# Topology driven quantum phase transitions

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**Simon Trebst**
Microsoft Station Q
UC Santa Barbara

**Charlotte Gils**

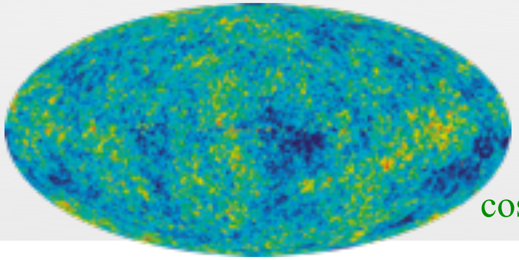
Alexei Kitaev
Andreas Ludwig

Matthias Troyer
Zhenghan Wang

# Topological quantum liquids

## Spontaneous symmetry breaking

- ground state has **less** symmetry than high-$T$ phase
- Landau-Ginzburg-Wilson theory
- **local** order parameter



cosmic microwave background

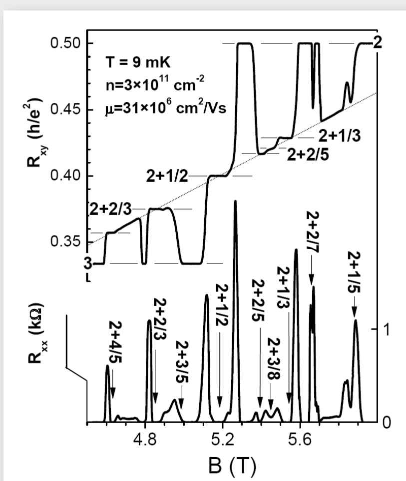
## Topological order

- ground state has **more** symmetry than high-$T$ phase
- degenerate ground states
- **non-local** order parameter
- quasiparticles have fractional statistics = **anyons**

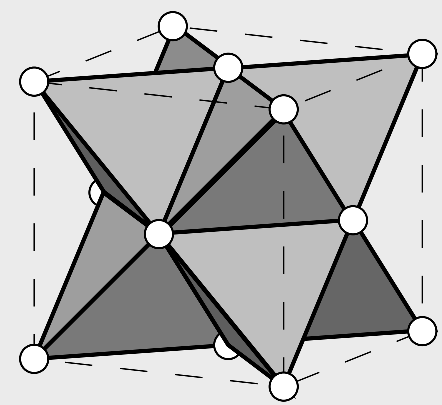
# Topological order

**time reversal symmetry**

**broken** → **quantum Hall liquids**

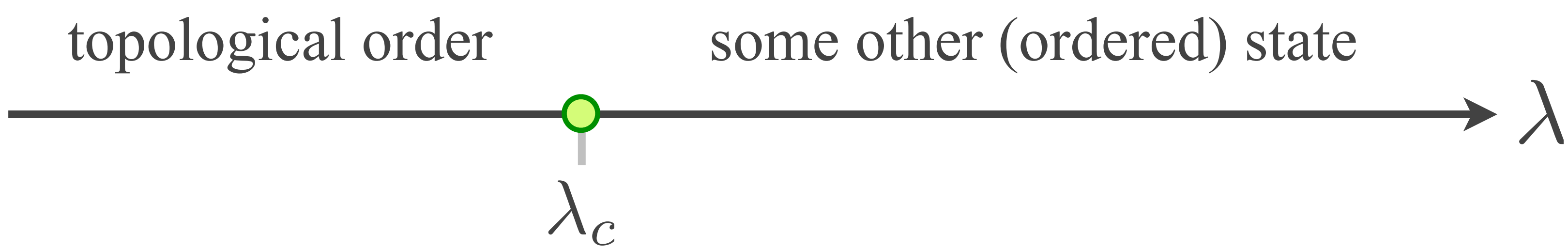



**invariant** → **magnetic materials**



# Quantum phase transitions

time-reversal invariant systems




**How can we describe these phase transitions?**

no local order parameter

↓

no Landau-Ginzburg-Wilson theory

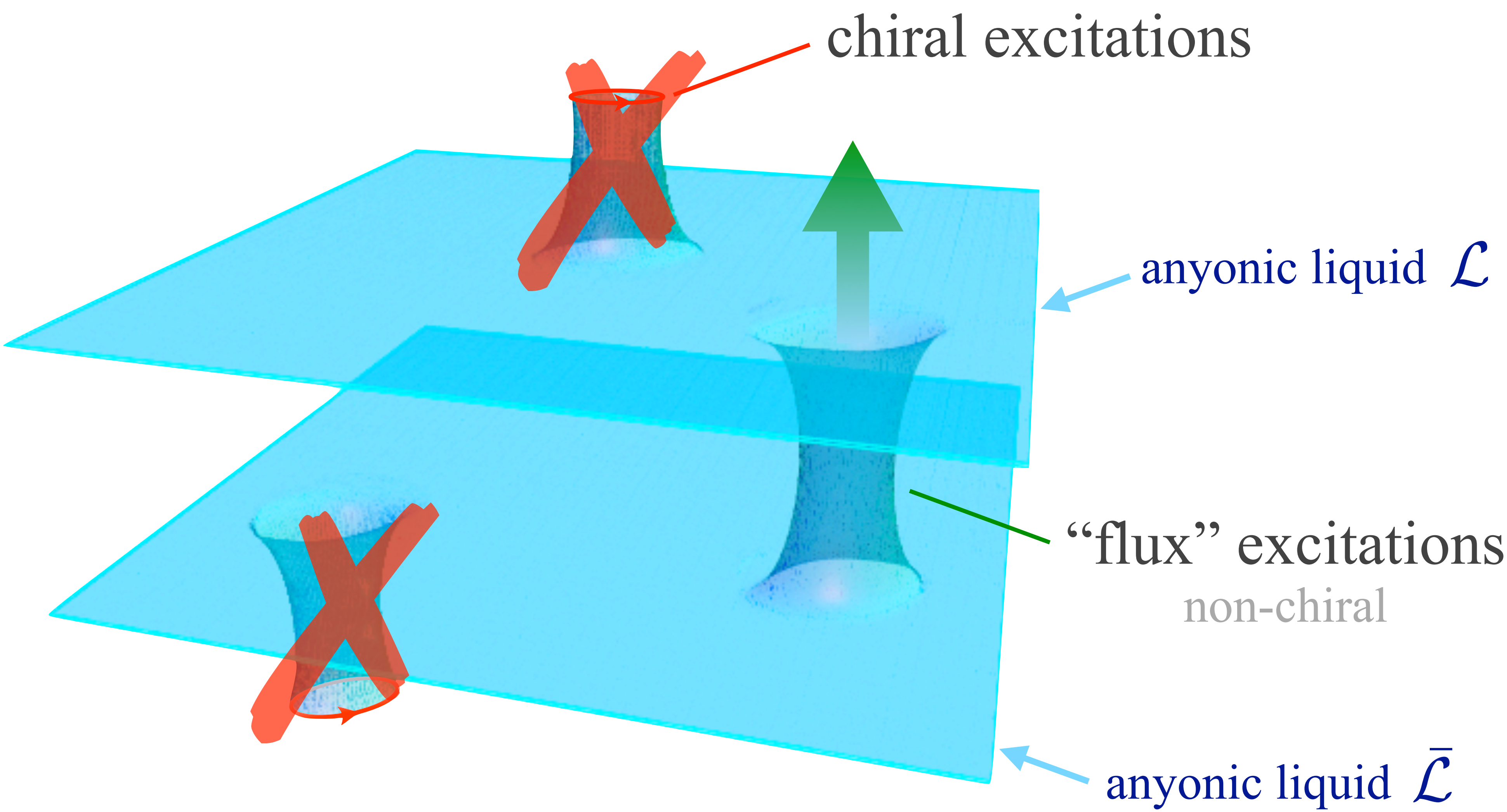
complex field theoretical framework

↓

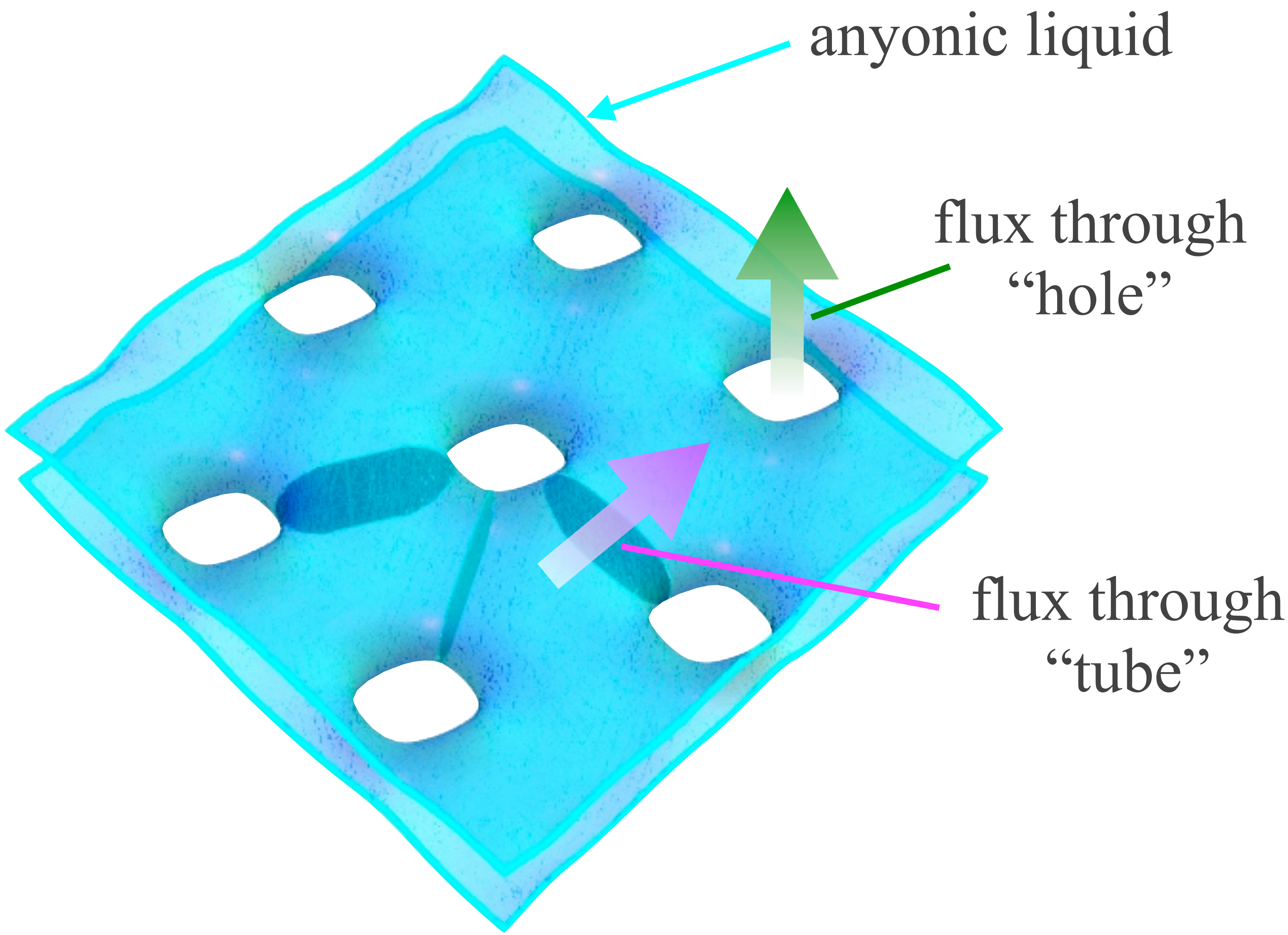
doubled (non-Abelian) Chern-Simons theories

Liquids on surfaces can provide a “topological framework”.

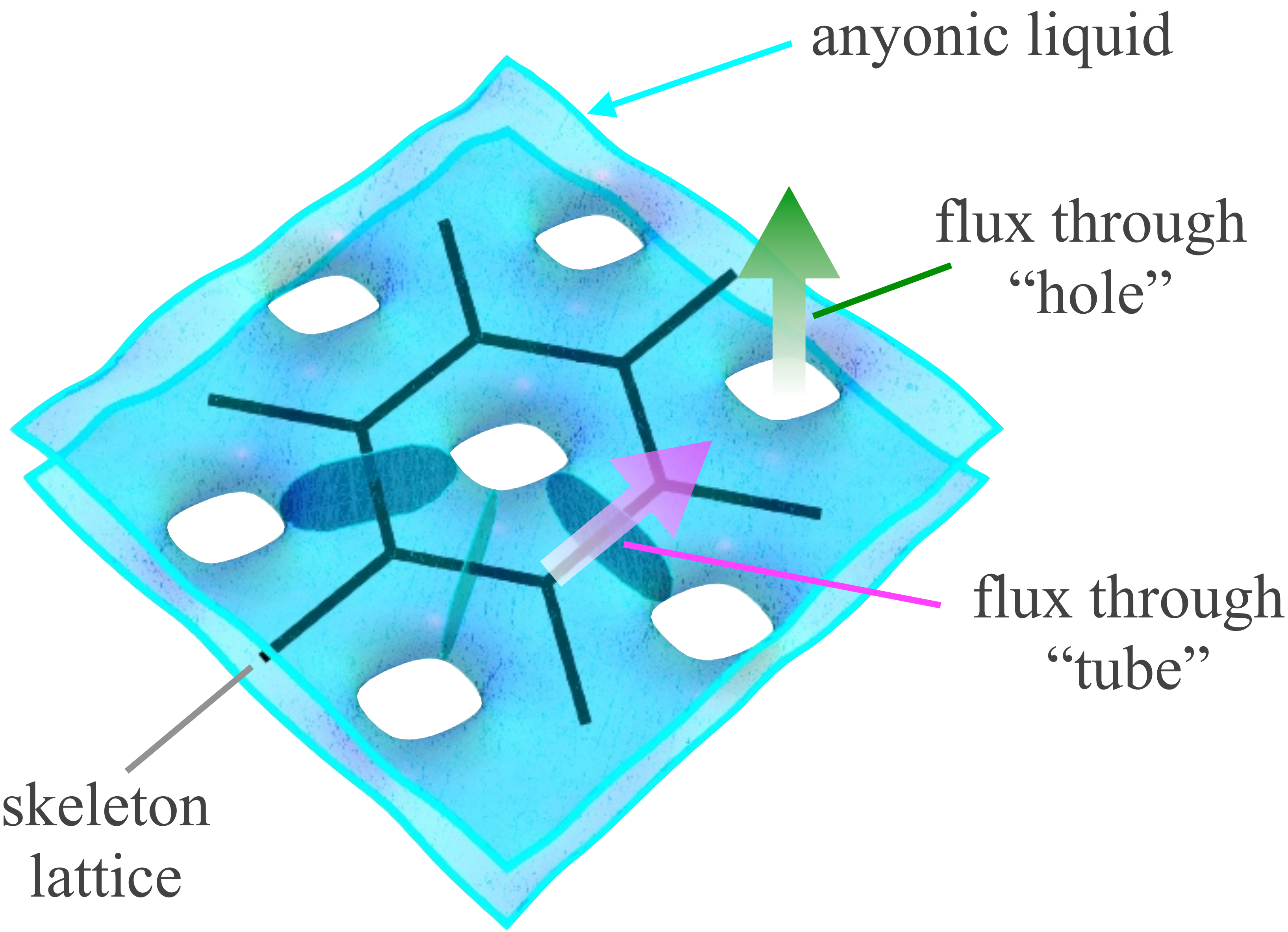
# Time-reversal invariant liquids



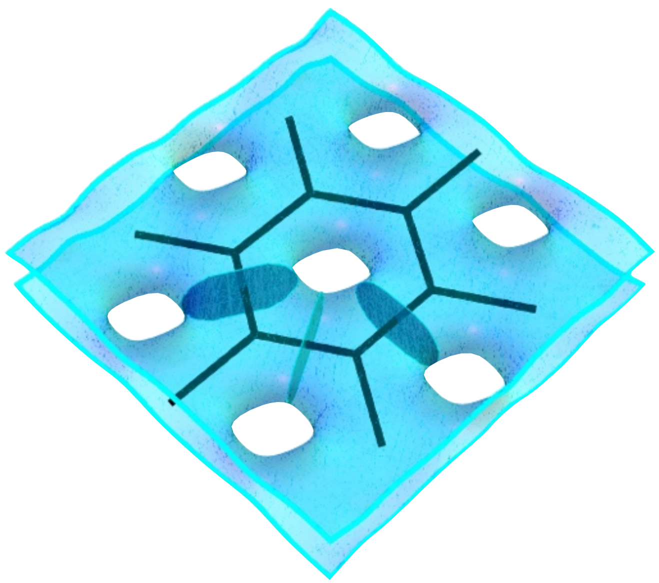
# Flux excitations

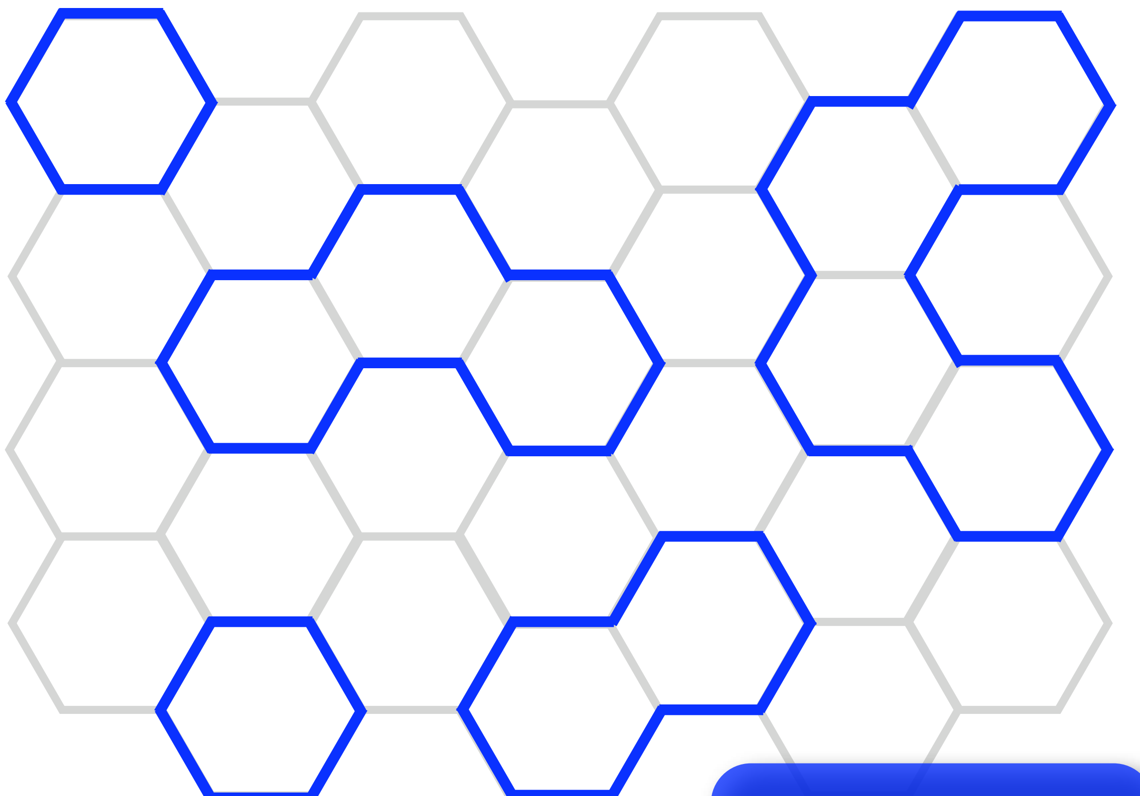


# Flux excitations



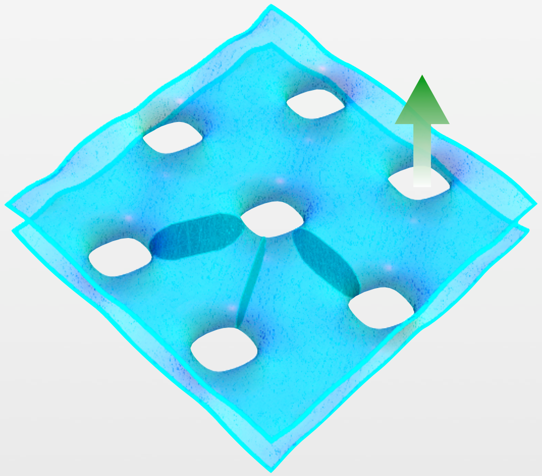
# Abelian liquids → loop gas / toric code






**Semion** fusion rules $s \times s = 1$

# Extreme limits



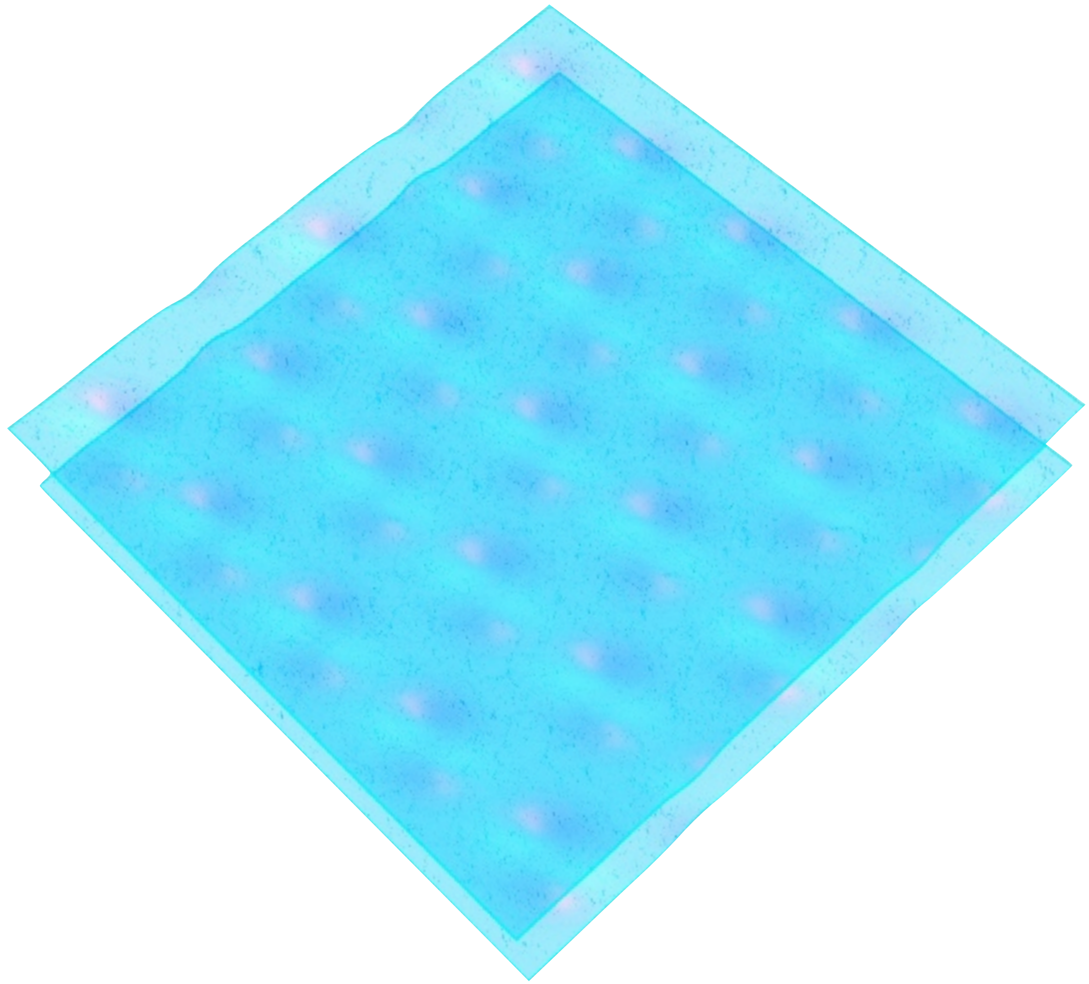
**no flux** through “holes”

↓

close holes

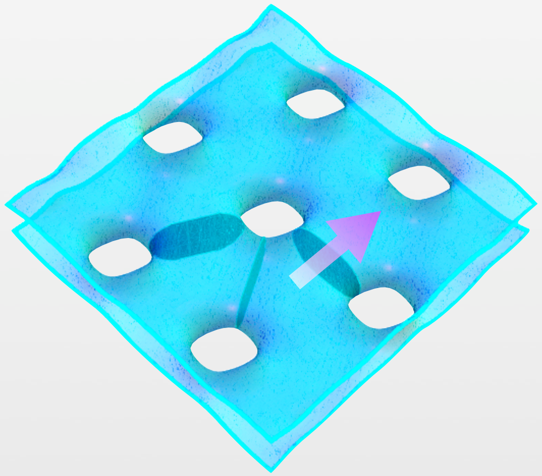
↓

**“two sheets”**

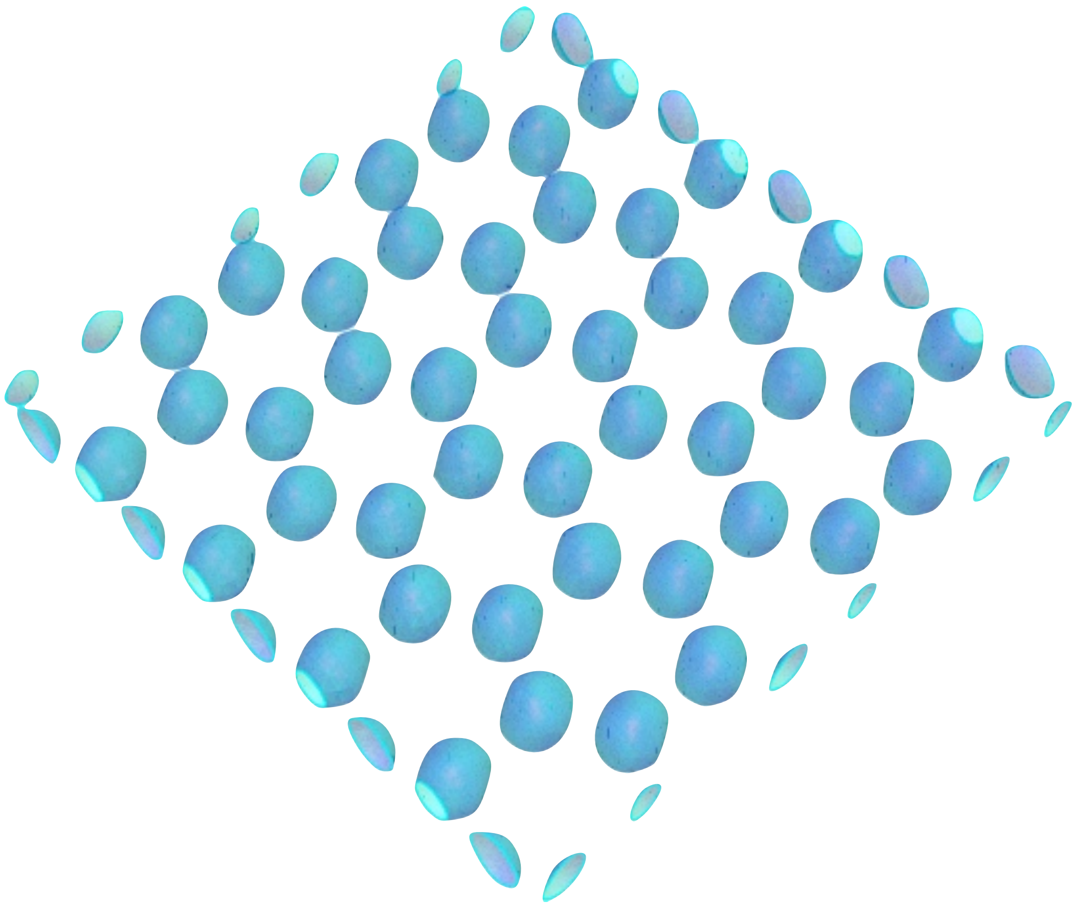


The “two sheets” ground state exhibits **topological order**.
In the toric code model this is the flux-free, loop gas ground state

# Extreme limits

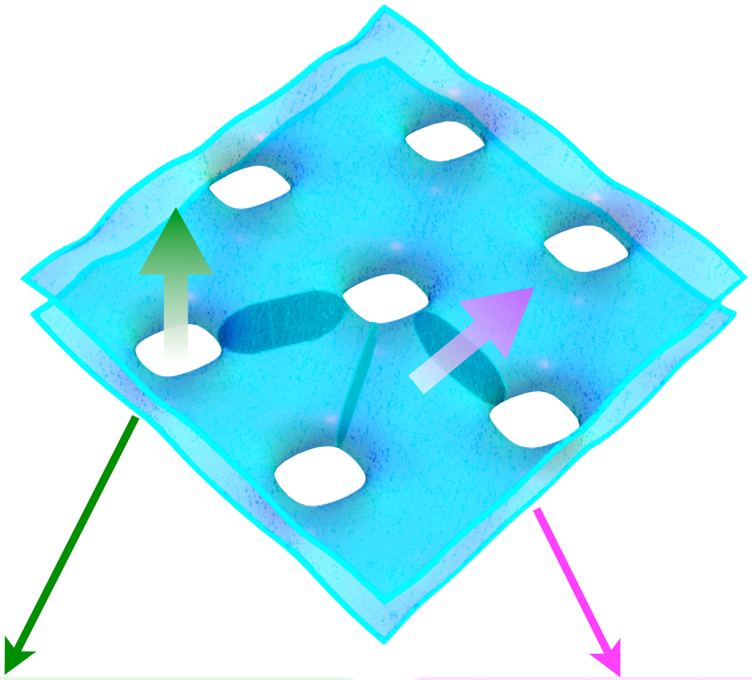


**no flux** through “tubes”
↓
pinch off tubes
↓
**“decoupled spheres”**



The “decoupled spheres” ground state exhibits **no topological order**.
In the toric code model this is the fully polarized, classically ordered state.

# Connecting the limits: a microscopic model

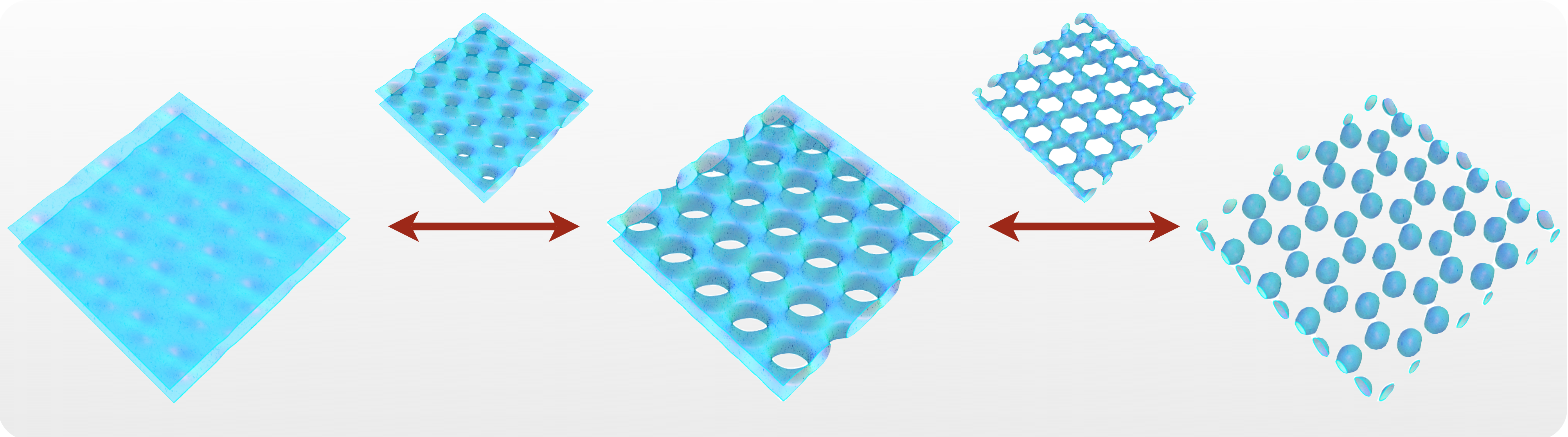


$$\mathcal{H} = -J_p \sum_{\text{plaquettes } p} \delta_{\phi(p),1} - J_e \sum_{\text{edges } e} \delta_{\ell(e),1}$$

Varying the couplings $J_e/J_p$ we can connect the two extreme limits.

**Semions (Abelian):**
Toric code in a magnetic field / loop gas with loop tension $J_e/J_p$.

# The quantum phase transition

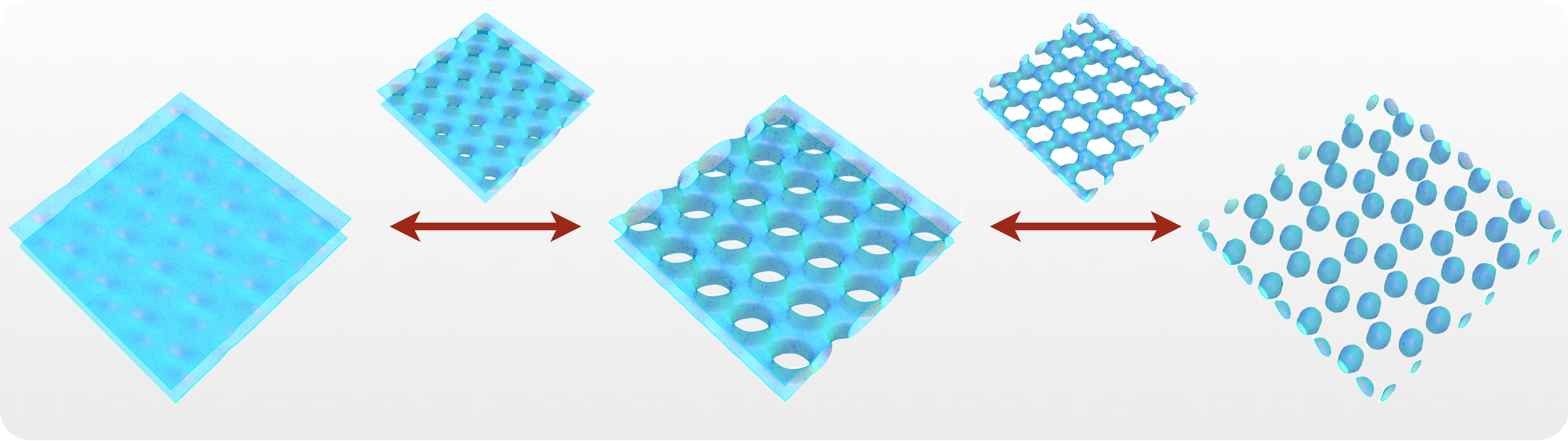


“two sheets”
**topological order**

“decoupled spheres”
**no topological order**

vary **loop tension** $J_e/J_p$

# The quantum phase transition



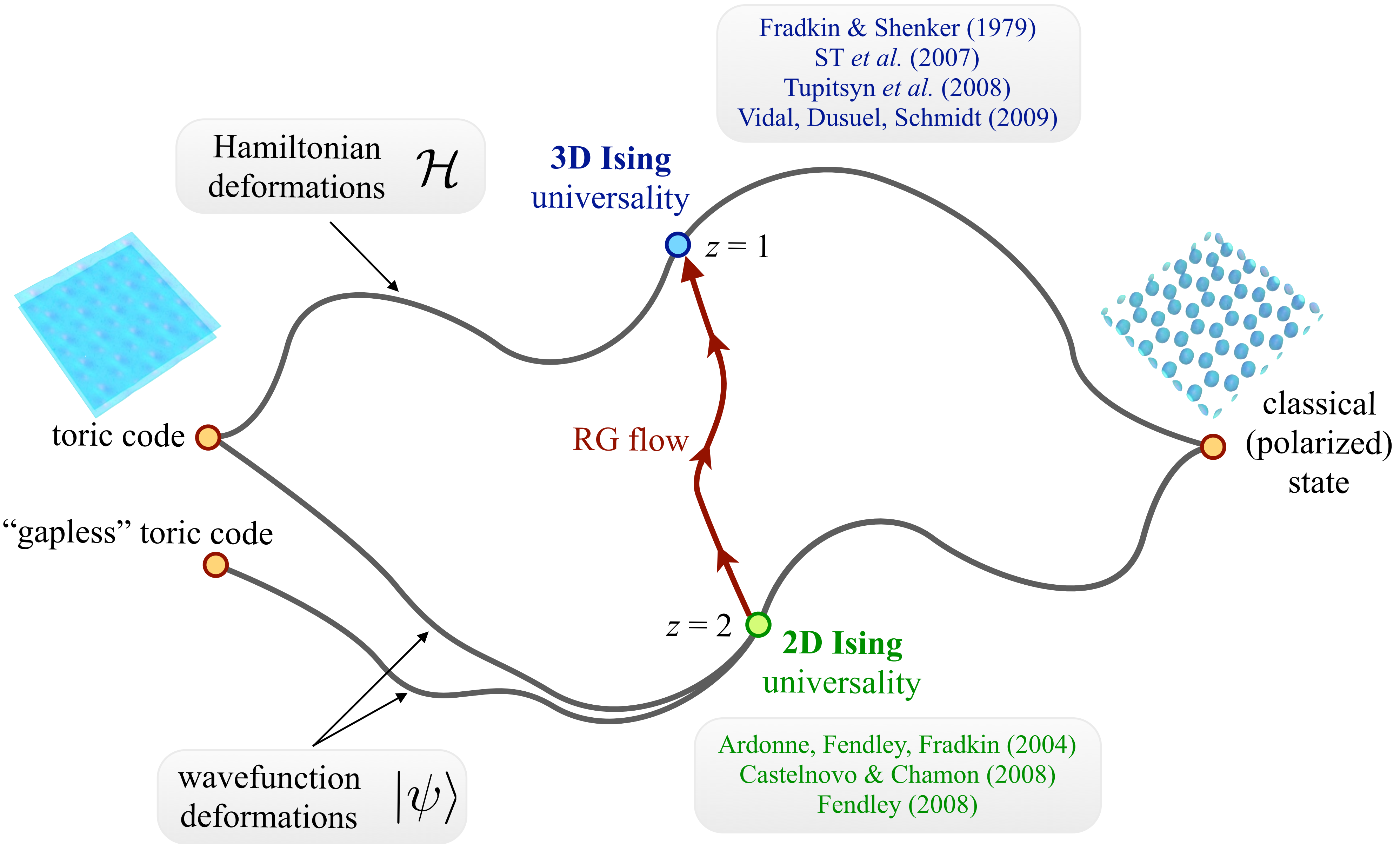
"two sheets"
**topological order**

"decoupled spheres"
**no topological order**

"quantum foam"
**topology fluctuations on all length scales**

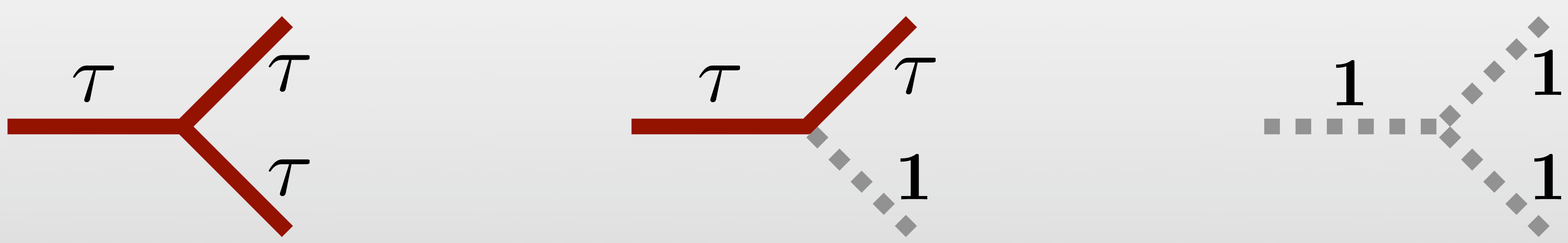
**continuous** transition

# Universality classes (semions)

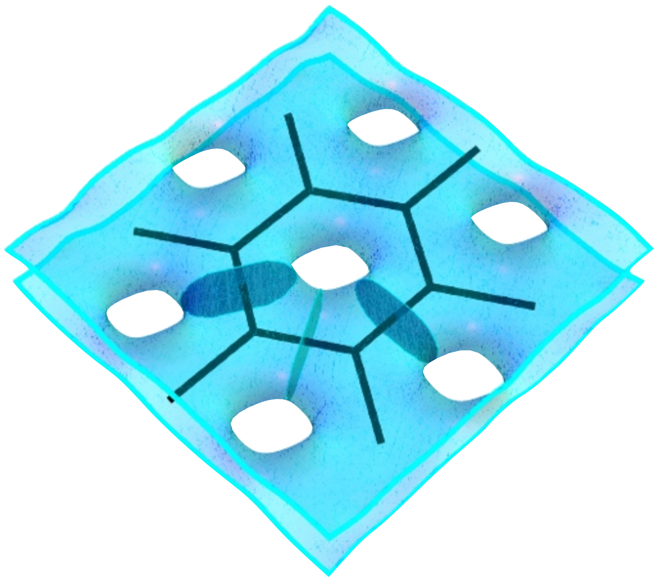


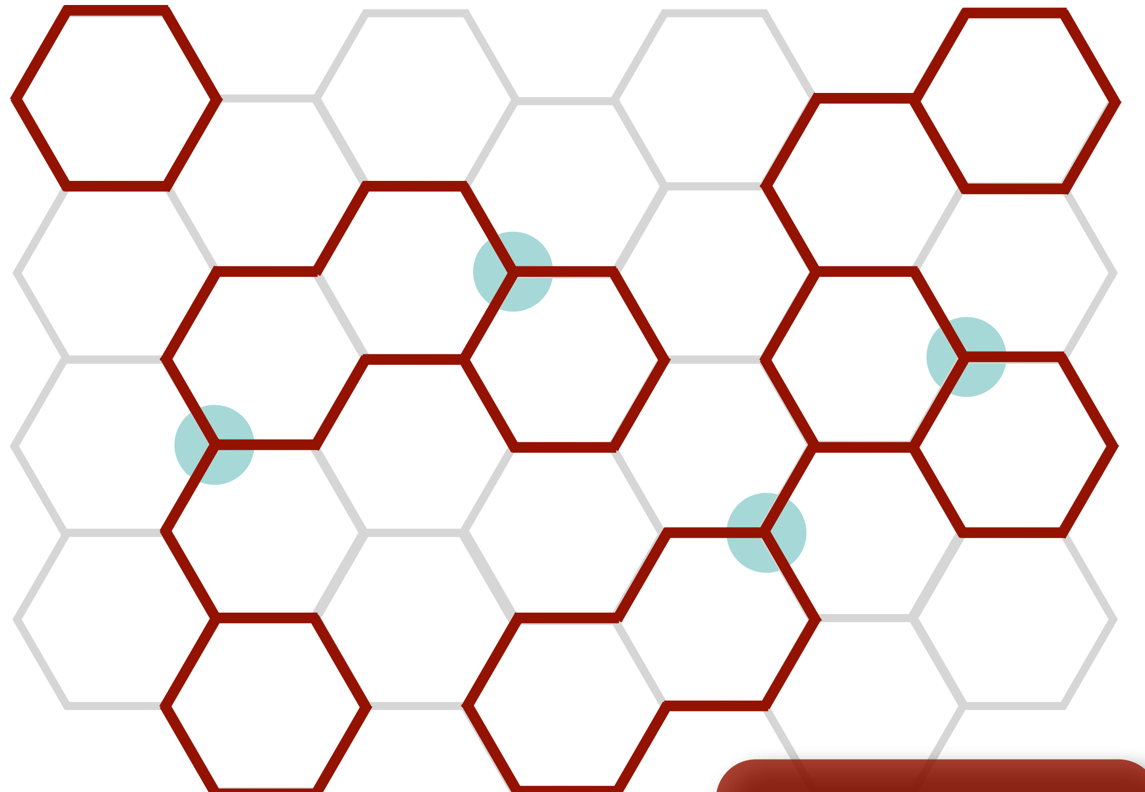
# The **non-Abelian** case

**Fibonacci anyon** fusion rules $\tau \times \tau = 1 + \tau$

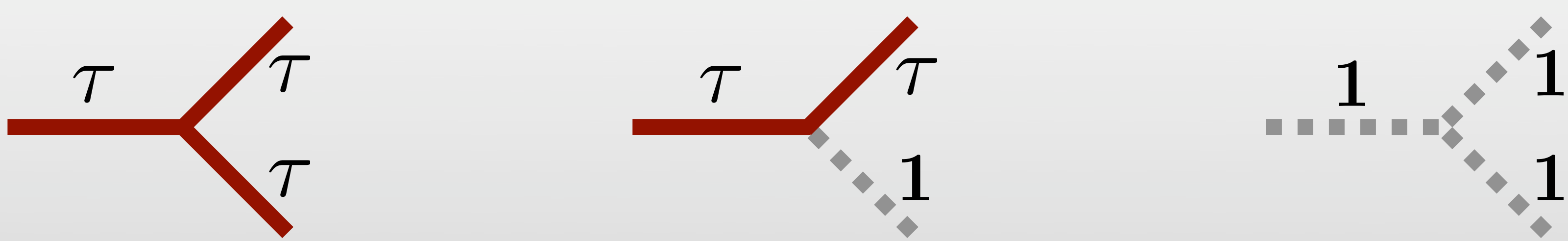


# Non-abelian liquids → string net

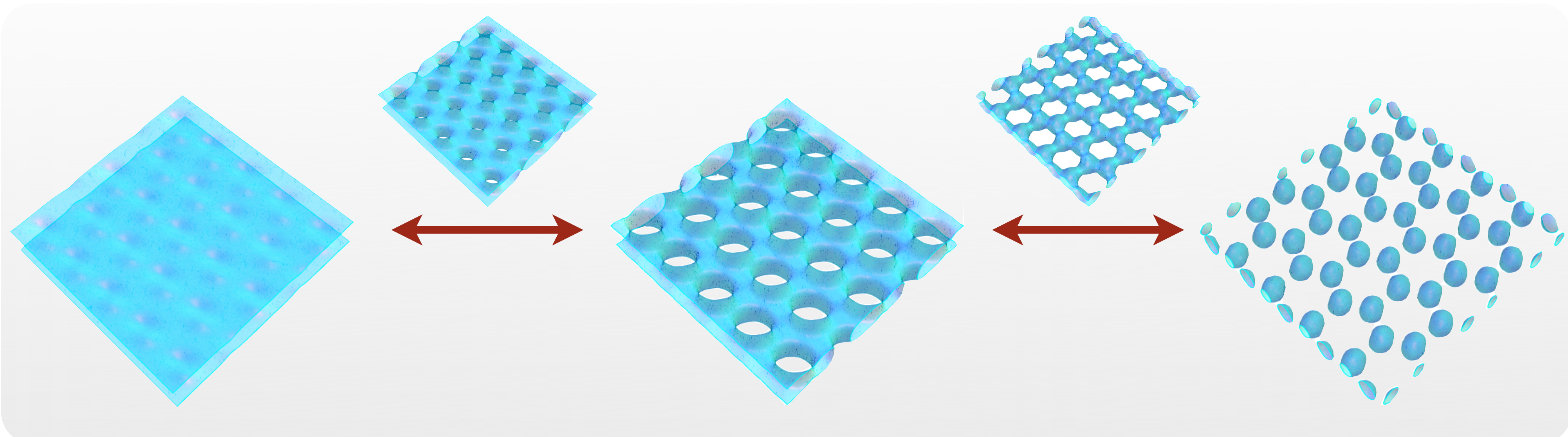





**Fibonacci anyon** fusion rules $\tau \times \tau = 1 + \tau$



# The quantum phase transition

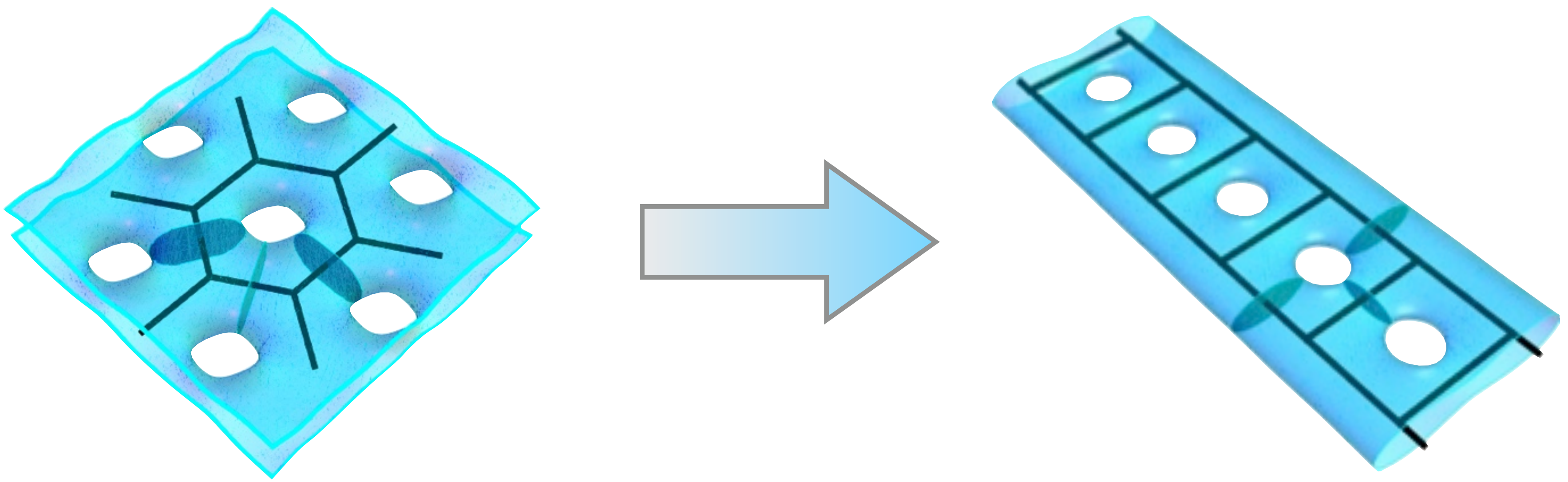


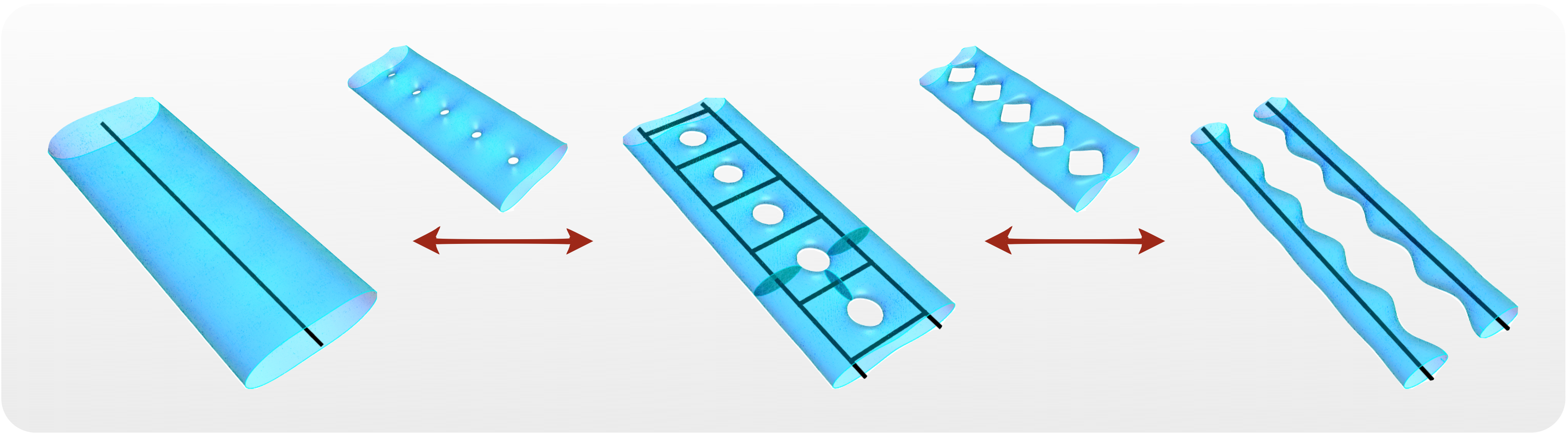
“two sheets”
**topological order**

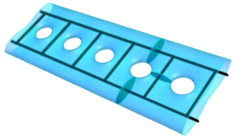
“decoupled spheres”
**no topological order**

vary **string tension** $J_e/J_p$

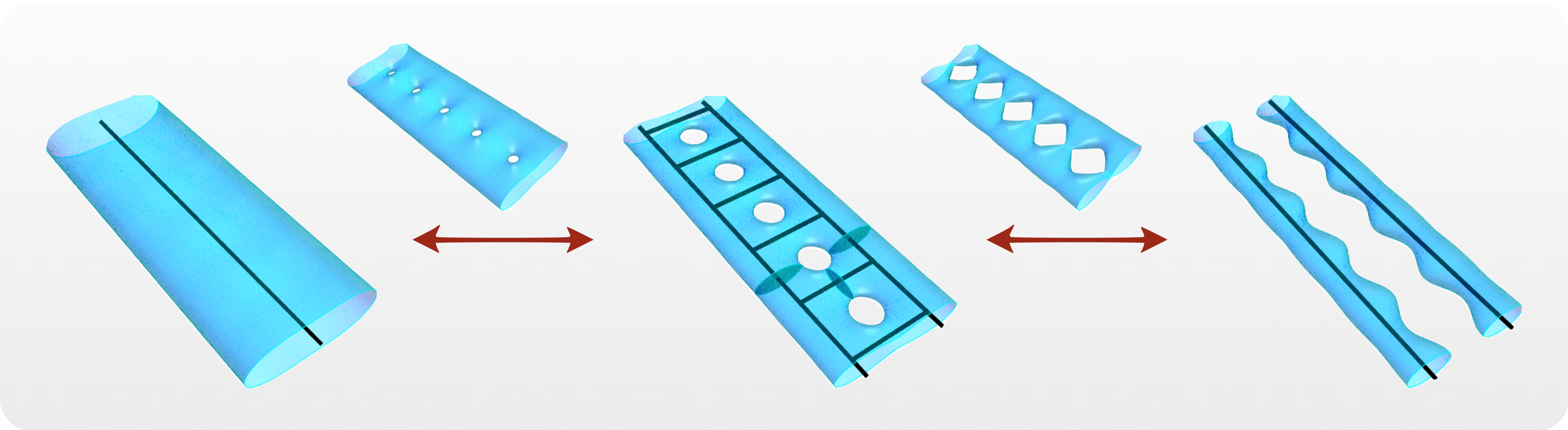
# One-dimensional analog





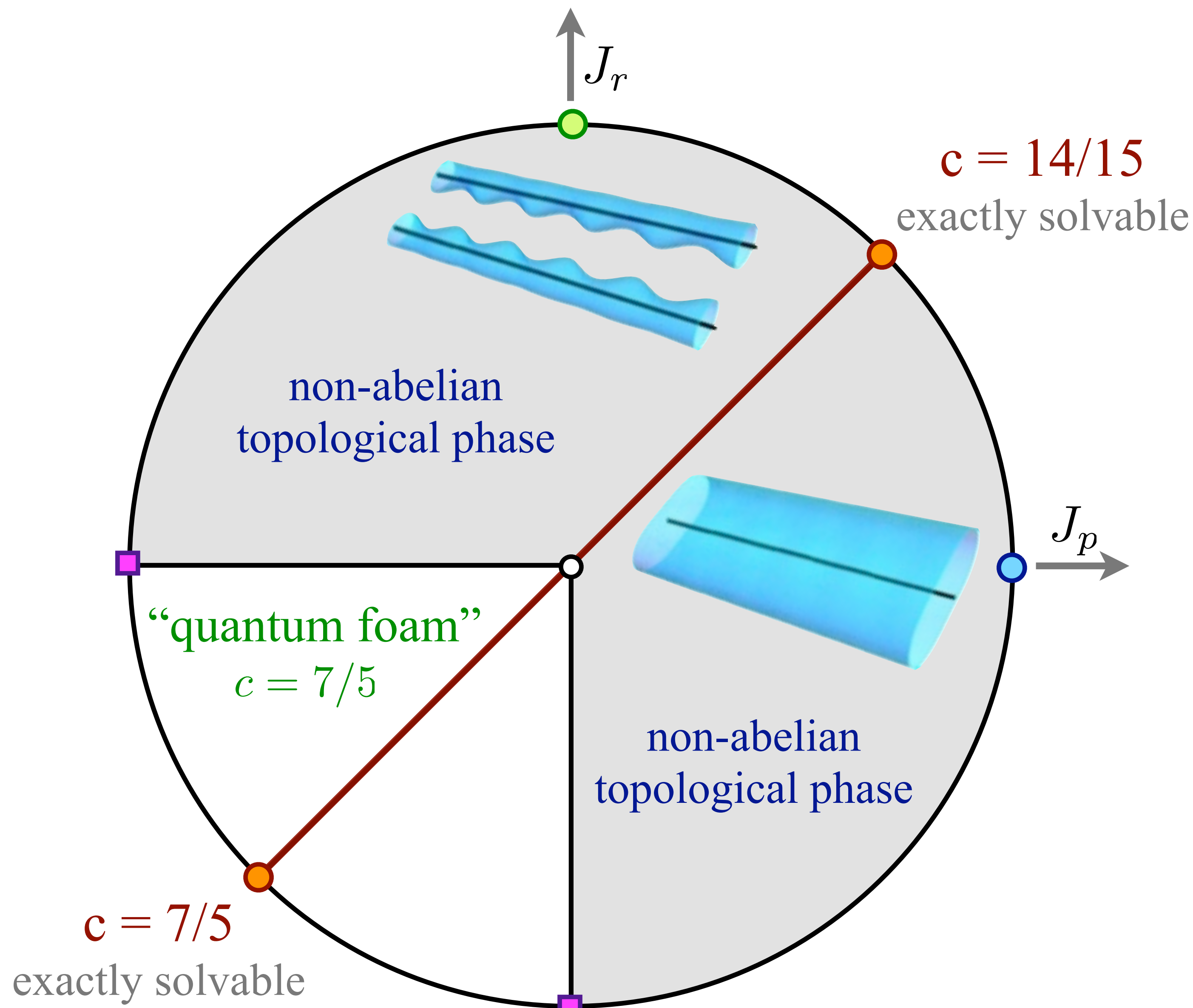


# A **continuous** transition



- A **continuous** quantum phase transition connects the two extremal topological states.
- The transition is driven by **topology fluctuations** on all length scales.
- The gapless theory is **exactly sovable**.

# Phase diagram



# Gapless theory & exact solution

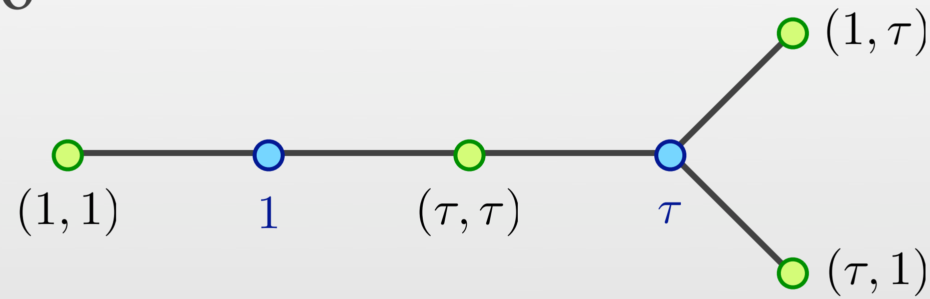
The operators in the Hamiltonian form a **Temperley-Lieb algebra**

$$(X_i)^2 = d \cdot X_i \qquad X_i X_{i\pm1} X_i = X_i \qquad [X_i, X_j] = 0$$

for $|i-j| \geq 2$

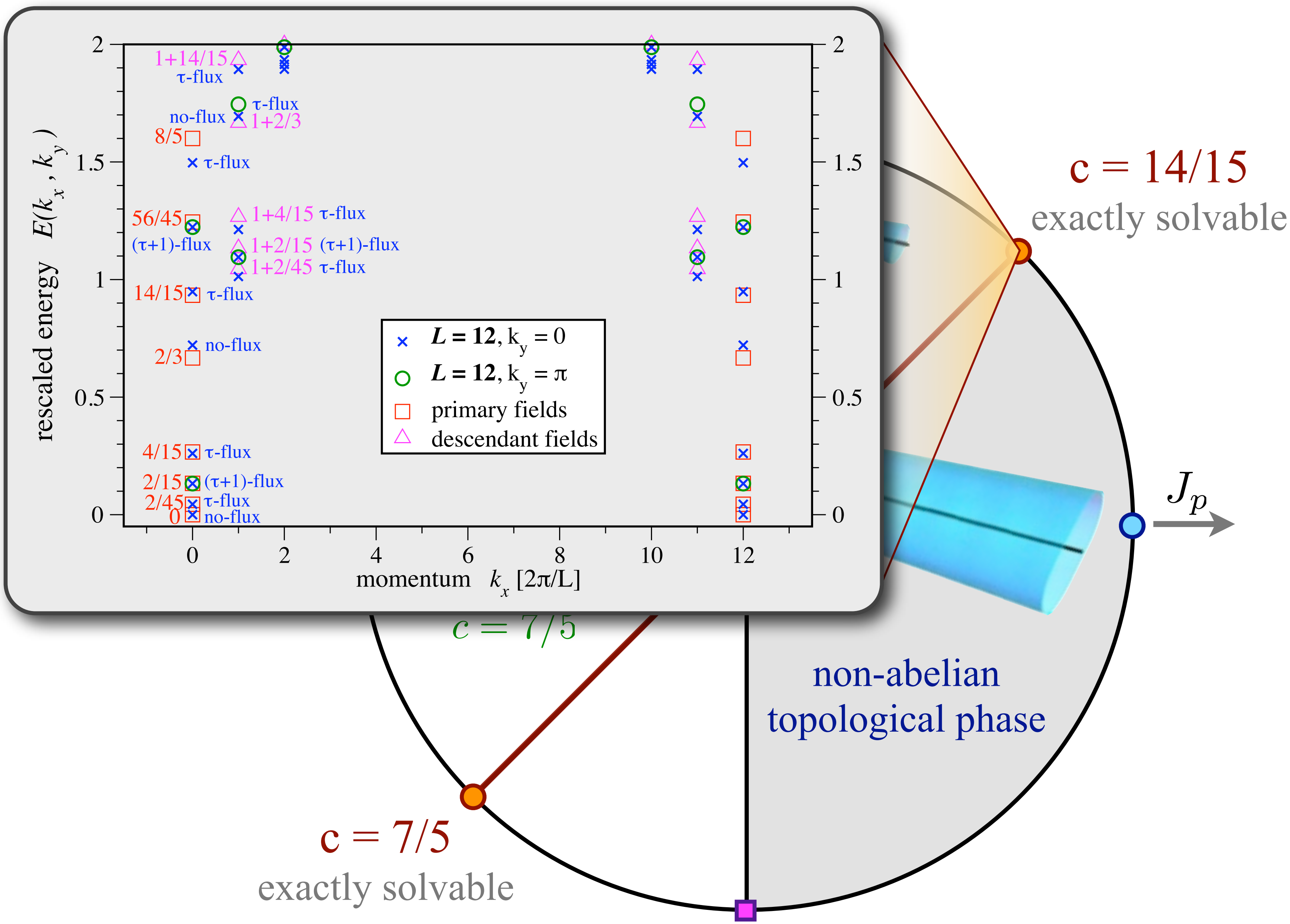
$$d = \sqrt{2+\phi}$$

The Hamiltonian maps onto the Dynkin diagram **D₆**

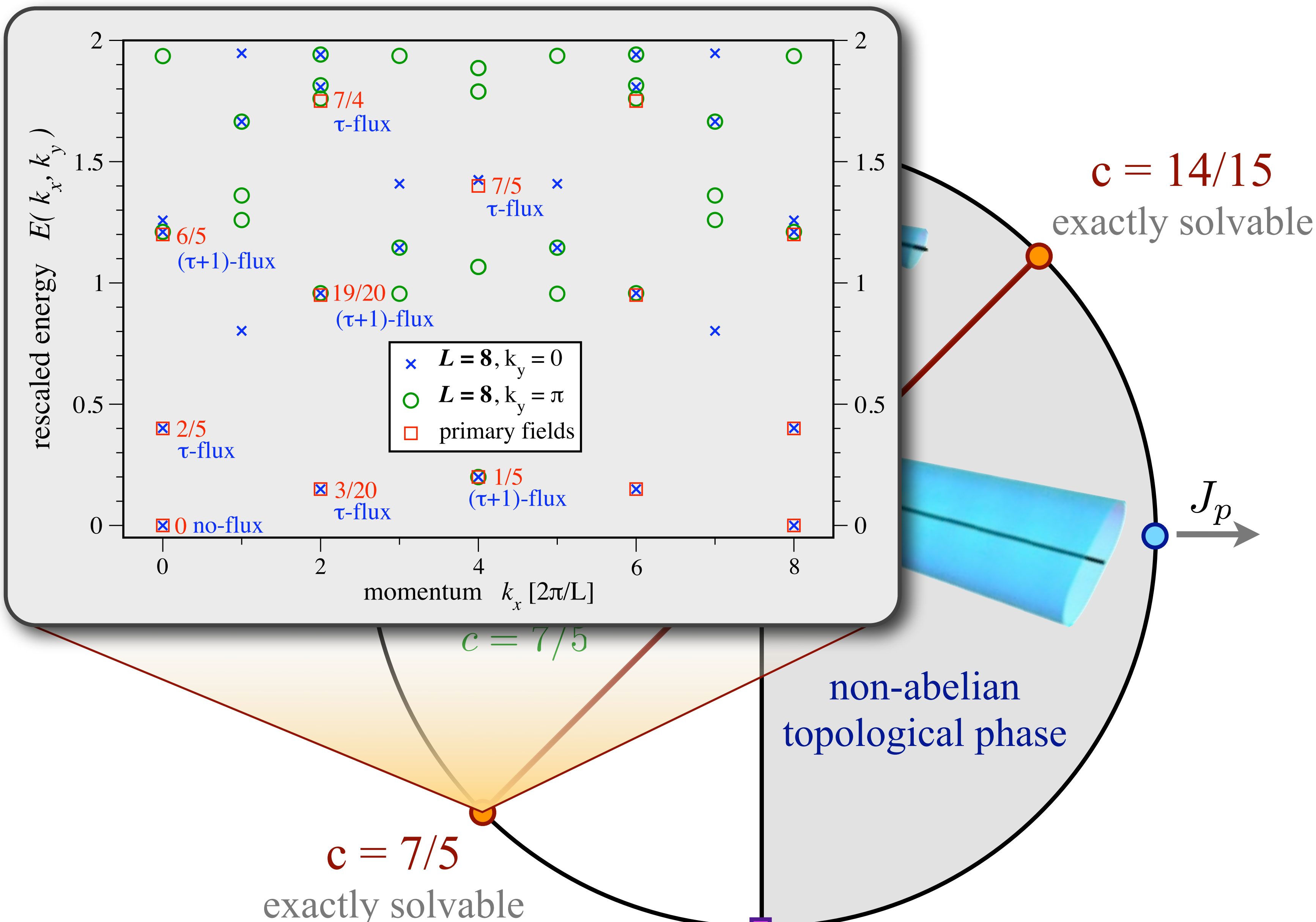


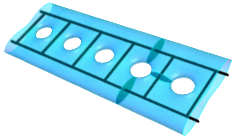

The gapless theory is a CFT with $c = 14/15$.

# Gapless theory & exact solution

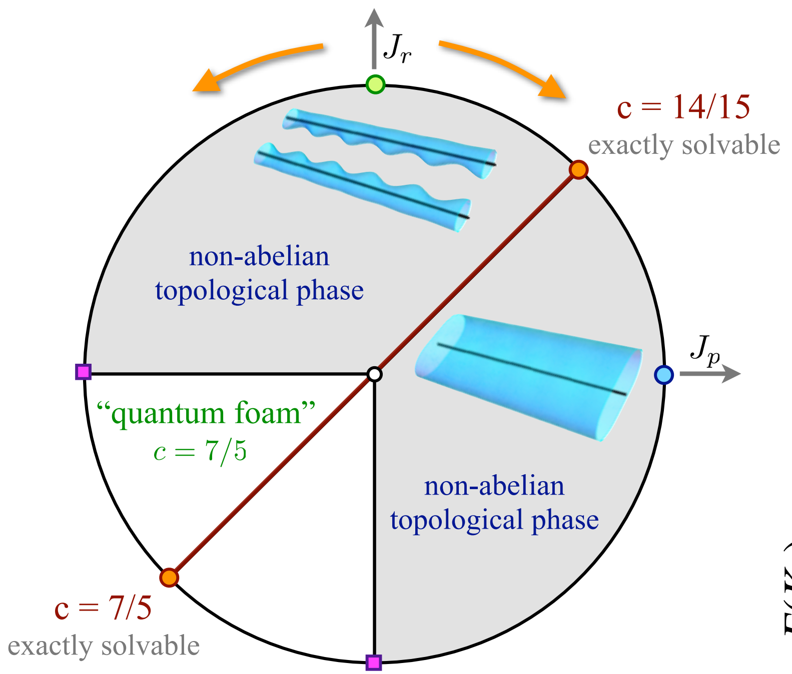


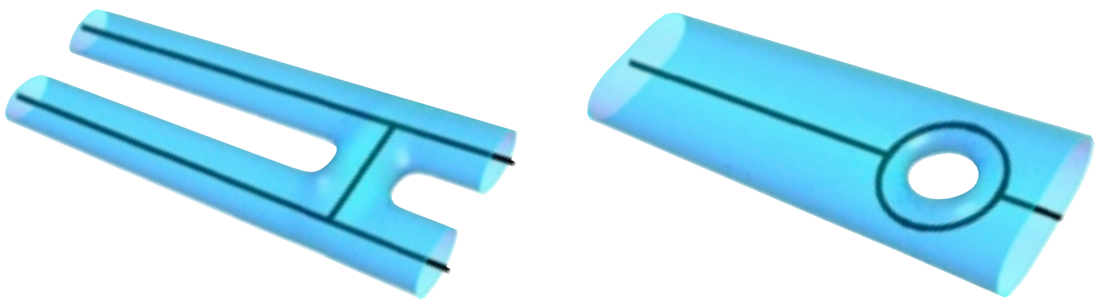
# Gapless theory & exact solution

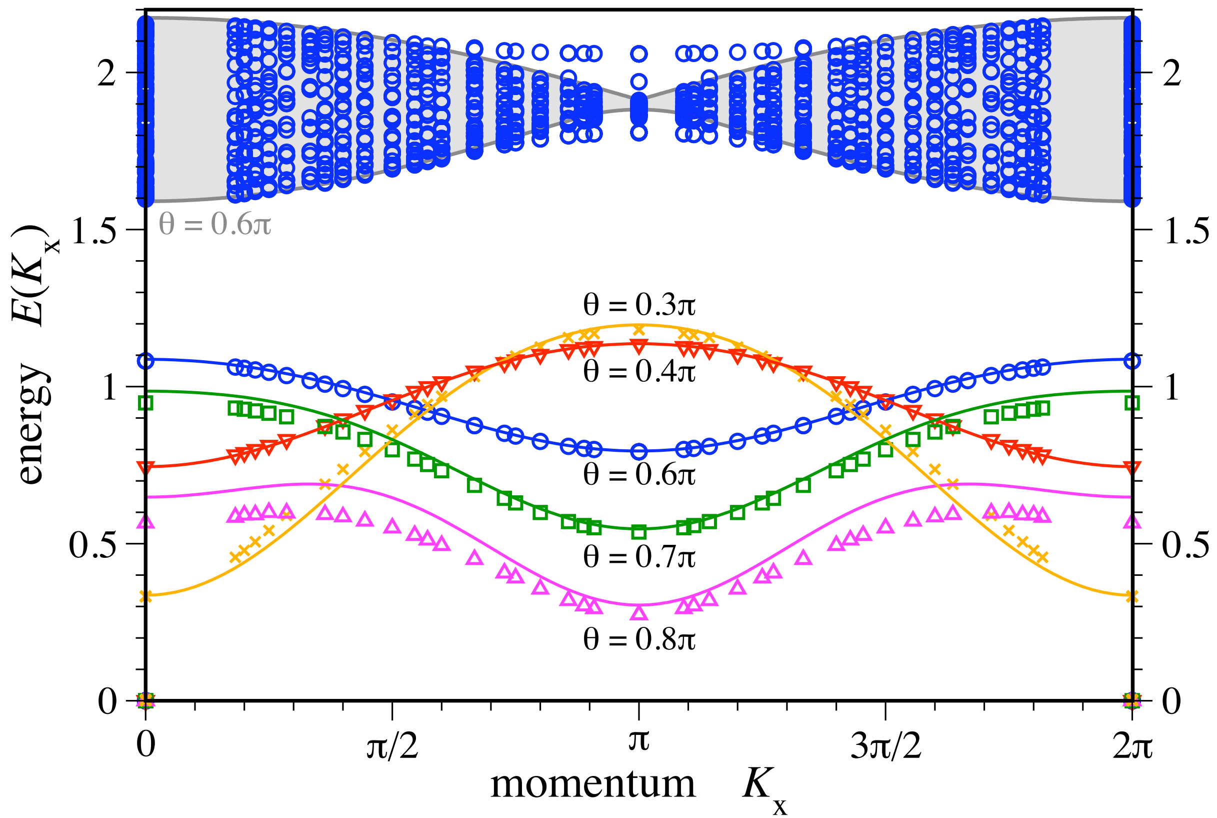




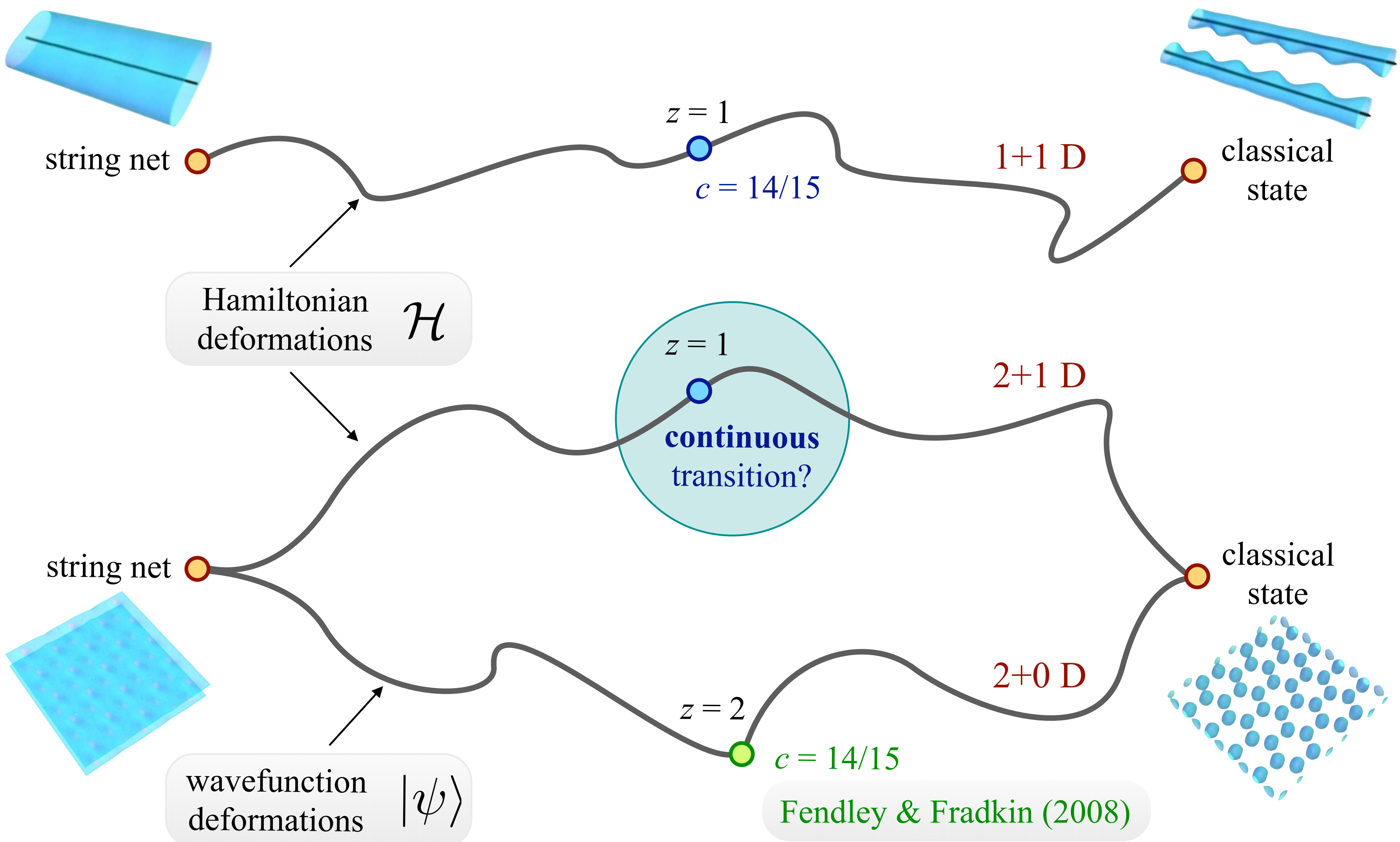
# Dressed flux excitations





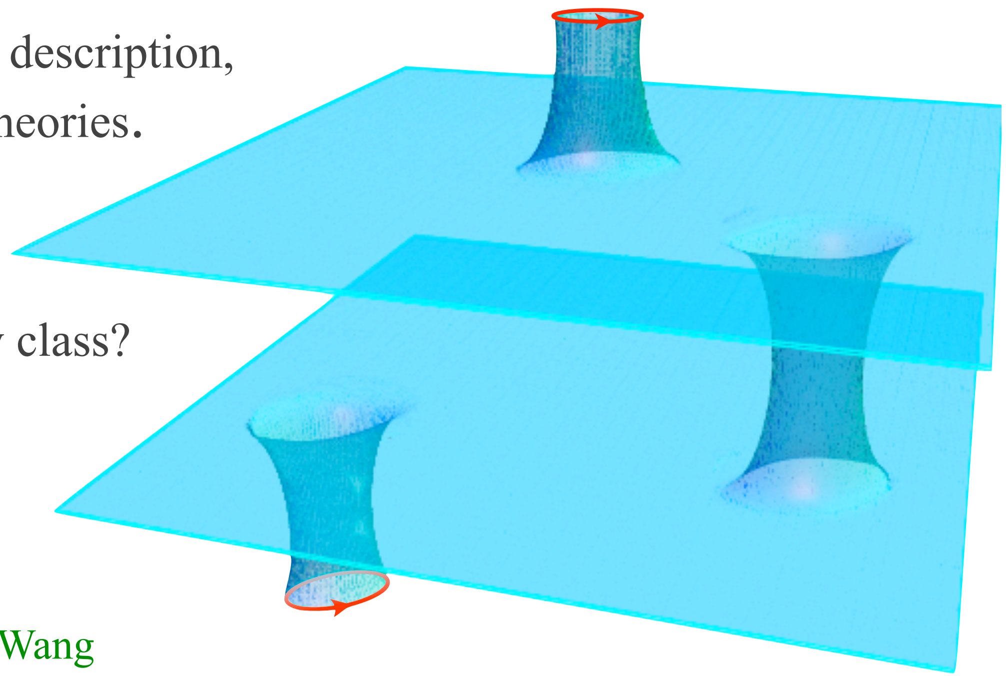
# Back to two dimensions



# Summary

- A "topological" framework for the description of topological phases and their phase transitions.
- Unifying description of loop gases and string nets.
- Quantum phase transition is driven by fluctuations of topology.

- Visualization of a more abstract mathematical description, namely doubled non-Abelian Chern-Simons theories.
- The 2D quantum phase transition out of a non-Abelian phase is still an open issue: A continuous transition in a novel universality class?



C. Gils, ST, A. Kitaev, A. Ludwig, M. Troyer, and Z. Wang
**arXiv:0906.1579** → Nature Physics (accepted)