Majorana metals and quantum spin liquids

Topology and entanglement conference
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PRB **89**, 235102 (2014)
more forthcoming ...

Collaborators

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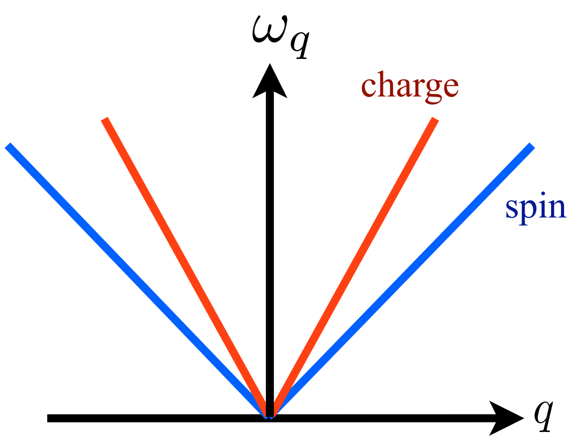
Fractionalization

Some of the most interesting phenomena in quantum many-body systems concur with the emergence of quasiparticles with fractional quantum numbers.

spin-charge separation

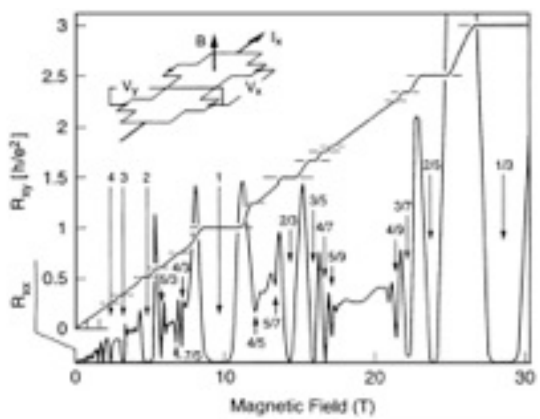
1D metals



1D

electron fractionalization

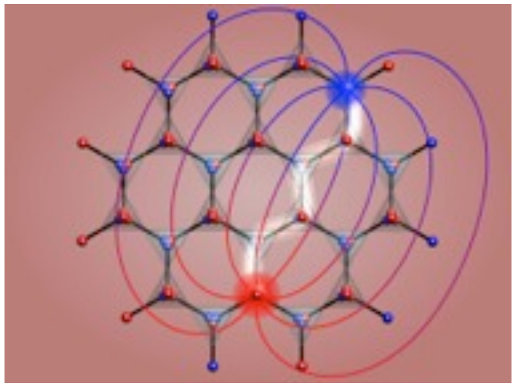
Quantum Hall liquids



2D

magnetic monopoles

Spin ice



3D

What this talk is going to be about

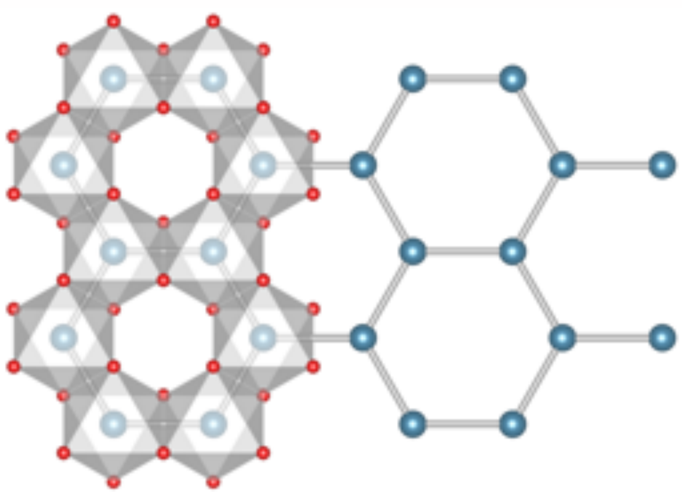
- We have found a family of **exactly solvable SU(2) spin-1/2 models**
  - in which the elementary excitations are fractionalized, i.e. **Majorana fermions** (spinons) and **Z_2 gauge fields** (visons)
  - in which the Majoranas take on their own lifes and form
    - a **Majorana metal** with full Majorana Fermi surface
    - a **topological semi-metal**

- The models are **Kitaev models** on 3D tri-coordinated lattices which are dubbed the **hyperoctagon lattice** and the **hyperhoneycomb lattice.**

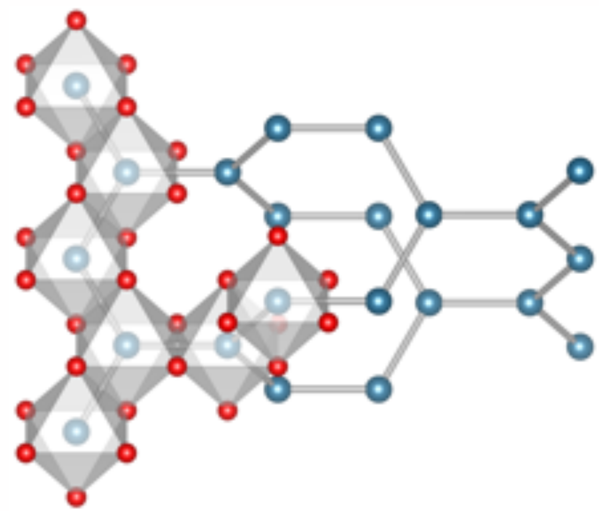
- Our original motivation to look into this is rooted in the physics of certain
  spin-orbit entangled Iridates.

family of Li_2IrO_3 compounds

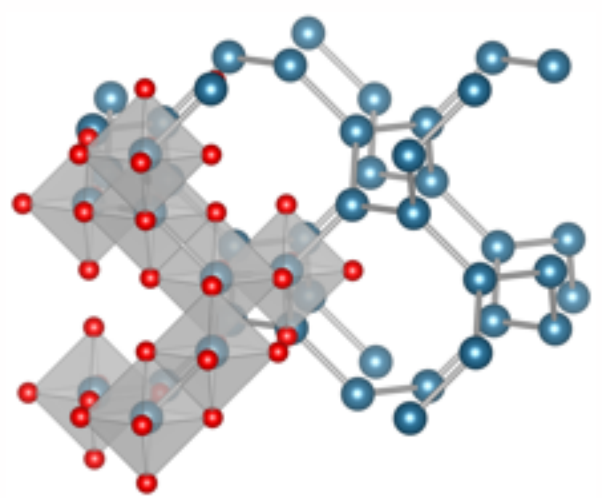
hexagonal layers



hyper-honeycomb

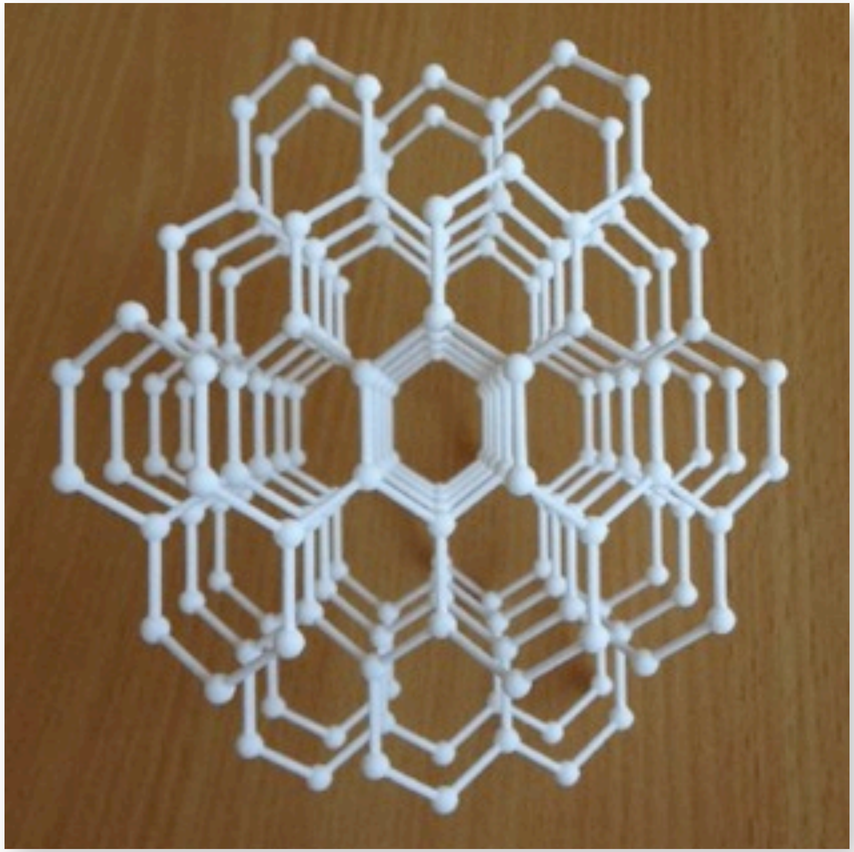


hyper-octagon

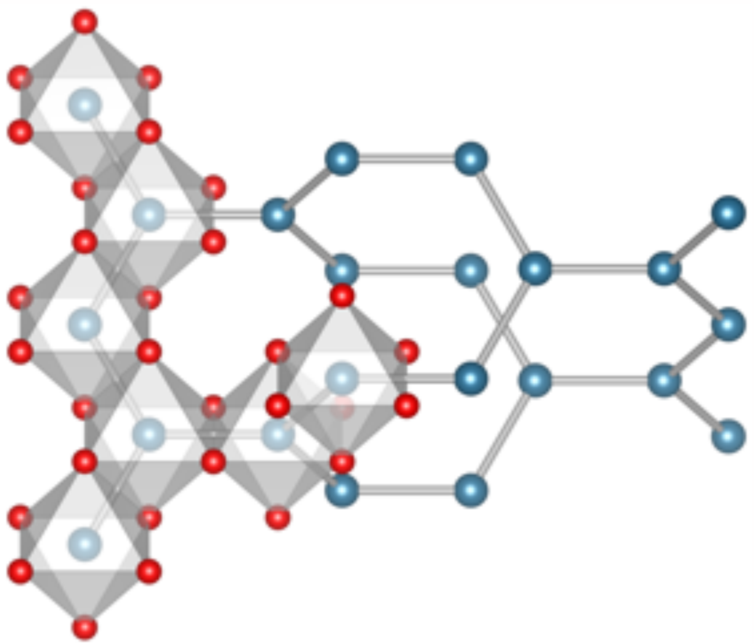


The hyperhoneycomb lattice

β-Li_2IrO_3



3D prints @ www.shapeways.com/designer/trebst



novel crystalline form of Li_2IrO_3

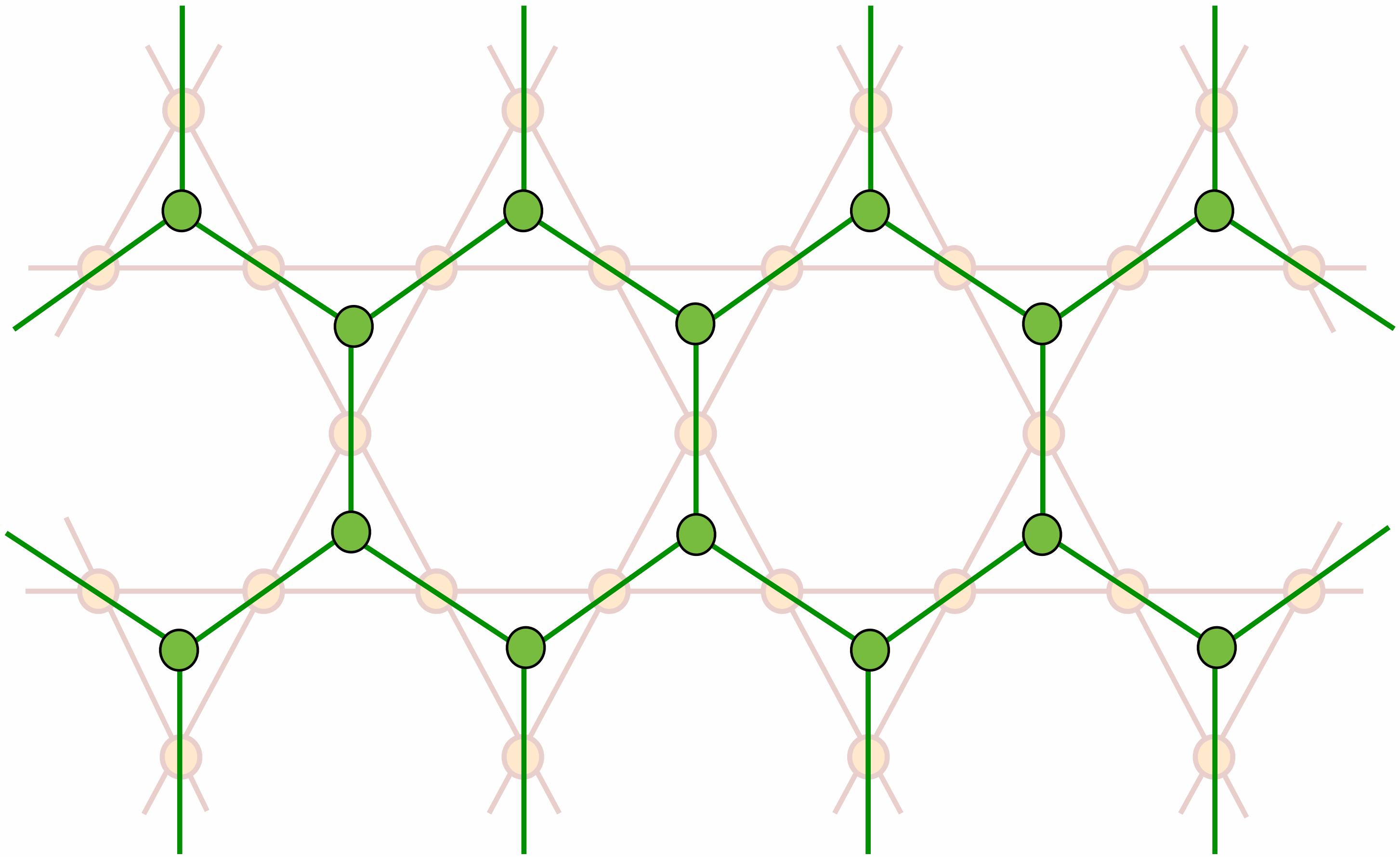
Hide Takagi's group, summer '13 arXiv:1403.3296

James Analytis's group, spring '14 arXiv:1402.3254

truly **3D tricoordinated** Ir lattice

space group **Fddd** (no. 70)

Medial and premedial lattices



kagome lattice

medial lattice

premedial lattice

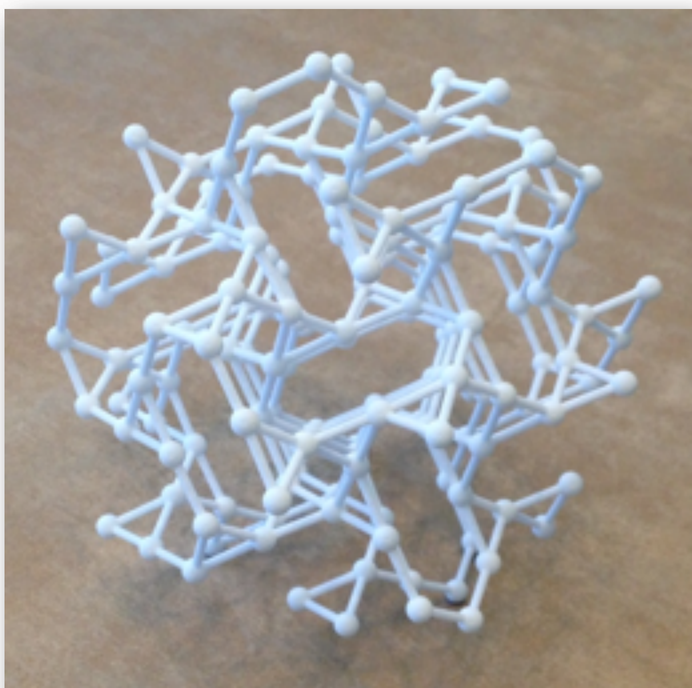
honeycomb lattice

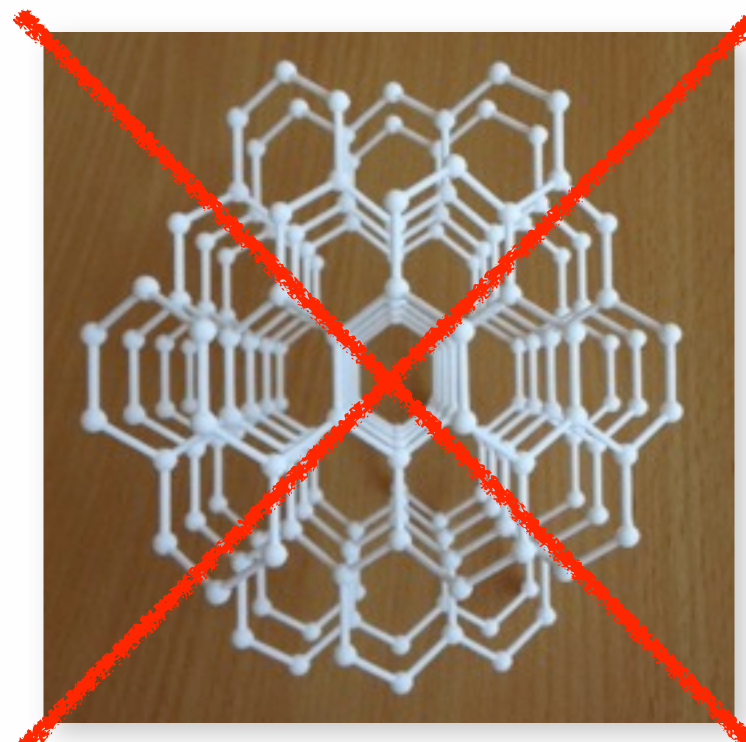
Medial and premedial lattices

hyperkagome

medial lattice
of triangles

hyperhoneycomb





The hyperkagome is **chiral**.

Square-octagon projection.

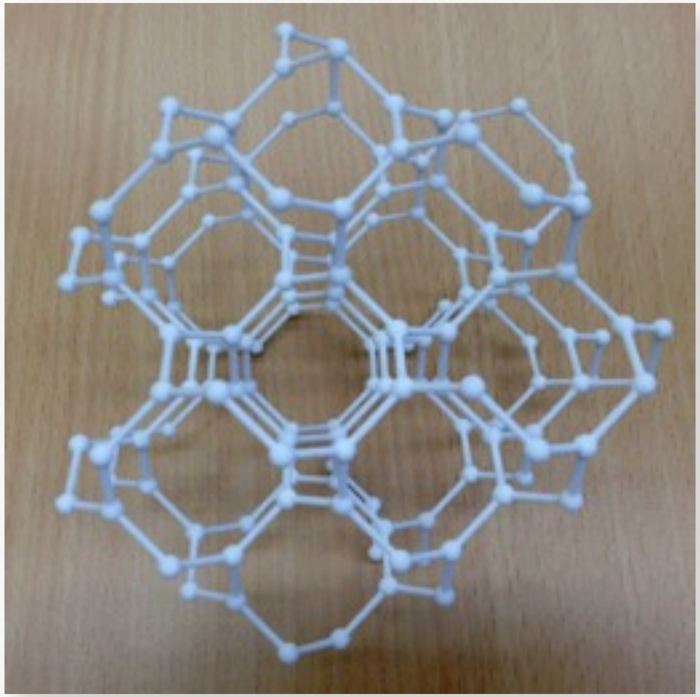
The hyperhoneycomb is **not chiral!**

Medial and premedial lattices

medial lattice of triangles

hyperkagome → hyperoctagon

square-octagon projection

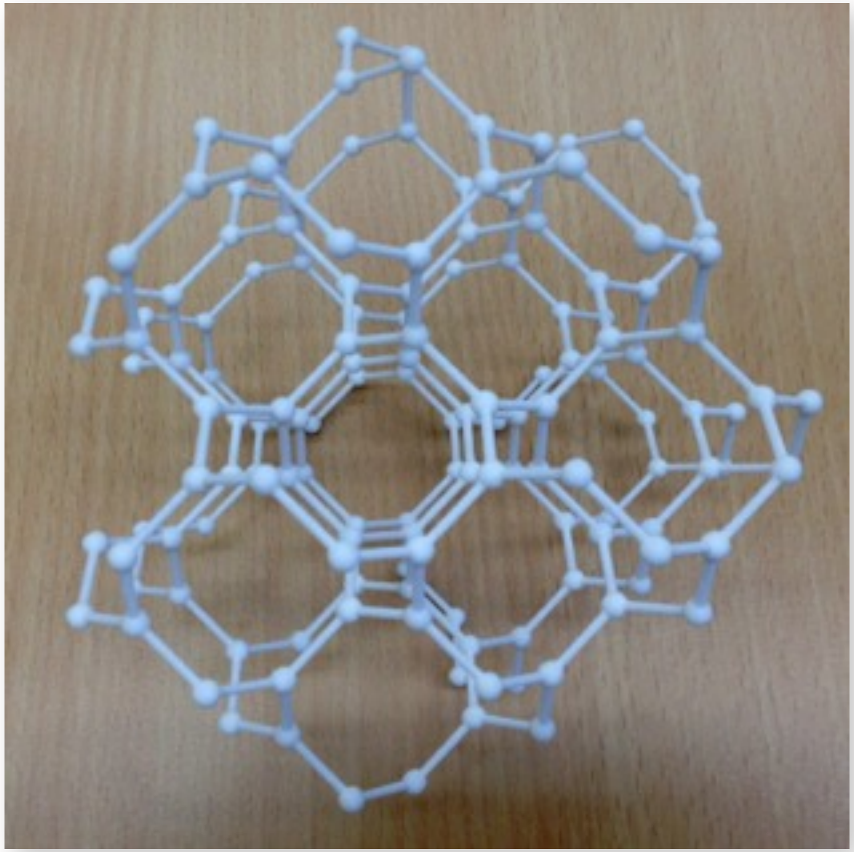


✓

square-octagon projection

The hyperoctagon lattice

γ-Li_2IrO_3



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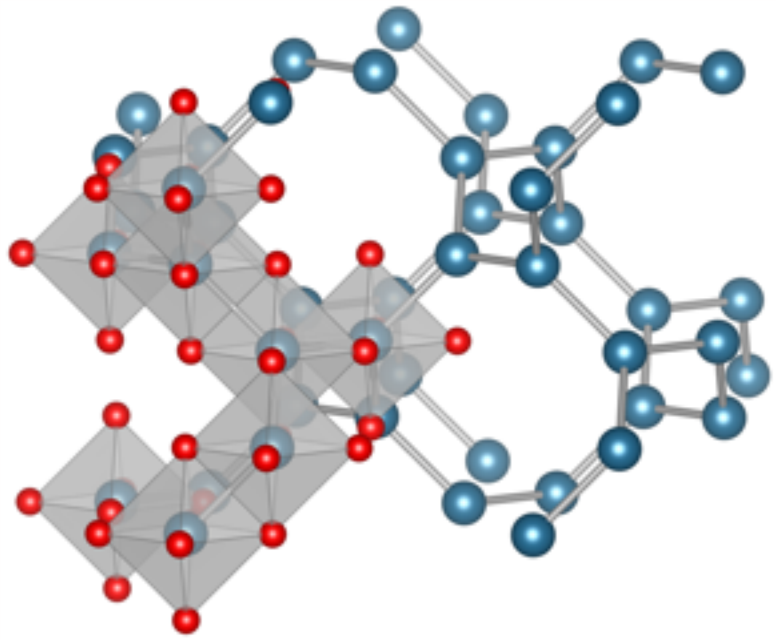
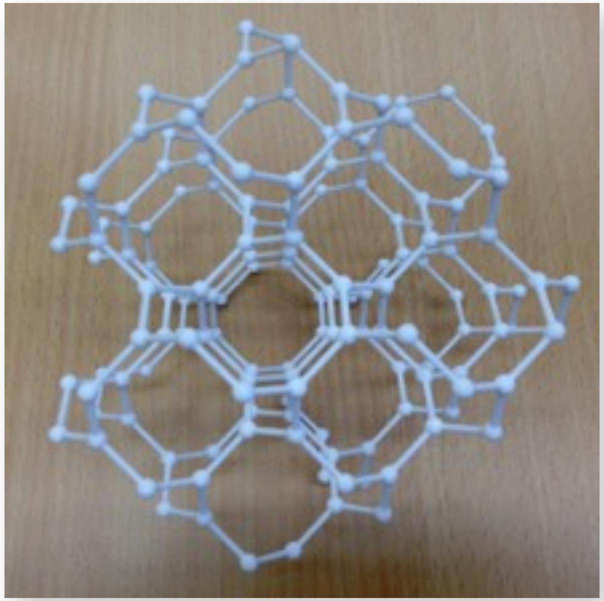


truly **3D tricoordinated** Ir lattice

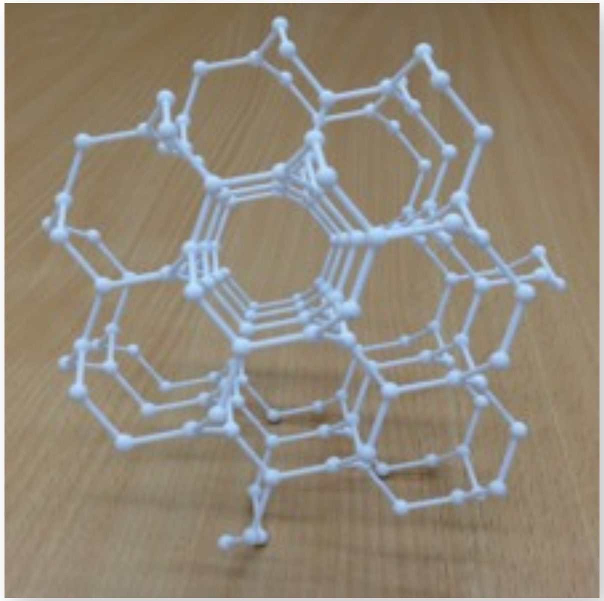
space group **$I4_132$** (no. 214)

possibly third crystalline form of Li_2IrO_3

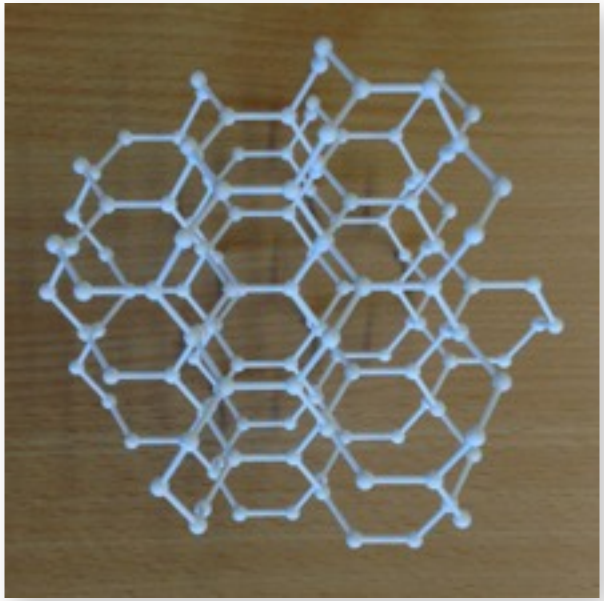
Hyperoctagon – space group symmetries



four-fold skew symmetry



three-fold symmetry



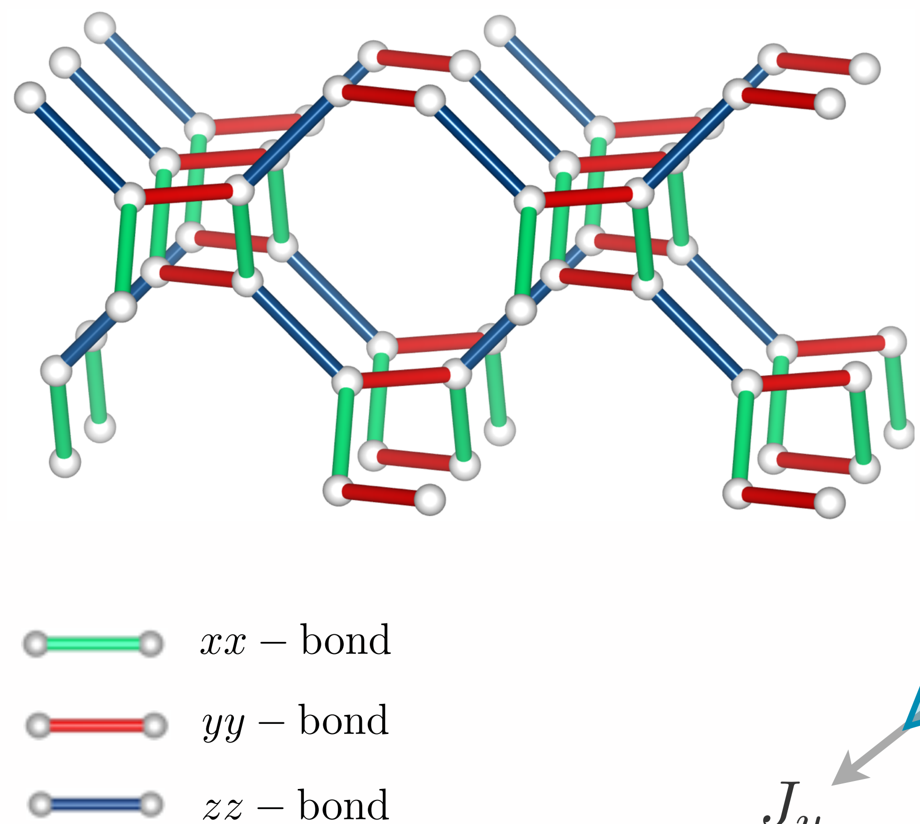
two-fold symmetry

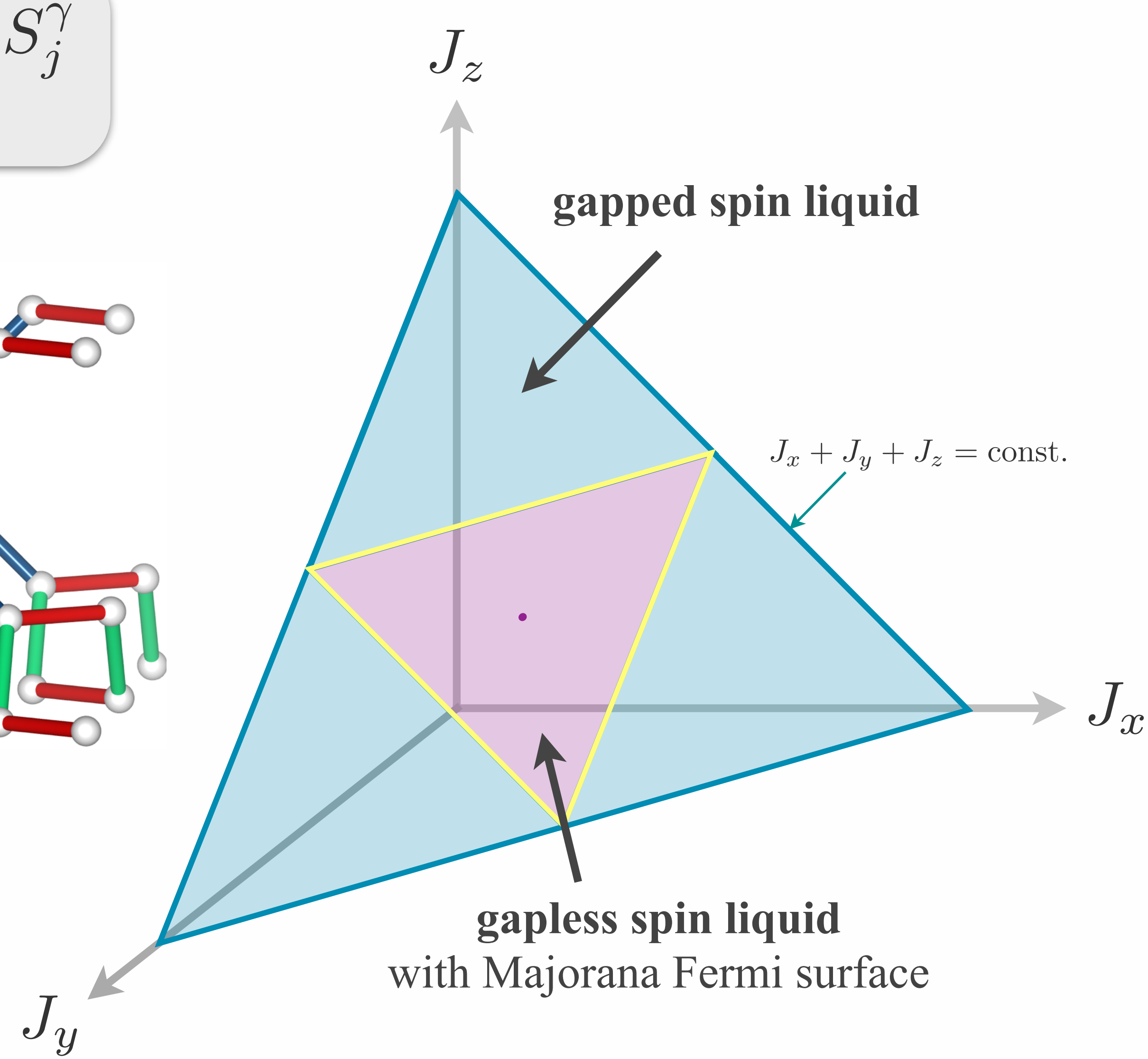
↓

space group **$I4_132$** (no. 214)

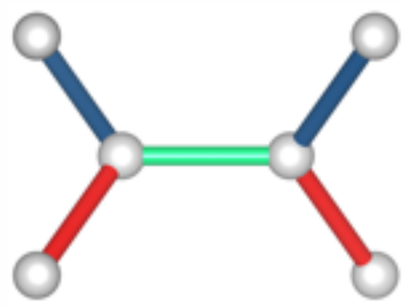
A three-dimensional Kitaev model

$$H_{\mathrm{Kitaev}} = \sum_{\gamma-\mathrm{links}} J_\gamma S_i^\gamma S_j^\gamma$$

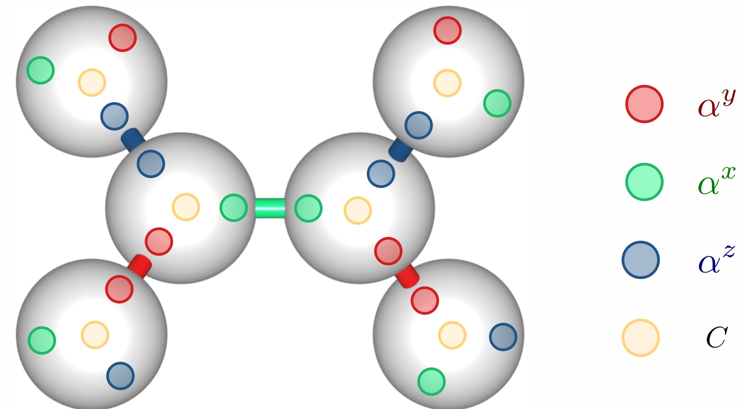




Let’s solve this model



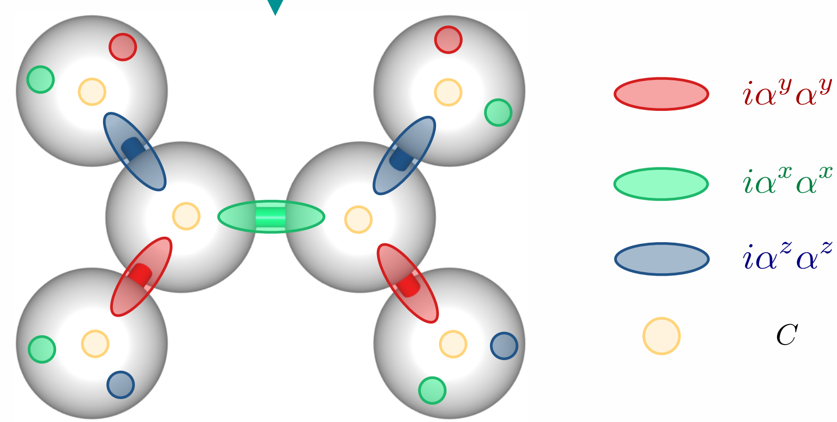
split spins →



↓ **regroup** Majoranas

Step 1: Represent spins in terms of four **Majorana fermions** $\sigma^\alpha = i a^\alpha c$

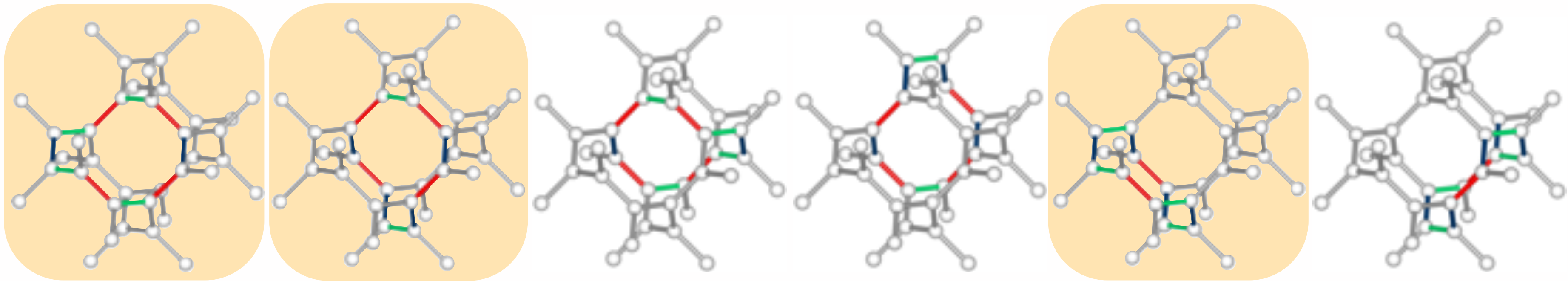
Step 2: Bond operators $\hat{u}_{jk} = i a_j^\alpha a_k^\alpha$ realize an emergent **Z_2 gauge field**



Physics of the Z_2 gauge field

Z_2 gauge field is **static** due to presence of additional conserved quantities

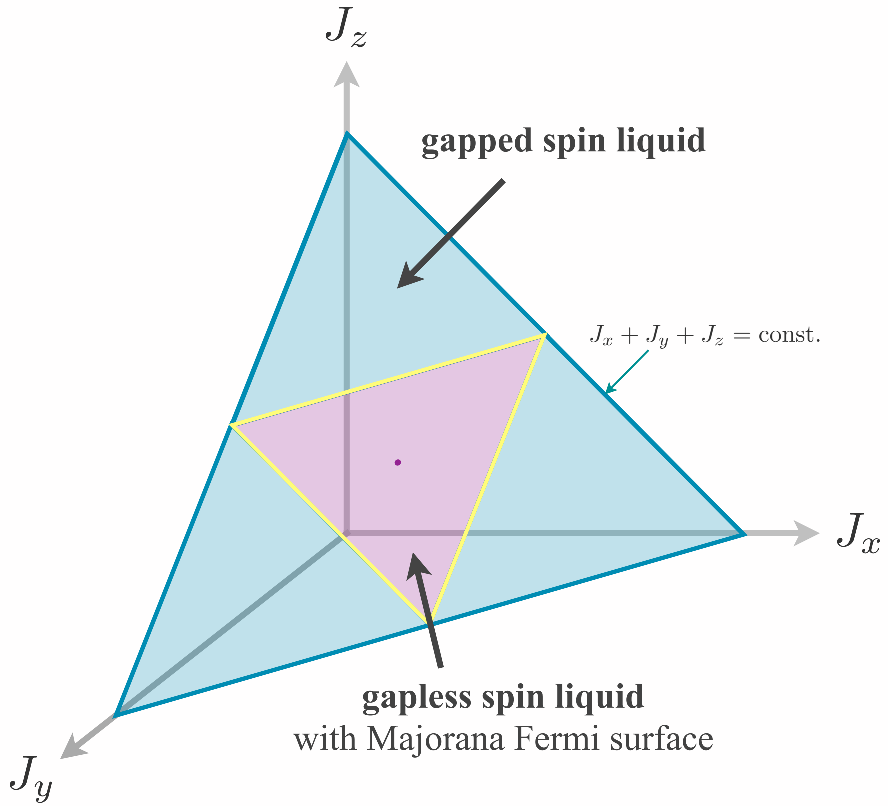
Six fundamental **loop operators** (per unit cell) $W_l = \prod_{\langle \alpha,\beta \rangle \in l} \sigma_\alpha^{\gamma_{\alpha\beta}} \sigma_\beta^{\gamma_{\alpha\beta}}$

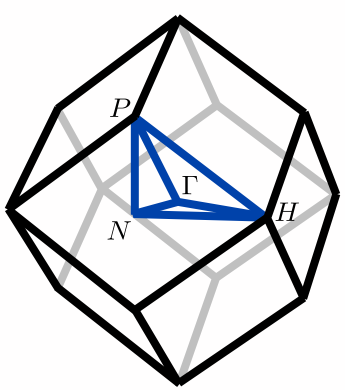


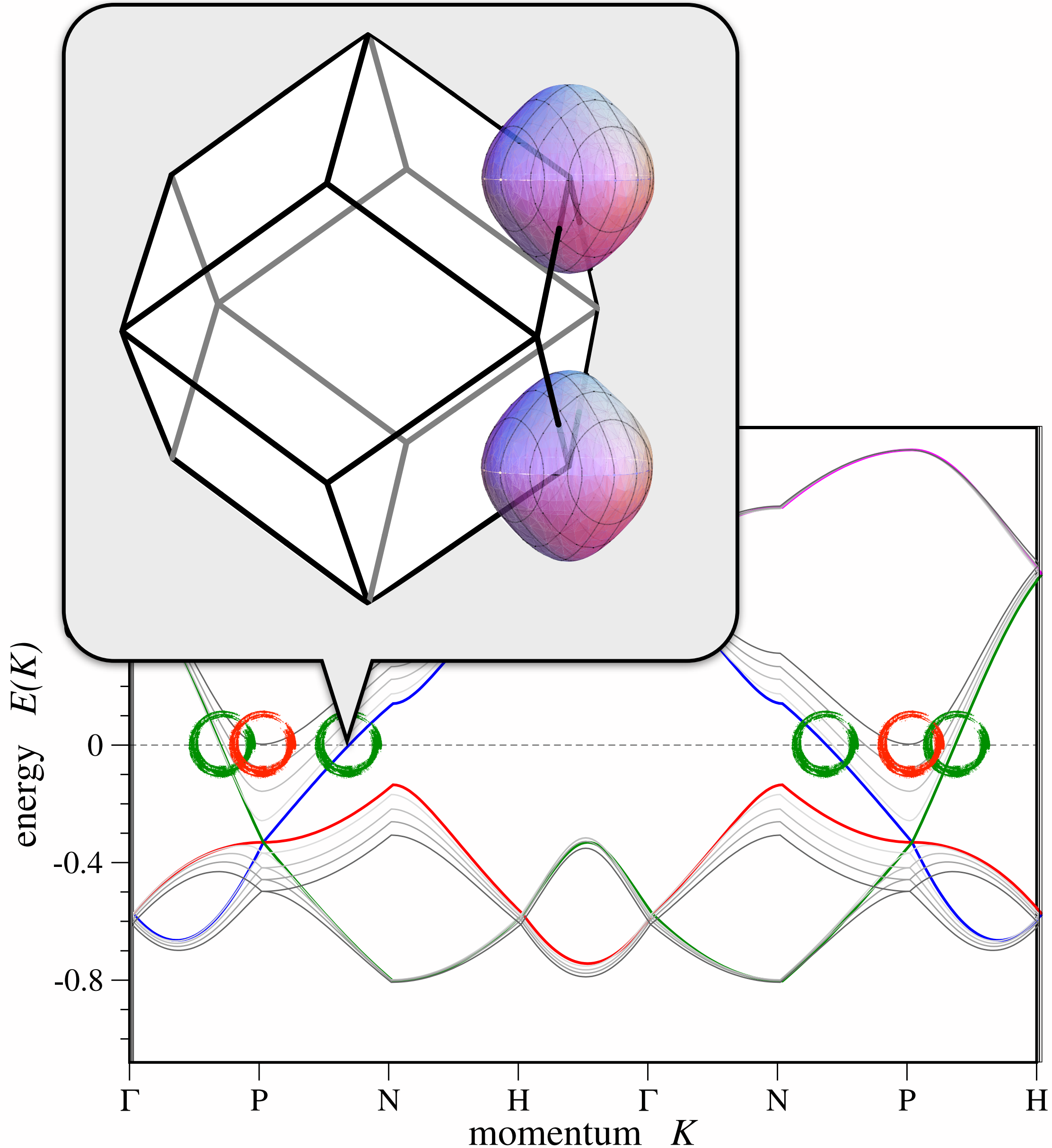
only two loop operators per unit cell are linearly independent

$$W_{l_a} W_{l_b} W_{l_c} = 1$$

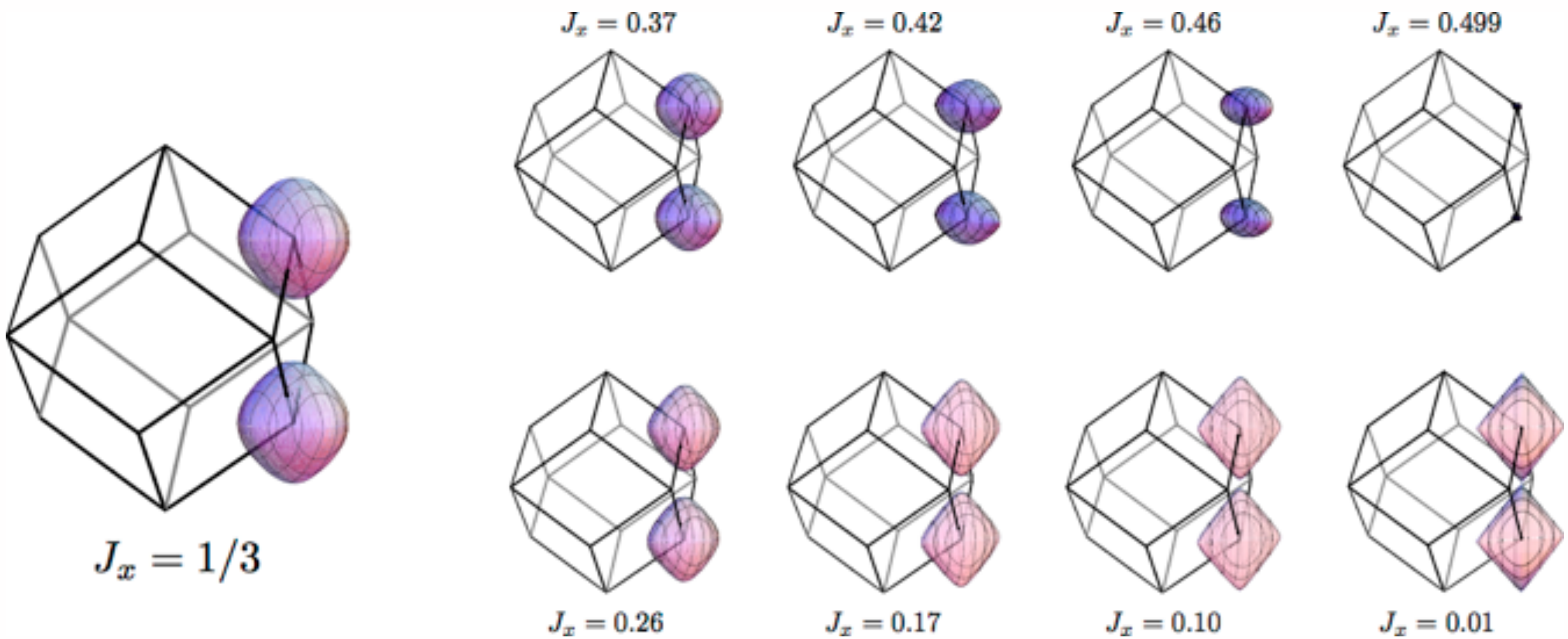
Majorana Fermi surface





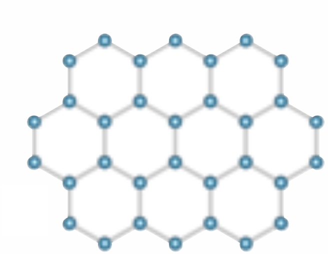


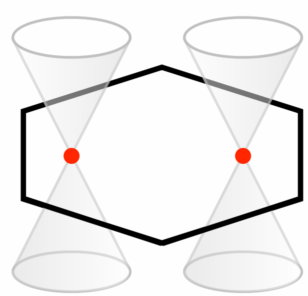
Majorana Fermi surface



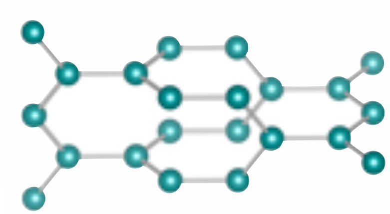
The hyperoctagon Kitaev model exhibits a full **two-dimensional Majorana Fermi surface**.

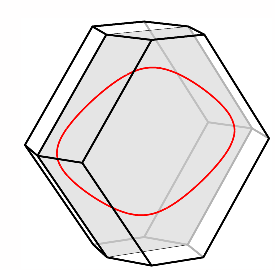
honeycomb – Fermi **points**





hyperhoneycomb – Fermi **lines**

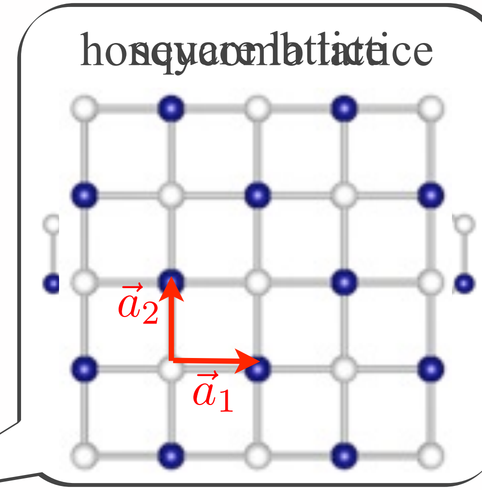


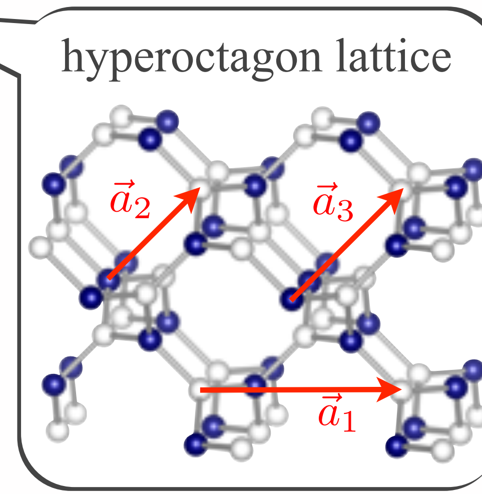


Why is the Fermi surface stable?

Symmetry relations

Particle-hole symmetry	$\epsilon(\mathbf{k}) = -\epsilon(-\mathbf{k})$
Sublattice symmetry	$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k} - \mathbf{k}_0)$
Time-reversal symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} + \mathbf{k}_0)$
Inversion symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} + \mathbf{k}_0)$





$\mathbf{k}_0$ is the reciprocal lattice vector
of the translation vector of the sublattice

Why is the Fermi surface stable?

Stability of gapless modes in the **honeycomb** model

$$H = \begin{pmatrix} \mathbf{0} & \boxed{if(\mathbf{k})} \\ -if^\star(\mathbf{k}) & \mathbf{0} \end{pmatrix} \longrightarrow$$

complex-valued function

$$E(\mathbf{k}) = \pm|f(\mathbf{k})|$$

Stability of gapless modes in the **hyperhoneycomb** model

$$H = \begin{pmatrix} \mathbf{0} & \boxed{\mathbf{A}} \\ \mathbf{A}^\dagger & \mathbf{0} \end{pmatrix} \longrightarrow$$

complex matrix

$$E(\mathbf{k}) = \pm|\lambda_j(\mathbf{k})|$$

Stability of gapless modes in the **hyperoctagon** model

$$H = \begin{pmatrix} 0 & & \boxed{\mathbf{A}} \\ & \ddots & \\ \mathbf{A}^\dagger & & 0 \end{pmatrix} \longrightarrow$$

generic band Hamiltonian with TR symmetry

However, there is only a **single** Majorana zero-mode at a given momentum.

Spin liquid with a spinon Fermi surface

Recasting our result in the language of spin liquids,
what we have found is the first exactly solvable microscopic model
of a spin liquid with a **spinon Fermi surface**.

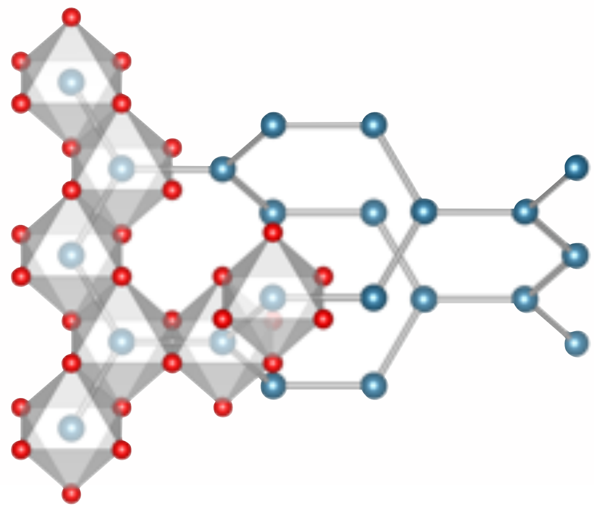
Spin-spin correlations $\langle S_i^z S_j^z \rangle$ decay exponentially.

Bond-bond energy correlations $\langle (S_i^z)^2 (S_j^z)^2 \rangle$ exhibit
algebraic divergence on spinon Fermi surface.

U(1) spin liquid	$C(T) \propto T \ln(1/T)$	$\gamma = C/T$ diverges
Z_2 spin liquid with spinon Fermi surface	$C(T) \propto T$	$\gamma = C/T$ constant
Z_2 spin liquid with spinon Fermi line	$C(T) \propto T^2$	$\gamma = C/T$ vanishes

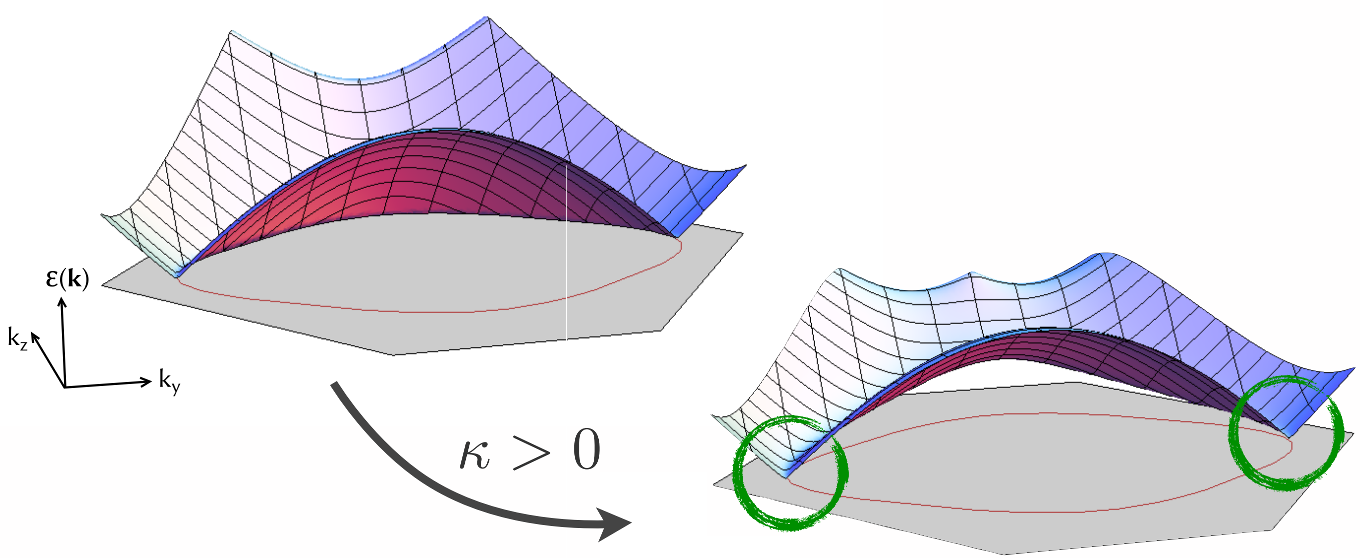
A Weyl spin liquid

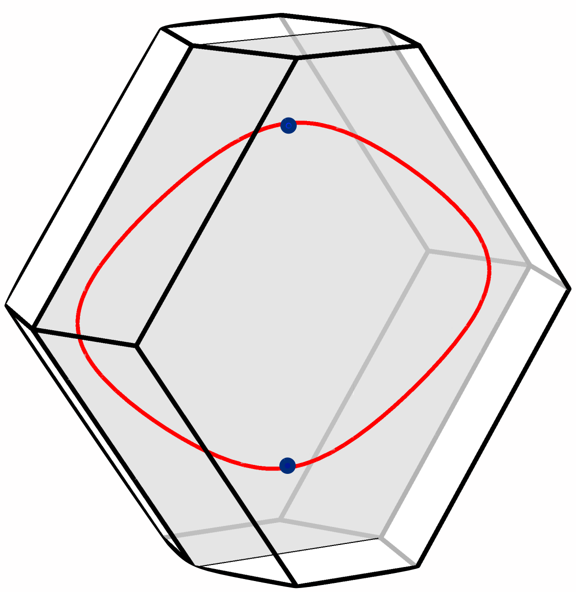
hyper-honeycomb



$$H = \sum_{\gamma-\text{links}} J_\gamma S_i^\gamma S_j^\gamma - \kappa \sum_{\langle j,k,l \rangle} S_j^\alpha S_k^\beta S_l^\gamma$$

Fermi lines gap out, but 2 **Weyl points** remain
Majorana semi-metal





Weyl nodes + semi-metals

Touching of two bands in 3D is generically **linear**

$$\hat{H} = \vec{v}_0 \cdot \vec{q}\,\mathbb{1} + \sum_{i=1}^{3} \vec{v}_j \cdot \vec{q}\,\sigma_j$$ **Weyl nodes**

Weyl nodes are **sources/sinks of Berry flux**

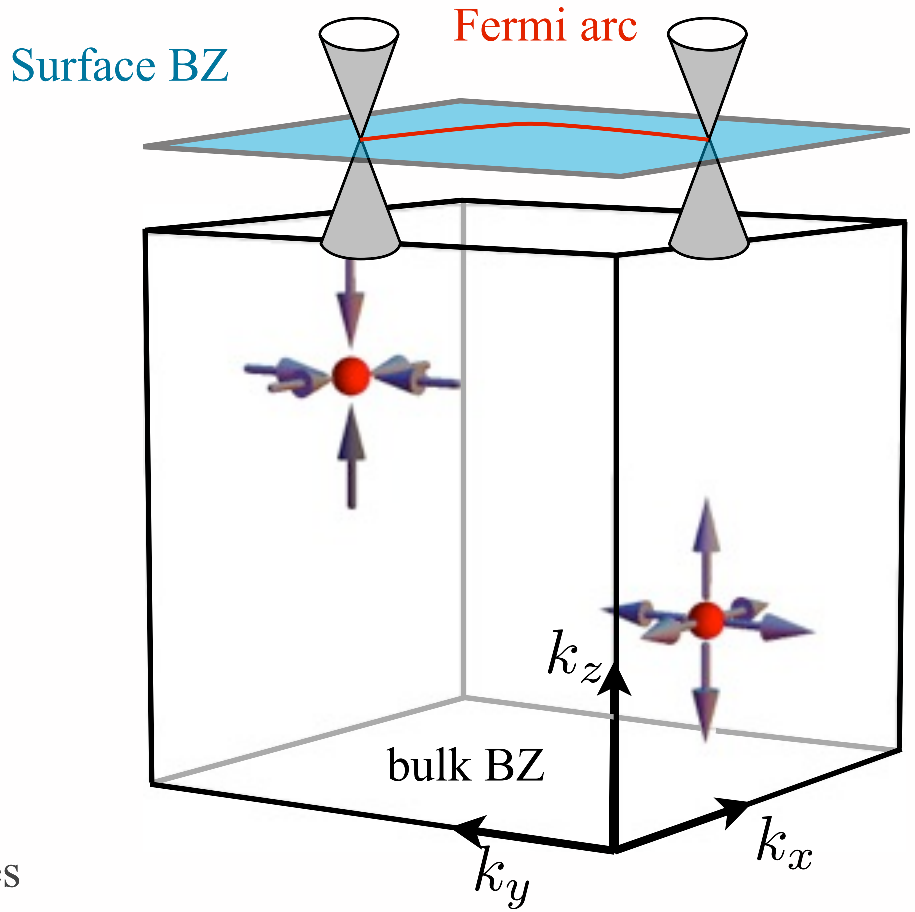
$$\vec{B}_n(\vec{k}) = \nabla_{\vec{k}} \times \left(i \langle n(\vec{k}) | \nabla_{\vec{k}} | n(\vec{k}) \rangle \right)$$

with chirality $\mathrm{sign}[\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)]$

here: Weyl nodes pinned at zero energy!

unusal surface states: **Fermi arcs**

topological semi-metal with **protected** surface states
(metallic cousin of the topological insulator)



Wan et al., PRB **83**, 205101 (2011)

Summary

- We have found a family of **exactly solvable SU(2) spin-1/2 models**
  - in which the elementary excitations are fractionalized, i.e. **Majorana fermions** (spinons) and **Z_2 gauge fields** (visons)
  - in which the Majoranas take on their own lifes and form
    - a **Majorana metal** with full Majorana Fermi surface = a quantum spin liquid with a spinon Fermi surface
    - a **topological semi-metal** = a Weyl spin liquid

- The models are **Kitaev models** on 3D tri-coordinated lattices which are dubbed the **hyperoctagon lattice** and the **hyperhoneycomb lattice.**
- Our original motivation to look into this is rooted in the physics of certain **spin-orbit entangled Iridates**.

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