Monitored Kitaev Models

Quantum circuits, entanglement dynamics, and synthetic fractionalization

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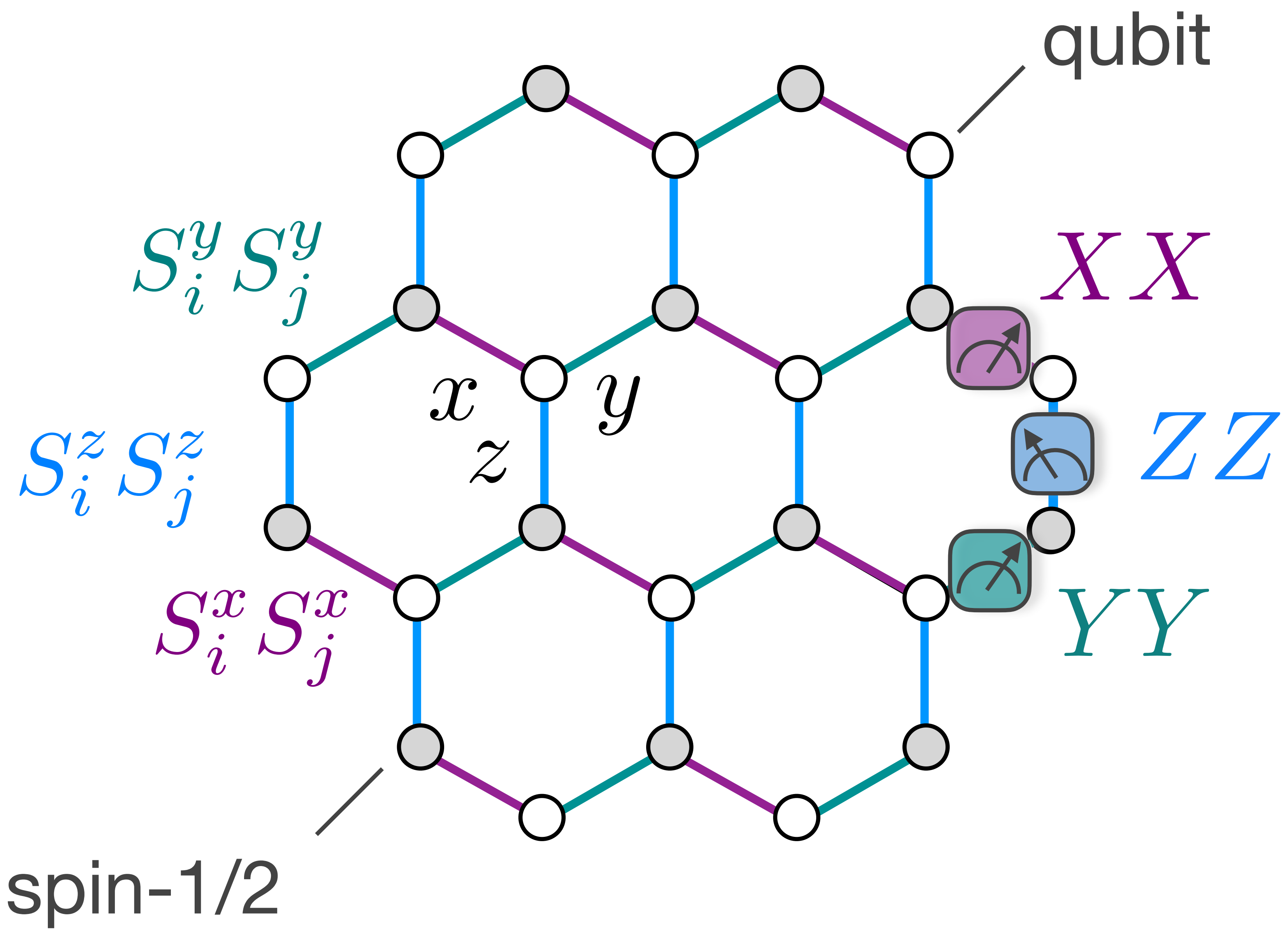
University of Cologne

Correlated Quantum Materials + *beyond*

ISSP Tokyo, November 2024

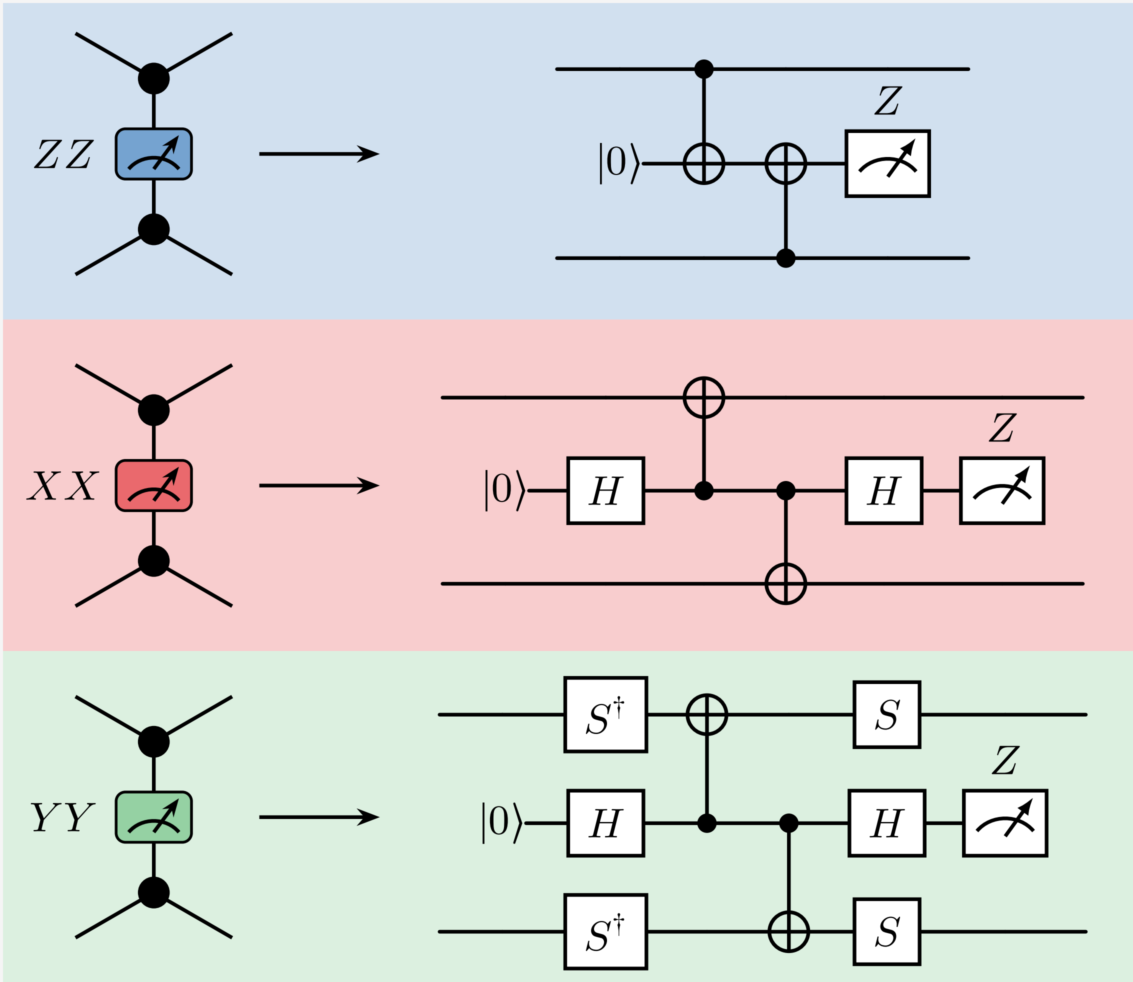
Kitaev physics

fractionalization
long-range etangelement

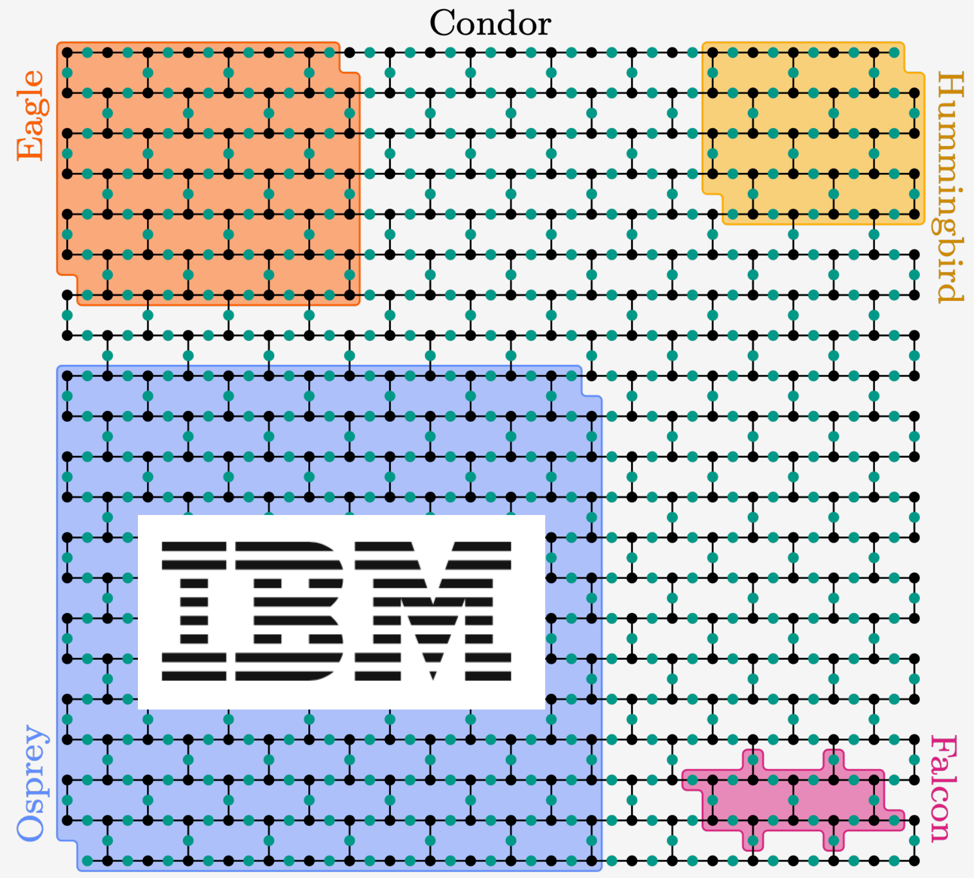


quantum
spin liquids

monitored
circuits



NISQ
devices



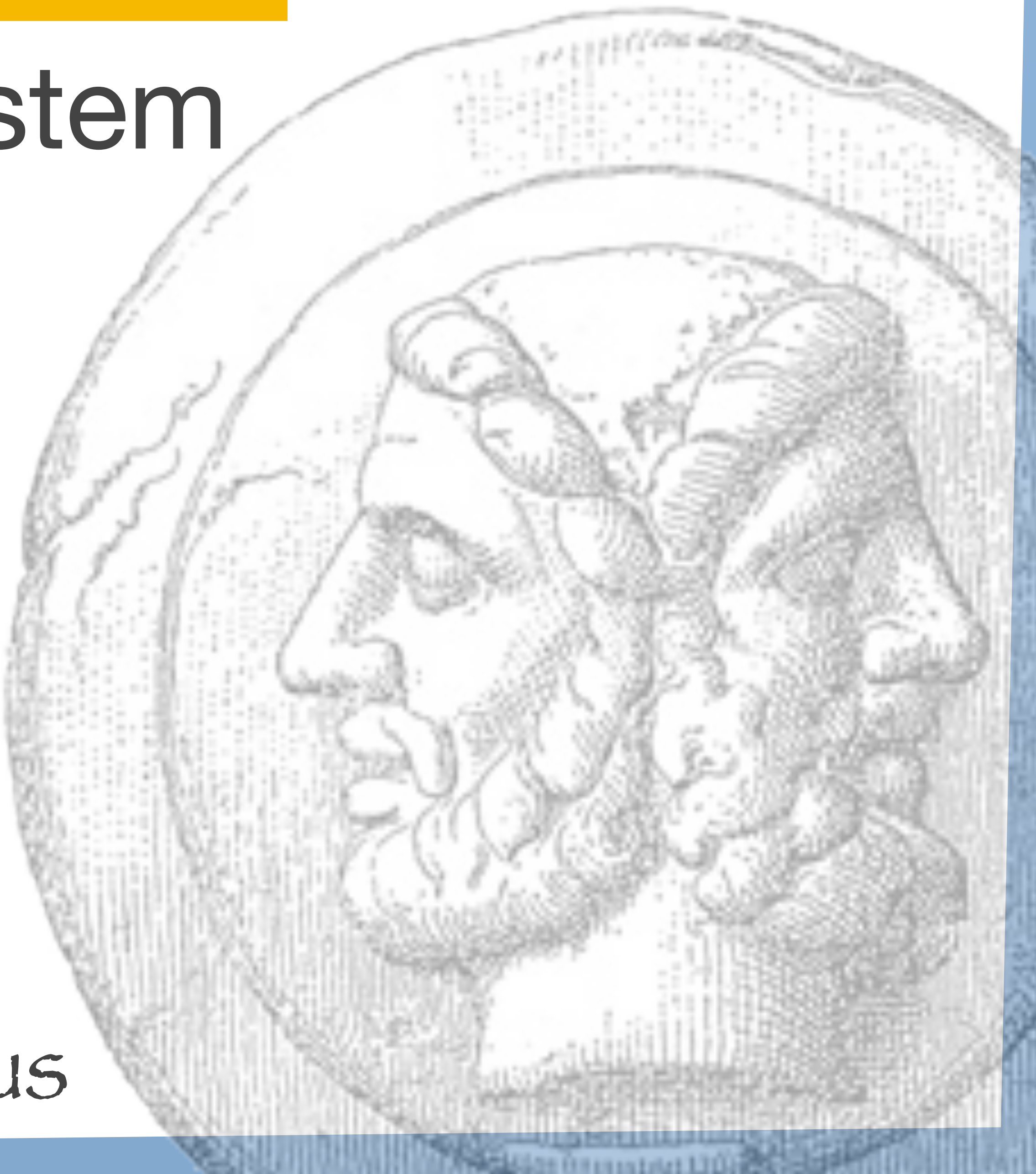
quantum measurements

quantum measurements

"About your cat, Mr. Schrödinger — I have good news and bad news."

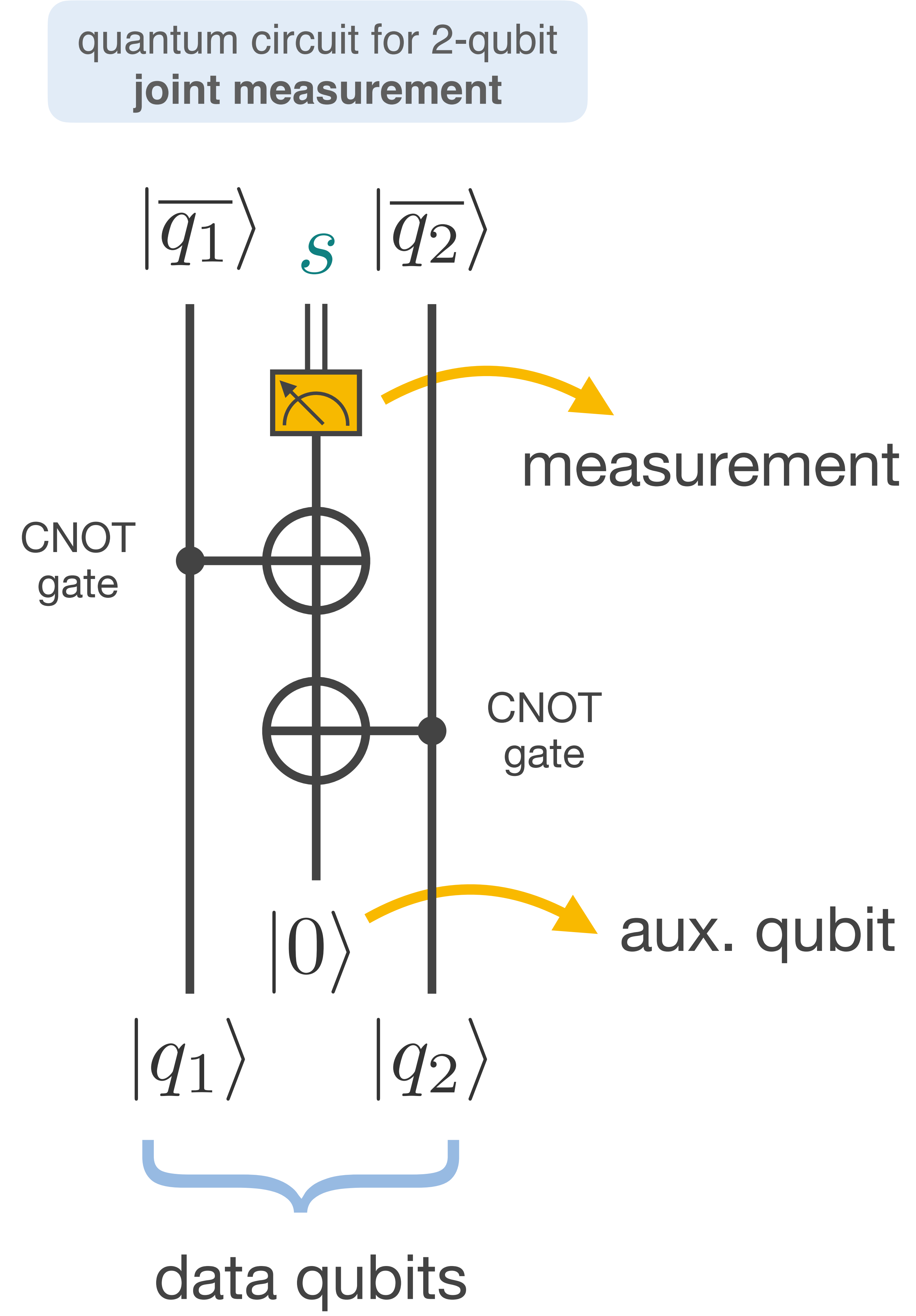
Quantum measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system



double-faced Janus

joint measurements



extract information

$\lvert q_1\rangle\lvert q_2\rangle$	$\lvert\overline{q_1}\rangle\lvert\overline{q_2}\rangle$	s
$\lvert 0\,0\rangle$	$\lvert 0\,0\rangle$	0
$\lvert 0\,1\rangle$	$\lvert 0\,1\rangle$	1
$\lvert 1\,0\rangle$	$\lvert 1\,0\rangle$	1
$\lvert 1\,1\rangle$	$\lvert 1\,1\rangle$	0

syndrome measurement

ZZ **parity check**

shape entanglement

product state

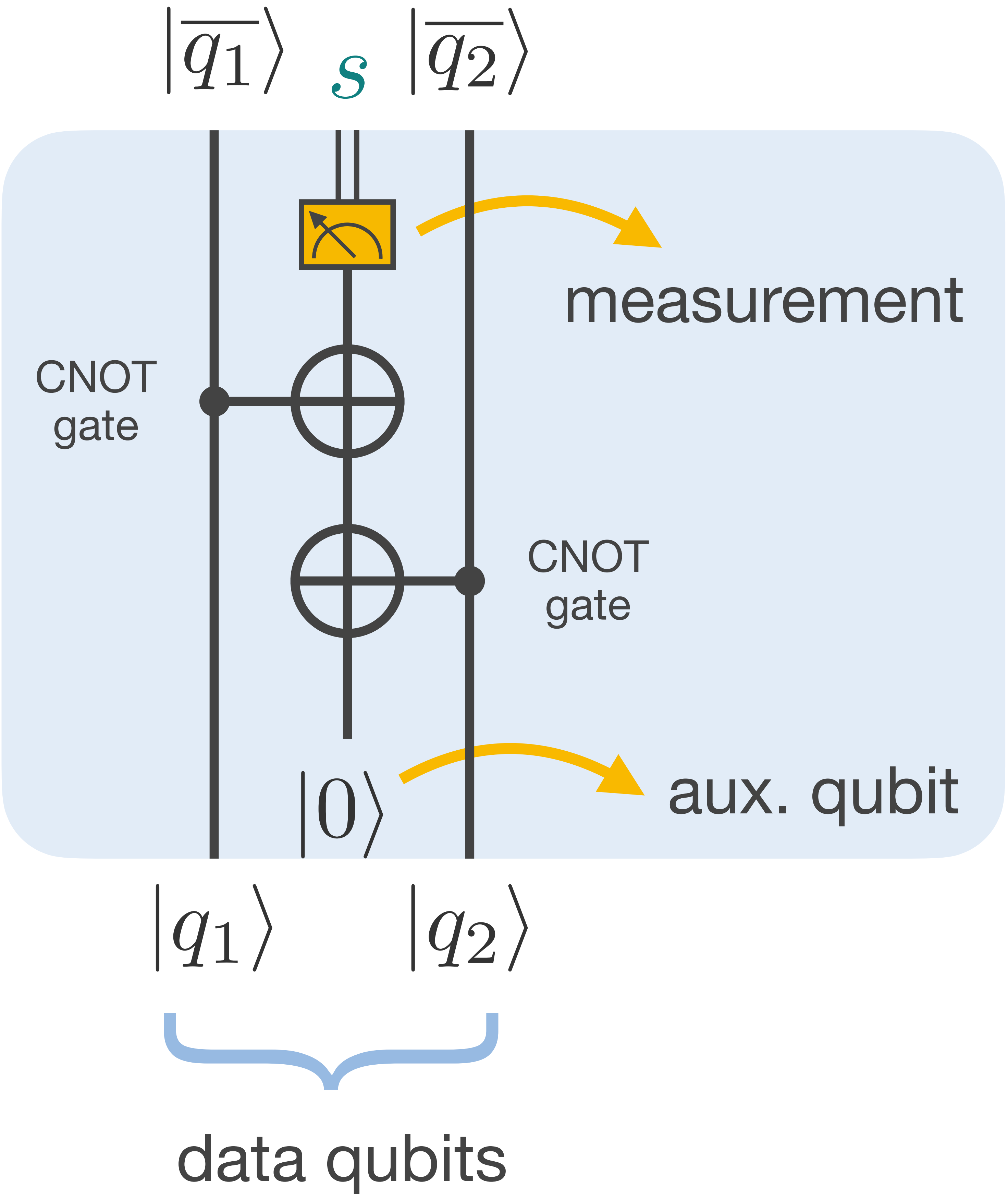
$\lvert q_1\rangle\lvert q_2\rangle$		$\lvert\overline{q_1}\rangle\lvert\overline{q_2}\rangle$	s
$\lvert{+}{+}\rangle$	p = 1/2	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2}$	0
	p = 1/2	$(\lvert 01\rangle + \lvert 10\rangle)/\sqrt{2}$	1

Bell state

$$|{+}{+}\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

joint measurements

Ippoliti, Gullans, Gopalakrishnan, Huse & Khemani, PRX (2021)



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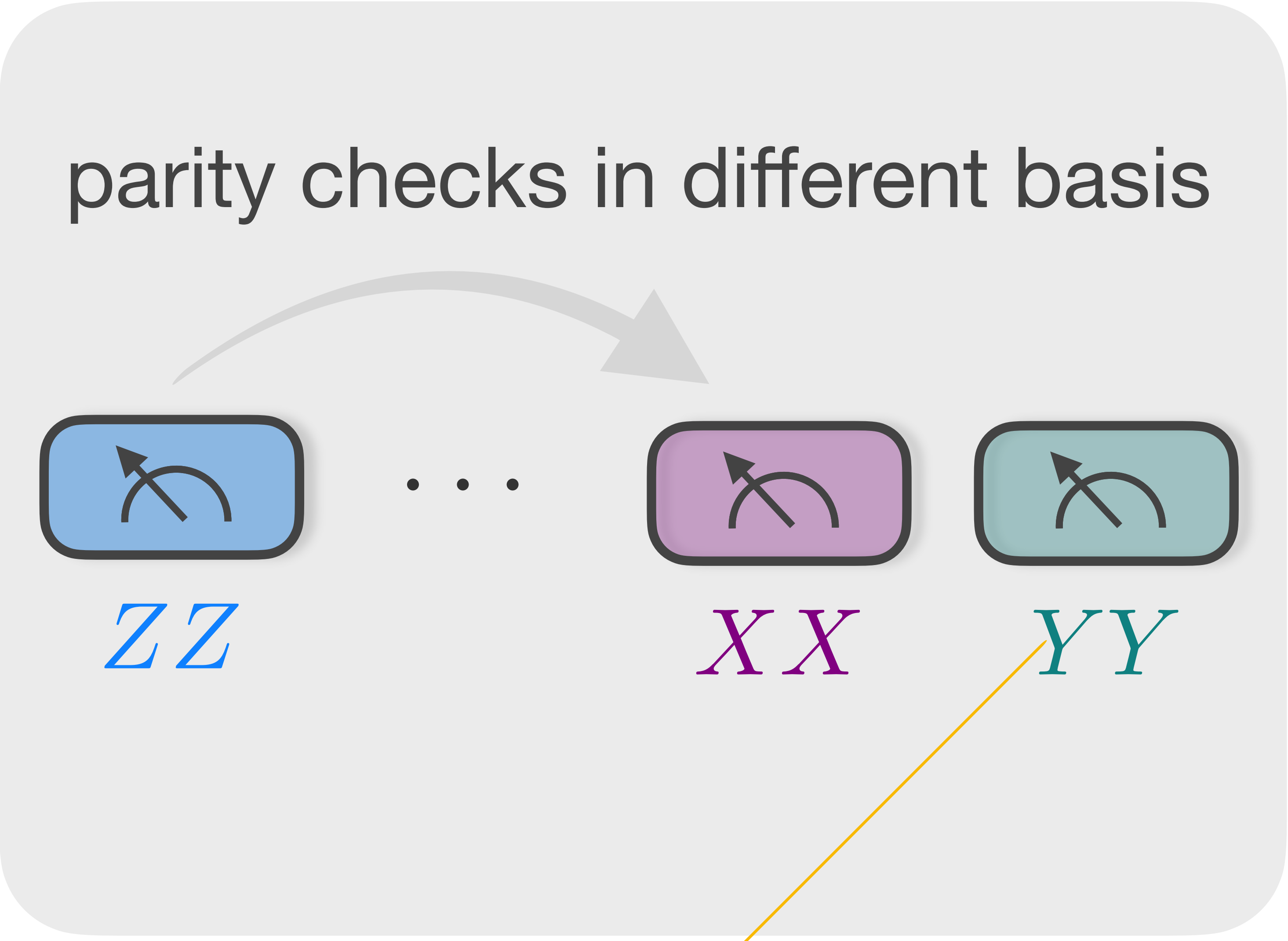
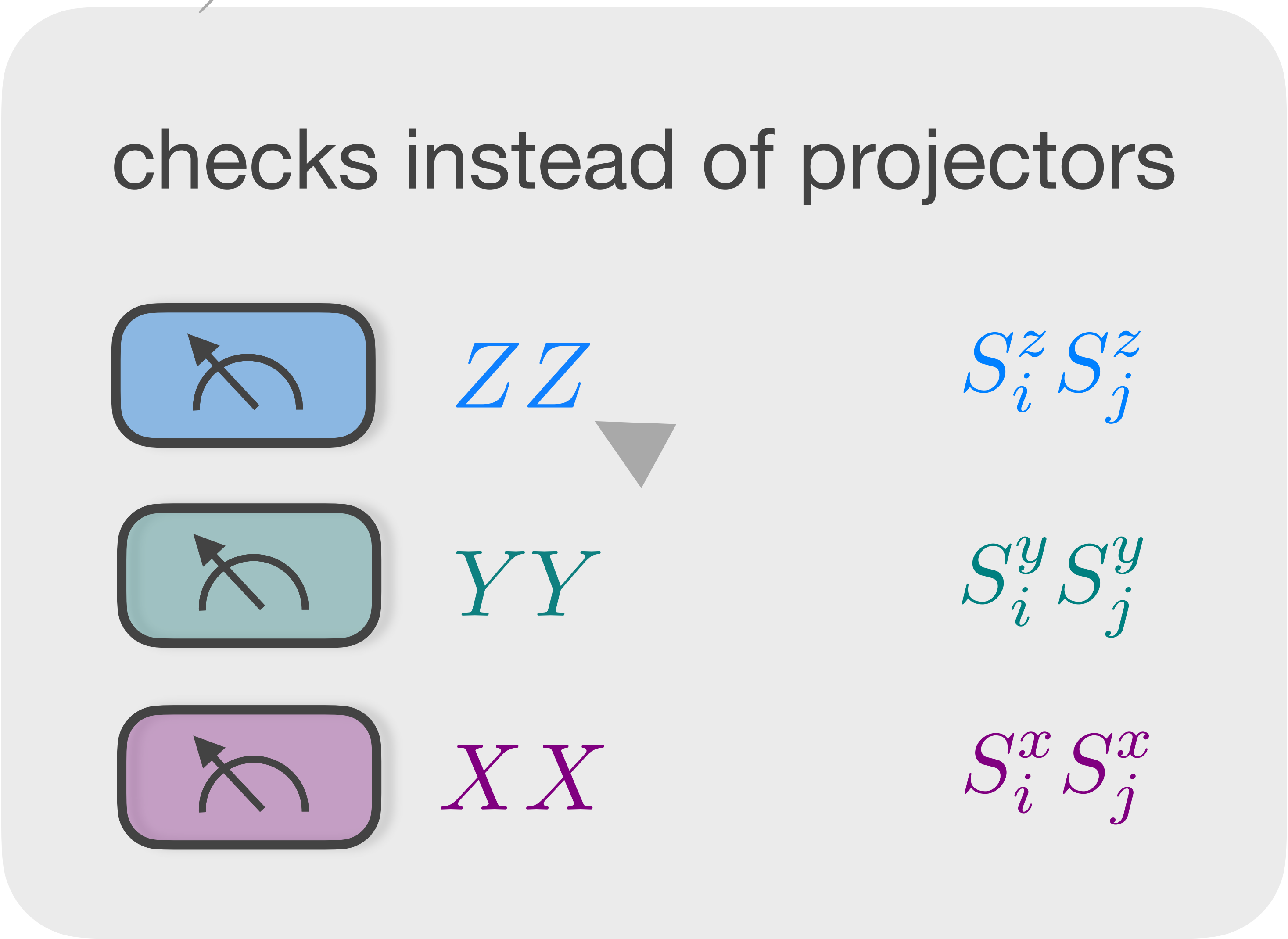




measurement dynamics

Hamiltonian dynamics

new toolbox for quantum many-body physics

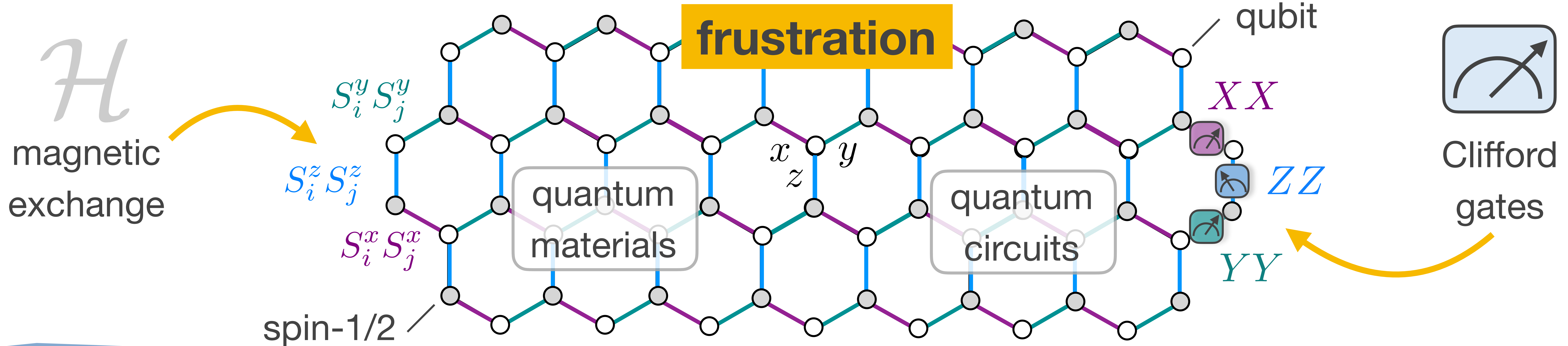
Hamiltonian vs. monitored dynamics

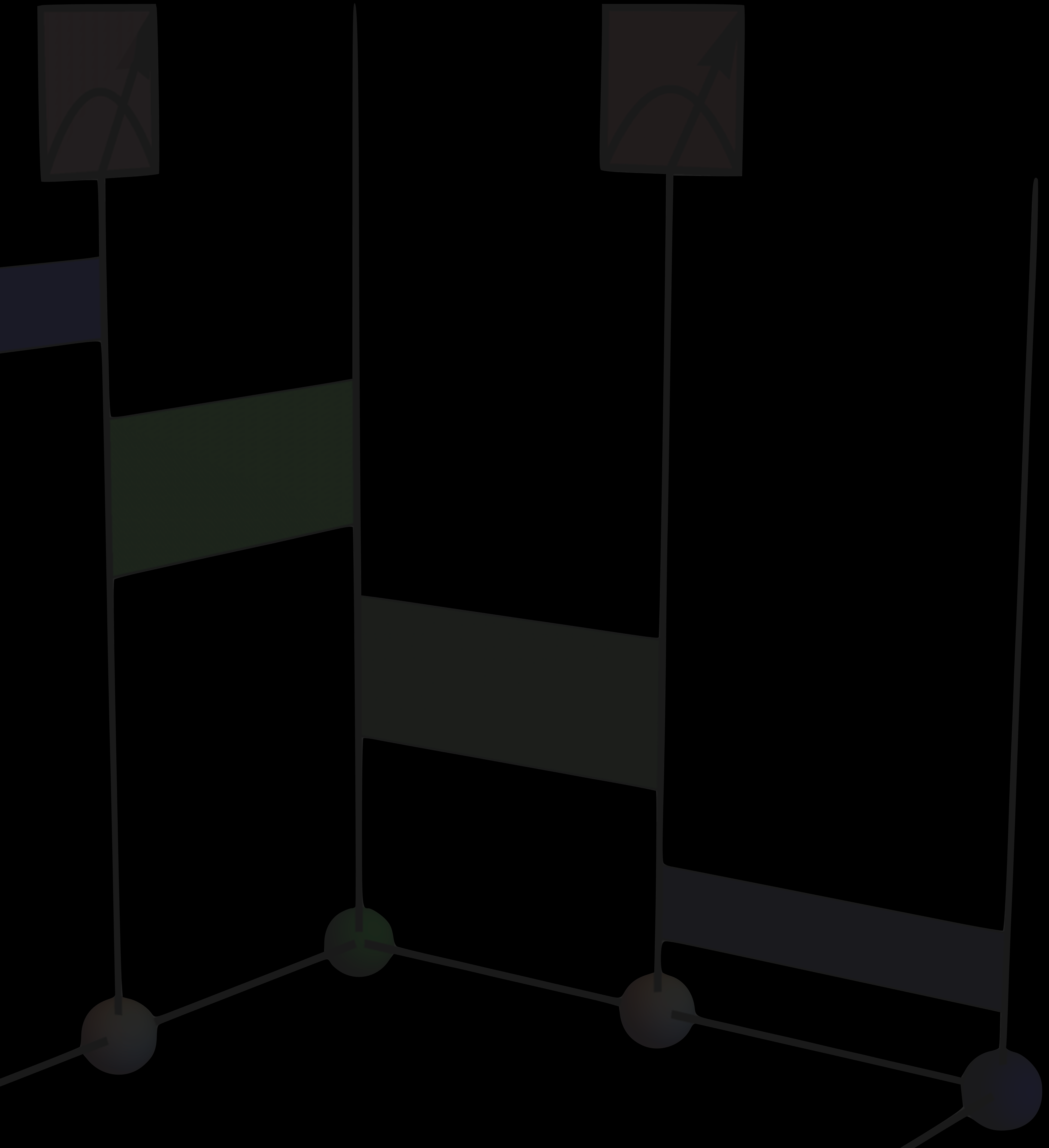
Hamiltonian dynamics

- **equilibrium** dynamics of **isolated** systems
- **unitary** evolution
- energy **conserved**
- quantum **ground states**
- **area-law entanglement** structures
- macroscopic entanglement (spin liquids)

measurement dynamics

- **out-of-equilibrium** dynamics of **open** systems
- **non-unitary** evolution
- energy **not** conserved
- long-time **steady states**
- **plethora of entanglement** structures
- macroscopic entanglement (spin liquids)



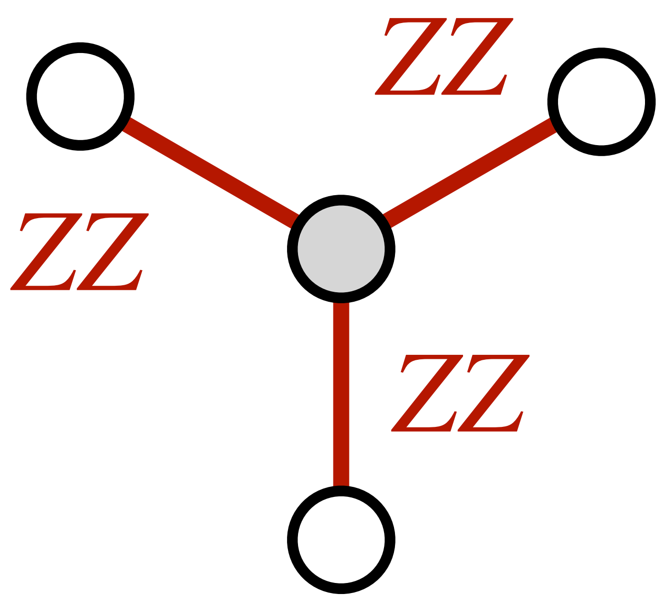


Nishimori physics

warm-up

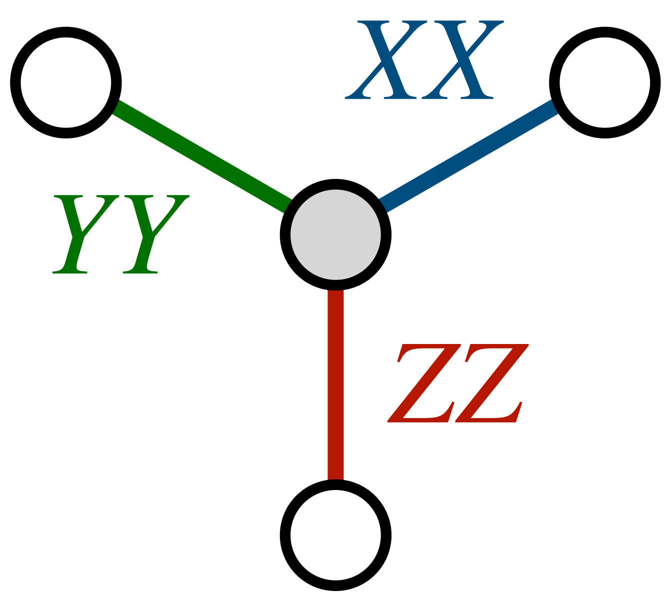
commuting vs non-commuting measurements

Guo-Yi Zhu



Nishimori's cat

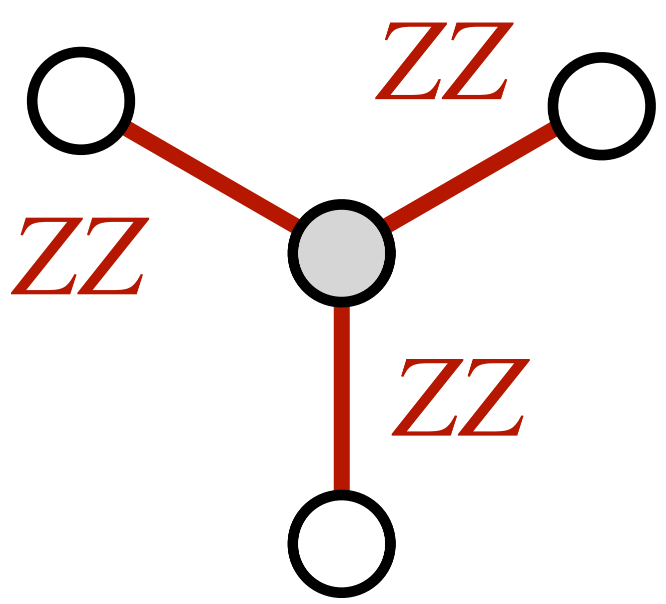
- **commuting**
- parallelized
- no dynamics



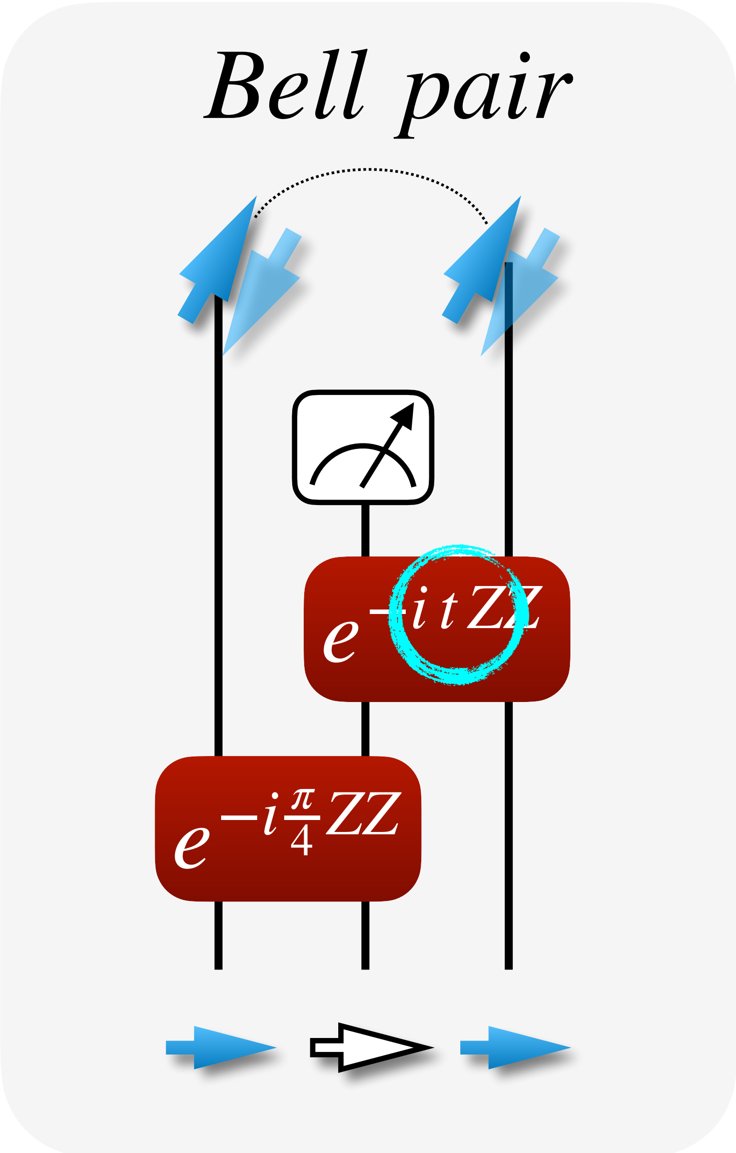
Kitaev spin liquid

- **non-commuting**
- sequential
- dynamics

Nishimori's cat



Nishimori's cat



$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2}\sum_{ij} J_{s_{ij}} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

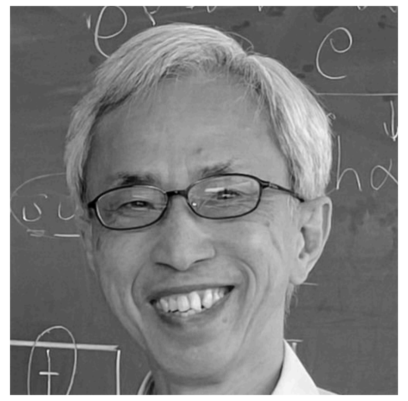
Born's rule

interpret as
classical
stat mech model

$$\tanh \frac{\beta J_\pm}{2} = \pm \tan t$$

thermal **fluctuations** and
disorder are **locked**

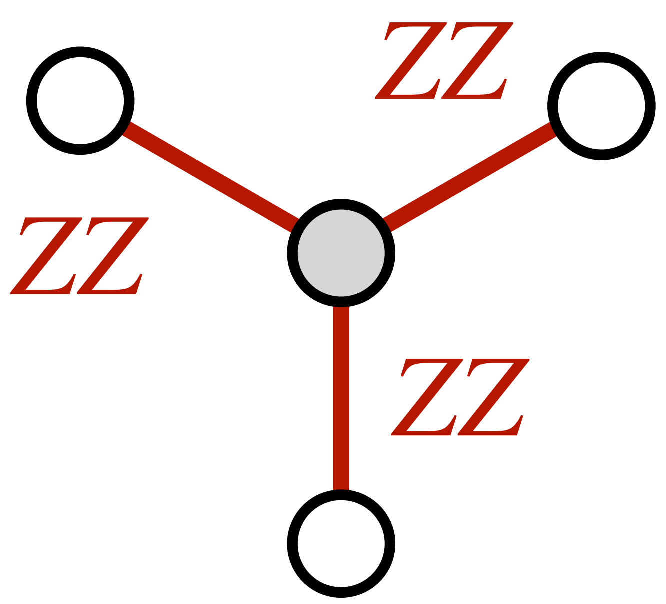
$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} J_{s_{ij}} \sigma_i \sigma_j}$$



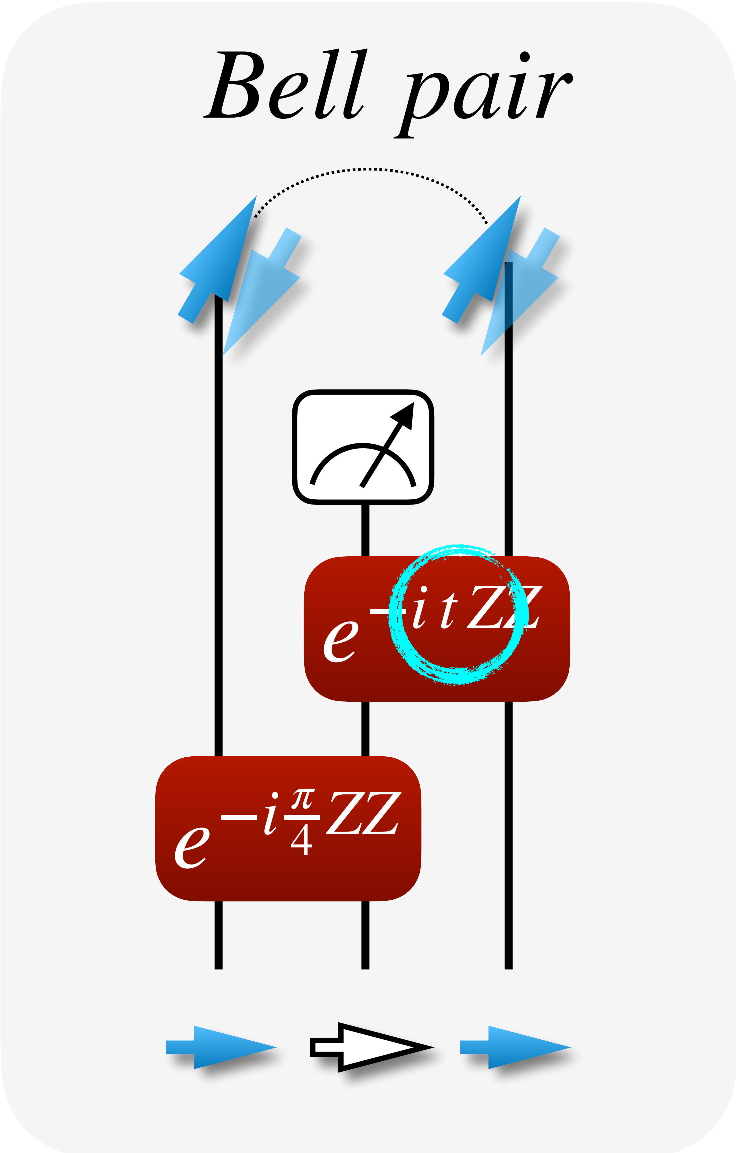
Nishimori (1981)

random bond Ising model

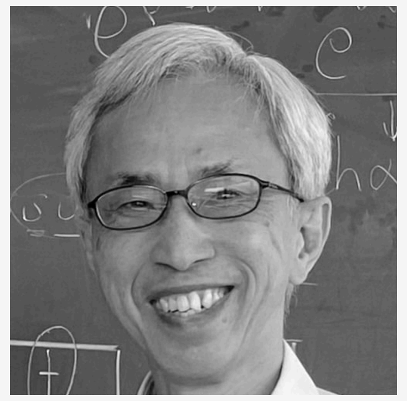
Nishimori's cat



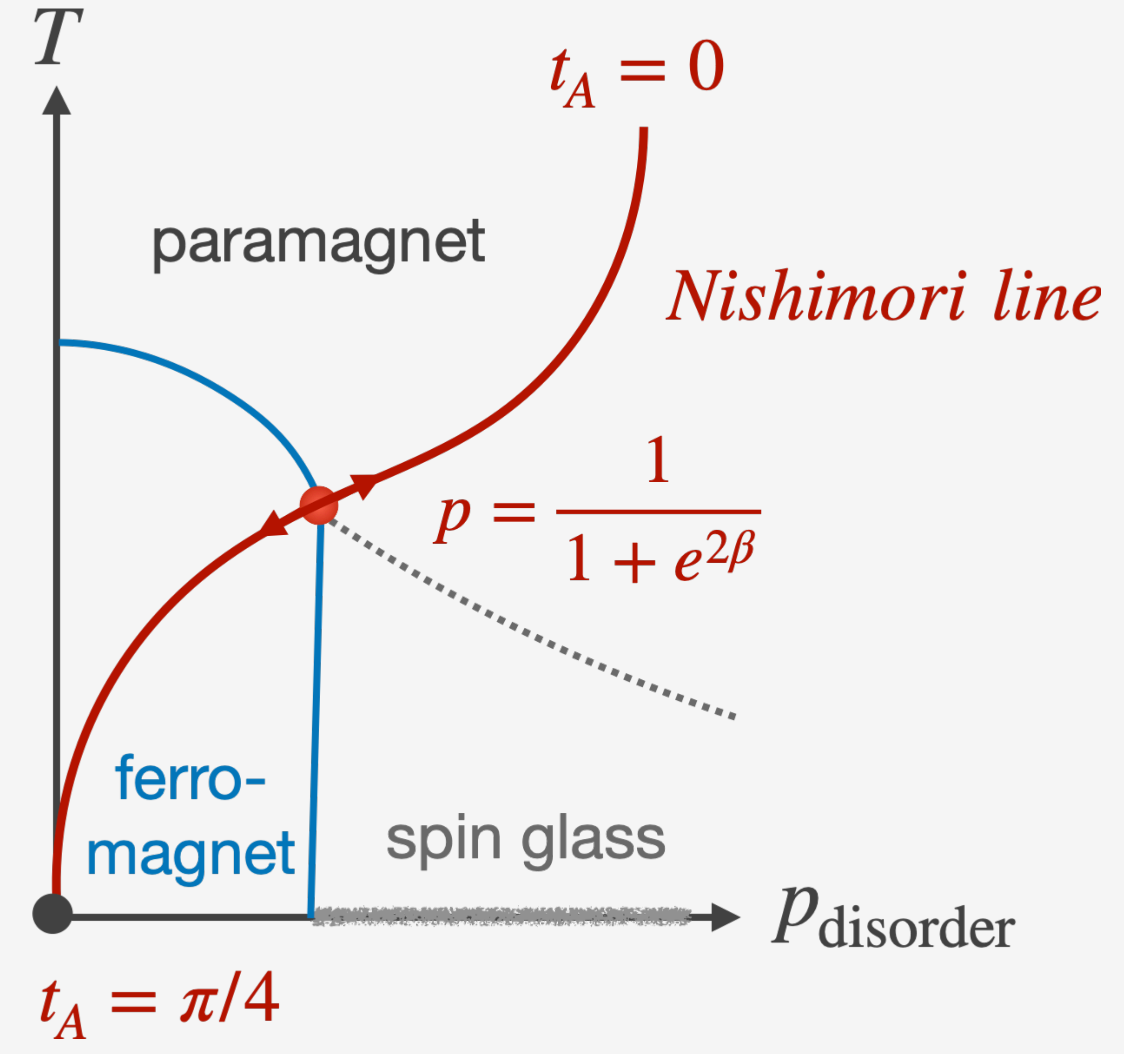
Nishimori's cat



thermal fluctuations and disorder are **locked**



Nishimori (1981)

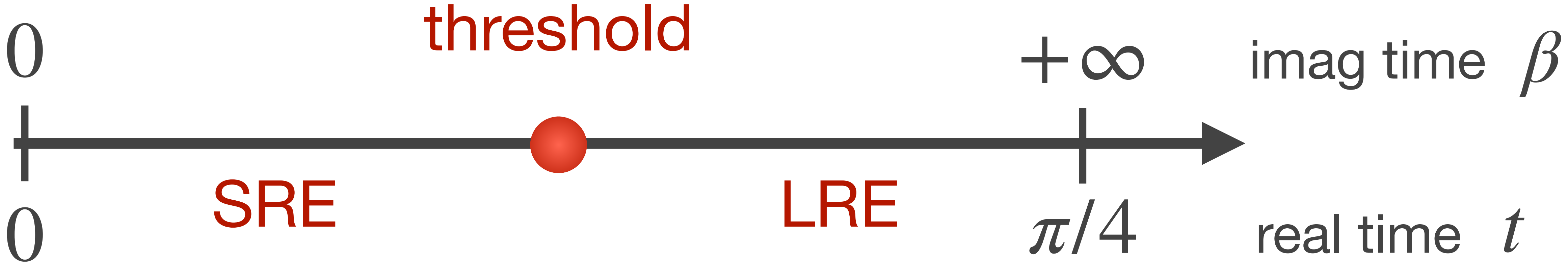


theory – Phys. Rev. Lett. 131, 200201 (2023)
experiment (IBM) – Nature Physics (2024)

"high temperature"

finite threshold

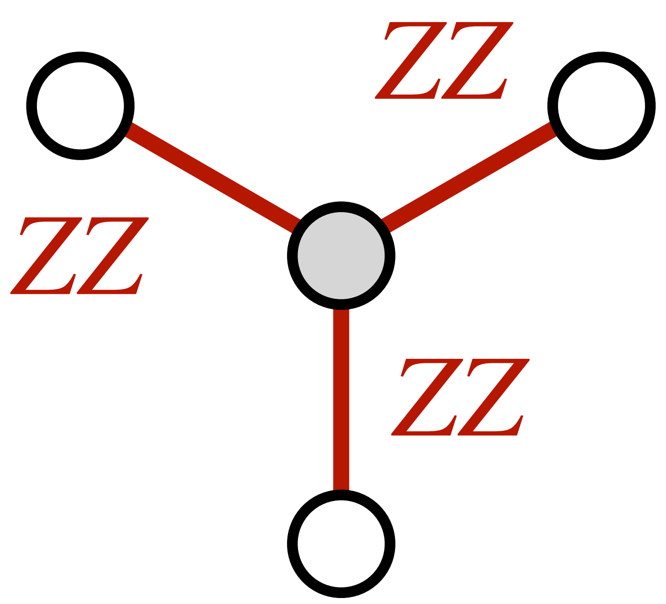
"low temperature"



weak measurement

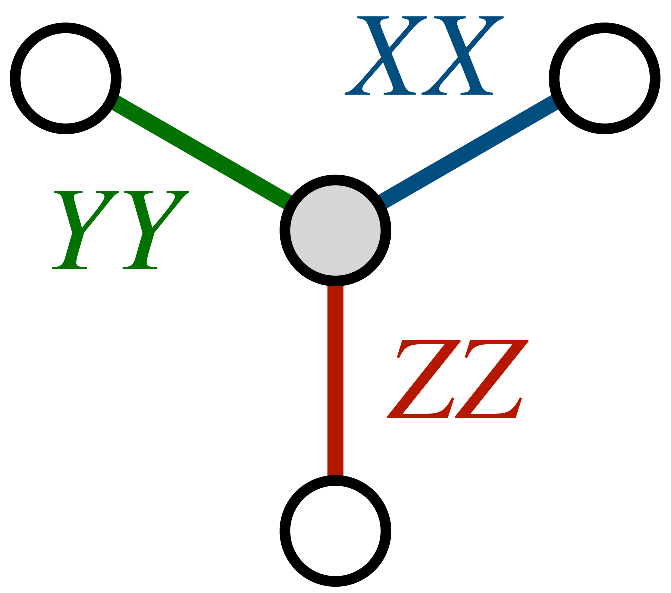
strong measurement

commuting vs non-commuting measurements



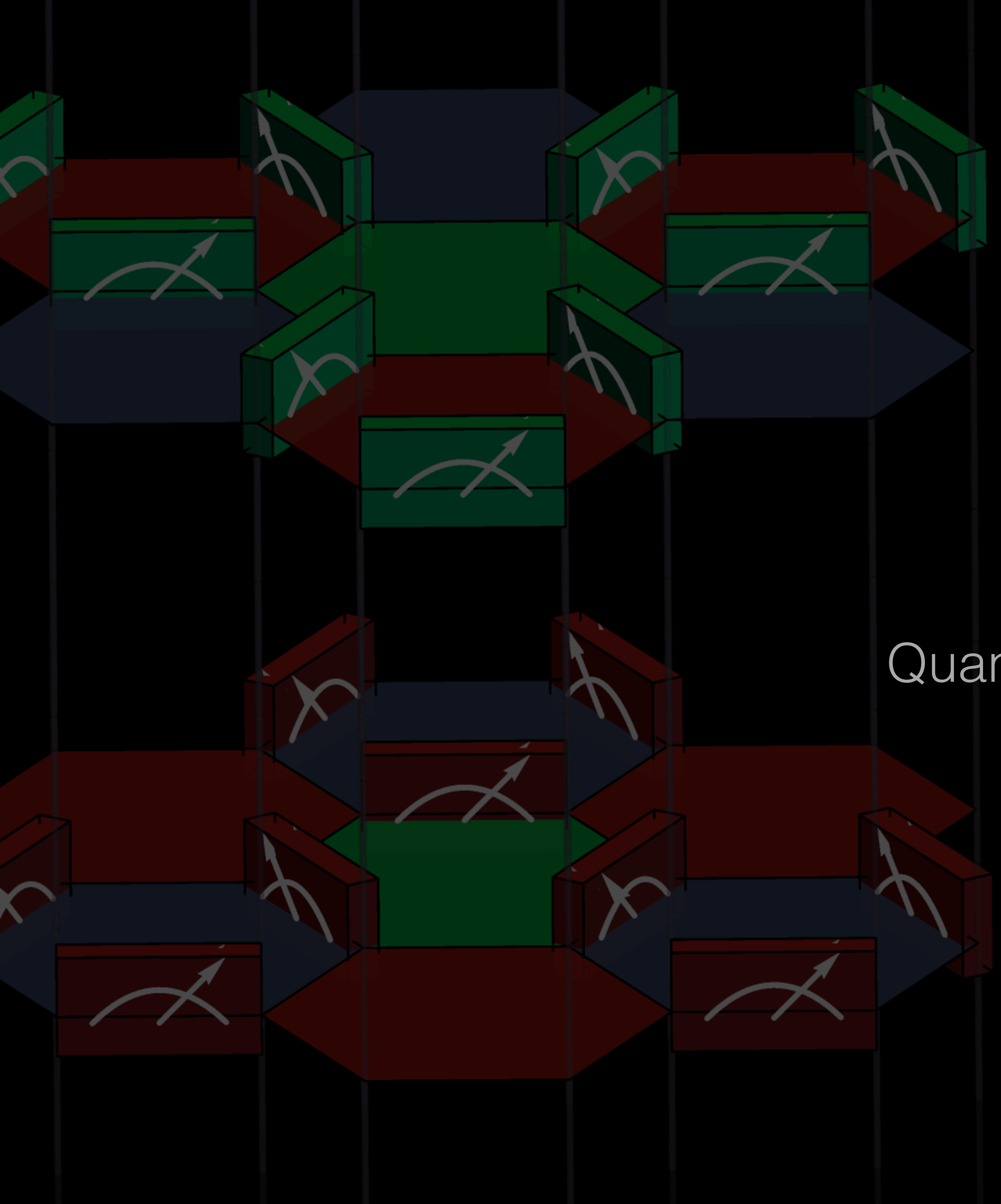
Nishimori's cat

- **commuting**
- parallelized
- no dynamics



Kitaev spin liquid

- **non-commuting**
- sequential
- dynamics



Kitaev circuits

Quantum Magnetism meets Quantum Computing

frustration and entanglement

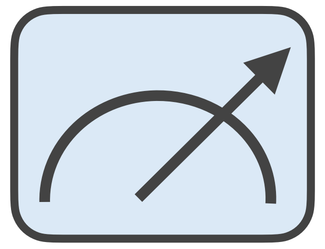
$\mathcal{H}$

magnetic exchange

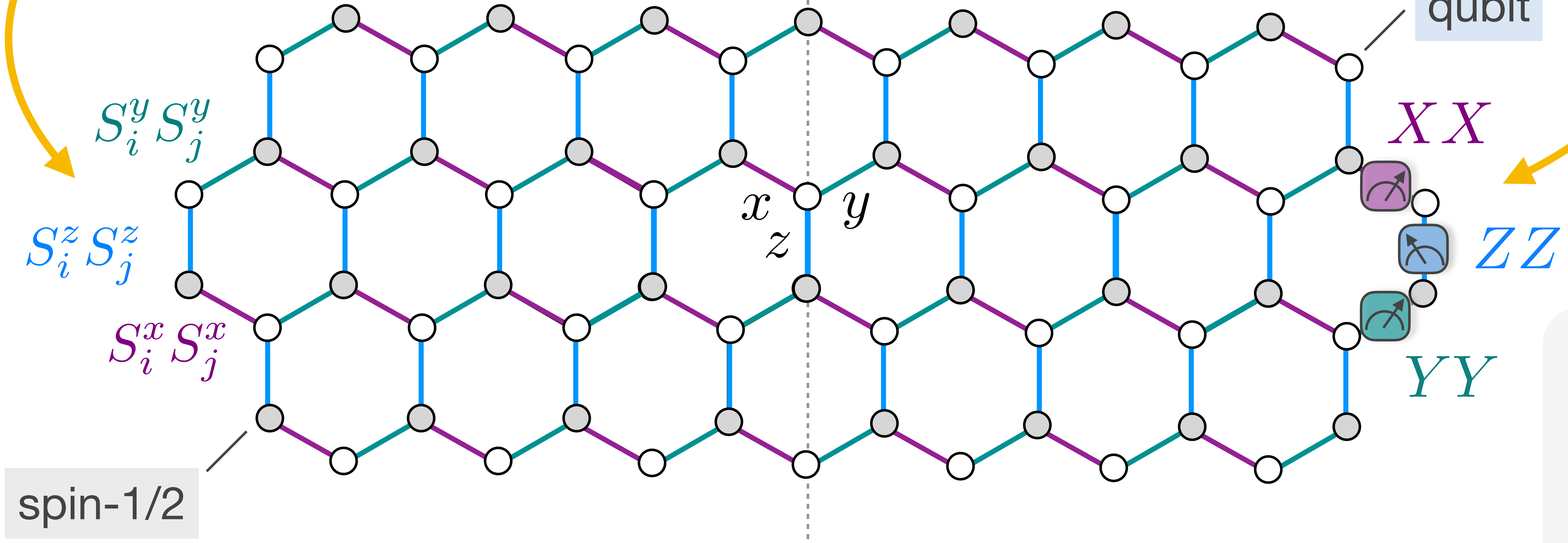

Kitaev (2006)

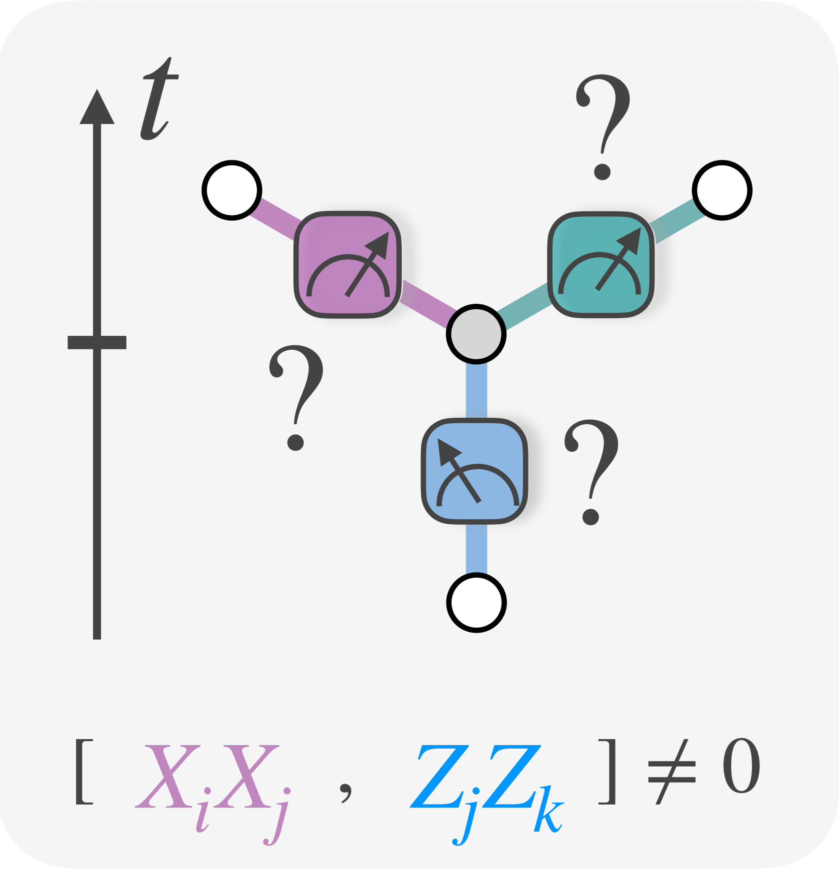
Hamiltonian

measurement



Clifford gates





ground state

minimizes global energy

dynamical state

eigenstate of measurements

no state can satisfy every local interaction

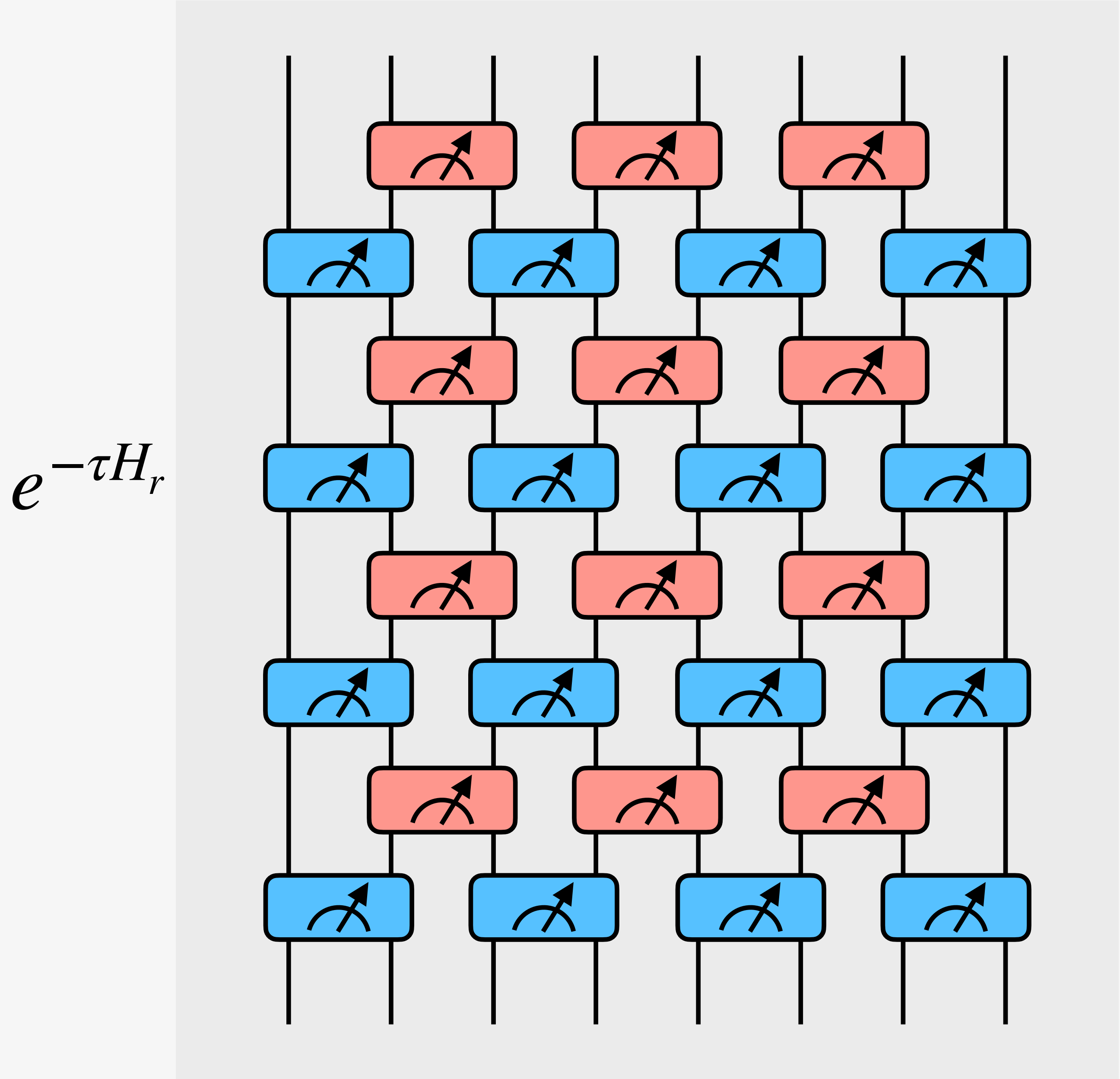
frustration

non-commuting operators
- cannot be measured simultaneously
- will be over-written

imaginary time vs. measurement-only dynamics

Hamiltonian ground state

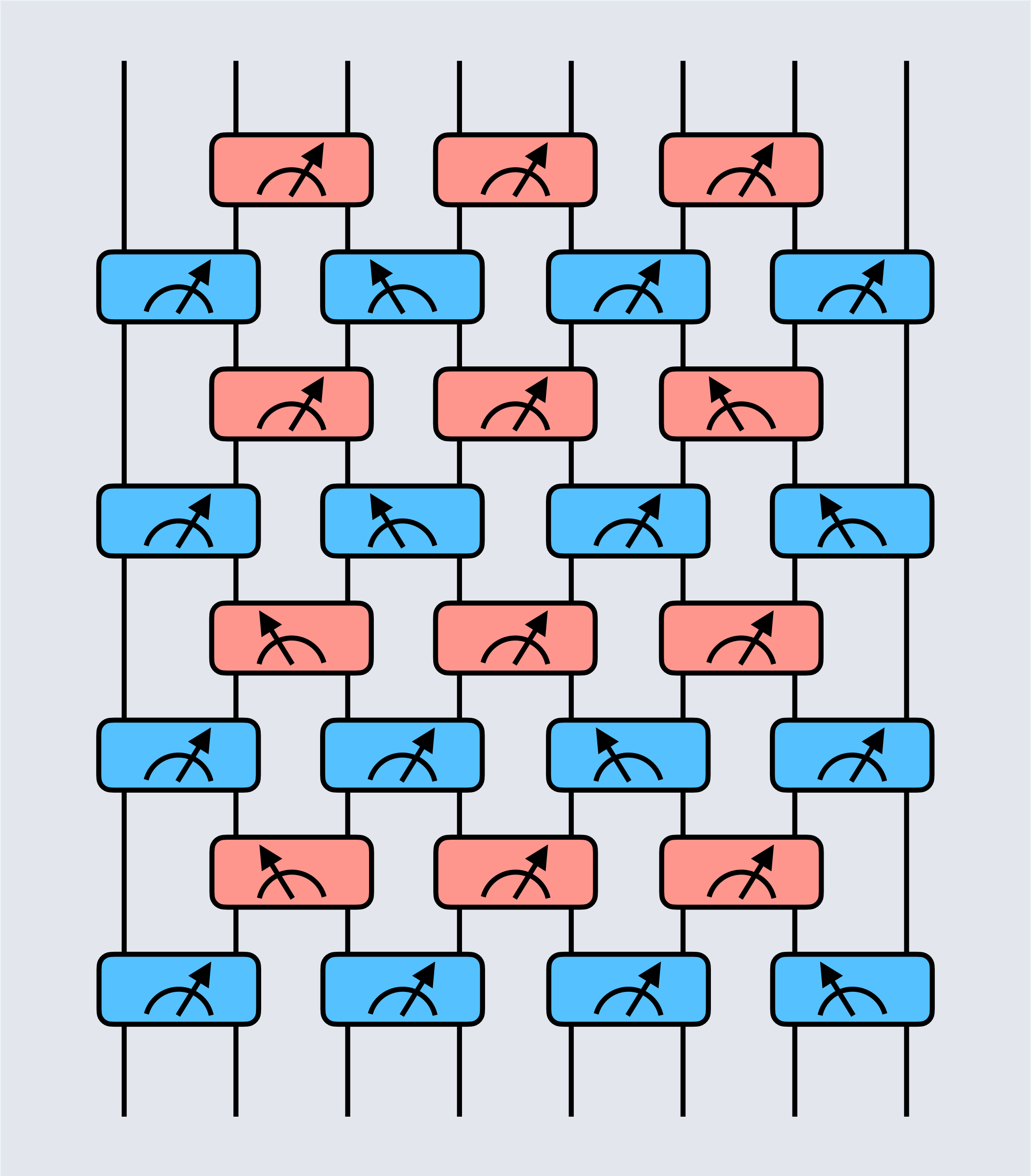
$$e^{-\beta H}\,|\psi_0\rangle$$



- **brickwall** circuit
- **no** disorder

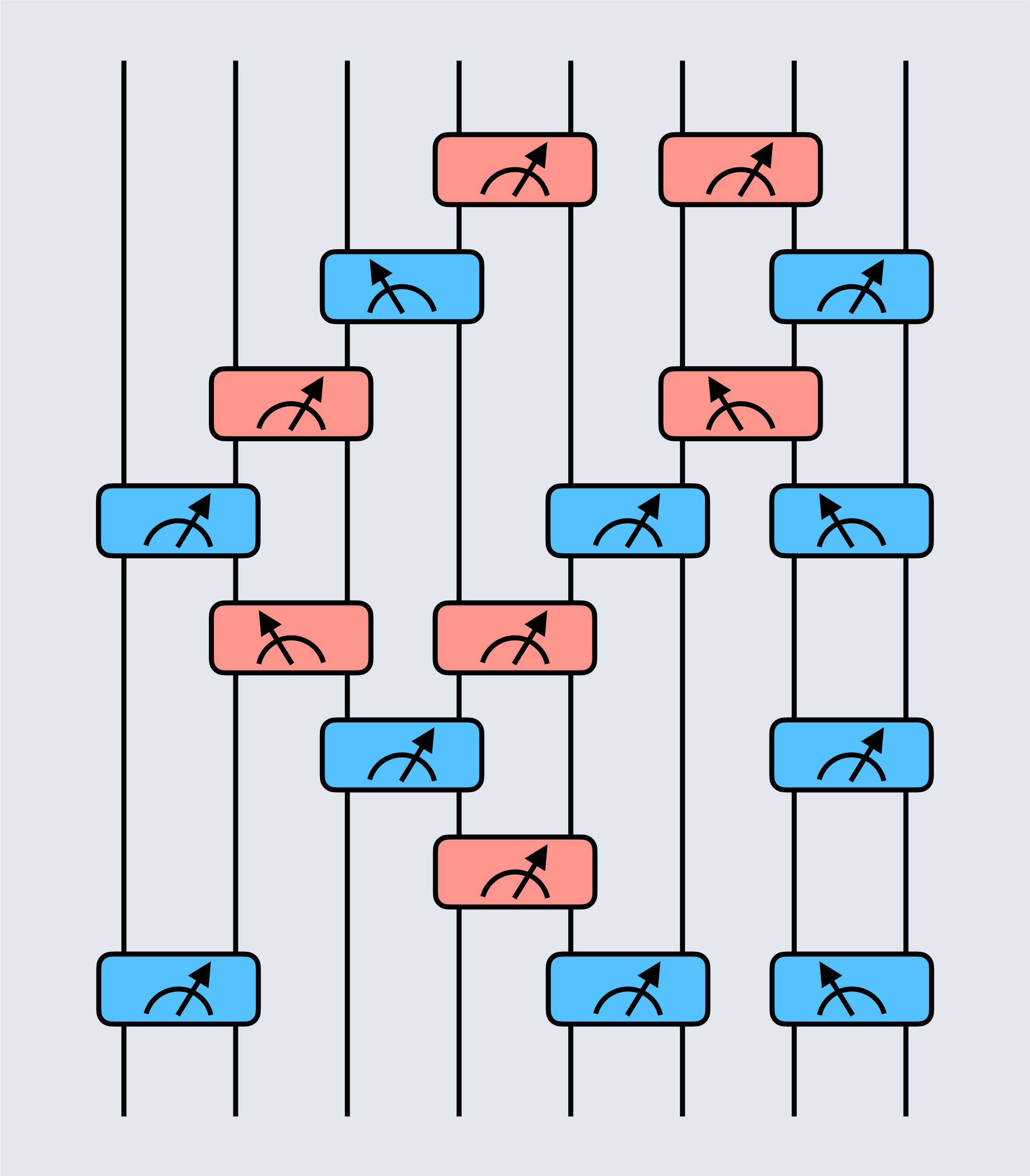
Floquet weak measurement

$$\left(e^{\mp\tau H_r}\cdots e^{\mp\tau H_0}\right)|\psi_0\rangle$$



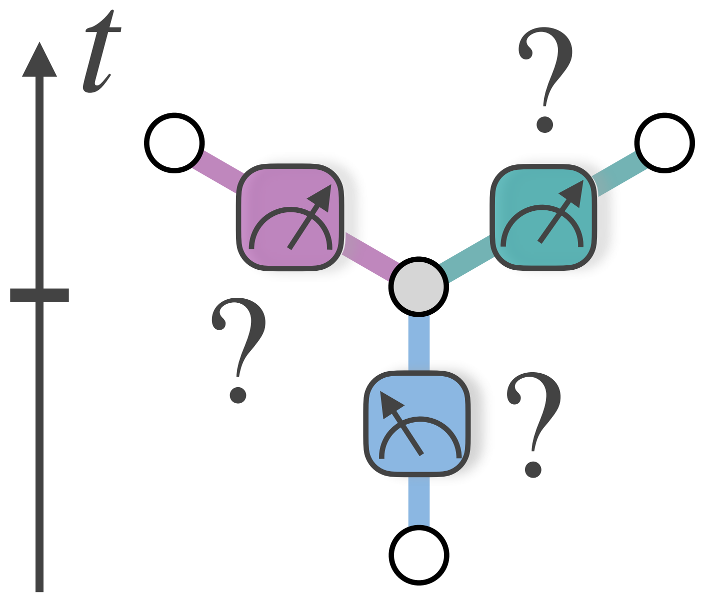
- **brickwall** circuit
- **Born** disorder

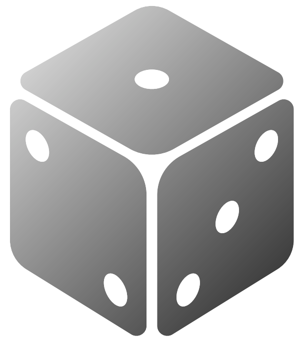
random weak/strong measurement

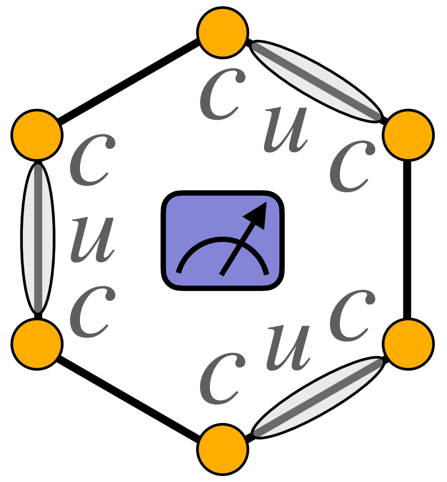


- **stochastic** circuit
- **Born** disorder

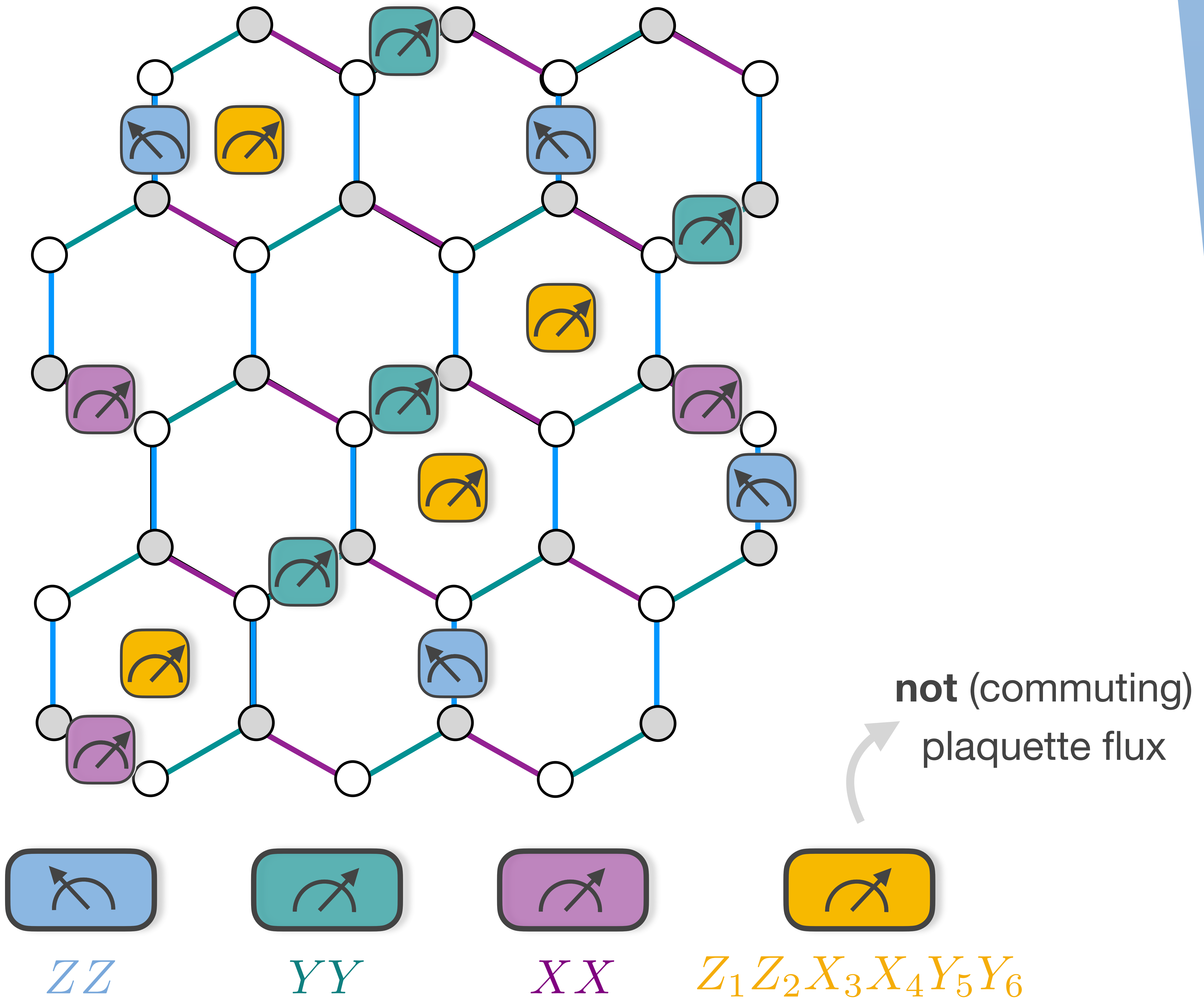
random projective Kitaev measurements







Majorana interaction
→ Majorana surface code

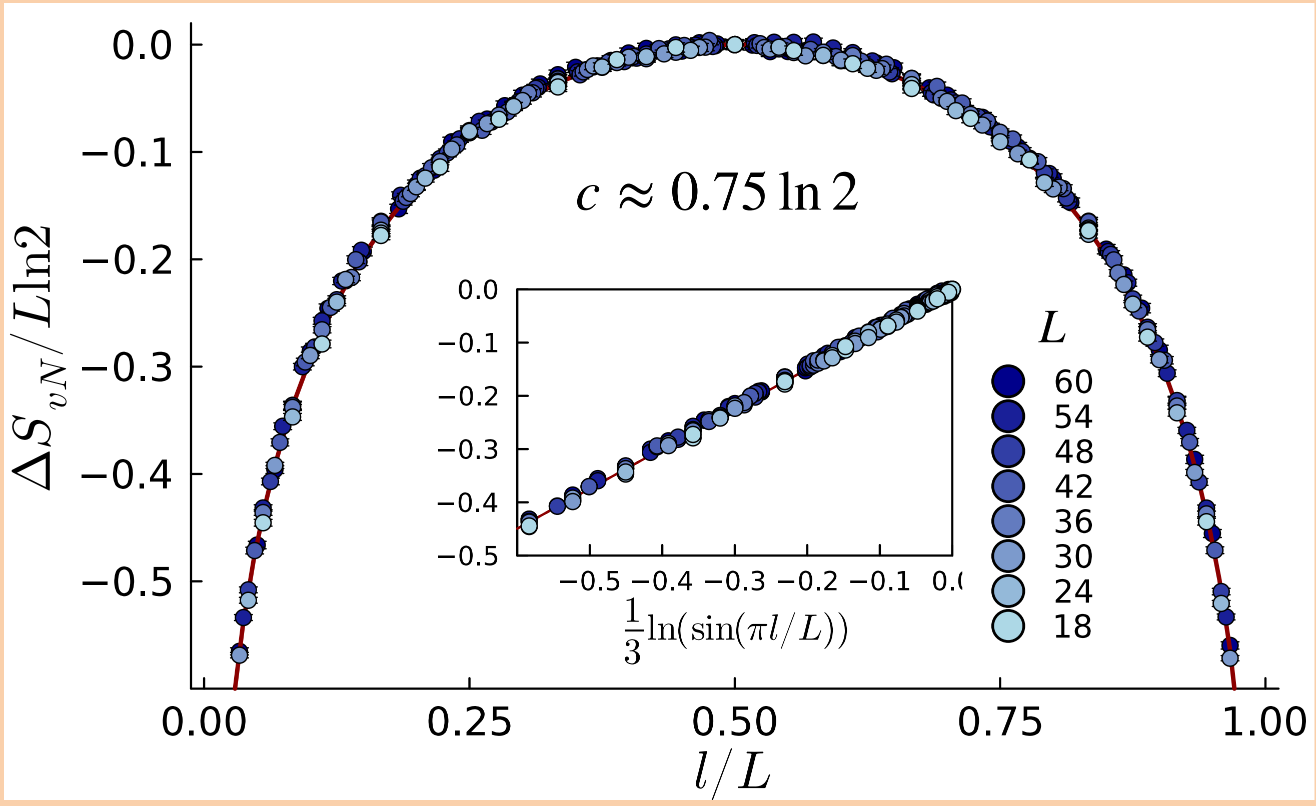


Clifford circuit

even **interacting** problem can be simulated in polynomial time (in Heisenberg picture)

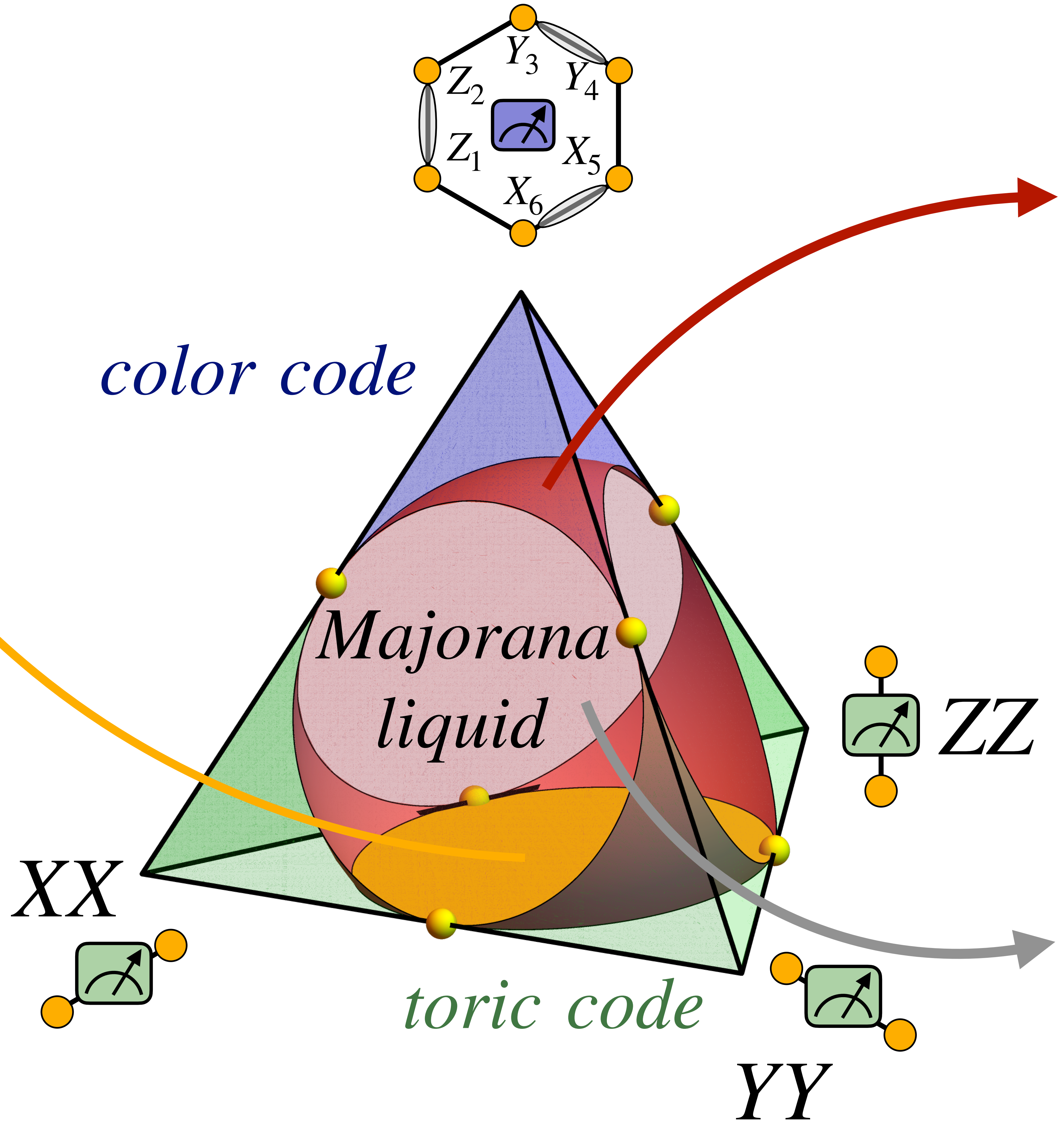
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023; **Zhu, Tantivasadakarn, ST 2023: + Majorana interaction**

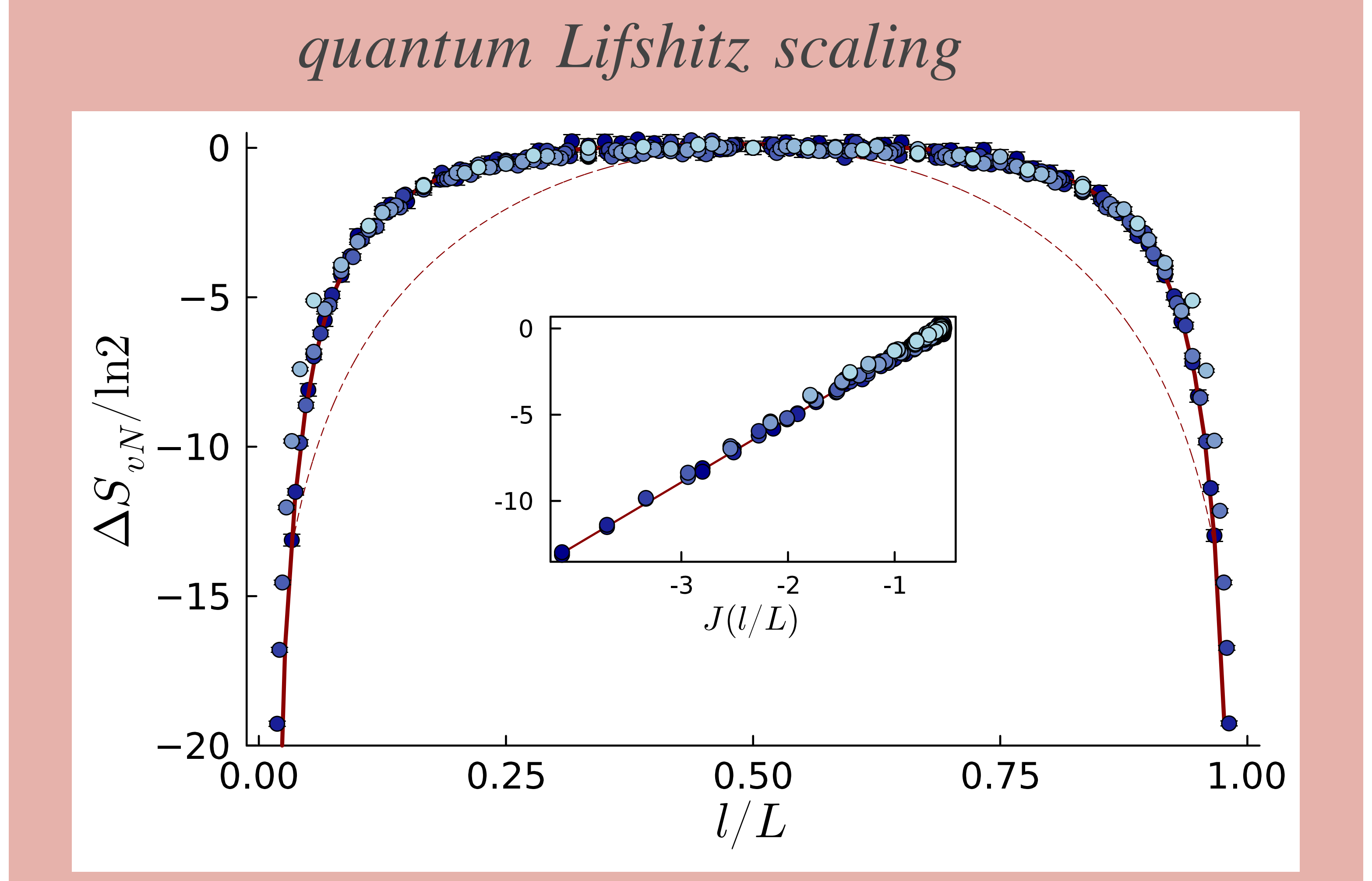
entanglement phase diagram

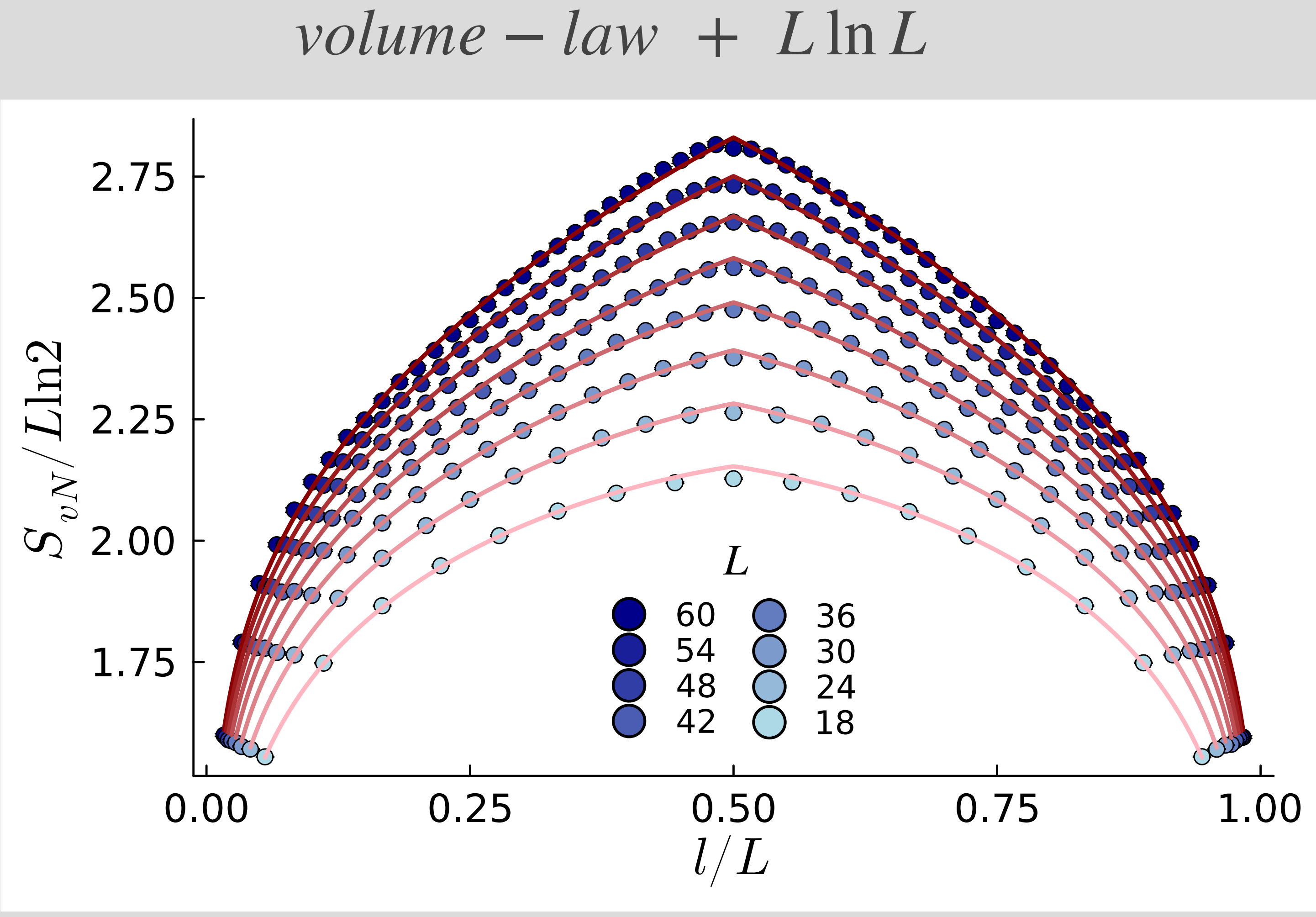


Fermi-surface scaling

$$S_{vN}(l,L) = \frac{cL}{3} \ln\left(\frac{L}{\pi} \sin\frac{\pi l}{L}\right) + \cdots$$



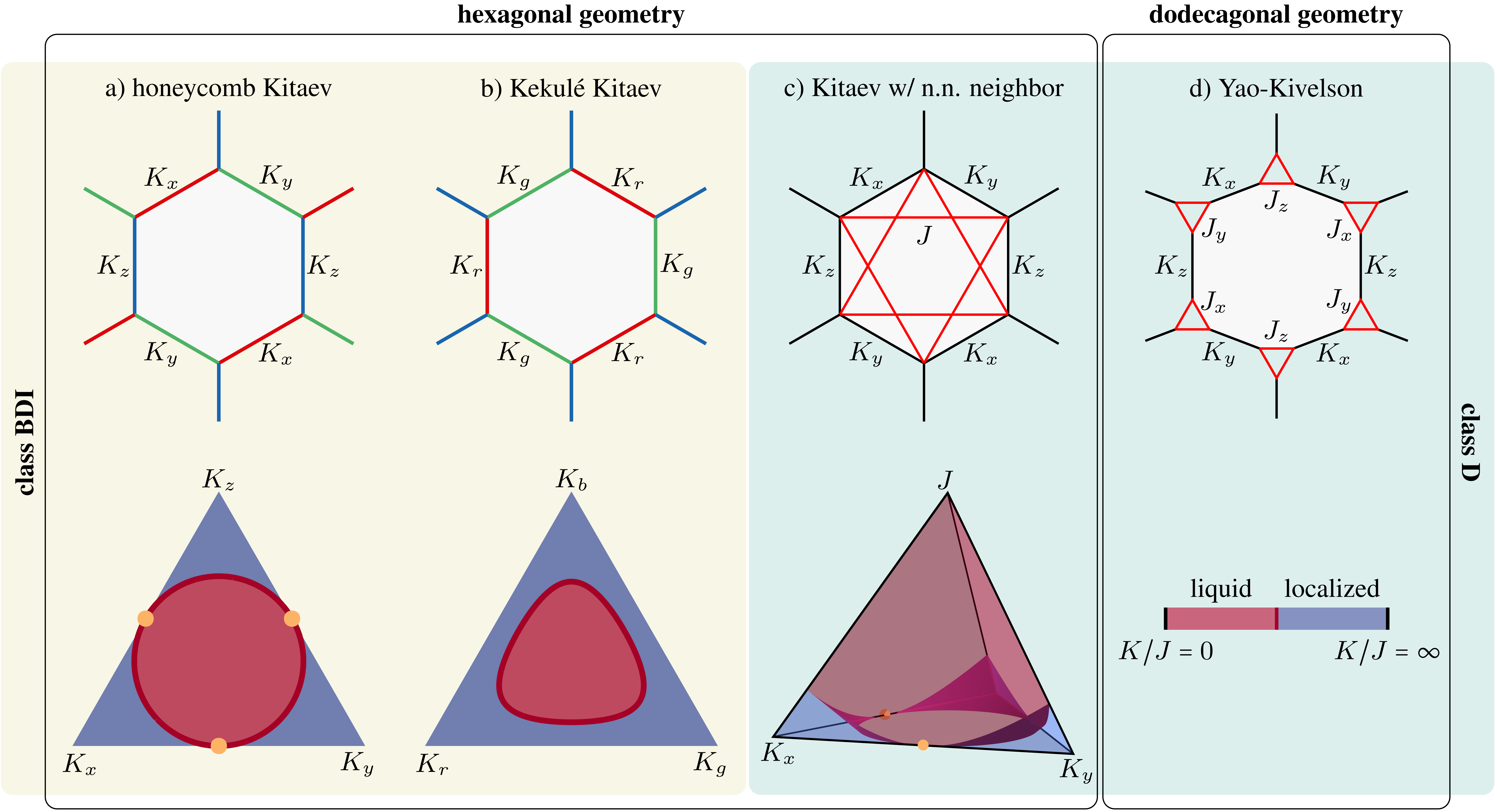




Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023

Zhu, Tantivasadakarn, ST 2023: + Majorana interaction

entanglement & circuit geometry



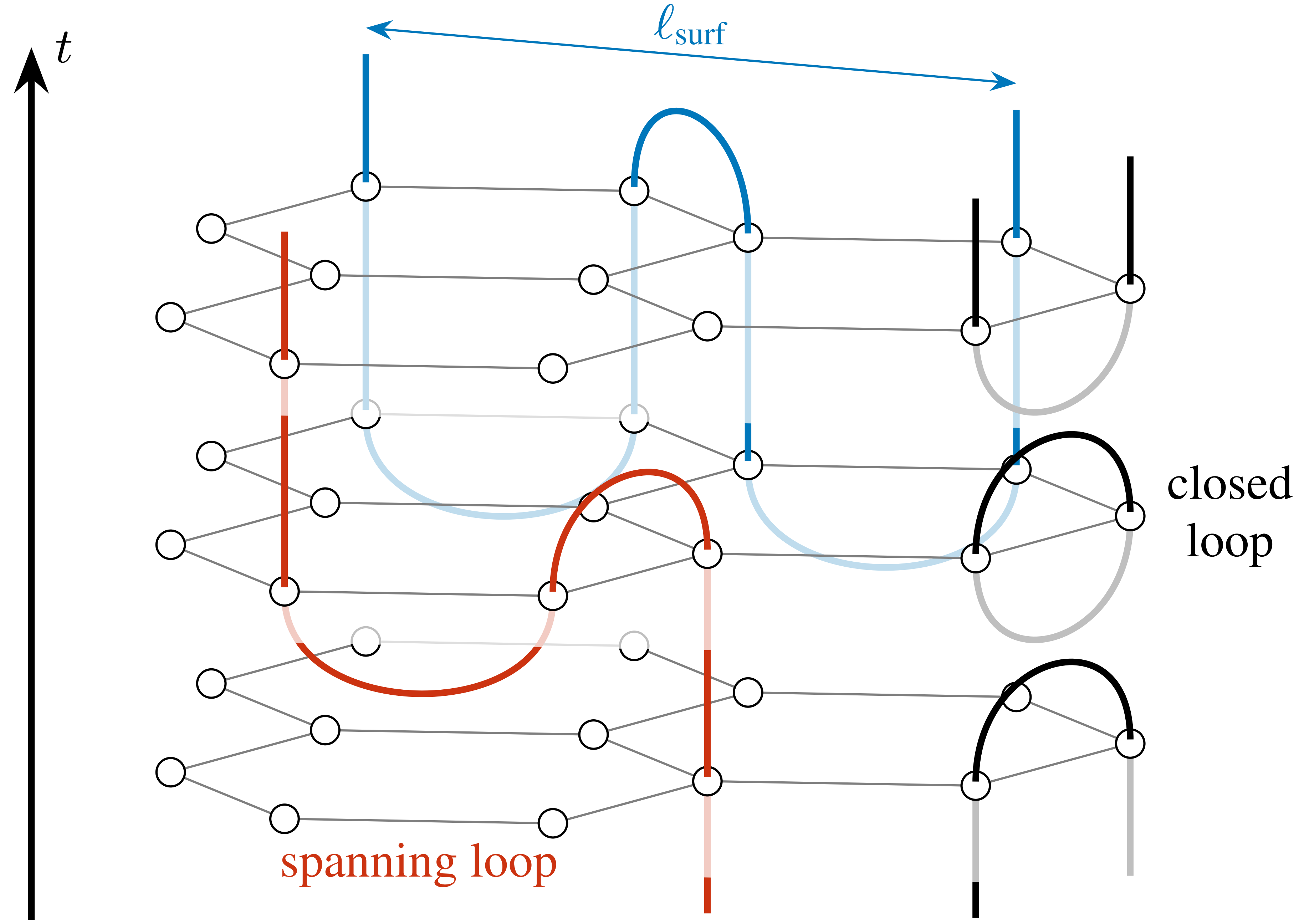
Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023

Klocke, Simm , Zhu, ST, Buchhold 2024: non-bipartite geometries

analytical connections to **loop models** and **NLσM models**

numerical simulations for 100,000,000 = 10^8 **qubits**

Majorana loop models



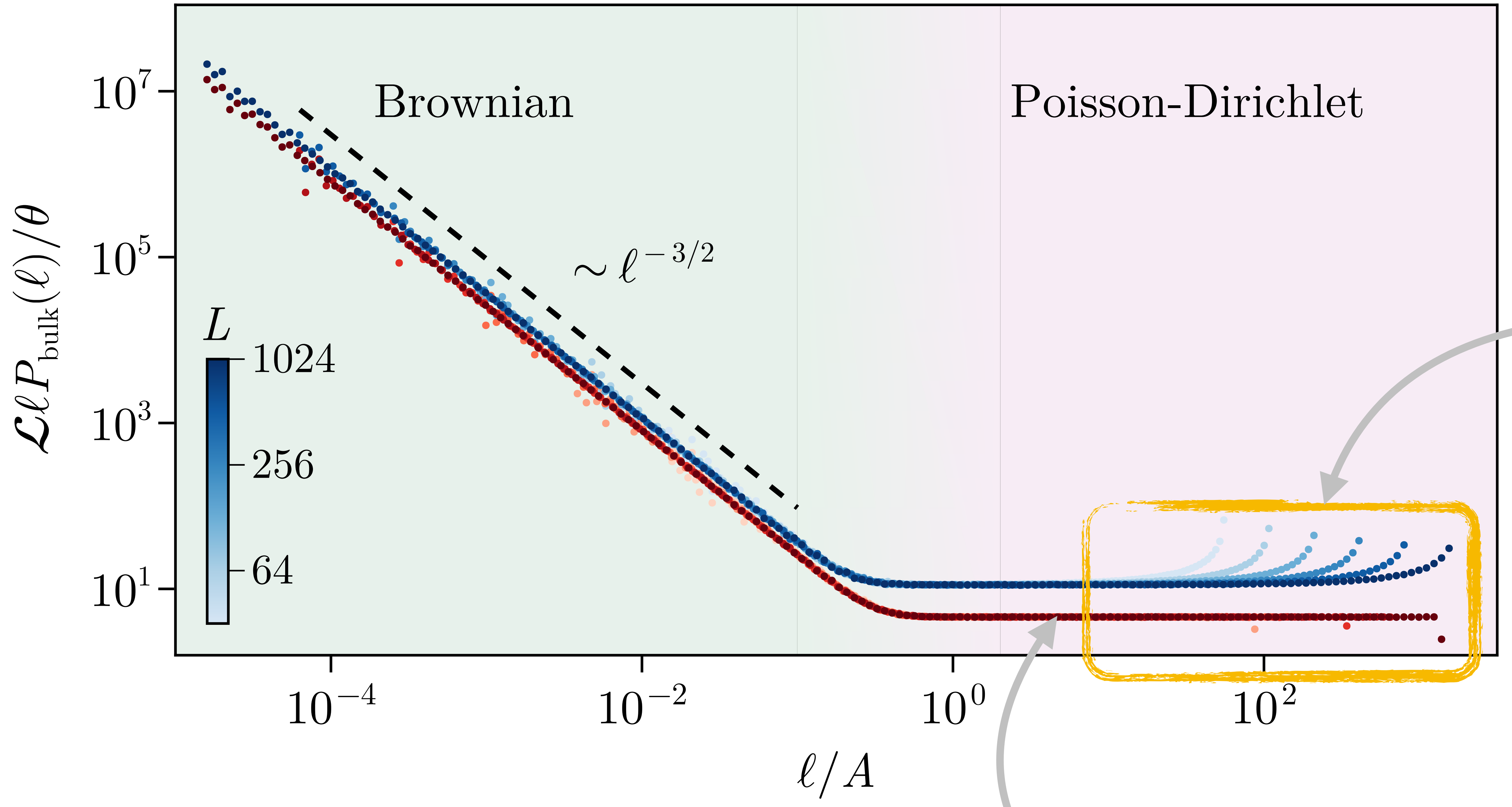
loop **fugacity**

circuit	Hamiltonian
$n = 1$	$n = \sqrt{2}$

	lattice /circuit **geometry**	**bipartite** (e.g. honeycomb)	**non-bipartite** (e.g. Yao-Kivelson)
	symmetry class	**BDI**	**D**
loop model	loop symmetry	orientable	non-orientable
	field theory	$\mathbb{CP}^{n-1}$	$\mathbb{RP}^{n-1}$
quantum **circuit**	entanglement scaling	$\sqrt{\mathcal{D}} \cdot L \log(L)$	
	dynamics	$P(\ell) \sim \text{const.}$	$P(\ell) \sim (\mathcal{L} - \ell)^{-\frac{1}{2}}$
quantum **Hamiltonian**	entanglement scaling	$L + \log L$	$L - \gamma_{\text{topo}}$
	Maj. spectrum	gapless Dirac	gapped Chern

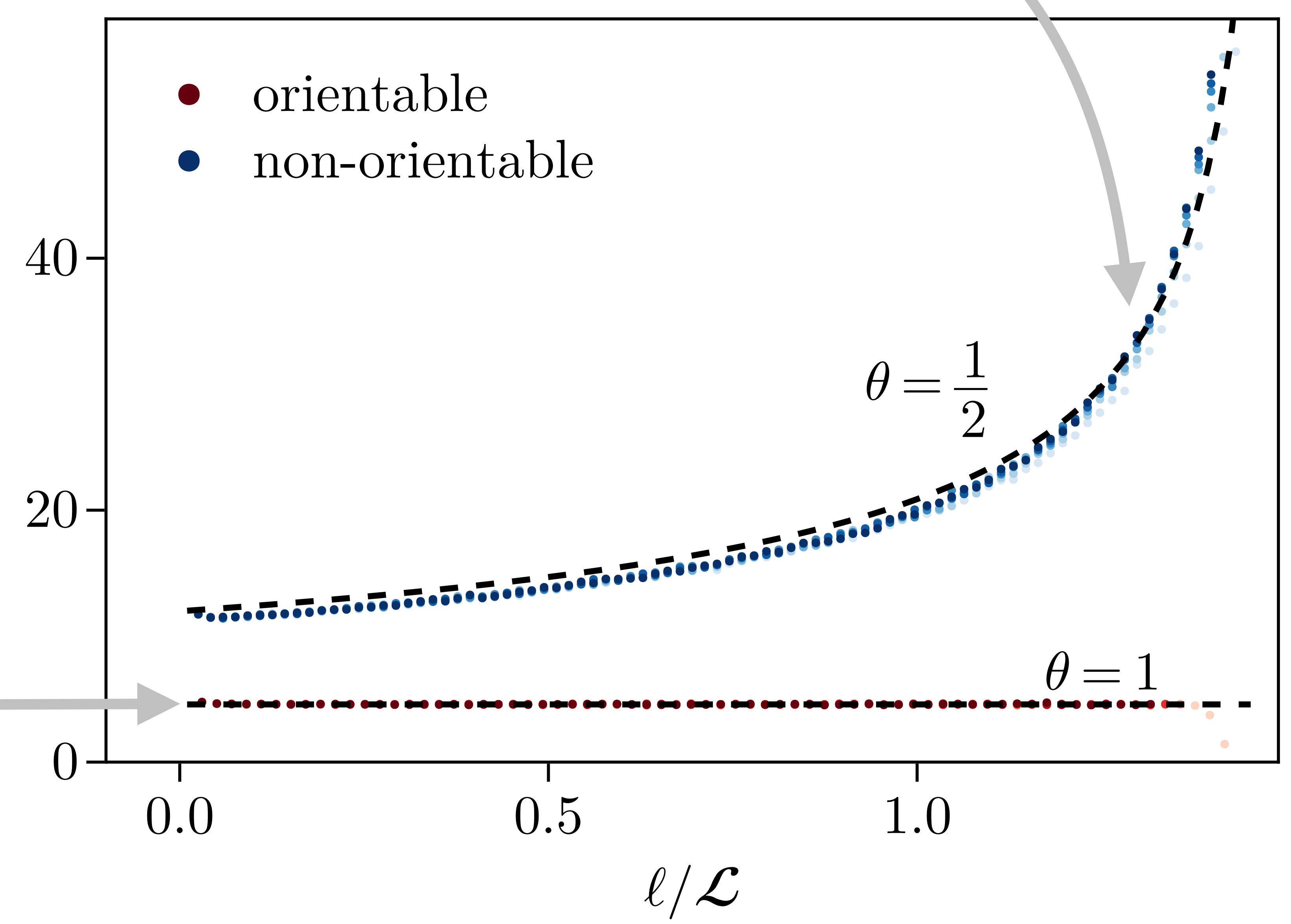
bulk loop statistics

distribution of **bulk loop lengths**



greater mobility of loops
w/o time-reversal symmetry constraint

non-orientable loops
(symmetry class **D**)

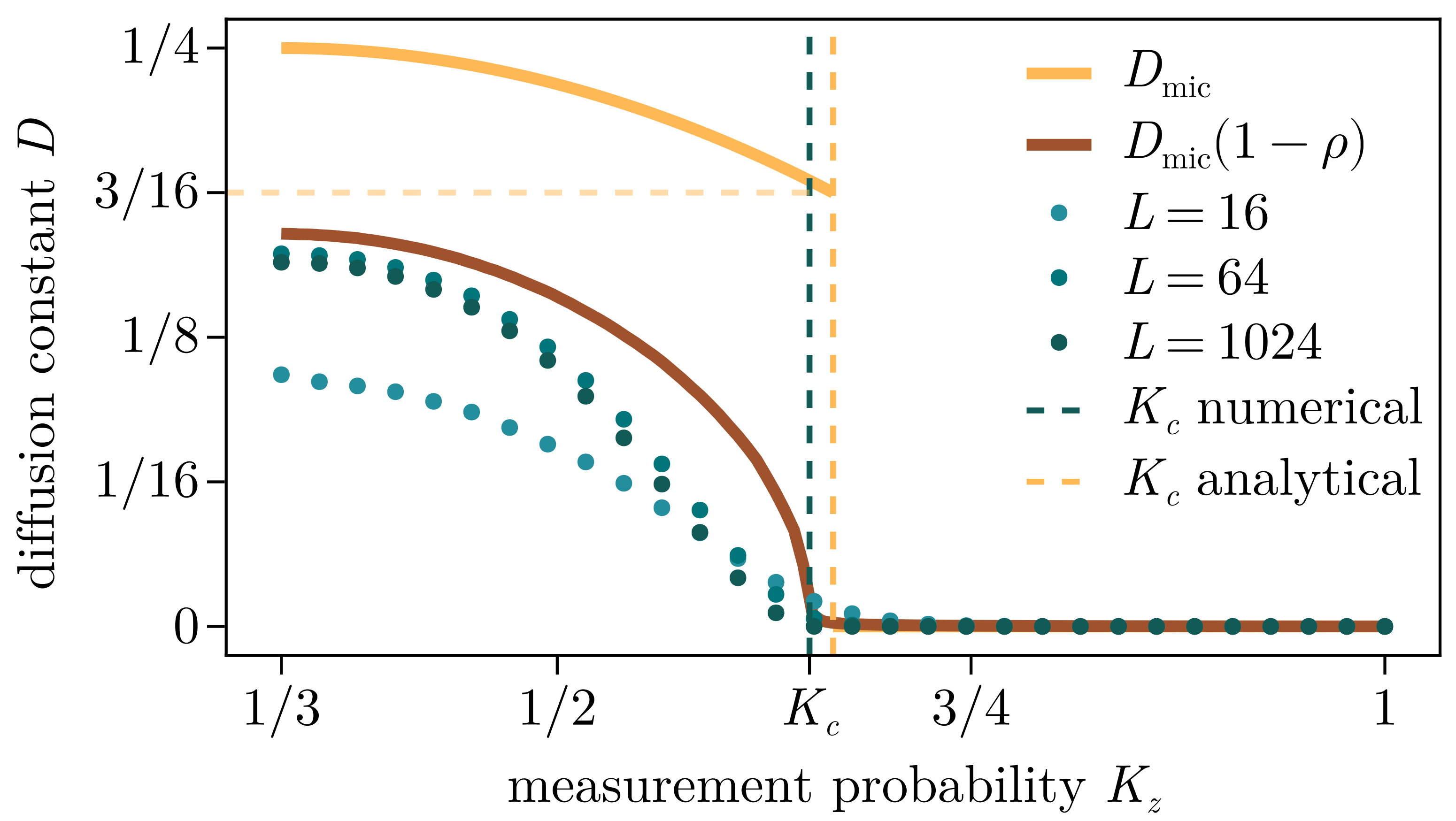


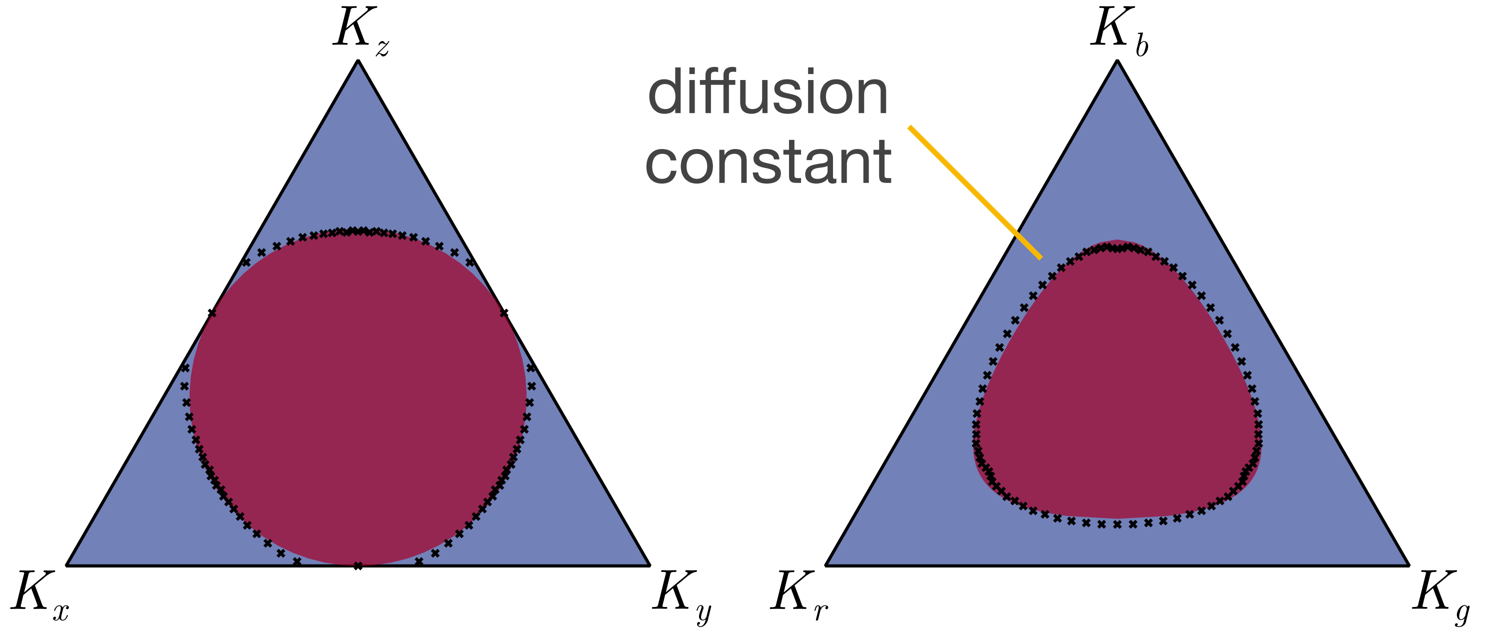
orientable loops
(symmetry class **BDI**)

short loops – area law phase
long loops – liquid phase

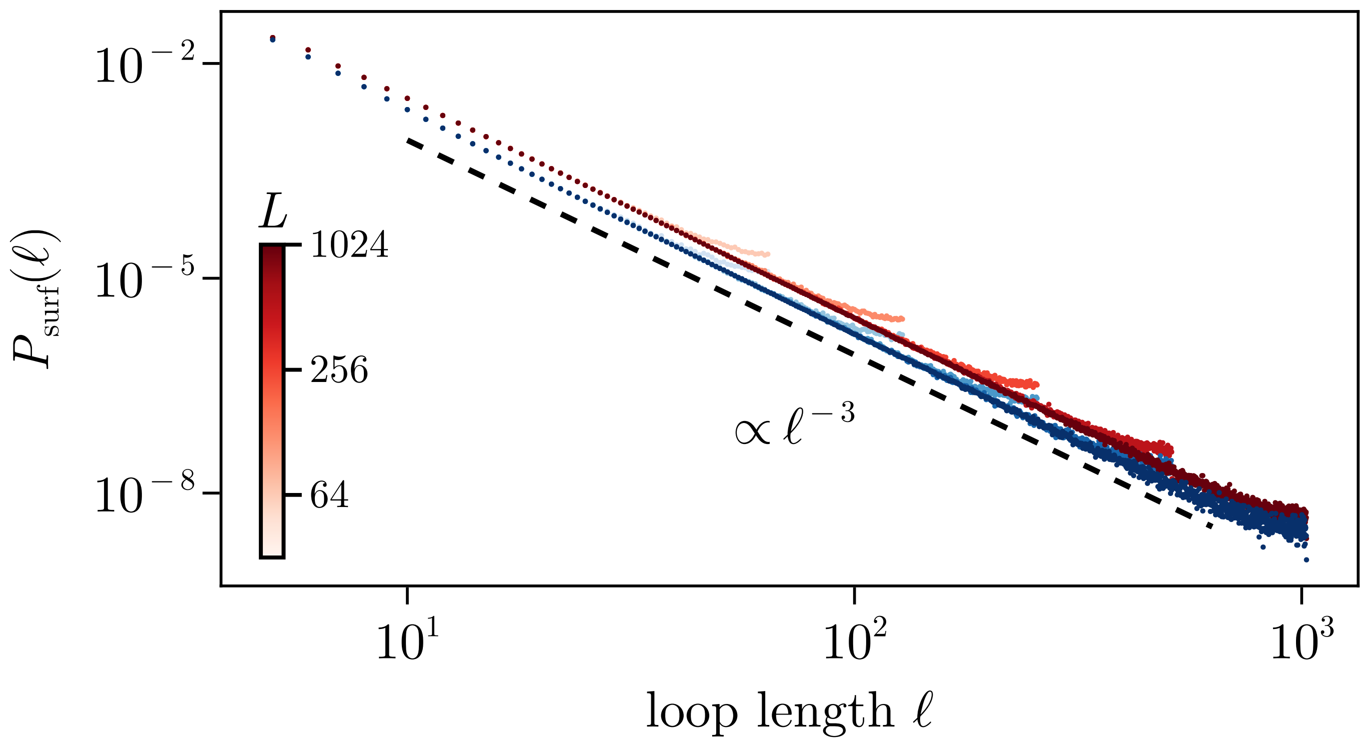
entanglement phase transitions

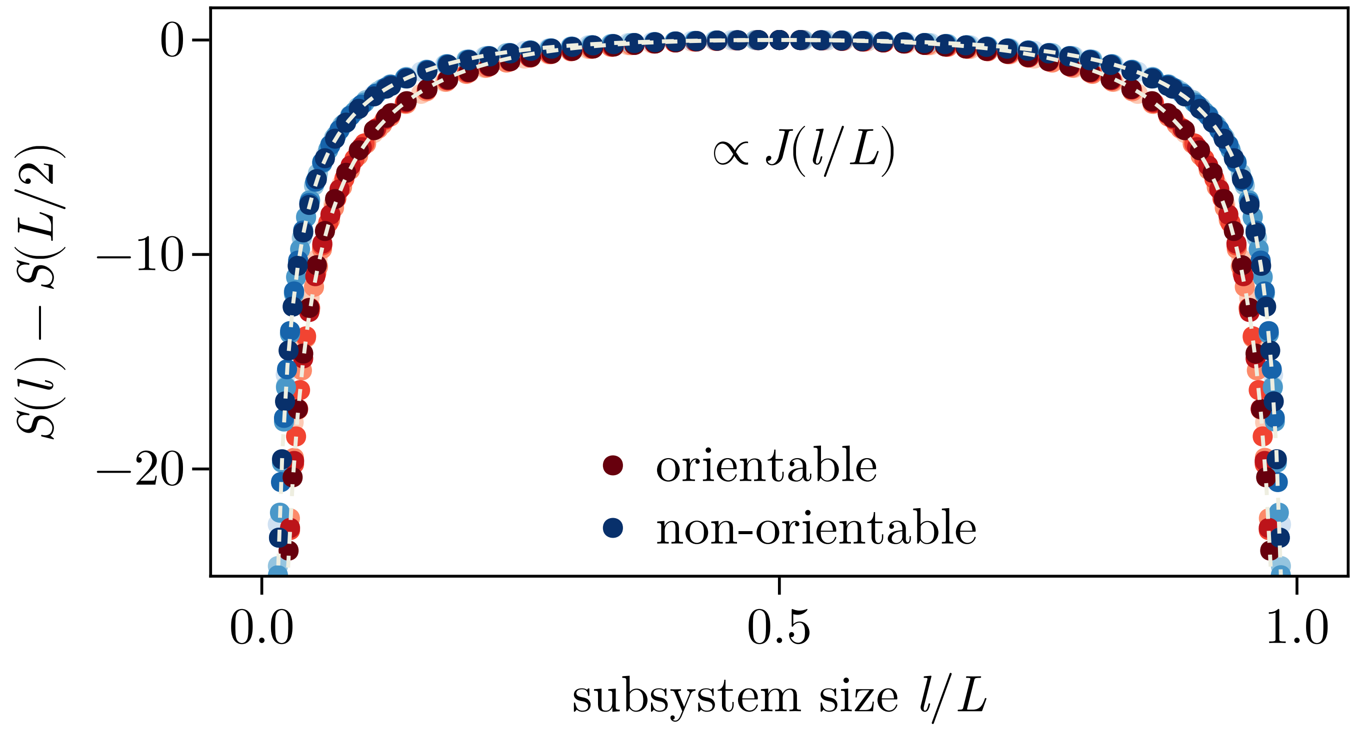
loop diffusion picture



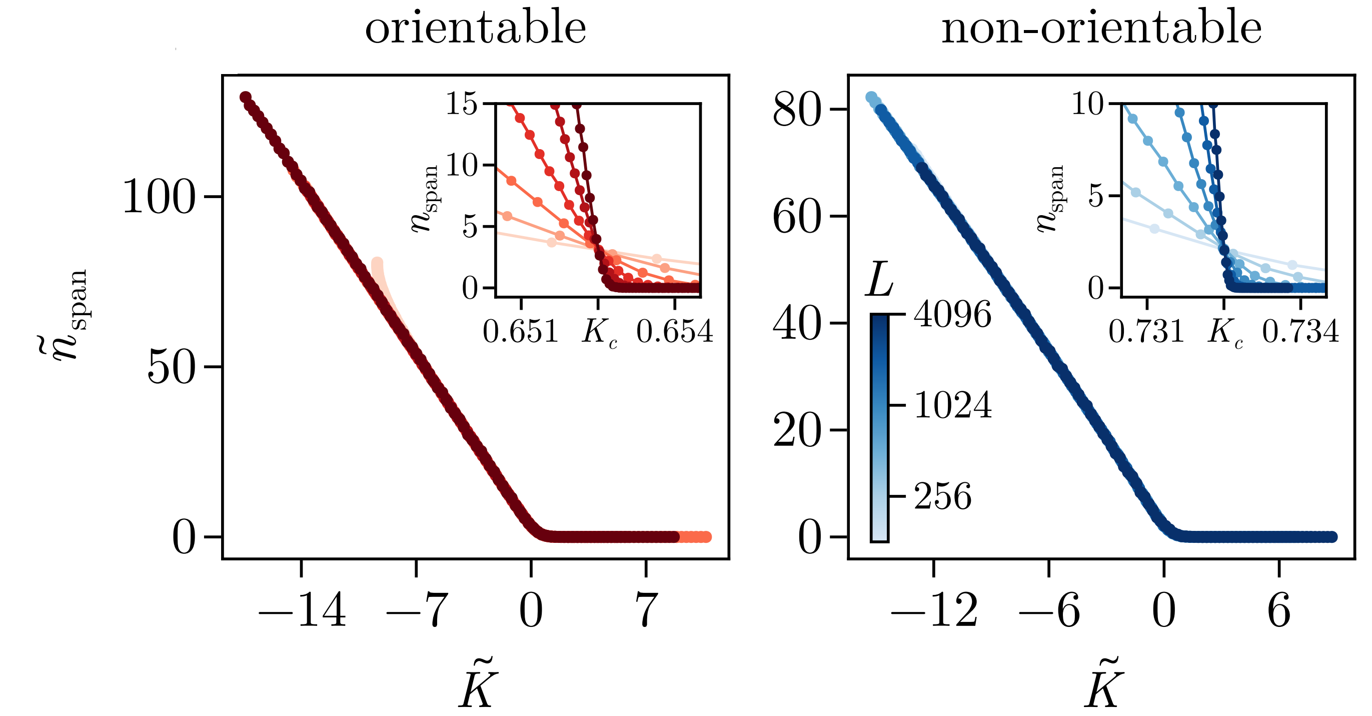


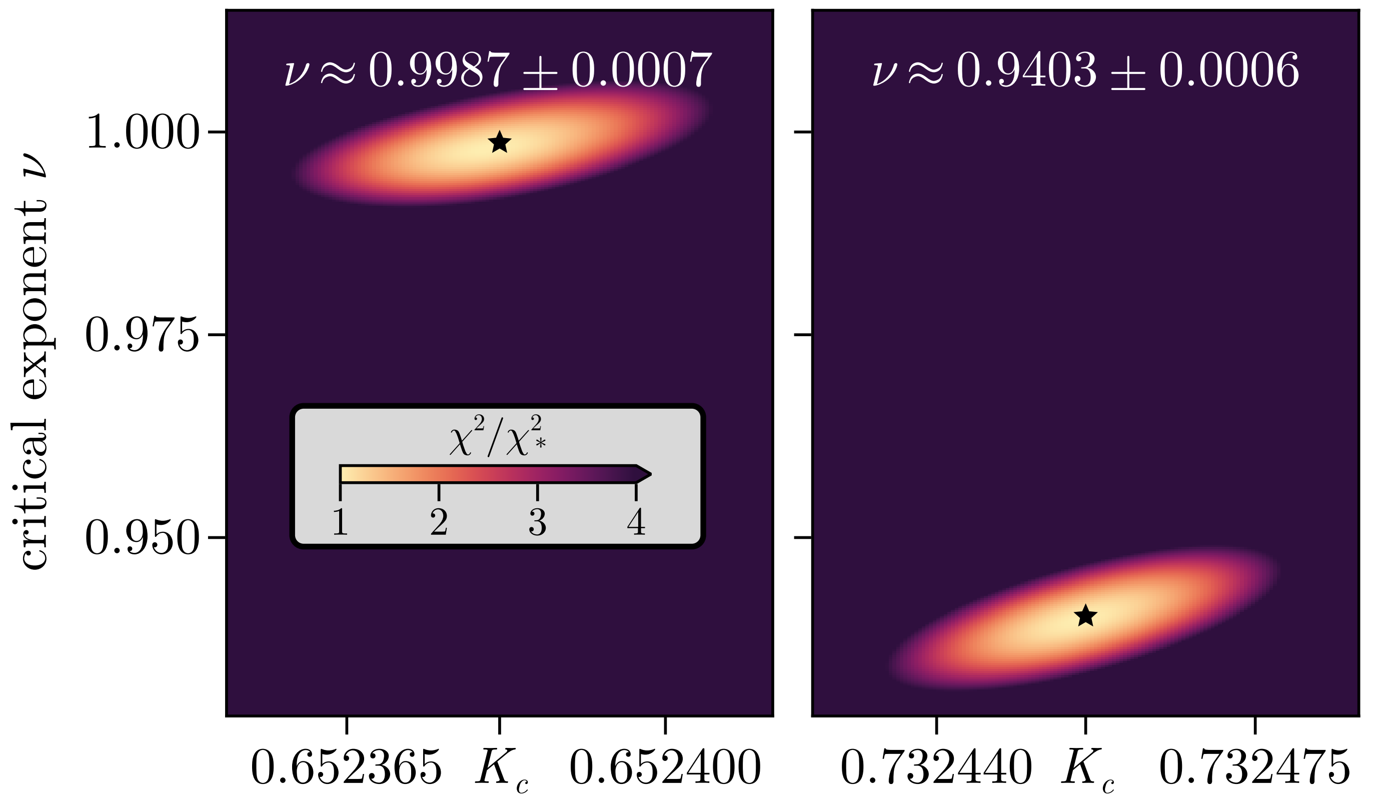
quantum Lifshitz criticality





critical **exponents**





entanglement phase transitions

critical exponent	ν	η	d_f	β
orientable loops (symmetry class **BDI**)	0.9987(7)	-0.084(4)	2.5383(11)	0.4590(20)
non-orientable loops (symmetry class **D**)	0.9403(6)	-0.066(7)	2.5263(11)	0.4400(40)

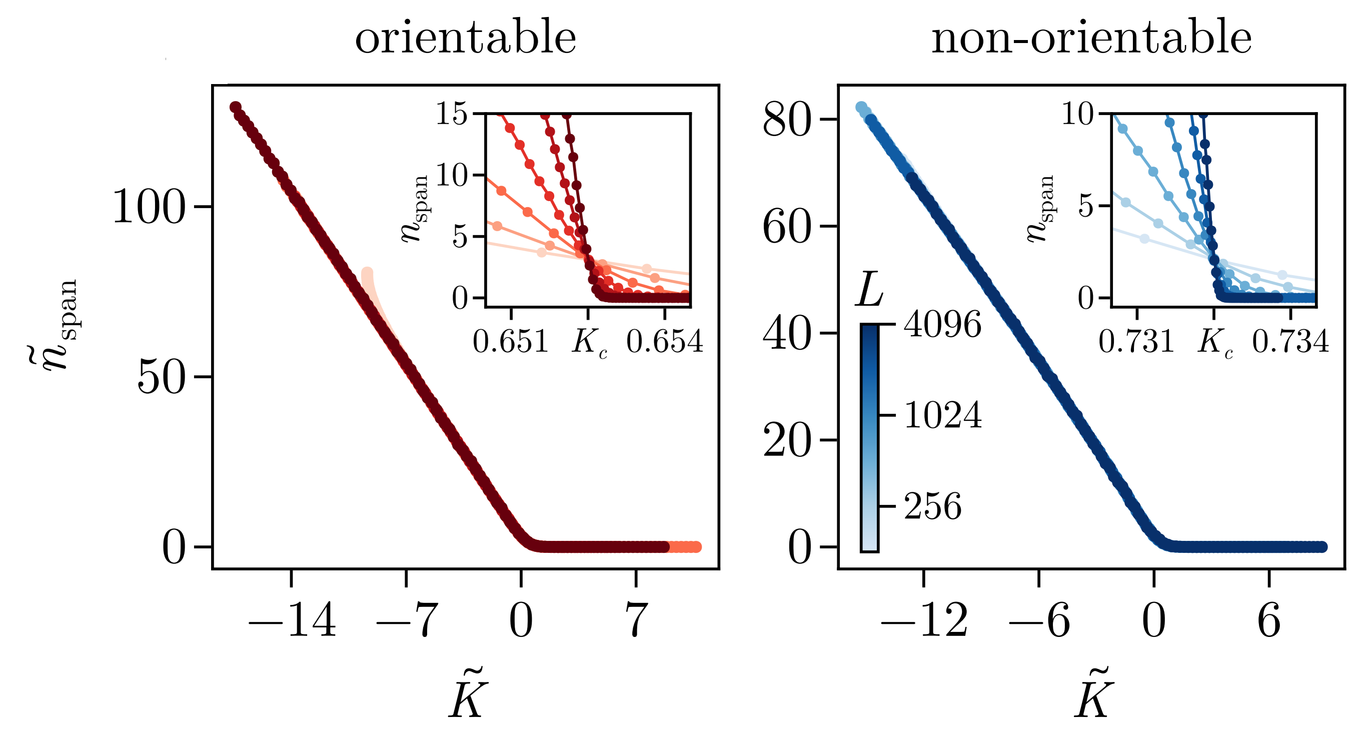
numerical simulations for 100,000,000 = **10^8 qubits**

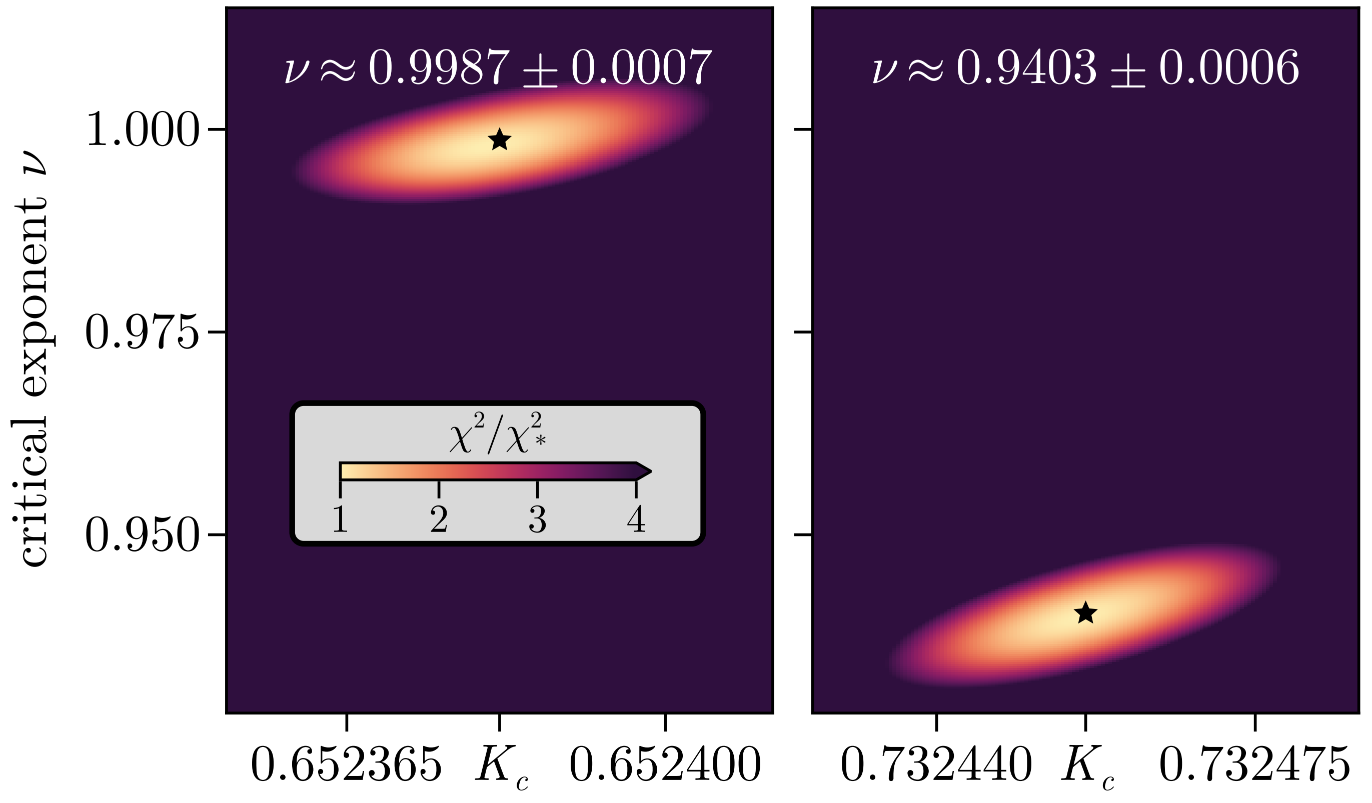
critical exponent	ν	η	d_f	β
orientable loops (symmetry class **BDI**)	0.999(2)	-0.068(18)	2.534(9)	0.4650(33)
non-orientable loops (symmetry class **D**)	0.918(5)	-0.091(9)	2.546(5)	0.4168(51)

numerical simulations for 1,000,000 = **10^6 sites**

M. Ortuño, A. M. Somoza, and J. T. Chalker, PRL **102**, 070603 (2009)
P. Serna, arXiv:2107.13366 (2021)

critical **exponents**



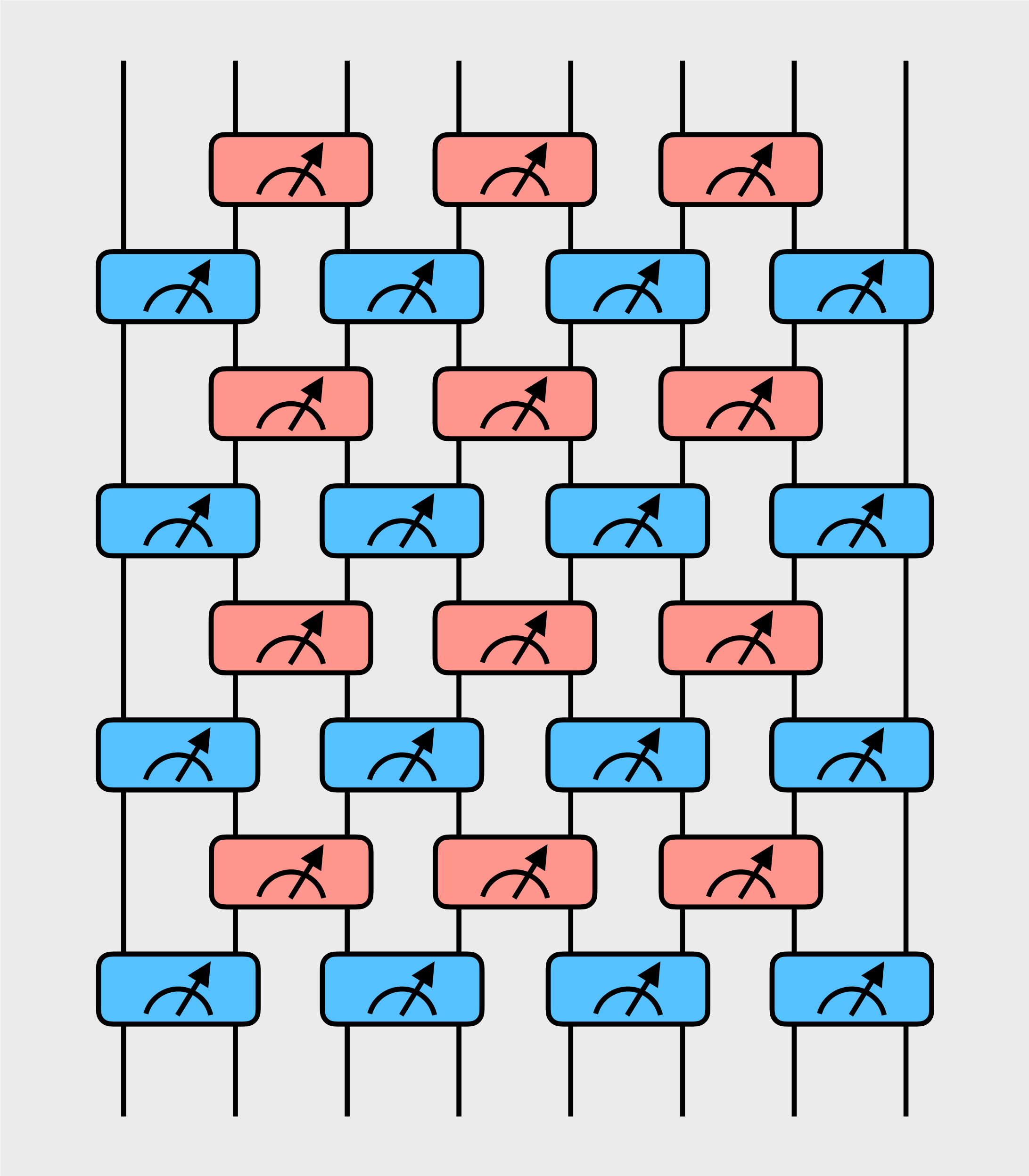


imaginary time vs. measurement-only dynamics

Hamiltonian ground state

$$e^{-\beta H}\,|\psi_0\rangle$$

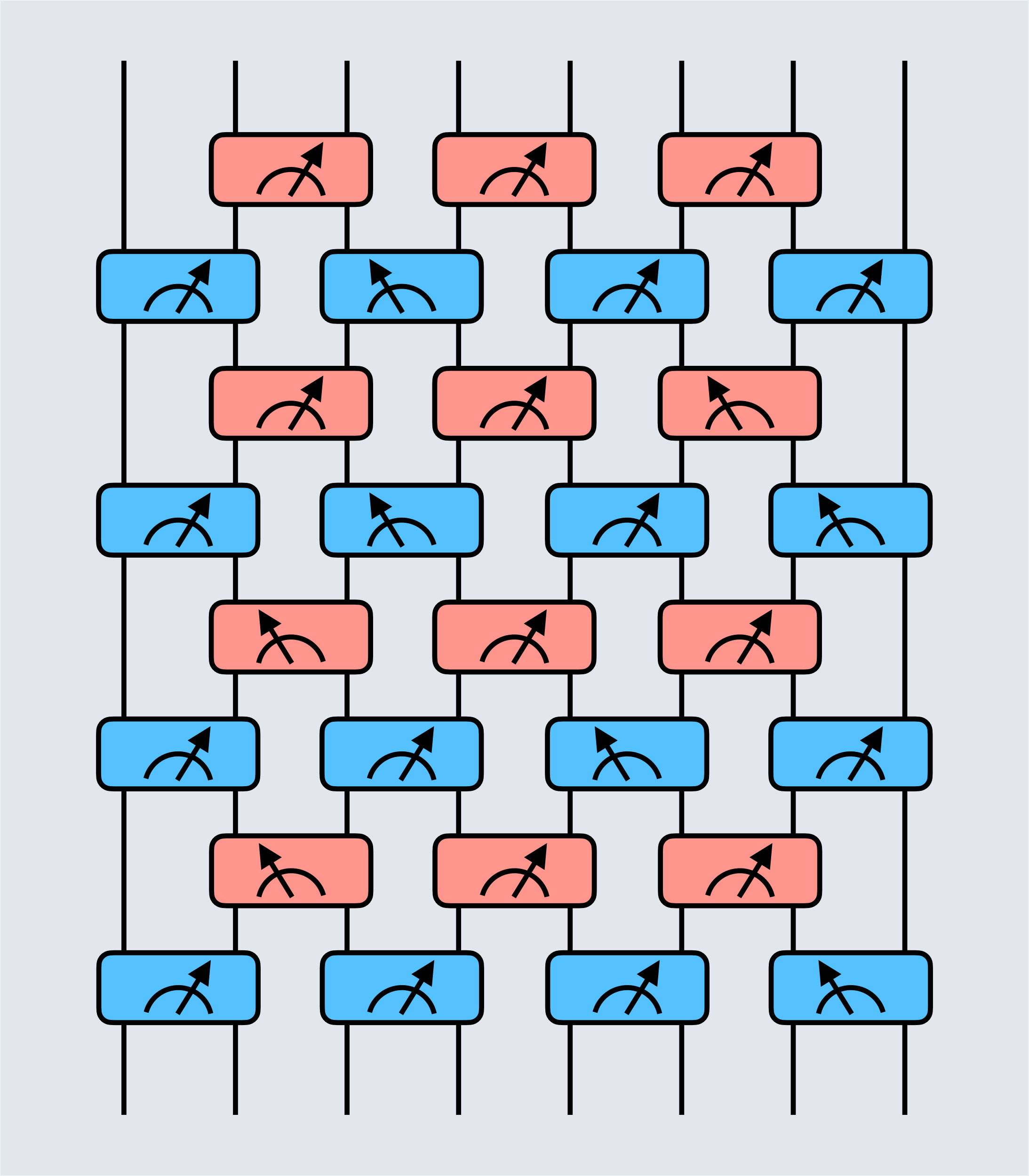
$e^{-\tau H_r}$



- **brickwall** circuit
- **no** disorder

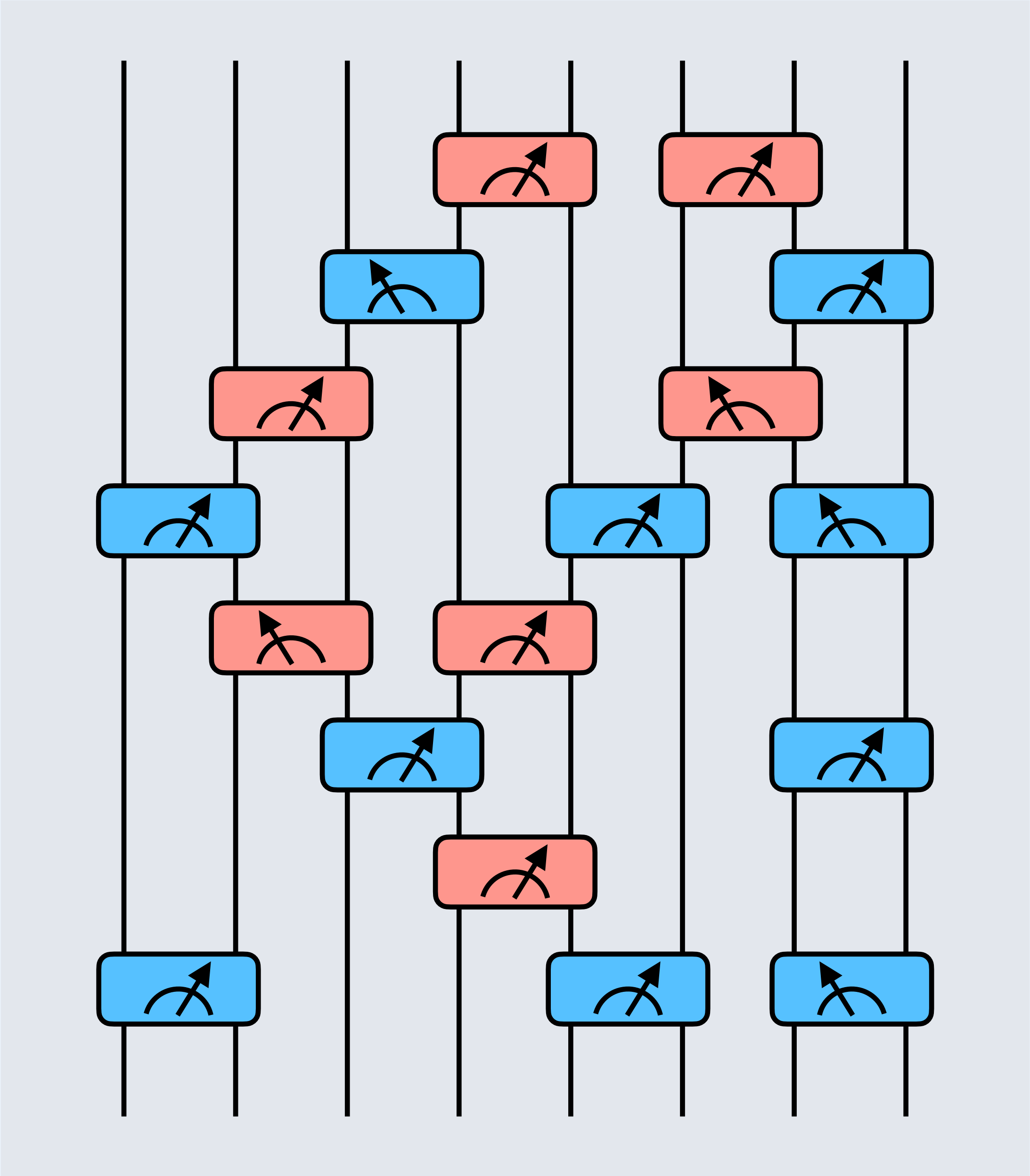
Floquet weak measurement

$$\left(e^{\mp\tau H_r}\ldots e^{\mp\tau H_0}\right)|\psi_0\rangle$$



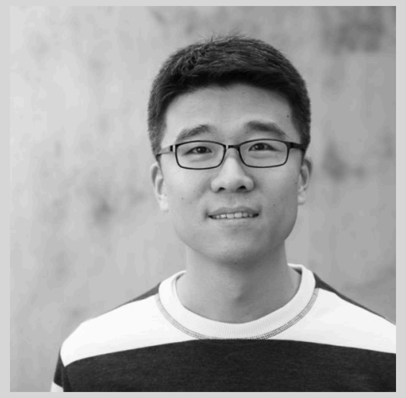
- **brickwall** circuit
- **Born** disorder

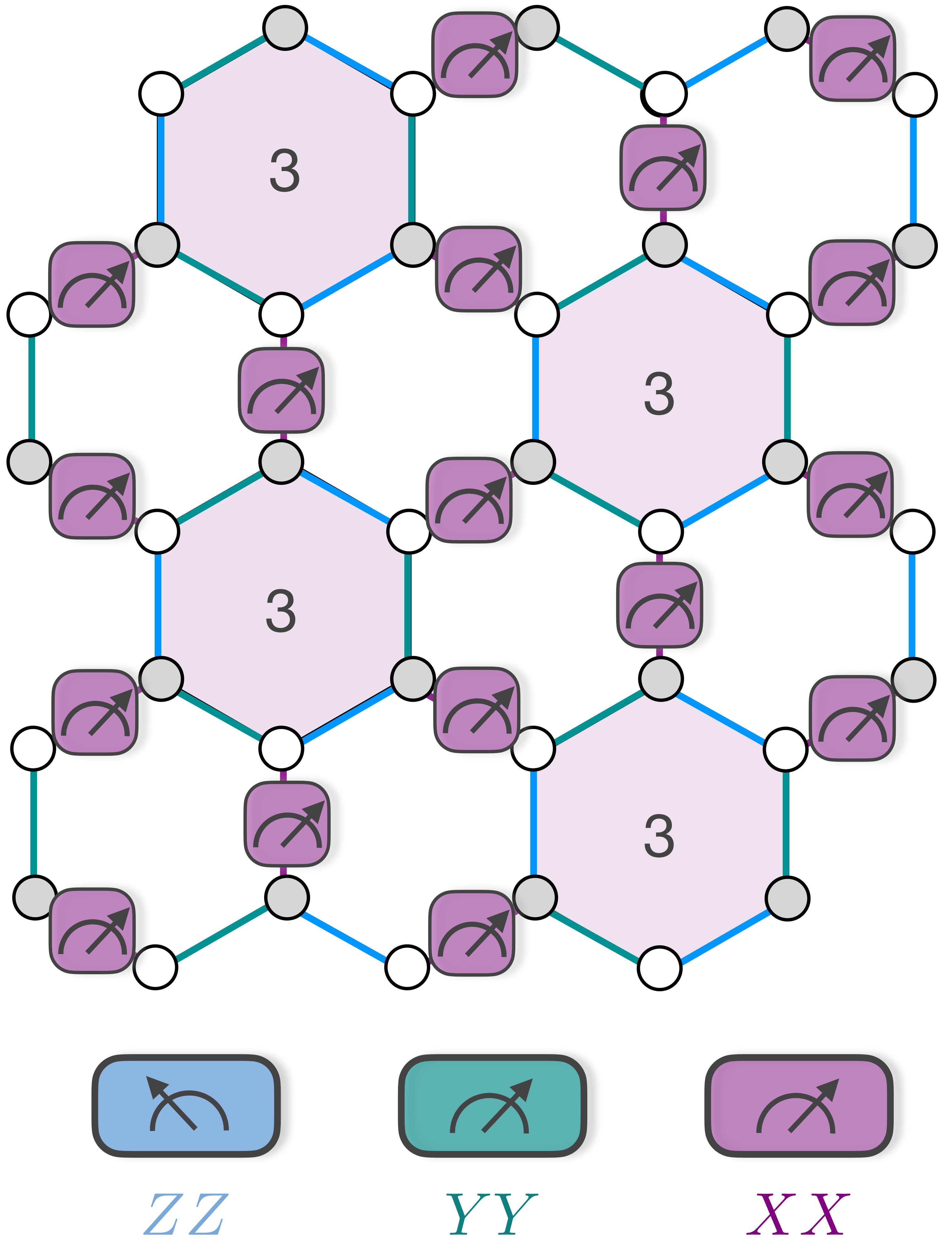
random weak/strong measurement



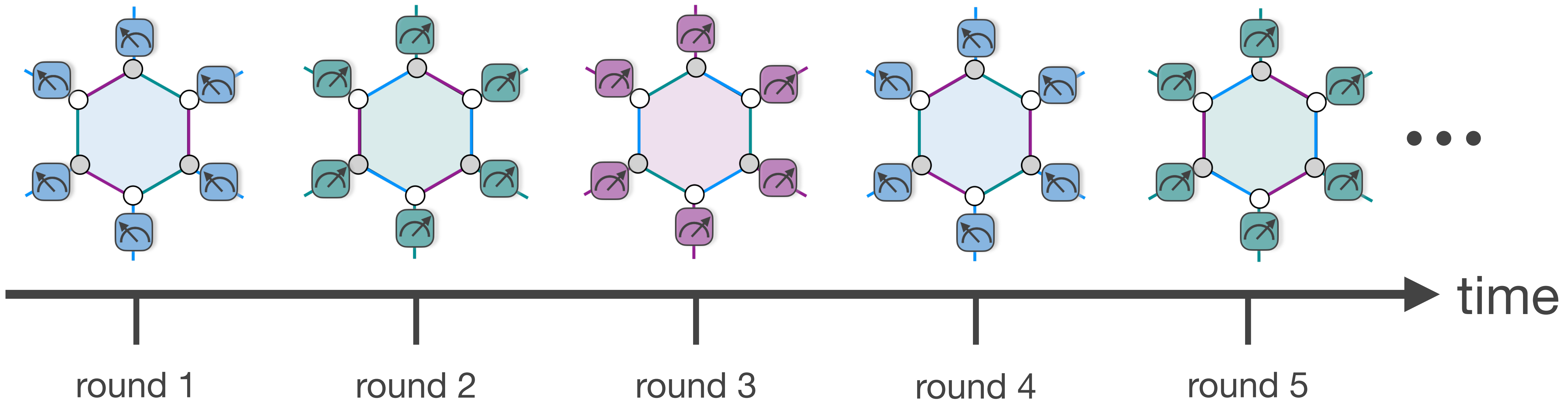
- **stochastic** circuit
- **Born** disorder

Hastings-Haah Floquet code



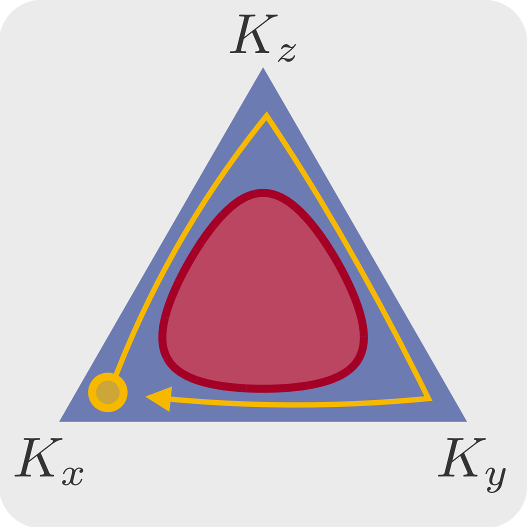


Kekulé pattern / color code



dynamically generated logical qubits

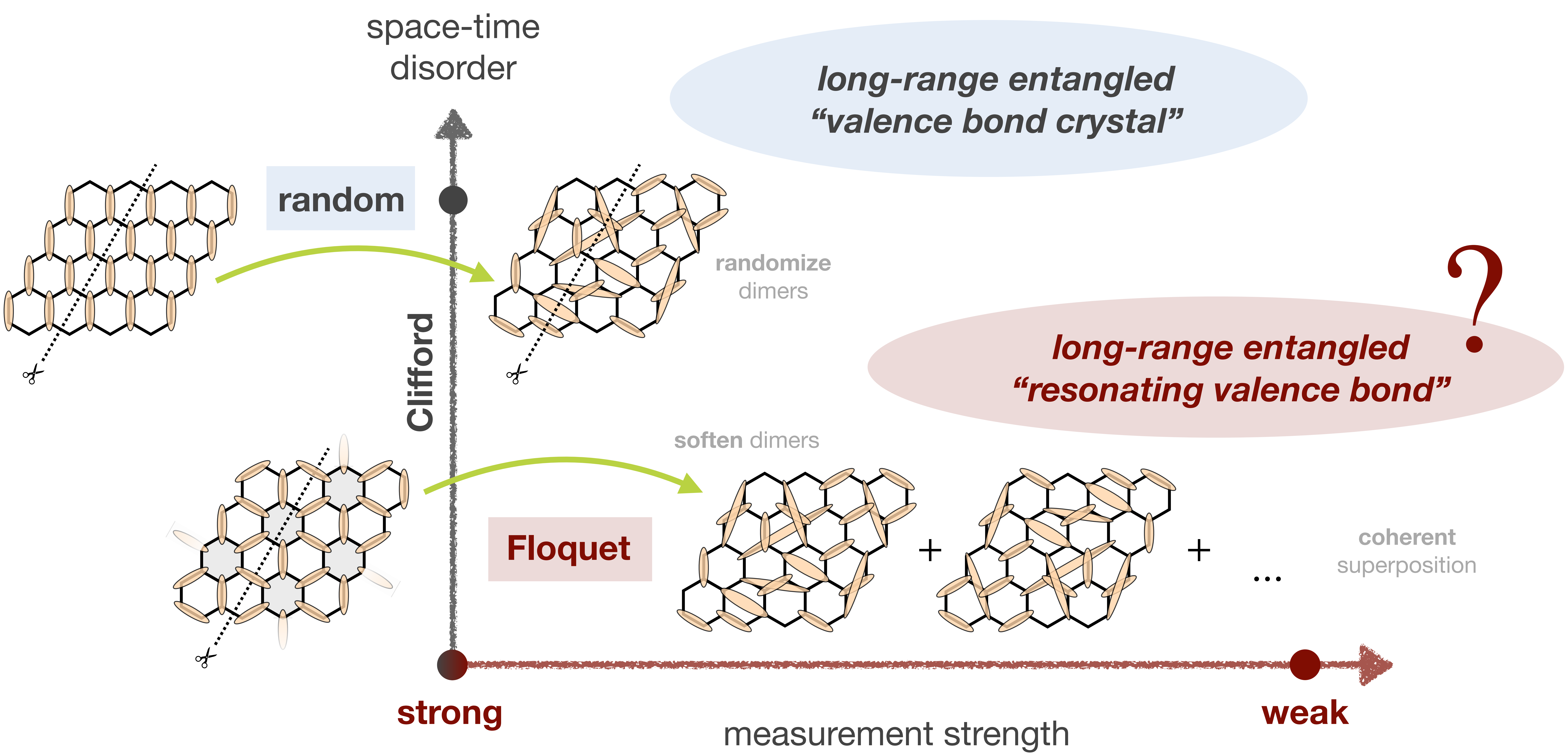
- **Floquet** dynamics
- **two-qubit** Pauli operators
- quantum **error correcting** code
- **two logical qubits**



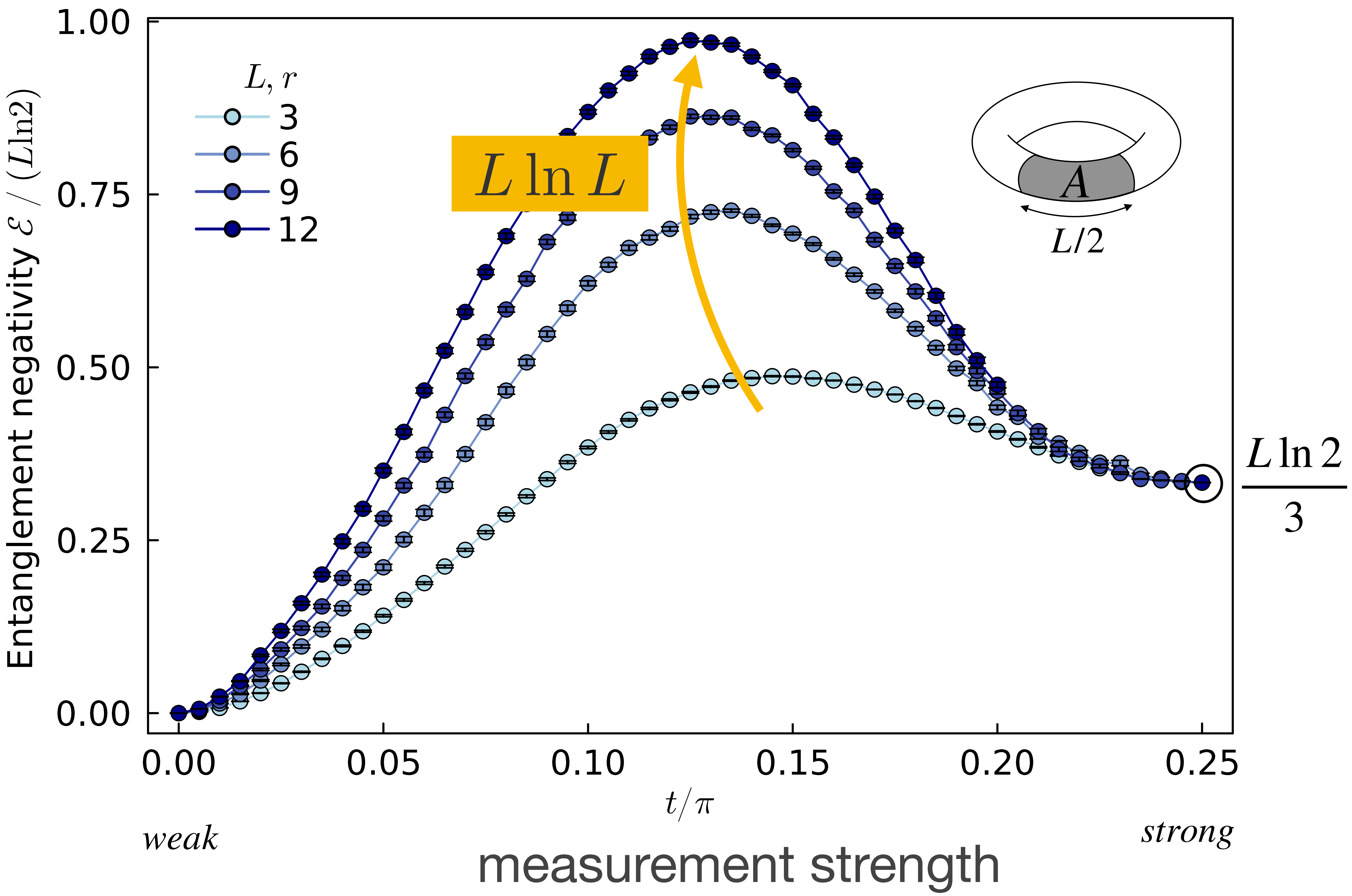
What happens when you turn stabilizers
from projective measurements into **weak measurements?**

Hastings & Haah, Quantum **5**, 564 (2021)

measurement-induced Majorana dimer crystals & RVB states



Majorana liquid



- fermionic **entanglement negativity**
  - definition: response under ***partial*** **time reversal**
  - property: **distill out thermal entropy**
  - diagnose: **mixed state entanglement**

  Shapourian, Shiozaki, Ryu, 2017

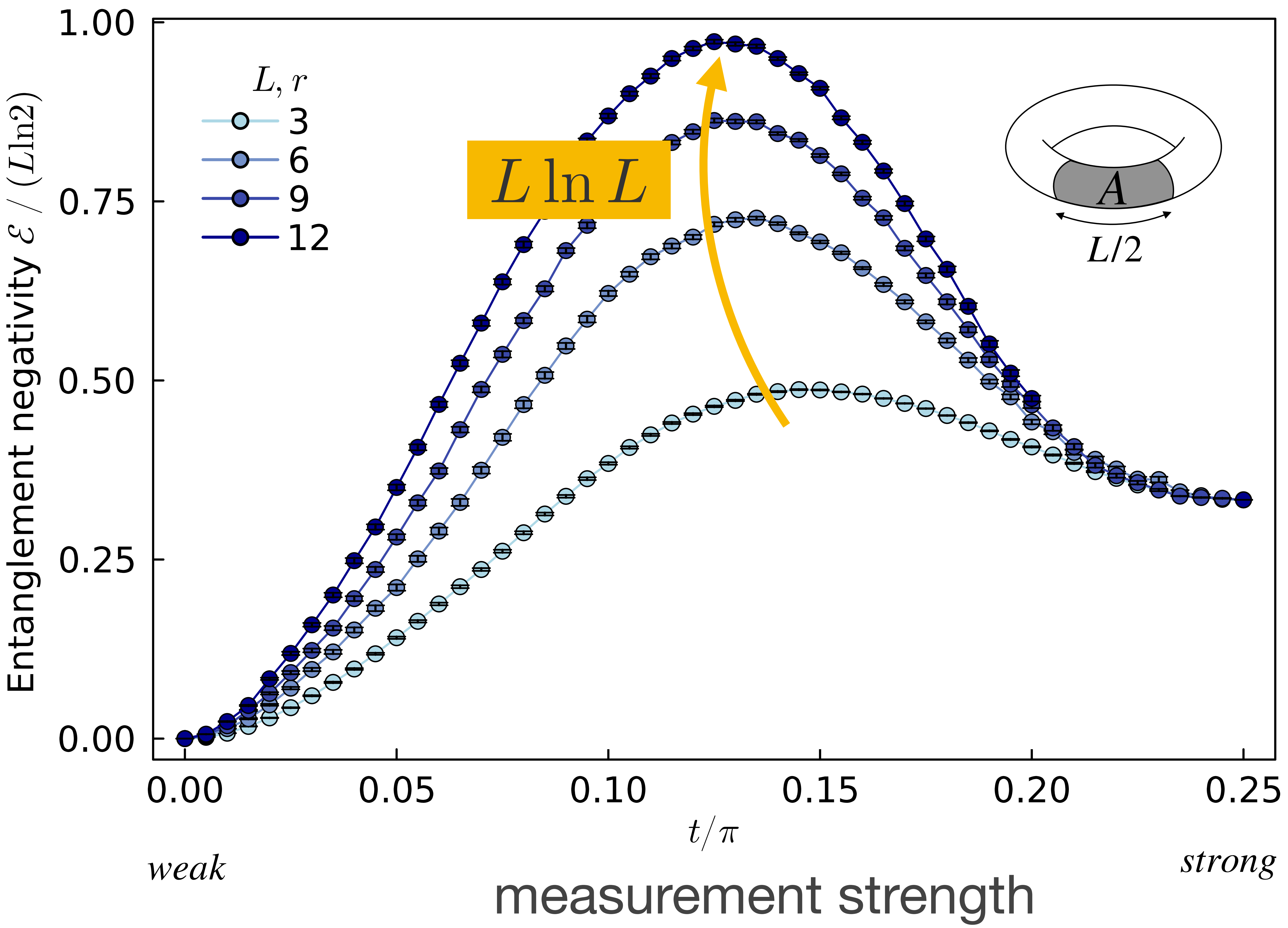
$$\mathcal{E} = \sum_{\mathbf{su}} p_{\mathbf{su}} \cdot \ln || \rho_{\mathbf{su}}^{R_A} ||_1$$

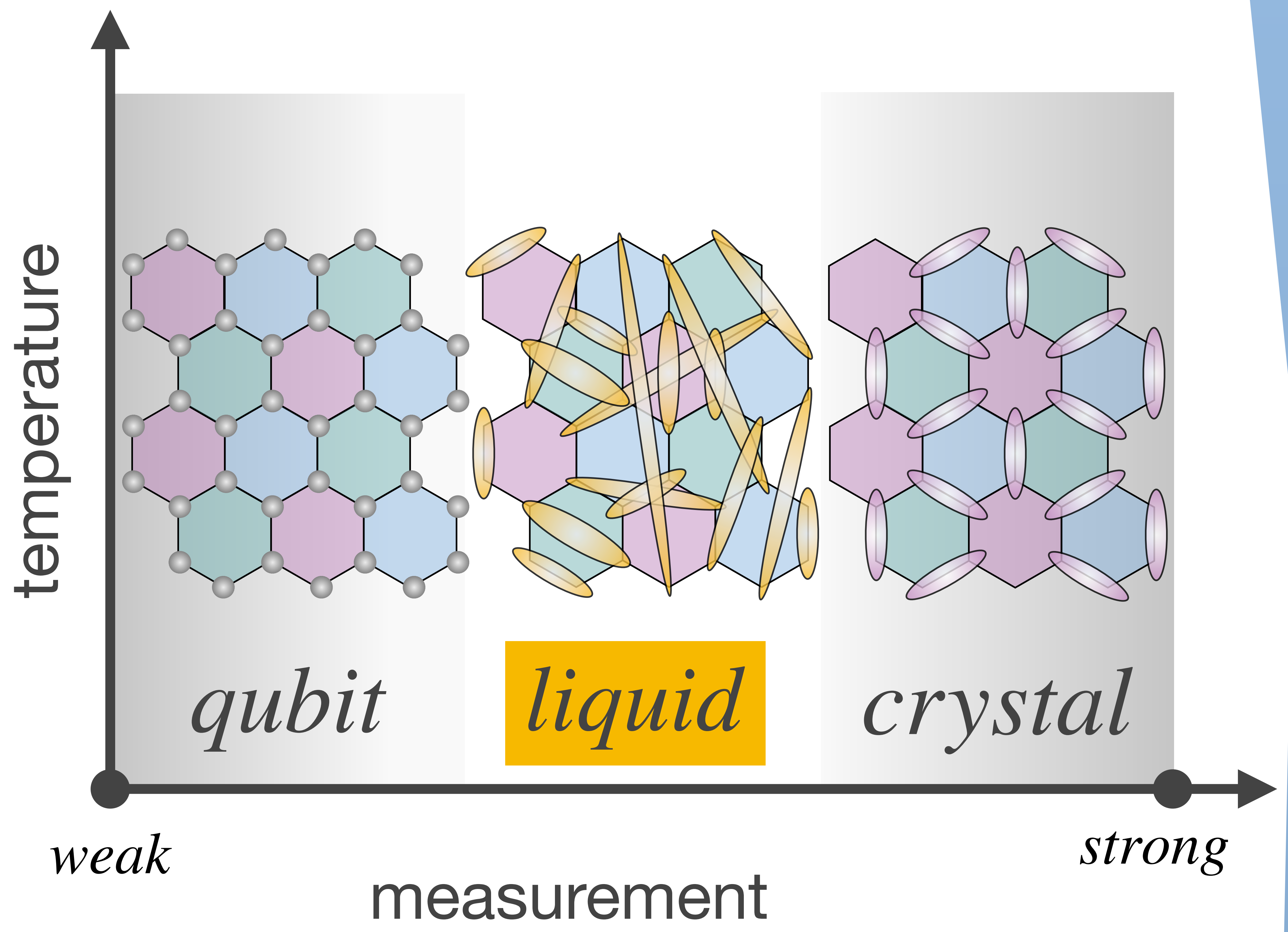
- **entanglement phase transition**

  stable fermion phase with $\mathbf{L} \ln \mathbf{L}$ **entanglement**

  Fava, Piroli, Swann, Bernard, Nahum, NLσM, 2023

Majorana liquid





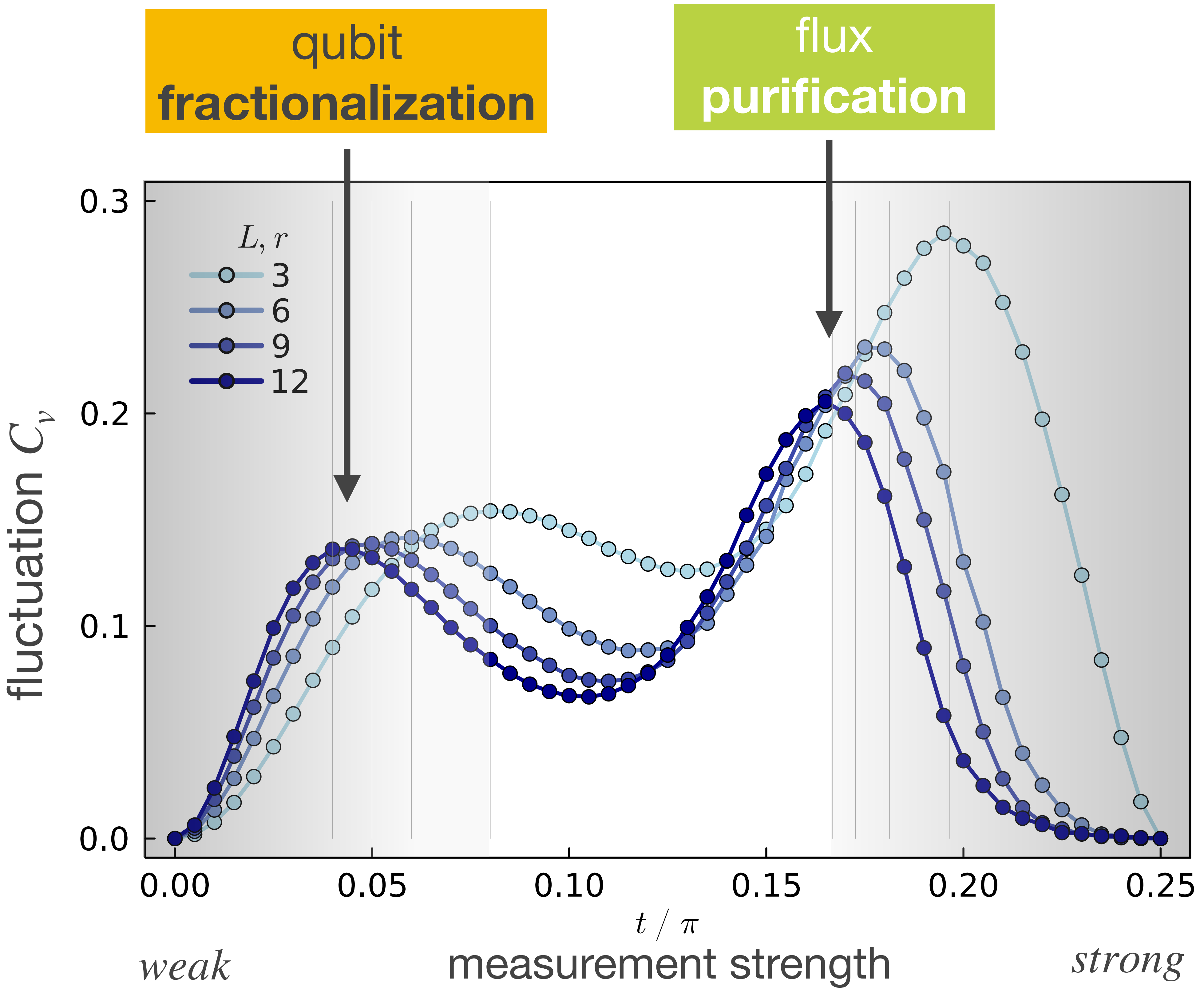
synthetic fractionalization – double-peaks

circuit depth

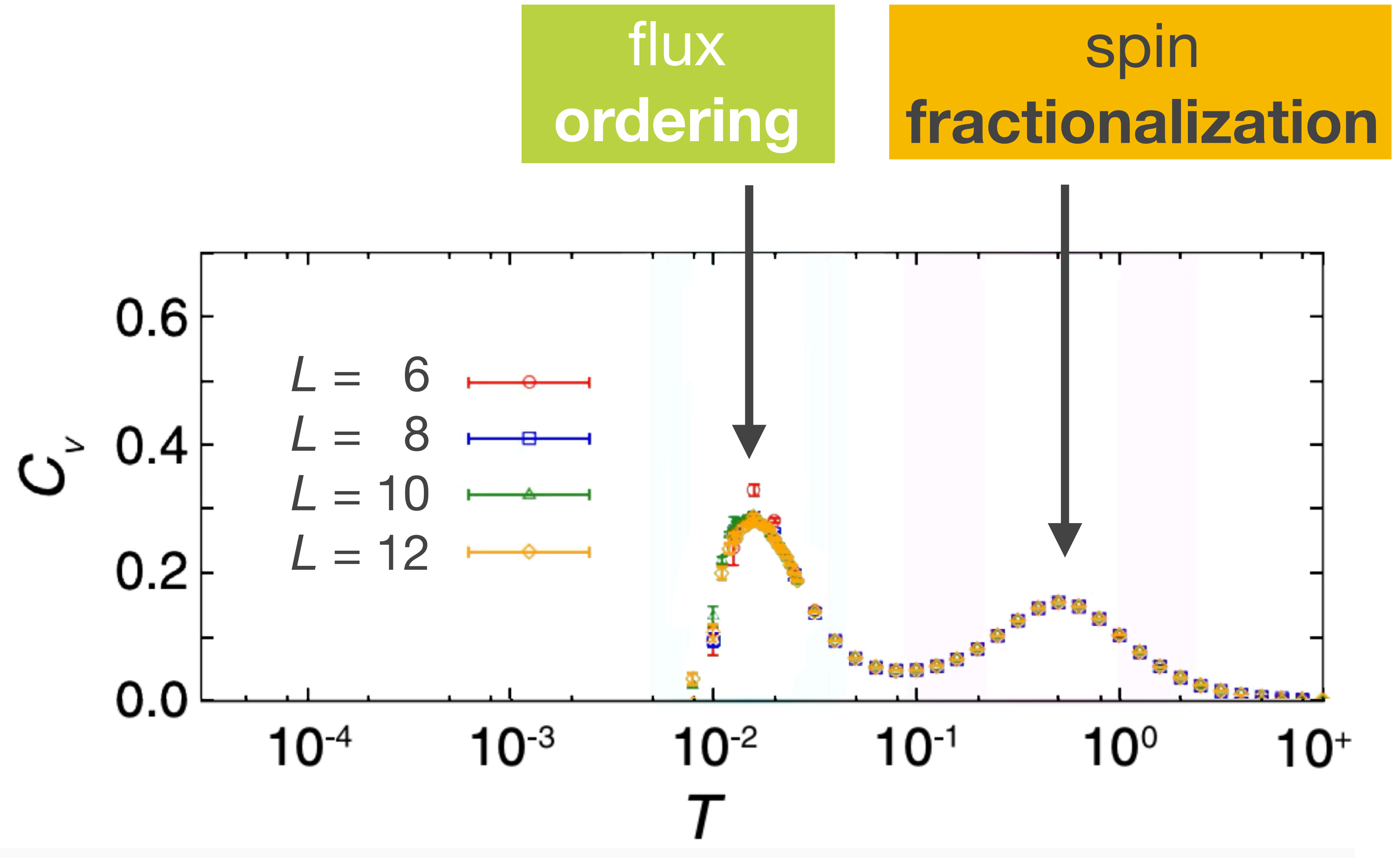
$$\rho_{\mathbf{su}} \propto \exp\left(-\frac{\beta}{4}\mathbf{c}H_{\mathbf{su}}\mathbf{c}\right)$$

effective Hamiltonian

weak measurement-only circuit



Hamiltonian at finite temperature



Nasu, Udagawa, Motome, 2014

PRL **113**, 197205 (2014) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2014

Vaporization of Kitaev Spin Liquids

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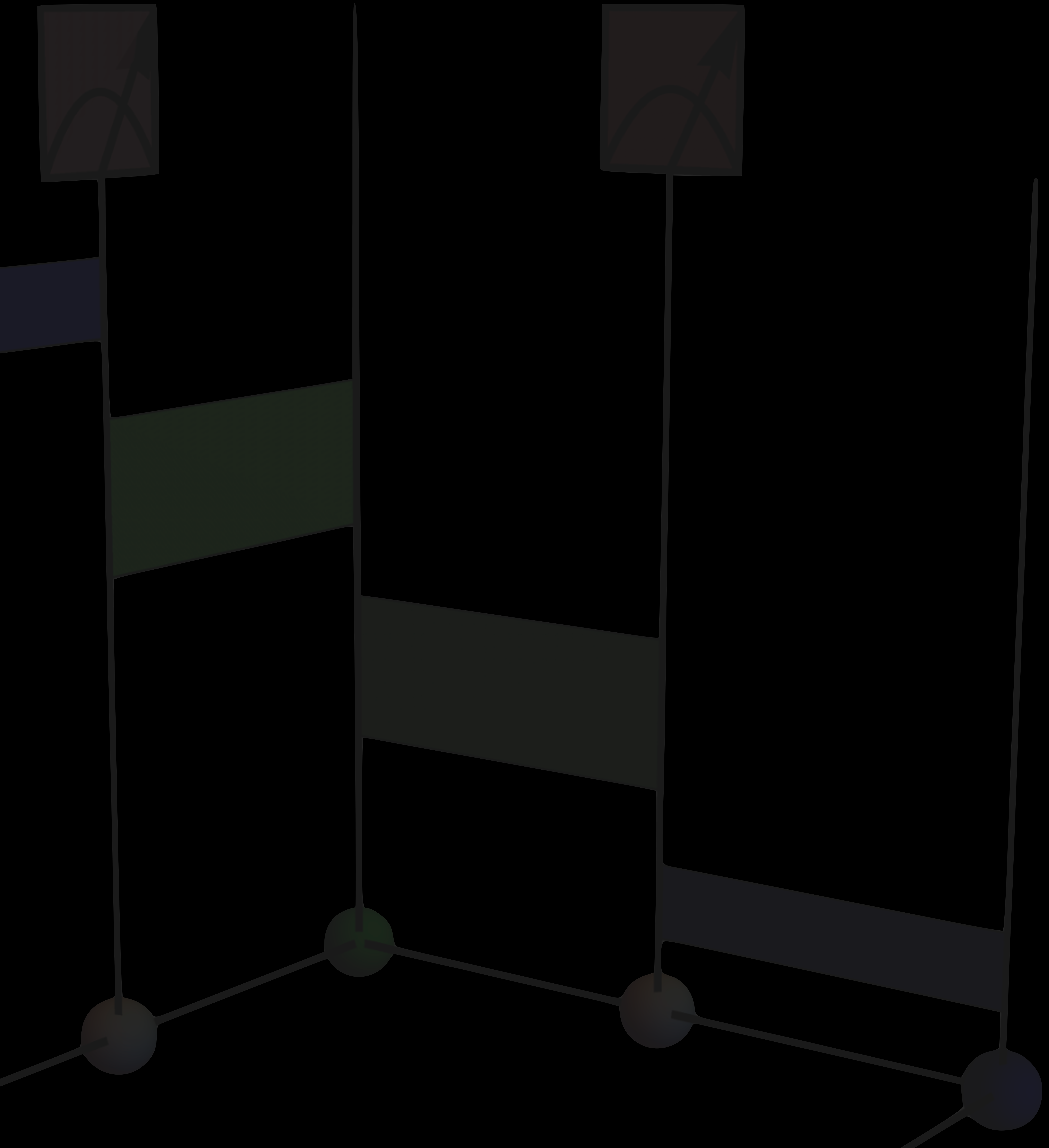

summary

Hamiltonian vs. monitored dynamics

Hamiltonian dynamics

- **equilibrium** dynamics of **isolated** systems
- quantum **ground states**
- **area-law entanglement** structures
- macroscopic entanglement (spin liquids)
- finite-temperature **fractionalization**

measurement dynamics

- **out-of-equilibrium** dynamics of **open** systems
- long-time **steady states**
- **plethora of entanglement** structures
- macroscopic entanglement (spin liquids)
- weak-measurement **fractionalization**