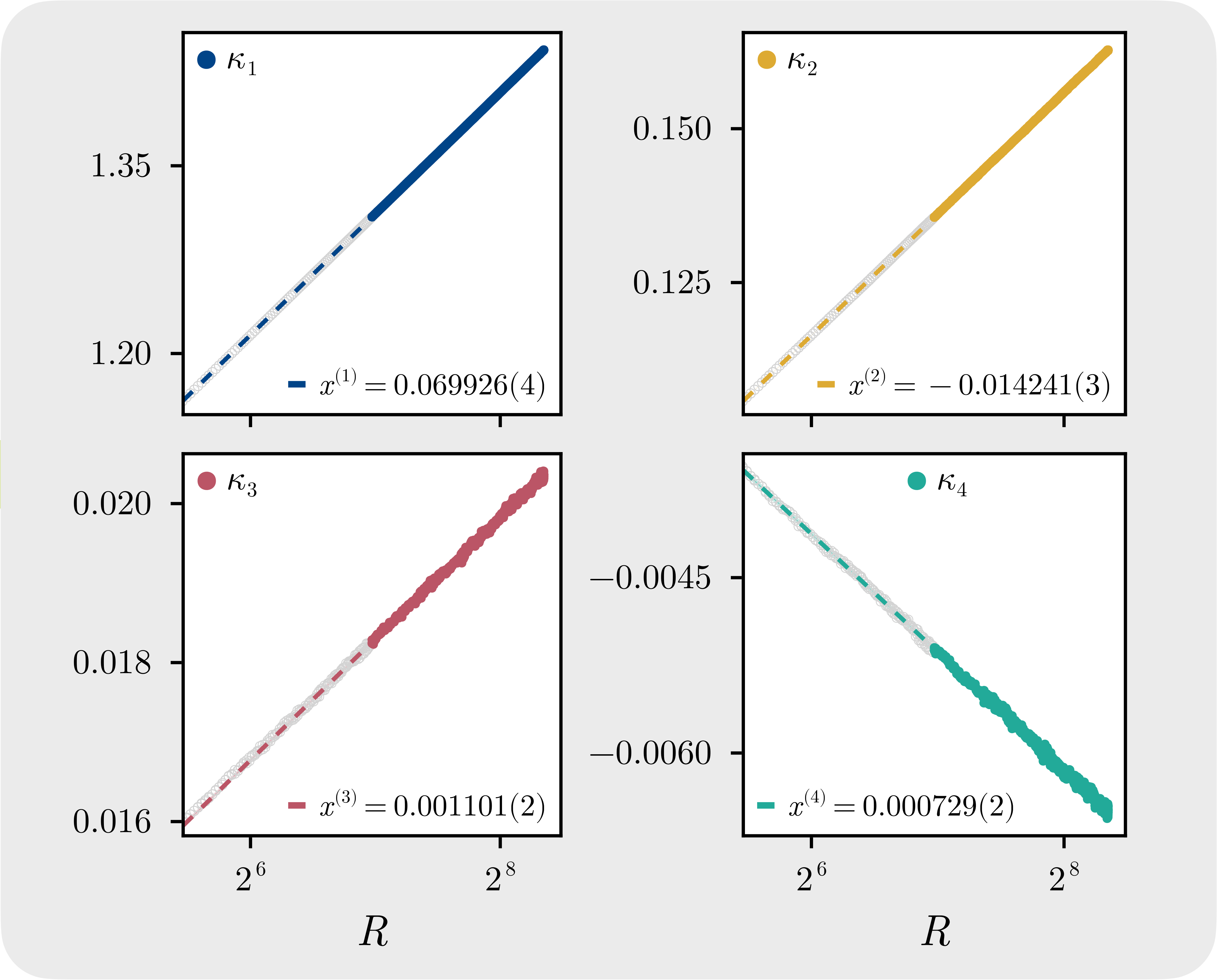


RG flow from percolation
to Nishimori criticality

Nishimori / percolation & **multifractality**



Both percolation and Nishimori criticality exhibit **multifractal spectra** of scaling dimensions.



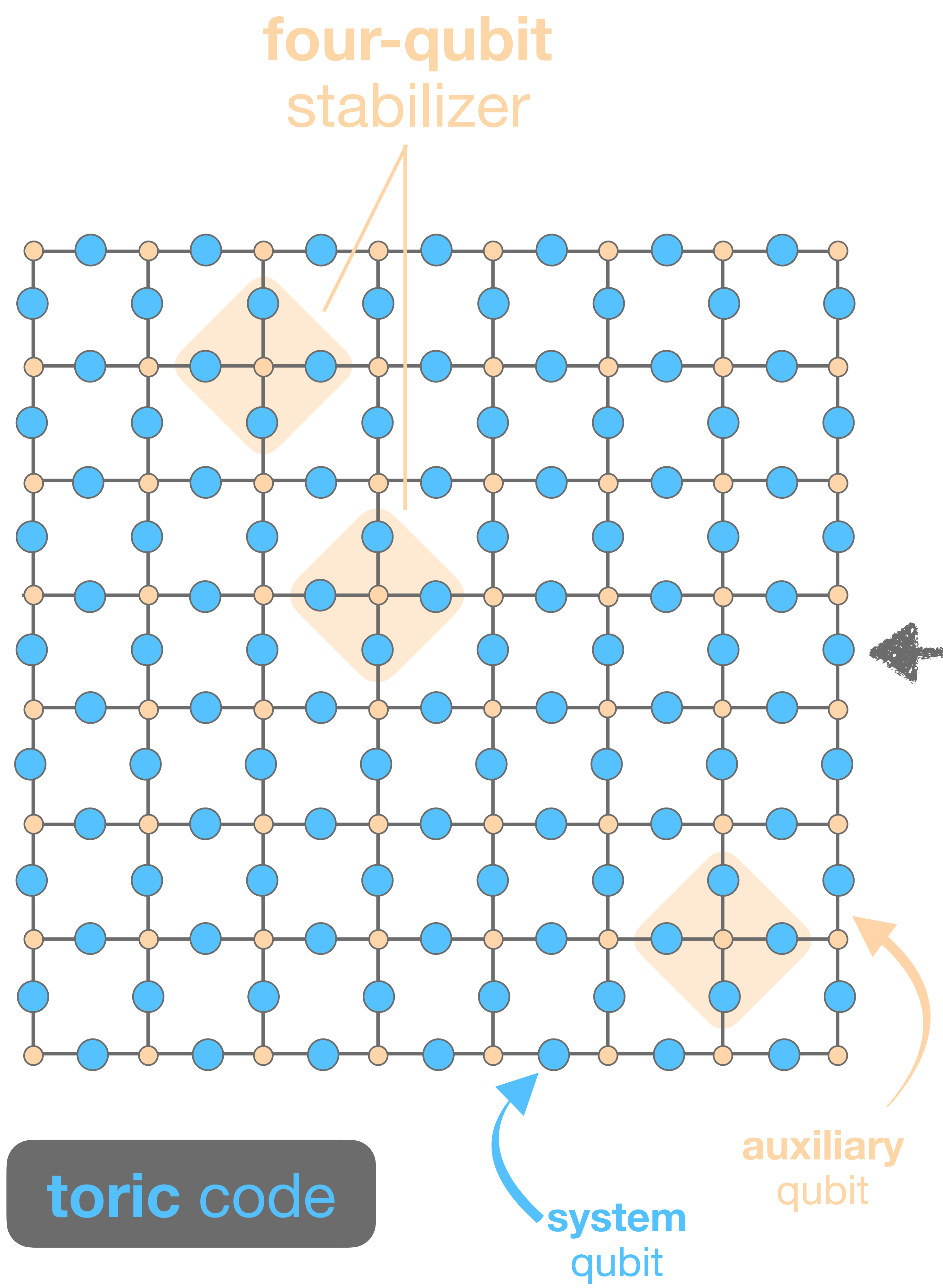
monitored toric codes

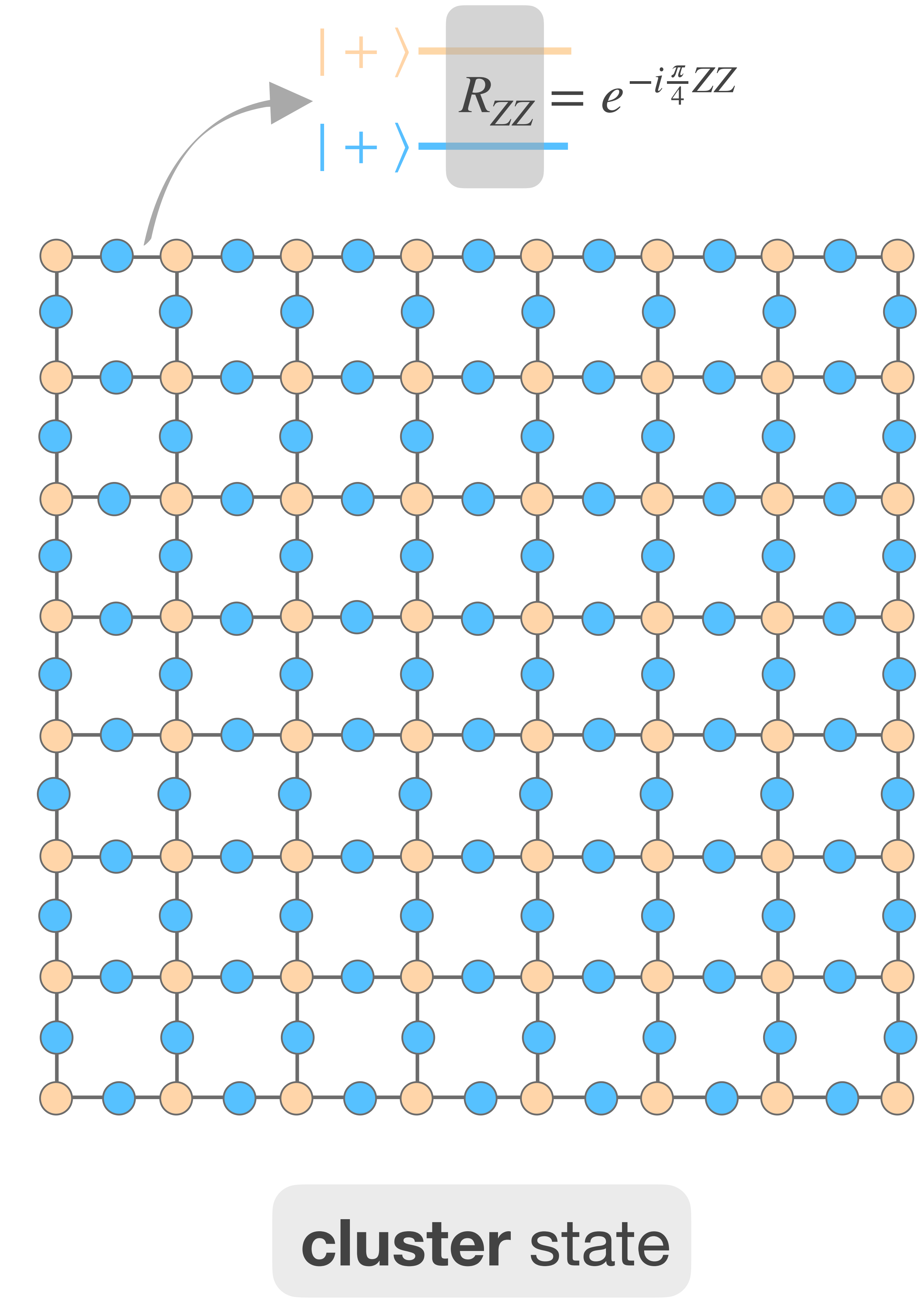
arXiv:2502.14034
PRX Quantum **5**, 040313 (2024)

F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu

entanglement by measurement: **surface code**

Kitaev (1997)



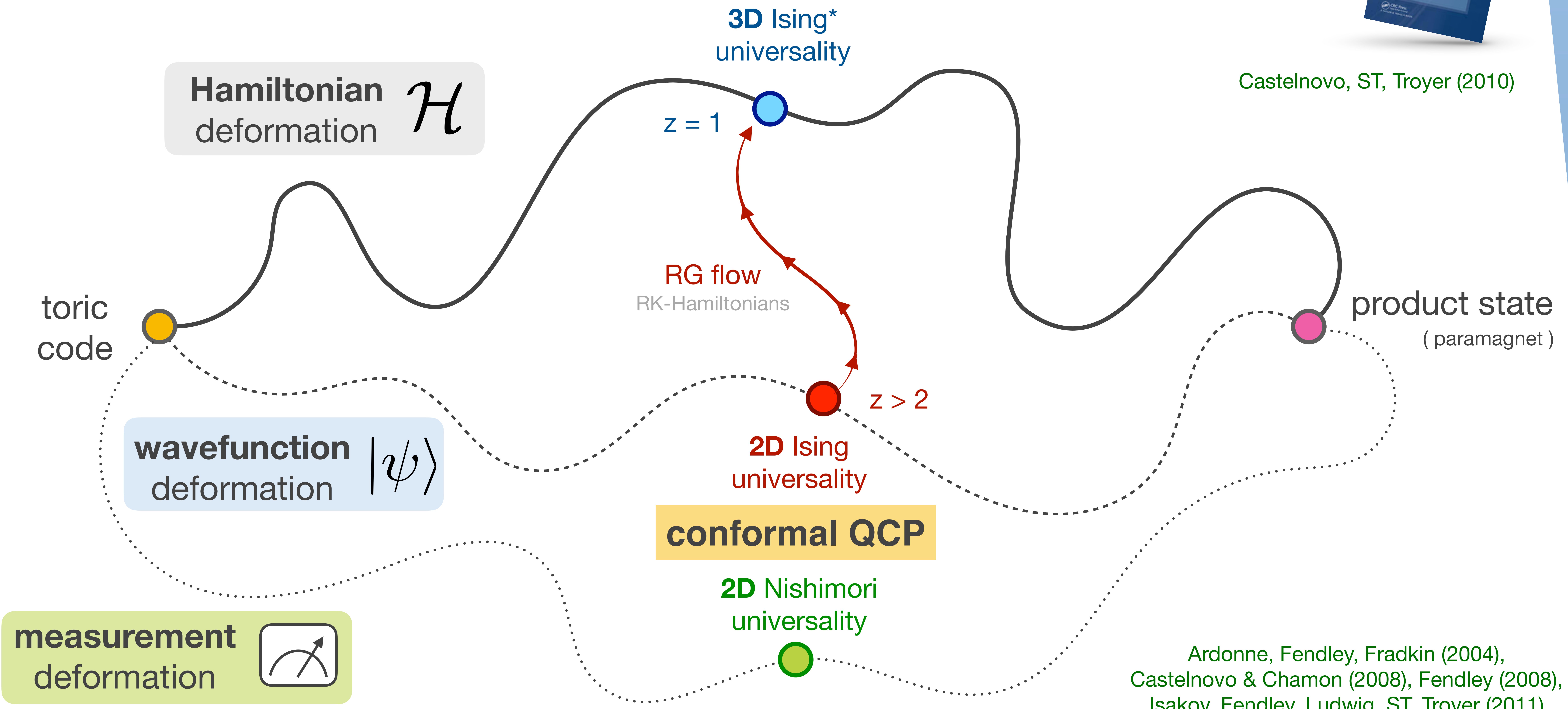


The toric/surface code is an ingenious **measurement protocol.**

The toric/surface code is an exactly solvable **Hamiltonian model.**

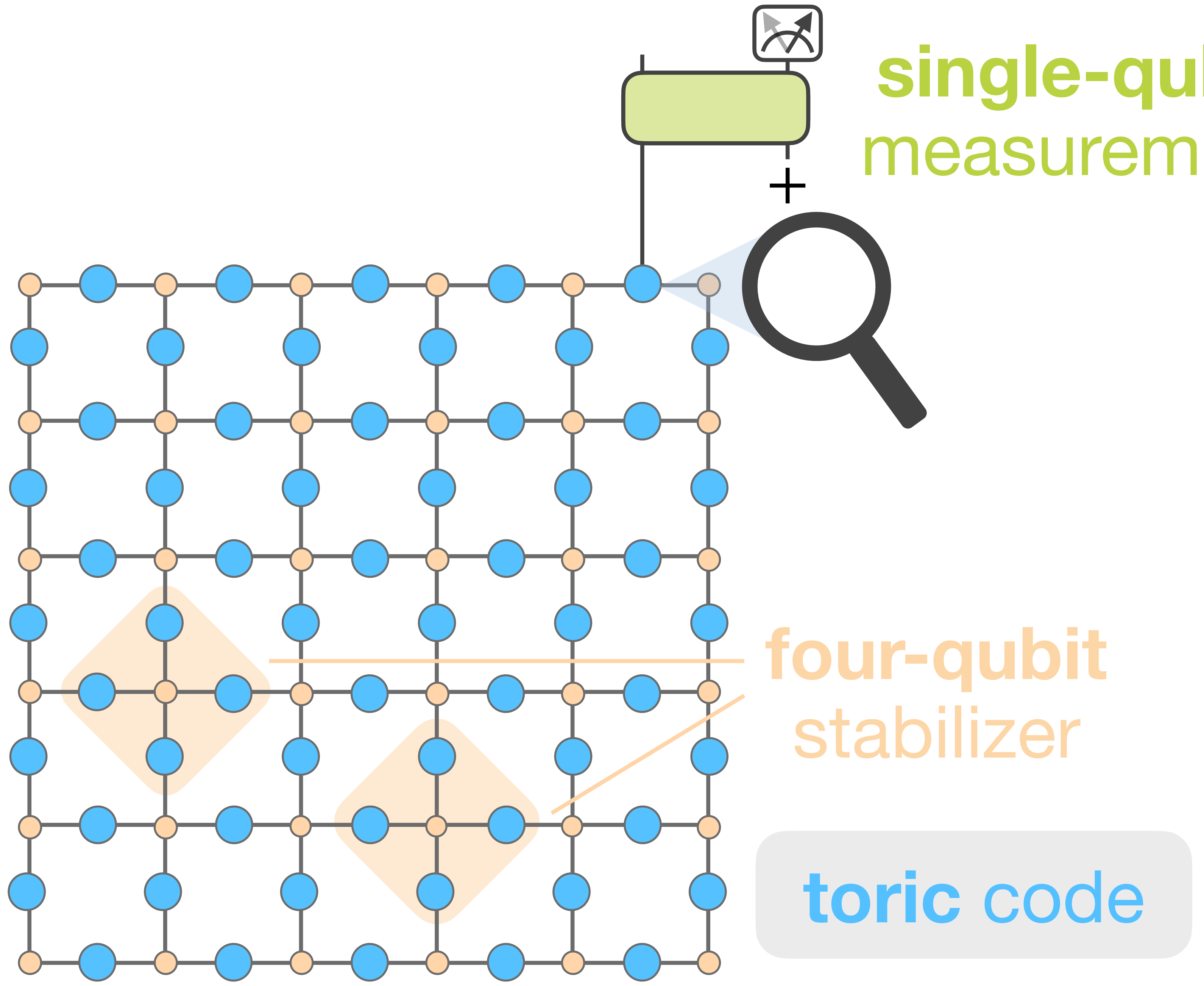
phase transitions & deformations

Castelnovo, ST, Troyer (2010)

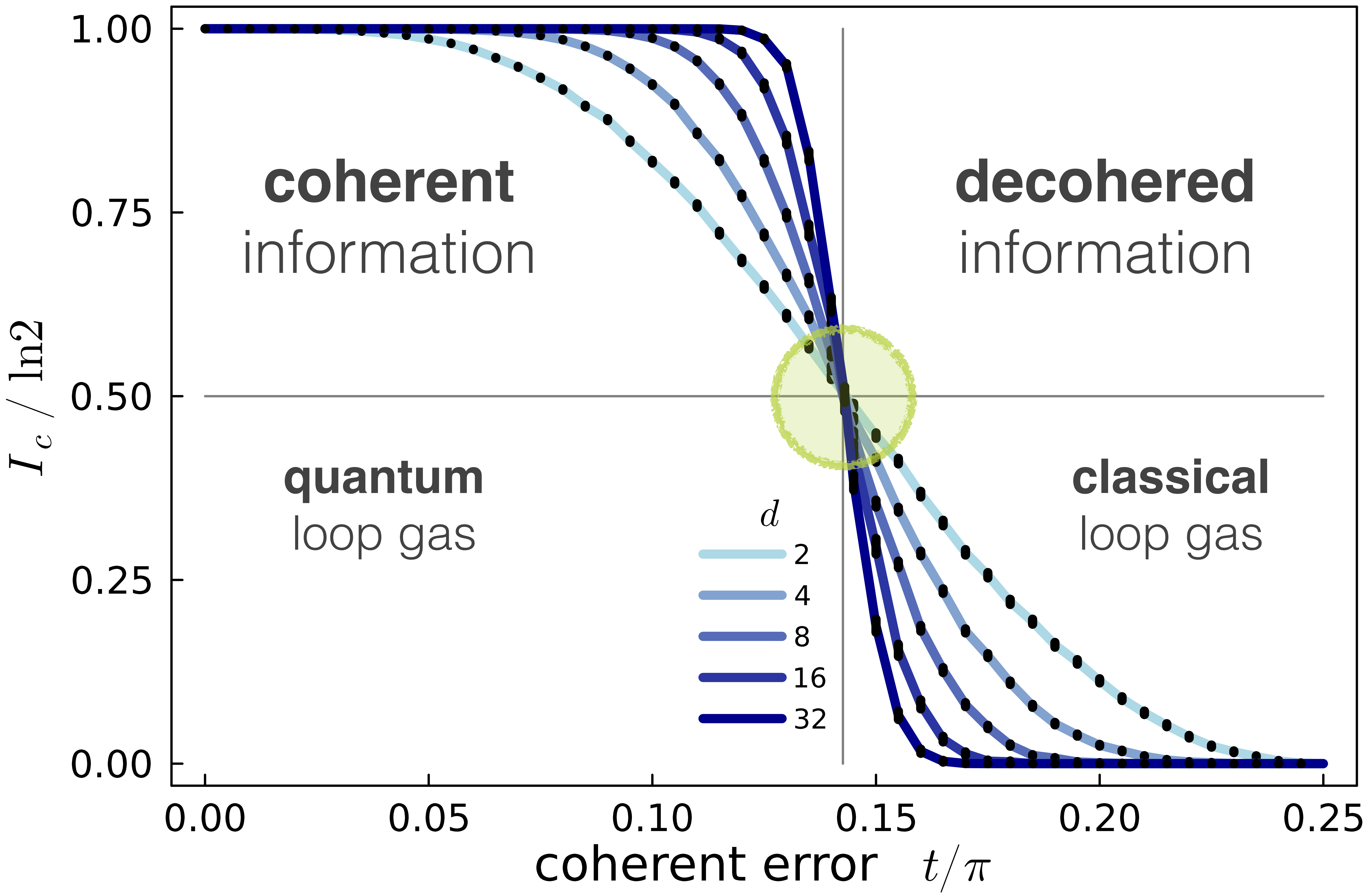


Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008),
Isakov, Fendley, Ludwig, ST, Troyer (2011)

toric code under **weak measurement**



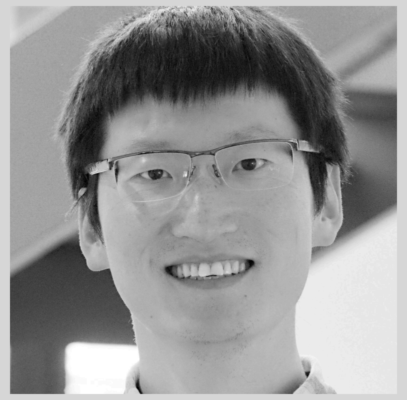
$$|\psi\rangle \mapsto \begin{cases} \left(1 + \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle \\ \left(1 - \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle \end{cases}$$



Nishimori transition

F. Eckstein, B. Han, ST, G.Y. Zhu
PRX Quantum **5**, 040313 (2024)

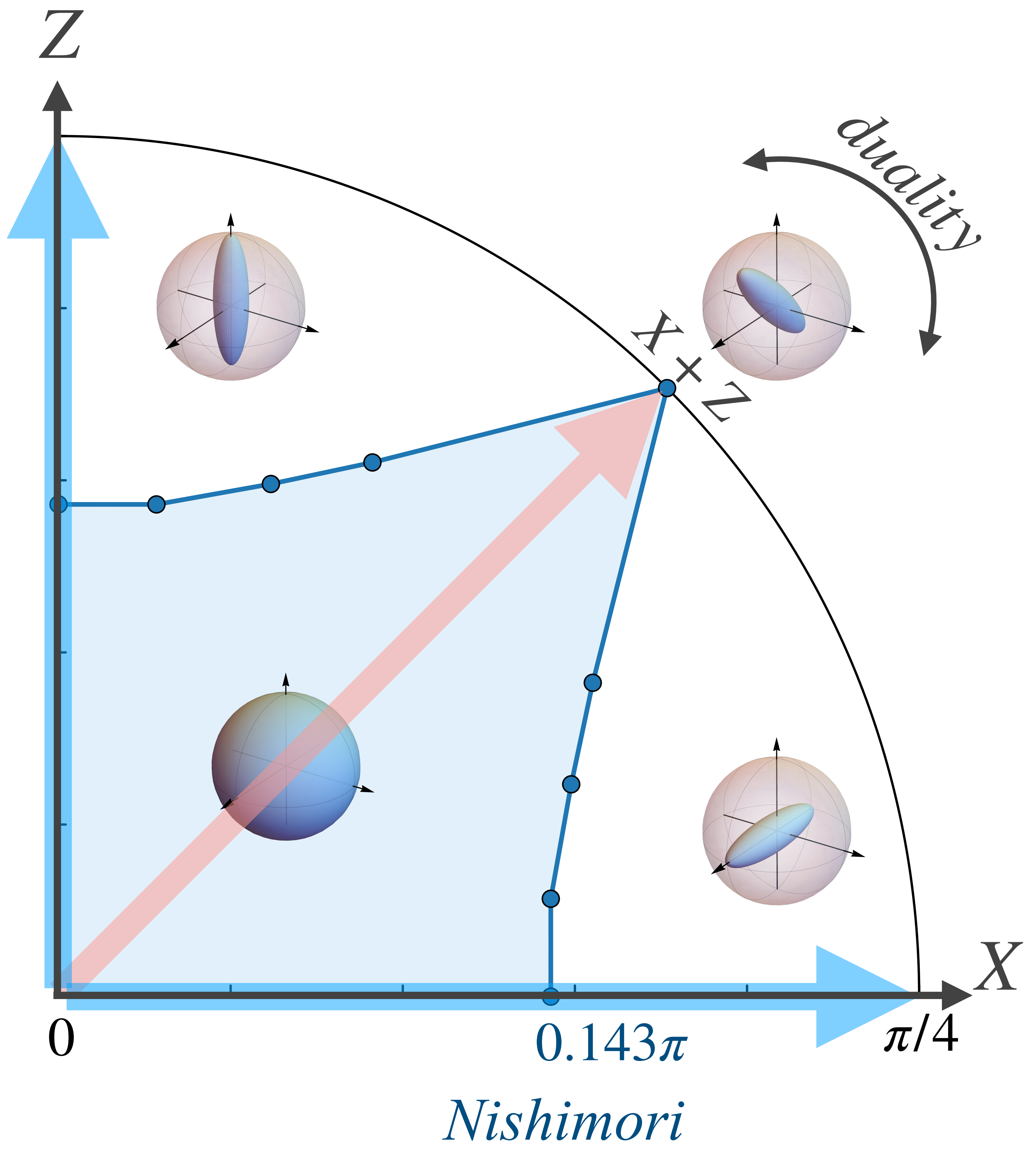
toric code under **weak measurement**

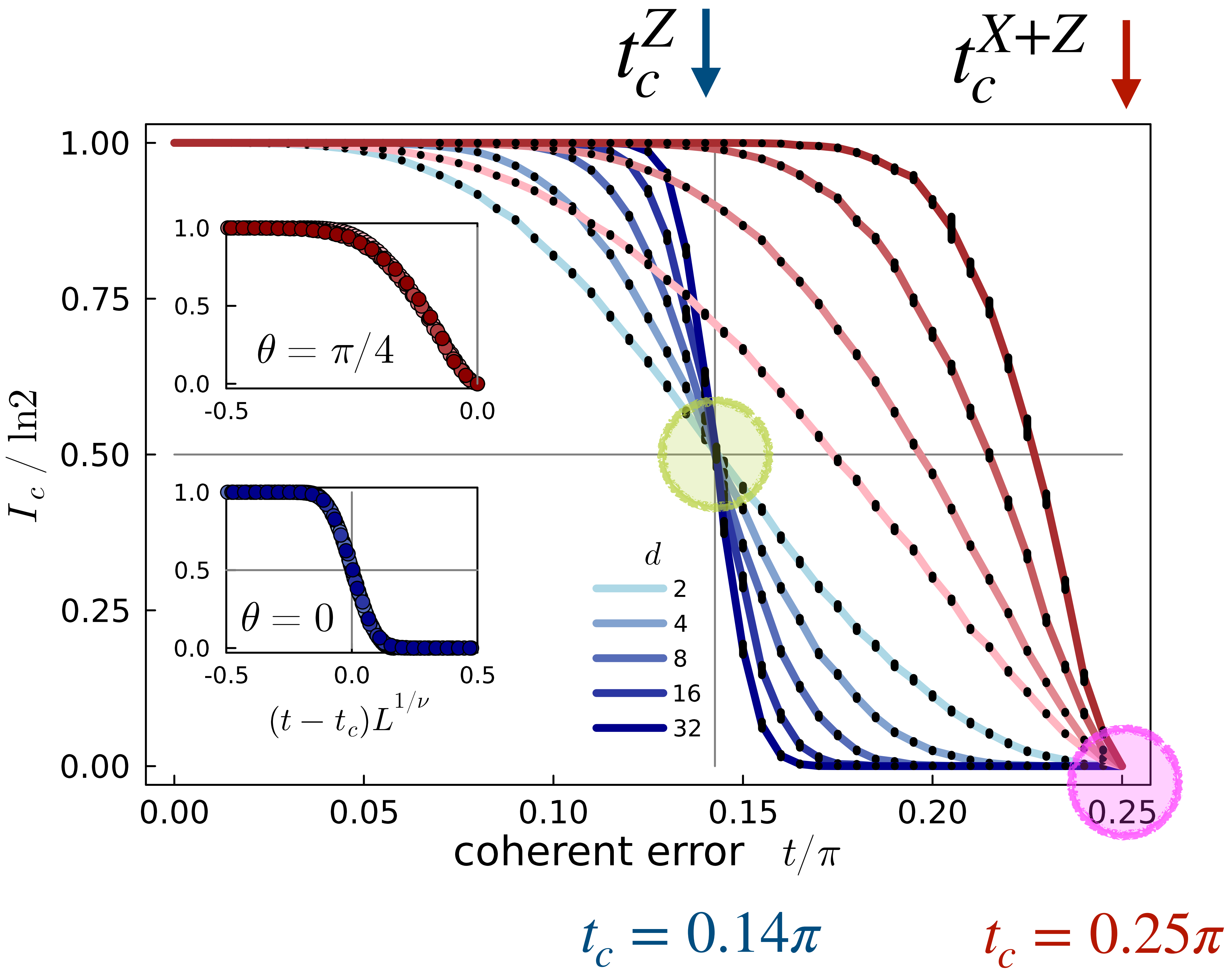


$$\Pi_{ij}[1 \pm \tanh\frac{\beta}{2}(\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

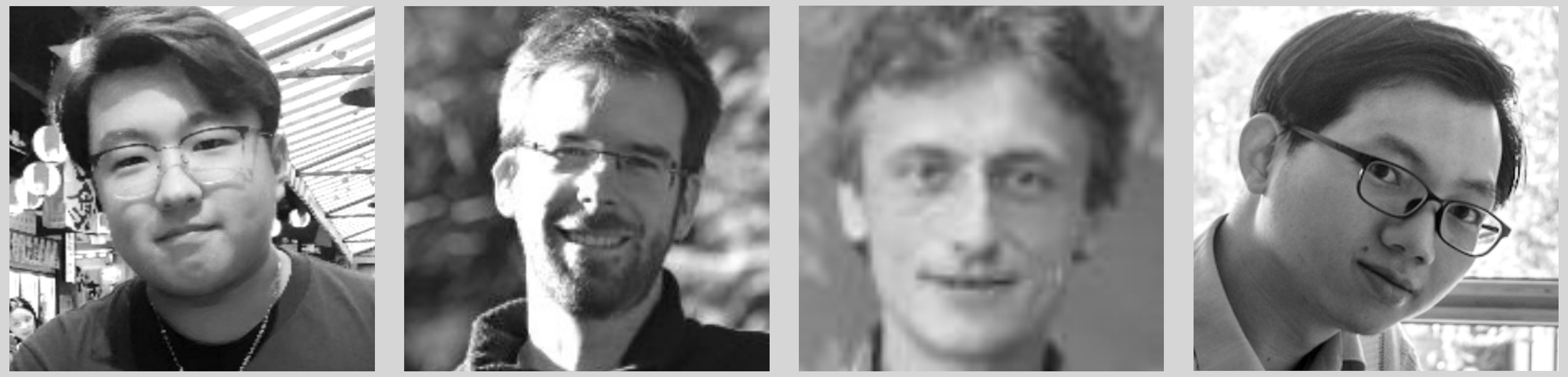
measurement **angle**

Self-duality makes the toric code absolutely **stable** against decoherence.



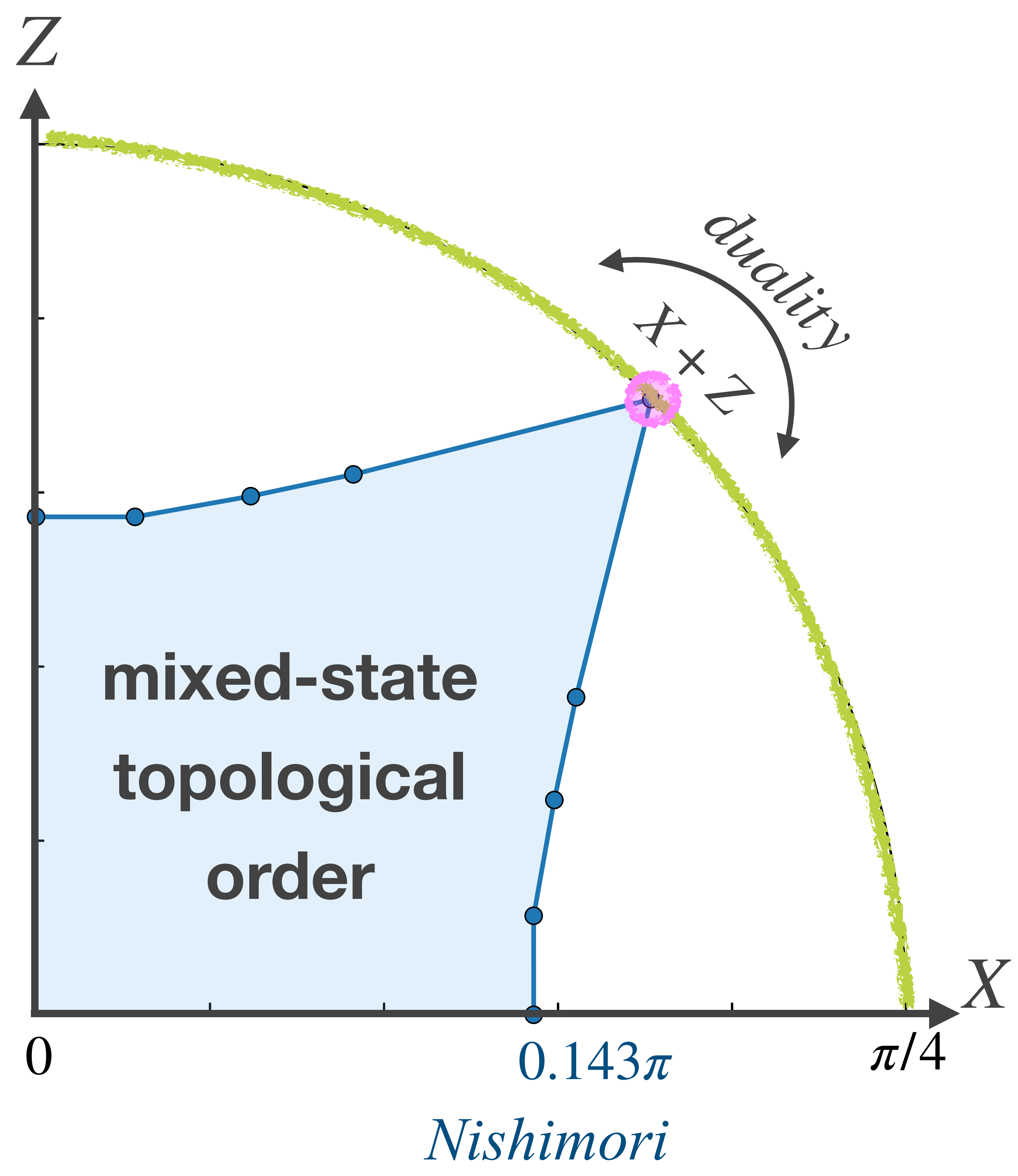


toric code & **strong measurement**

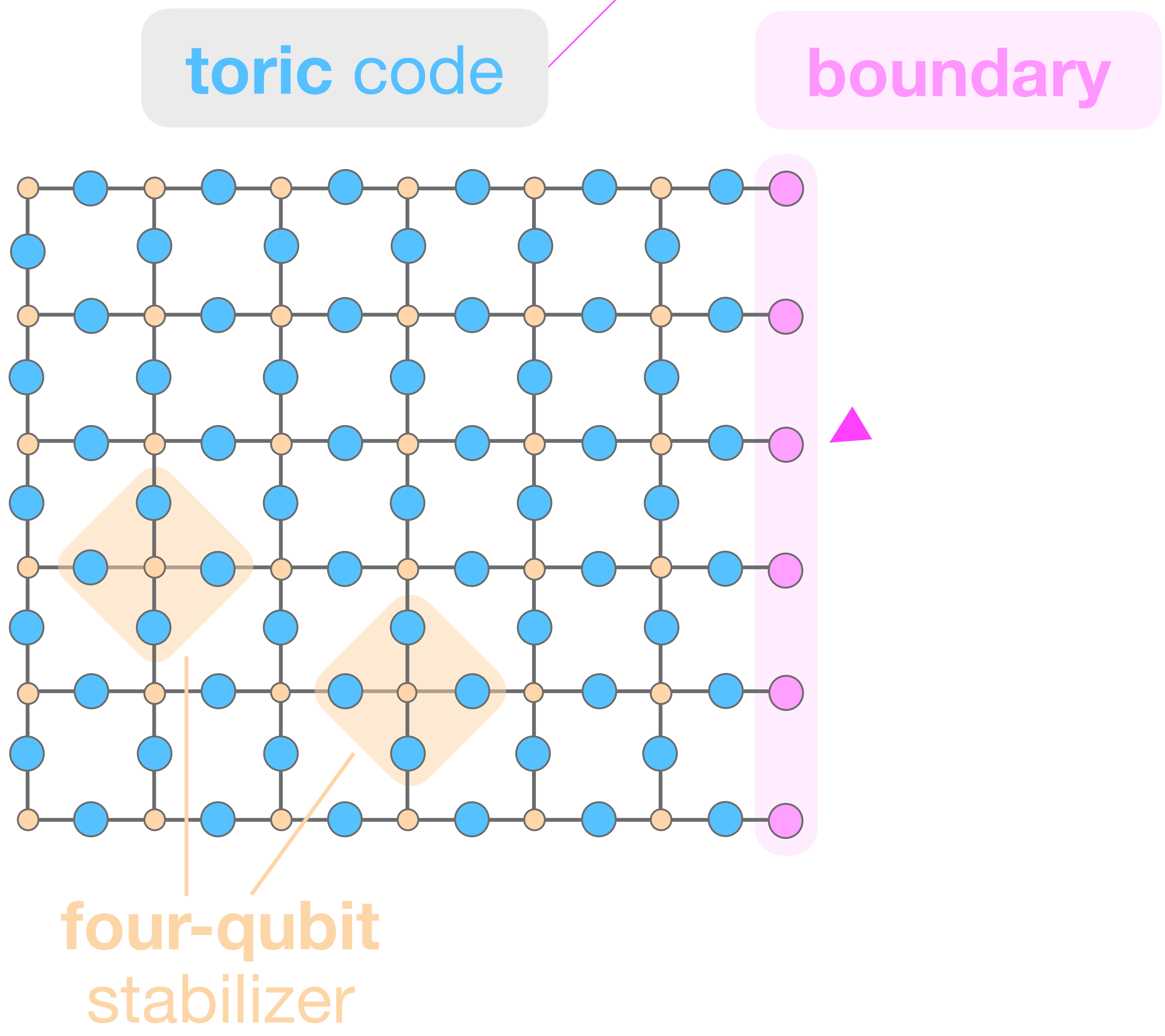


$$\Pi_{ij}[1 \pm \tanh\frac{\beta}{2}(\cos\theta\, Z_{ij} + \sin\theta\, X_{ij})]|\psi\rangle$$

measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?

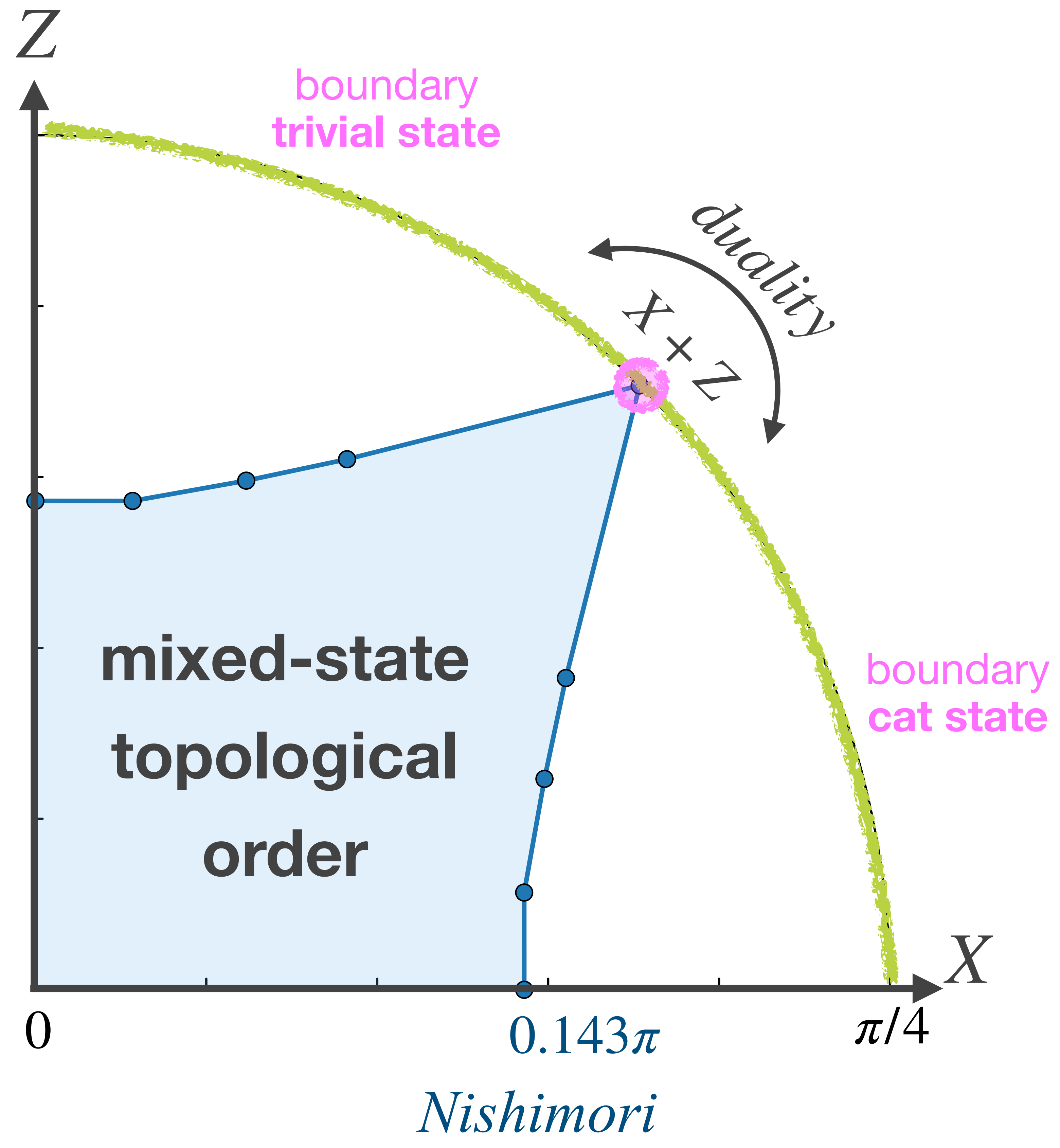


bulk-boundary correspondence




$$\Pi_{ij}[1 \pm \tanh \frac{\beta}{2} (\cos\theta \, Z_{ij} + \sin\theta \, X_{ij})]|\psi\rangle$$

measurement **angle**

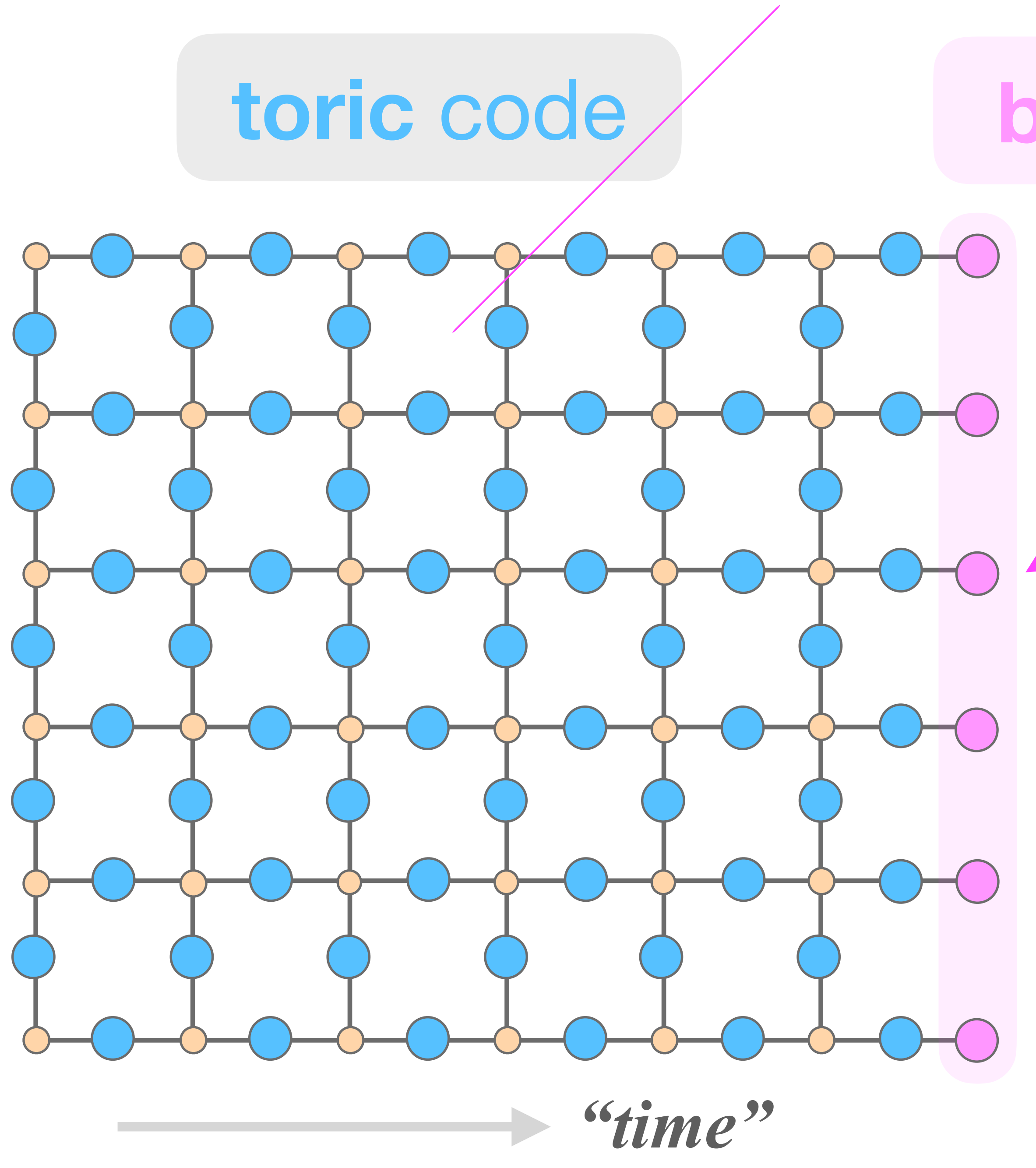


Why is there a **critical point** when all the bulk qubits **collapse**?

toric code

boundary

measure
bulk qubits



bulk

mixed-state
phase transition

measurement-induced
phase transition

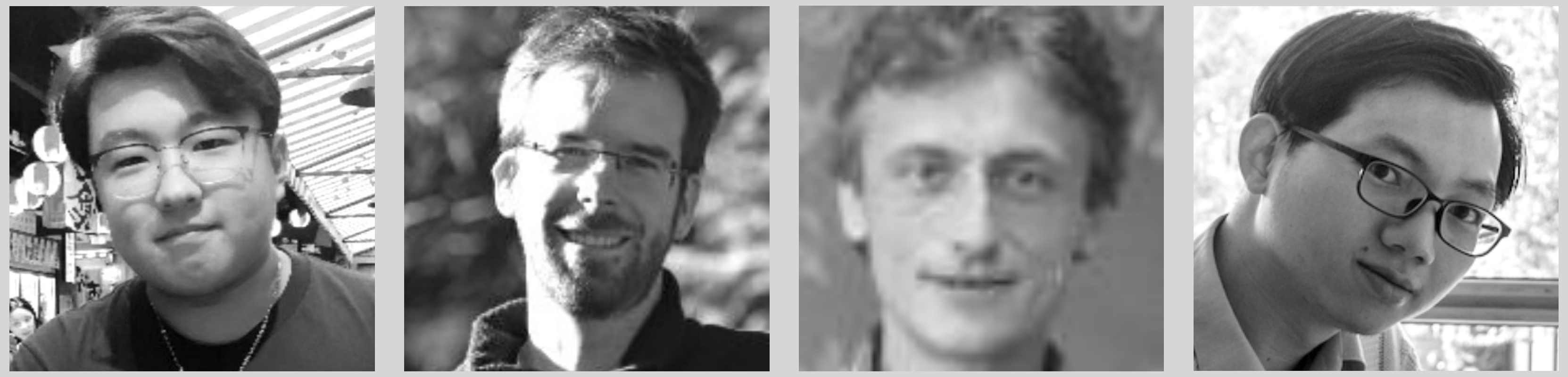
boundary

classical

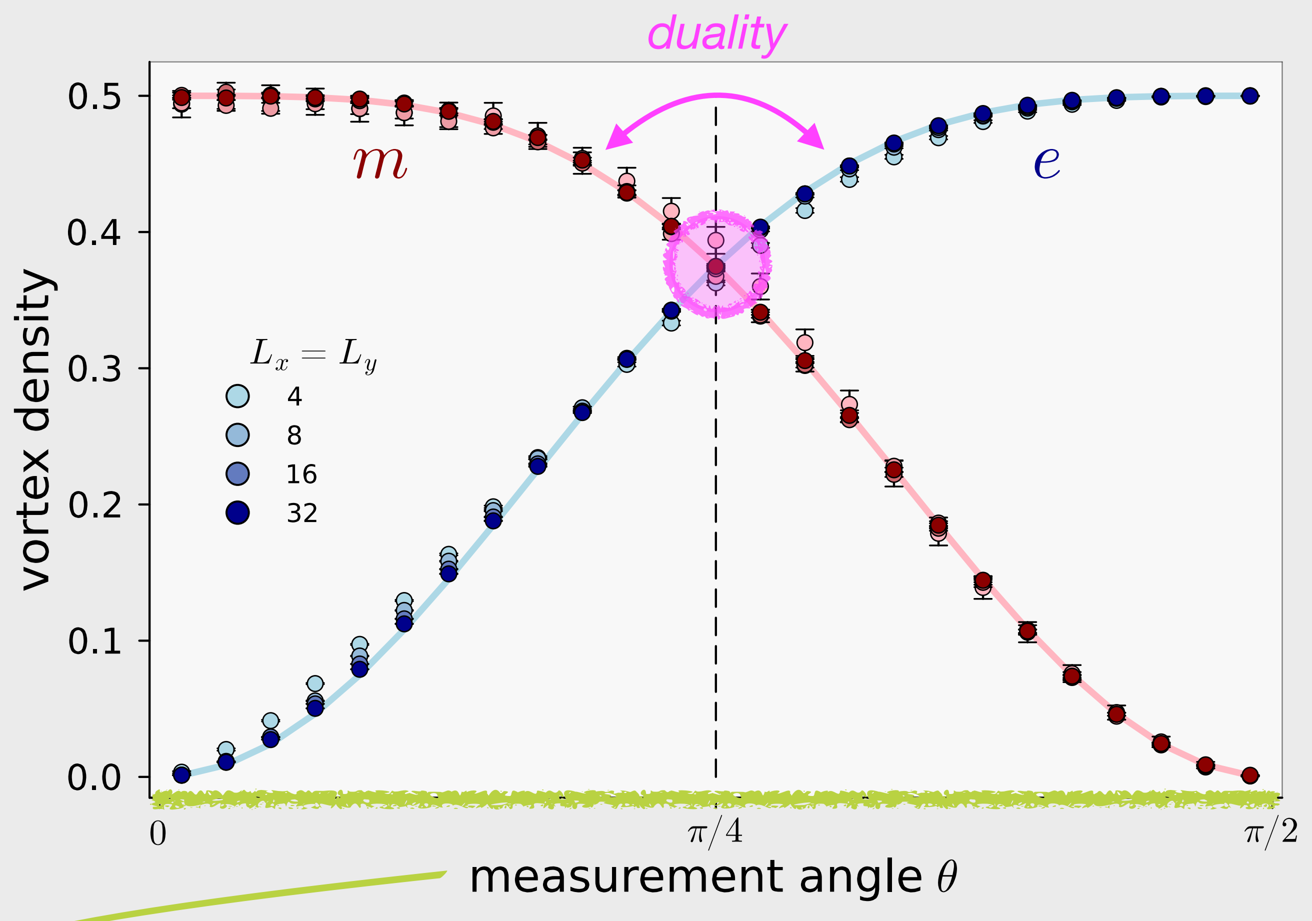
quantum

$$\rho = \sum_{em} |em\rangle\langle em|_C \otimes |\psi(em)\rangle\langle\psi(em)|_Q$$

bulk-boundary correspondence



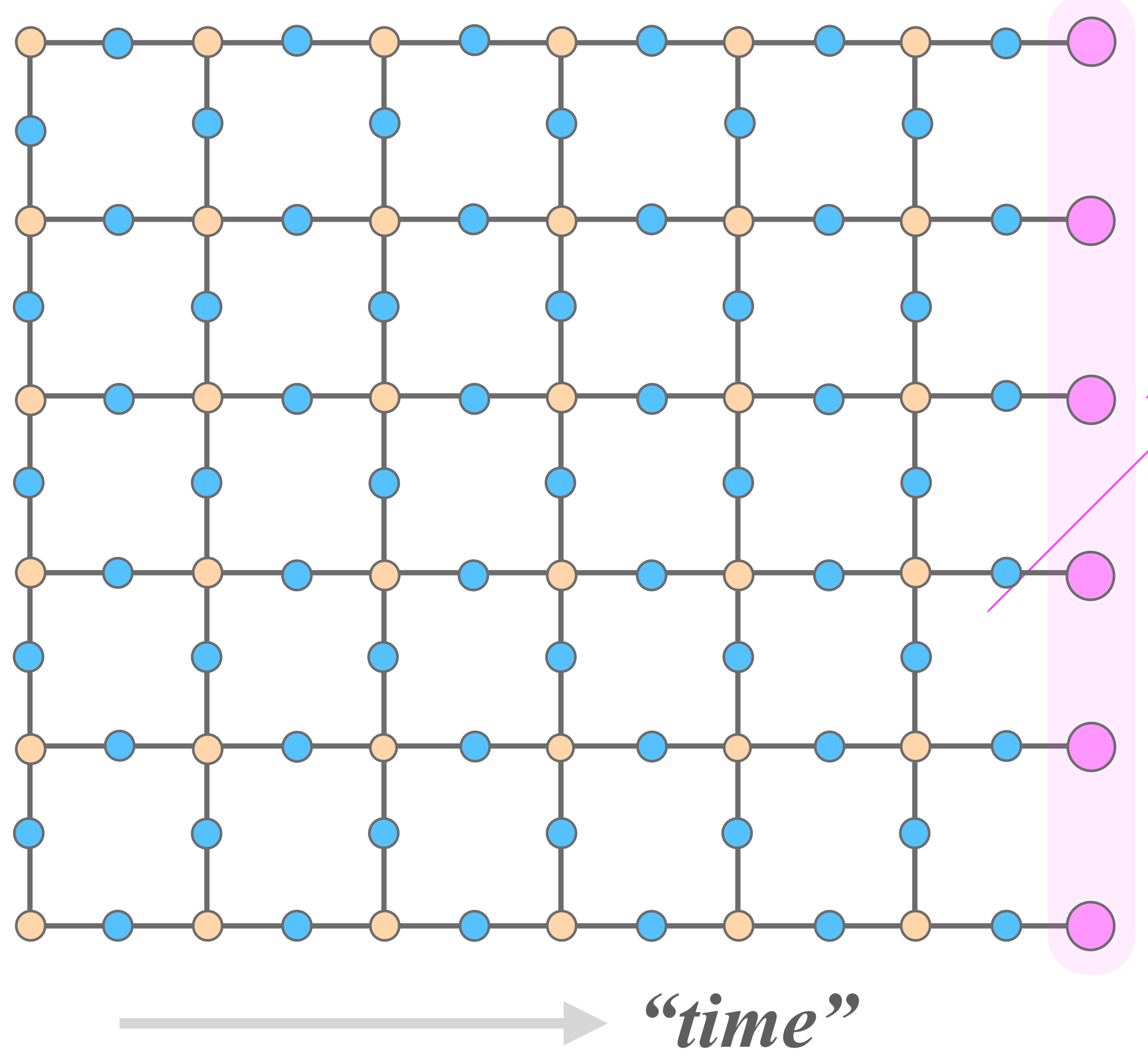
criticality is dictated by **self-duality**



tunes the **relative fluctuations of e & m** in toric code

toric code

boundary



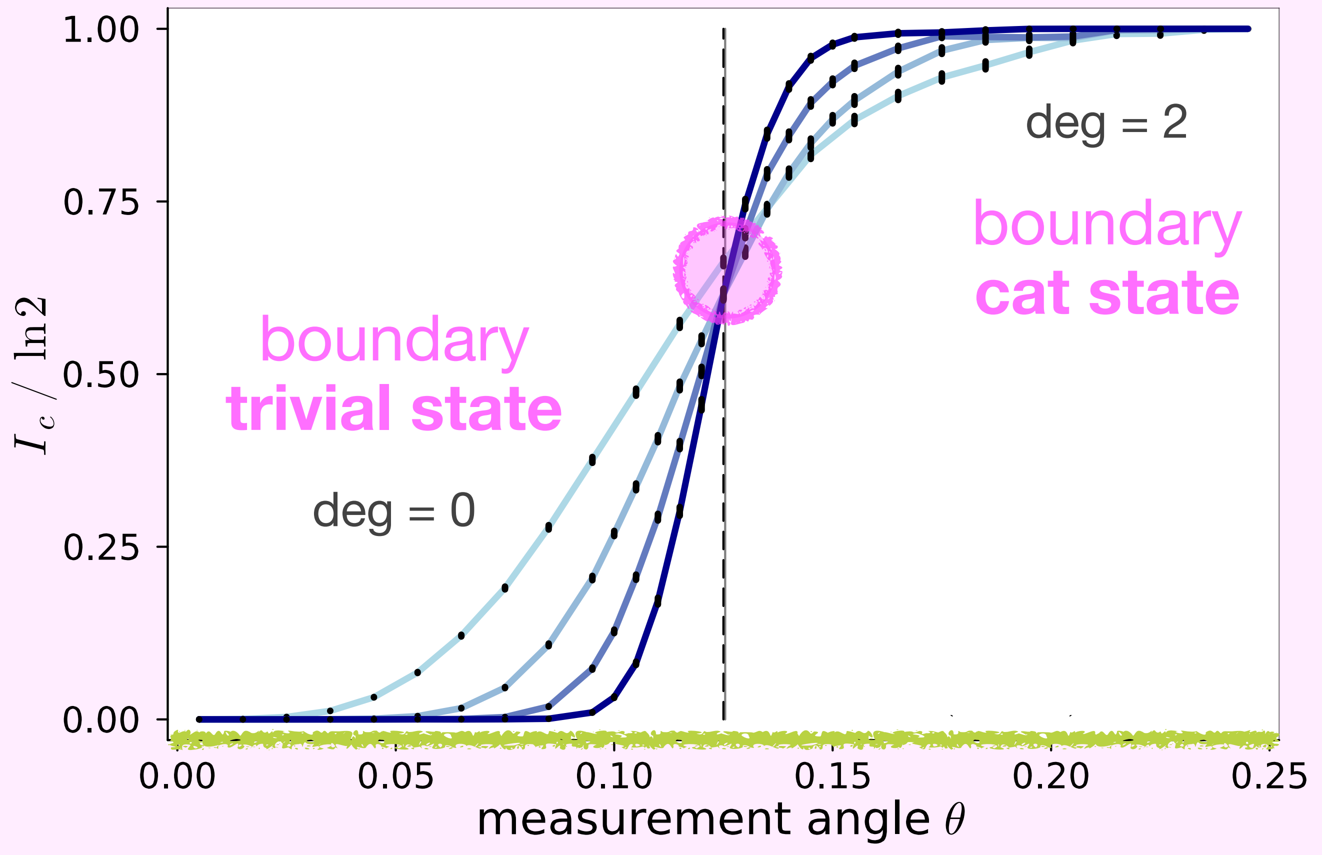
bulk

mixed-state
phase transition

measurement-induced
phase transition

boundary

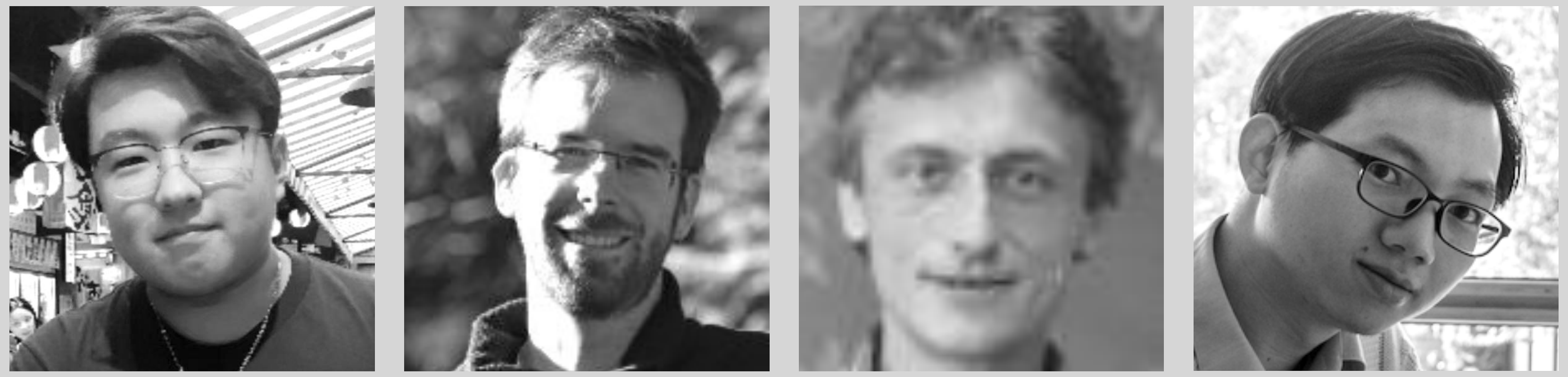
coherent information evaluates the size of the **degenerate space** of the boundary state



classical — quantum

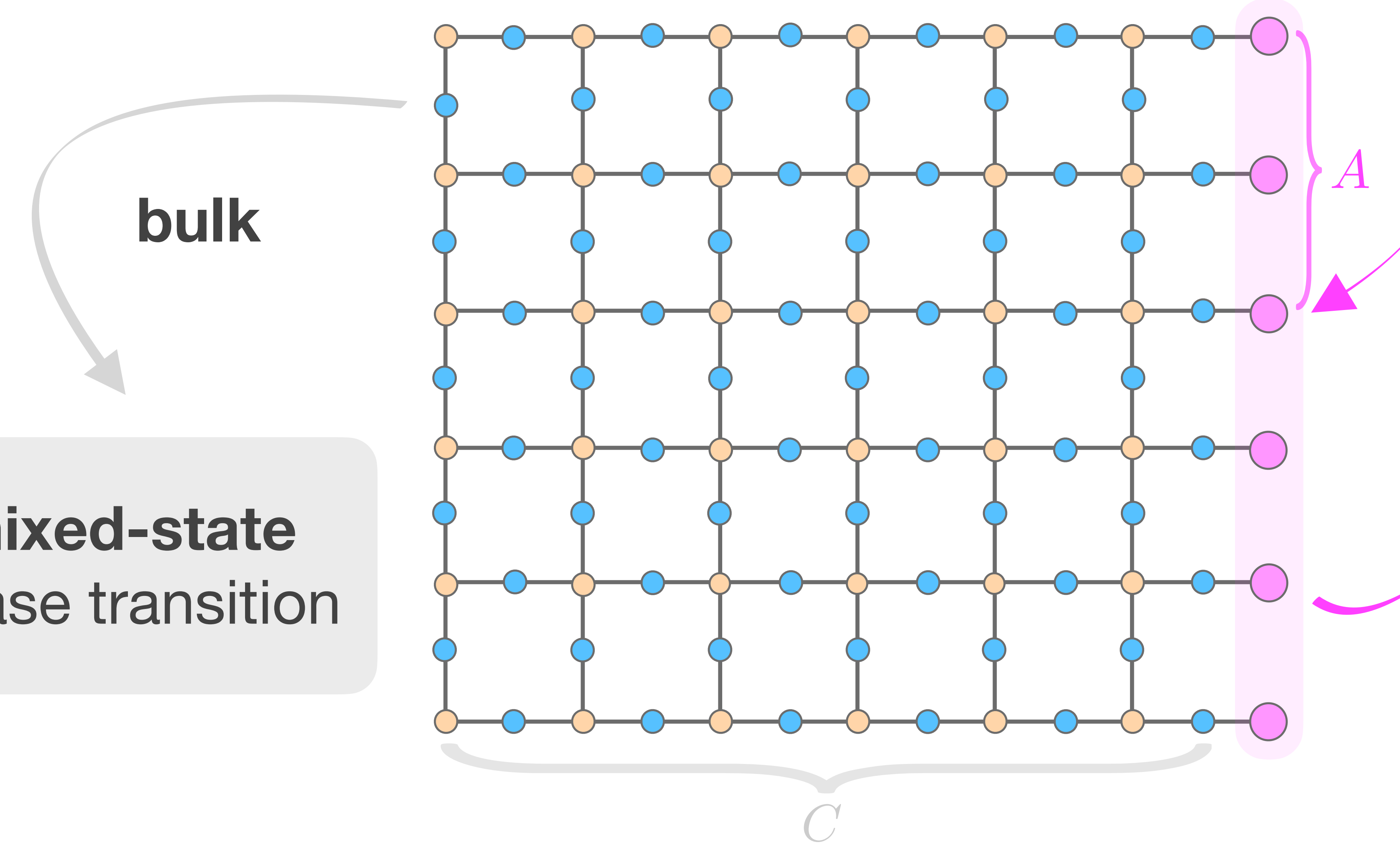
$$\rho = \sum_{em} |em\rangle\langle em|_C \otimes |\psi(em)\rangle\langle\psi(em)|_Q$$

bulk-boundary correspondence



toric code

boundary



mixed-state
phase transition

measurement-induced
phase transition

boundary

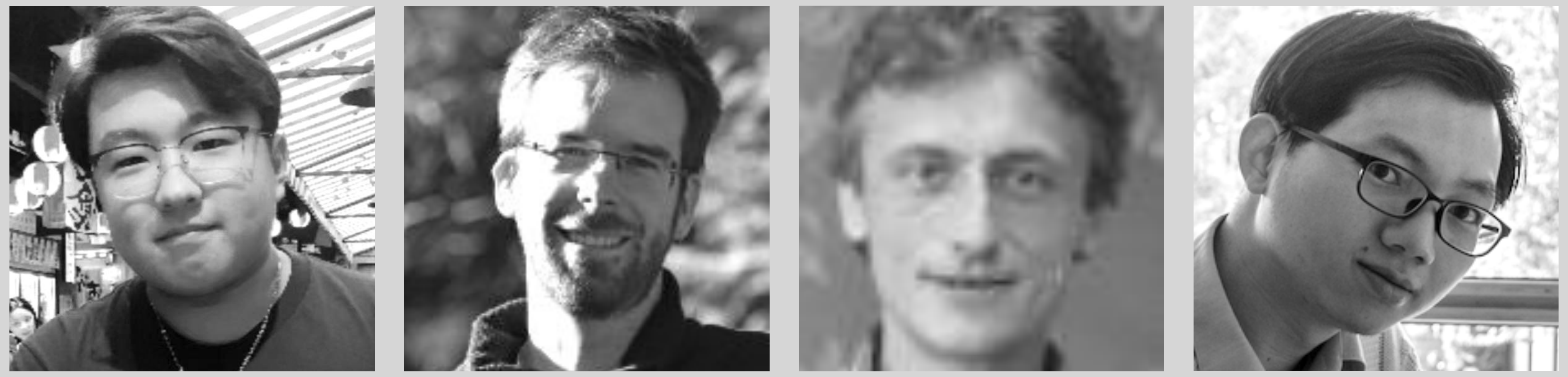
Shannon entropy
of mixed state

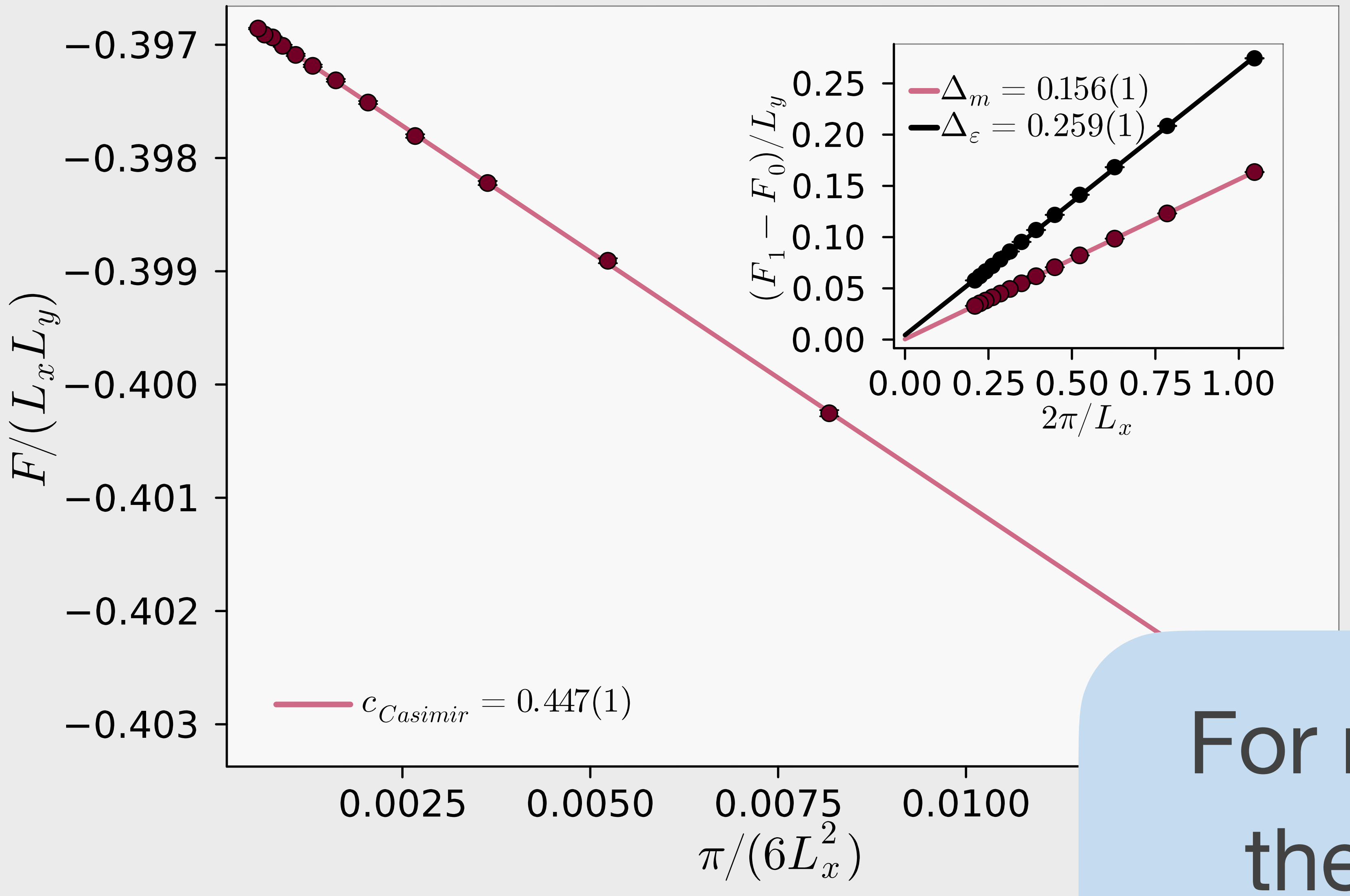
entanglement entropy
of boundary state

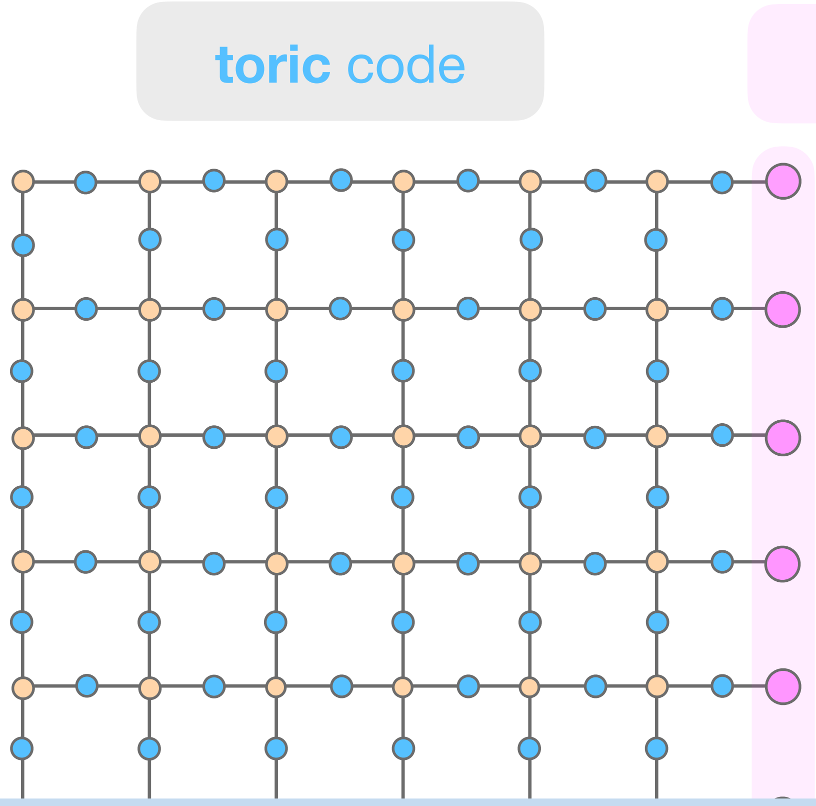
$$S_{AC} = S_C + S_{A|C}$$

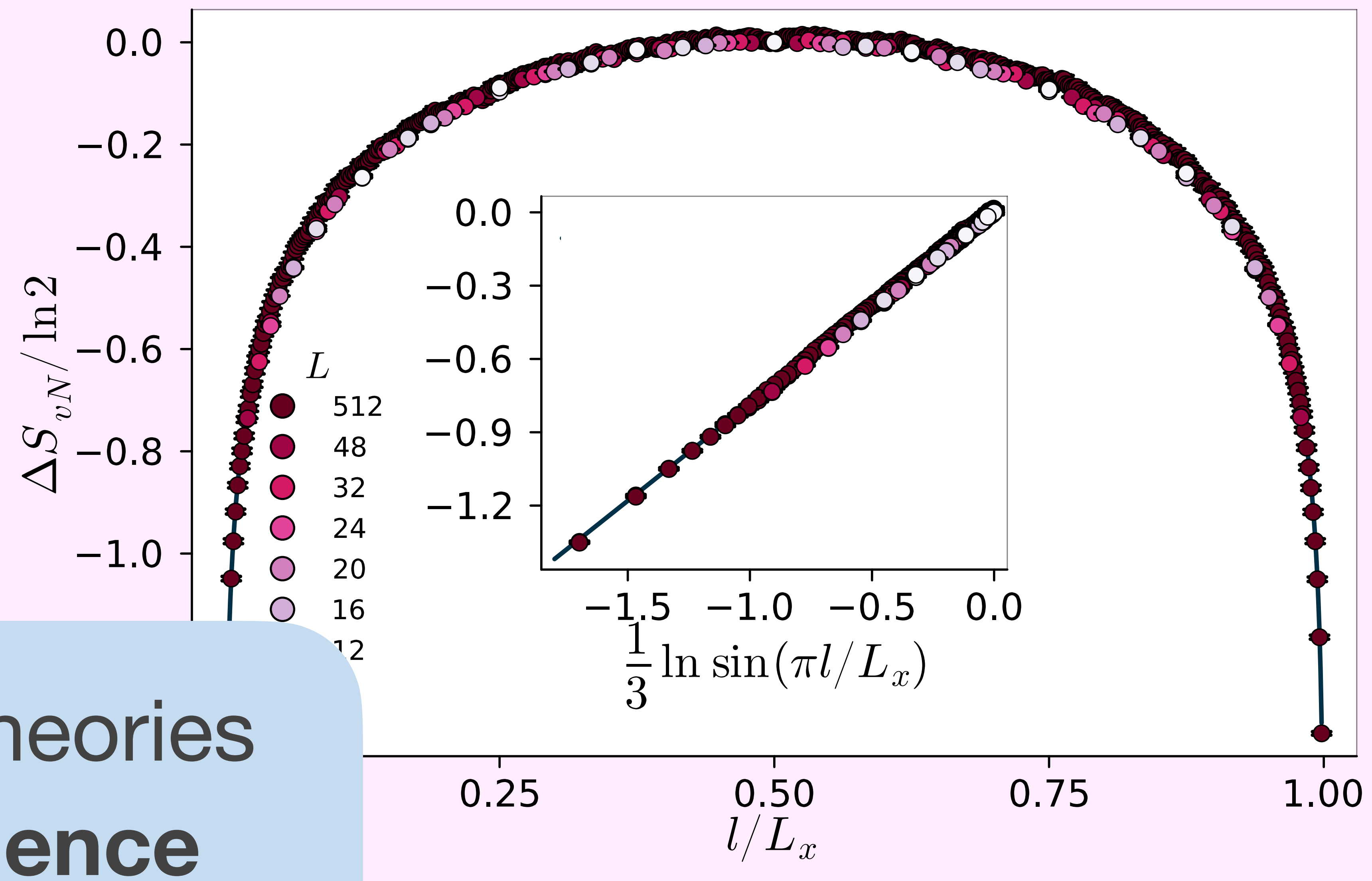
$$= -\frac{\pi c_{\text{Casimir}} L_y}{6 L_x} + \frac{c_{\text{ent}}}{3} \ln\left(\frac{L_x}{\pi} \sin \frac{\pi l}{L_x}\right) + \ldots$$

self-dual criticality









For **non-unitary** conformal field theories the **bulk-boundary correspondence** generally allows c_{Casimir} and c_{ent} to differ.

$c_{\text{Casimir}} = 0.447(1)$

$c_{\text{ent}}^{\text{vN}} = 0.795(1)$

Shannon entropy of mixed state

entanglement entropy of boundary state

$$S_{AC} = S_C + S_{A|C}$$

$$= -\frac{\pi c_{\text{Casimir}} L_y}{6L_x} + \frac{c_{\text{ent}}}{3}\ln\left(\frac{L_x}{\pi}\sin\frac{\pi l}{L_x}\right) + \dots$$

universality classes

Ising	**Nishimori**	**weak self-dual**
no disorder	Born-rule disorder (1-replica)	Born-rule disorder (1-replica)
Z_2 **SSB** spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking
Kramers-Wannier duality	**broken** duality	**weak** duality
$\mathrm{KW}\rho \propto \rho$		$\mathrm{KW}\,\rho\,\mathrm{KW} = \rho$

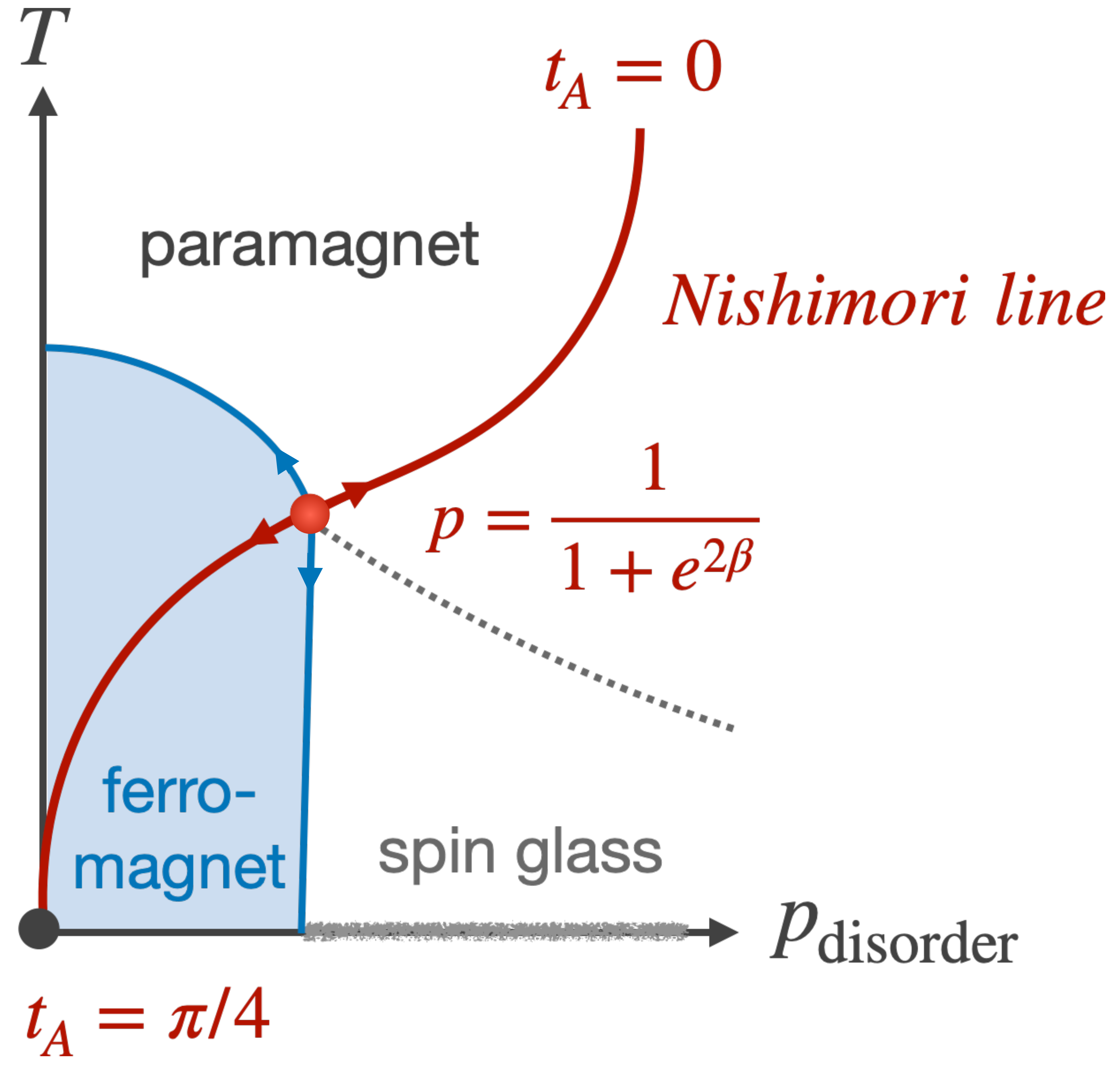
universality classes

Ising	Nishimori	weak self-dual	tricritical
no disorder	Born-rule disorder (1-replica)	Born-rule disorder (1-replica)	Born-rule disorder (1-replica)
Z_2 **SSB** spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	Z_2 **SW-SSB** strong-to-weak spontaneous symmetry breaking	**strong/weak/broken** Z_2 symmetry phases meet
Kramers-Wannier duality	**broken** duality	**weak** duality	learning transitions
$\mathrm{KW}\rho \propto \rho$		$\mathrm{KW}\,\rho\,\mathrm{KW} = \rho$	arXiv:2504.12385

perspectives

Nishimori physics

- a staple of classical **statistical physics**
- but **ubiquitous in quantum** physics
  - enforced by **Born's rule**
  - induced by **coherent** and **incoherent** errors
  - **RG flow** form percolation, self-dual, tricriticality
  - an emerging **fixed point universality class** for **non-unitary** conformal QCPs?



theory – Phys. Rev. Lett. **131**, 200201 (2023)
experiment (IBM) – Nature Physics **21**, 161 (2025)

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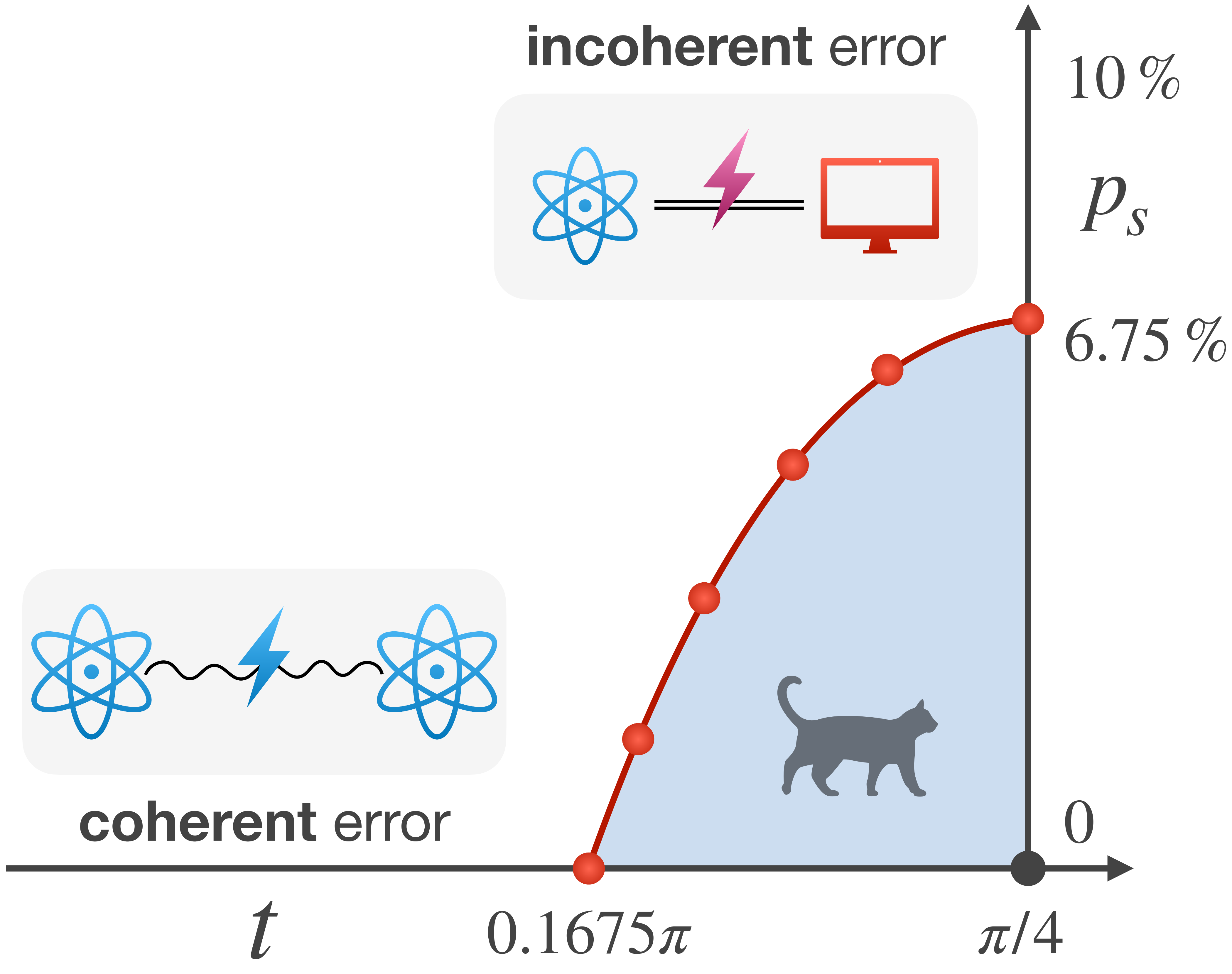

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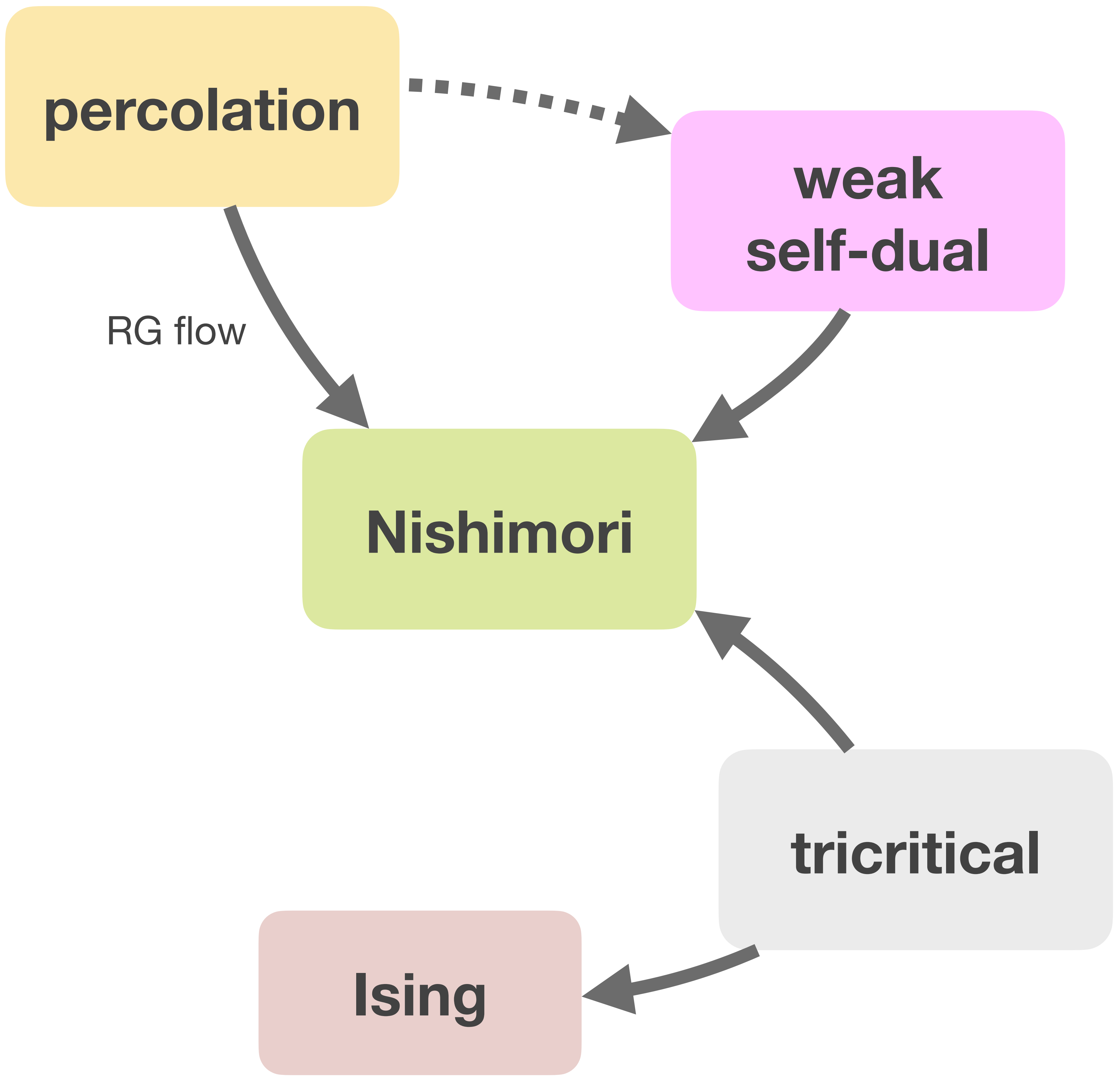

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percolation – arXiv:2505.22720
self-dual – arXiv:2502.14034, PRX Quantum 5, 040313 (2024)
tricritical – arXiv:2504.12385

Thanks!