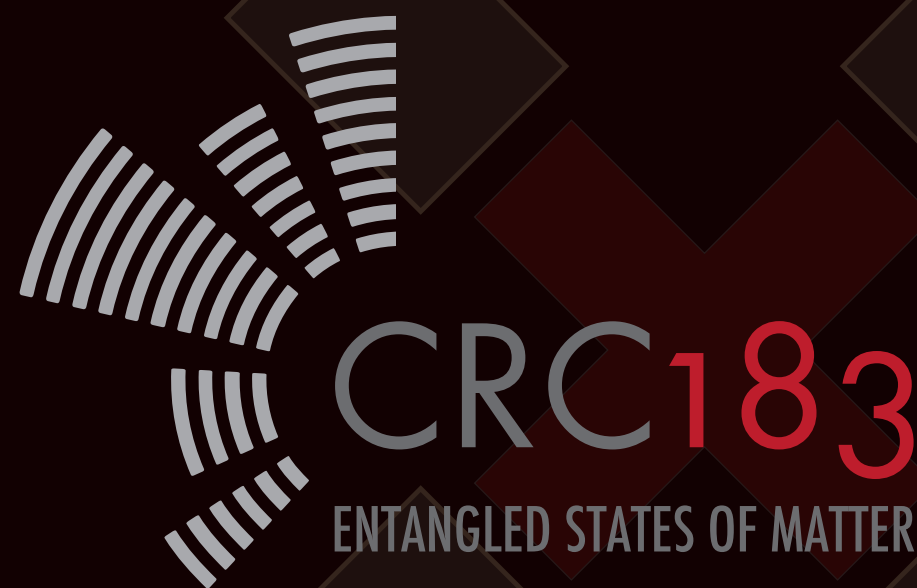


Quantum State Decoherence

Monitored quantum criticality, Nishimori physics, and beyond



Simon Trebst
University of Cologne

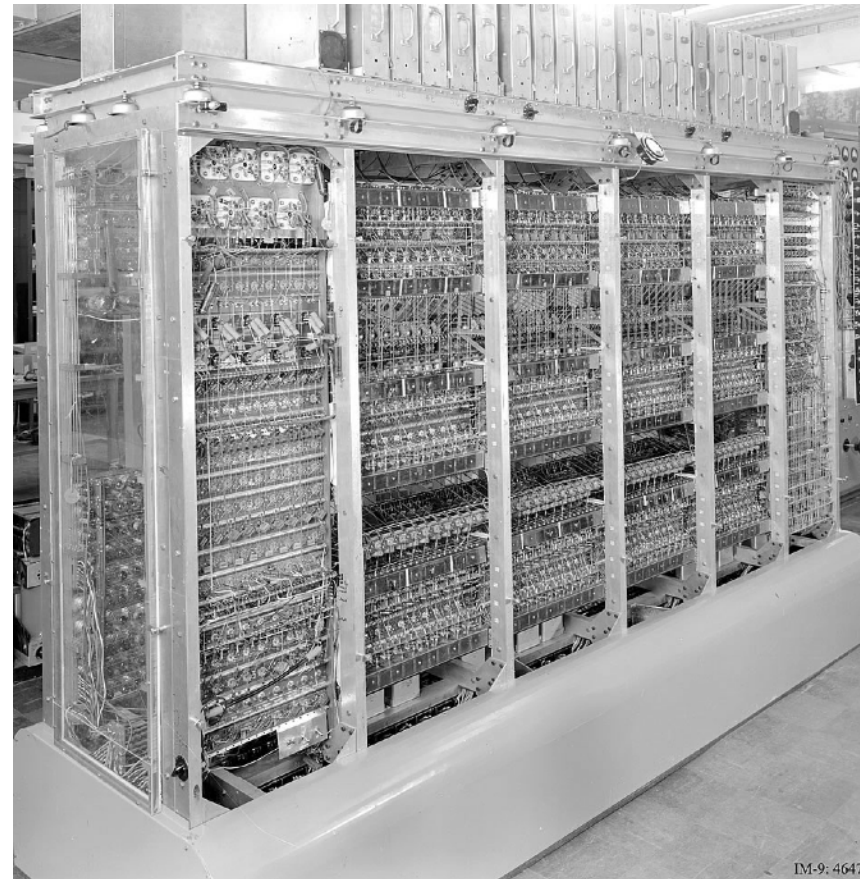


Advanced School and Conference on Quantum Matter

Trieste, December 2025

computational physics

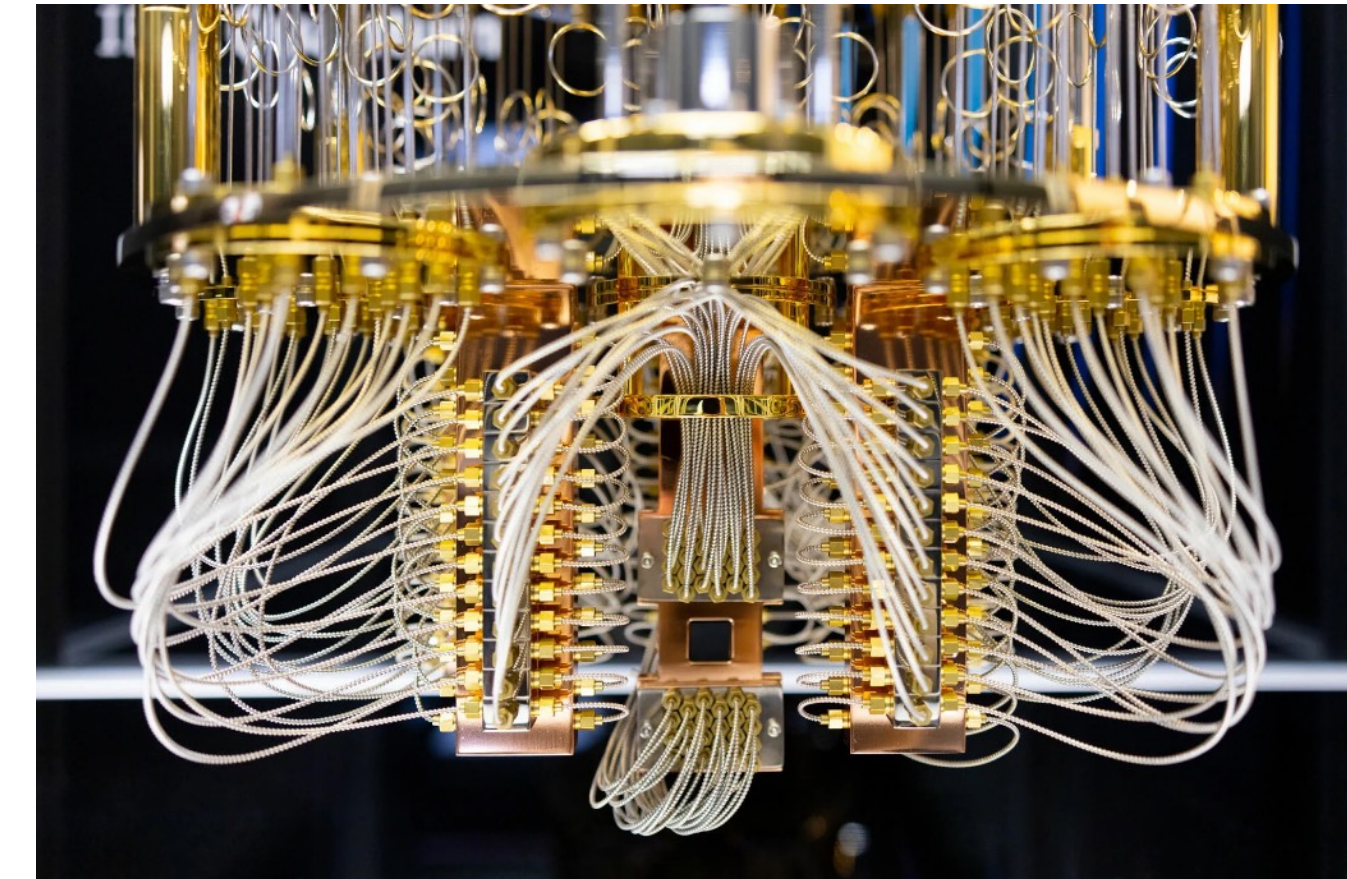
Maniac @ Los Alamos



Cray @ ETH Zurich



IBM Quantum (cloud)



———— (1953) ————— (1992) ————— (2019) —————>

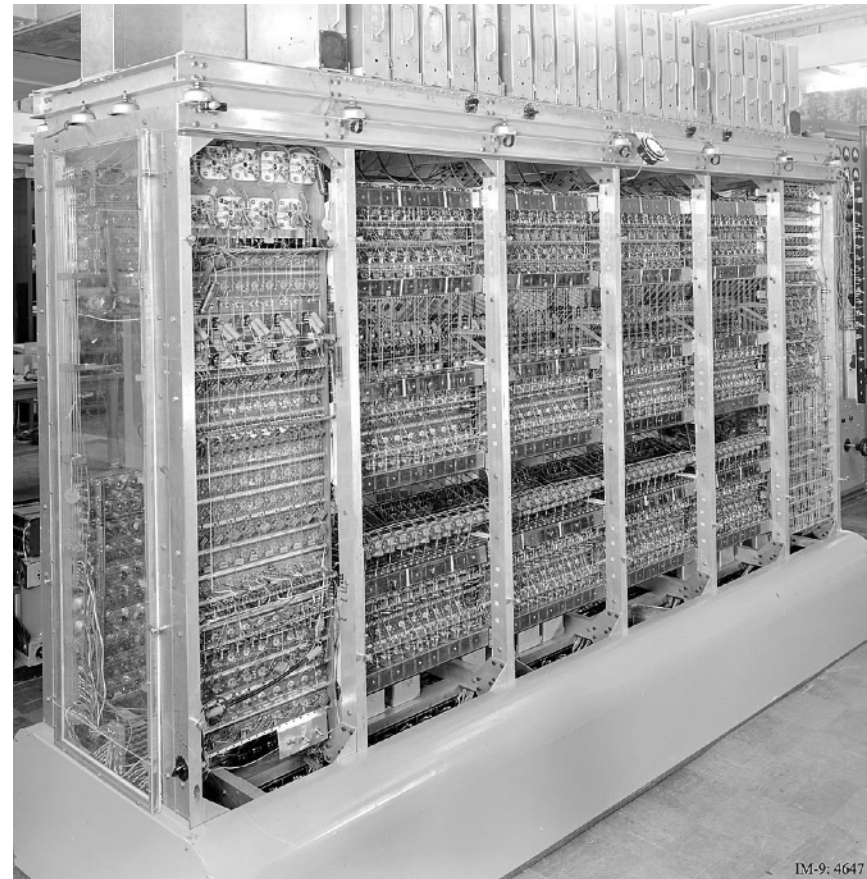
classical
many-body

quantum
many-body

$(\text{quantum})^2$
many-body

classical many-body physics

Maniac @ Los Alamos



(1953)

classical
many-body

THE JOURNAL OF CHEMICAL PHYSICS

VOLUME 21, NUMBER 6

JUNE, 1953

Equation of State Calculations by Fast Computing Machines

NICHOLAS METROPOLIS, ARIANNA W. ROSENBLUTH, MARSHALL N. ROSENBLUTH, AND AUGUSTA H. TELLER,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

AND

EDWARD TELLER,* *Department of Physics, University of Chicago, Chicago, Illinois*

(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.

I. INTRODUCTION

THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric.

II. THE GENERAL METHOD FOR AN EQUATION OF STATE BETWEEN THE PARTICLES

In order to reduce the problem to a feasible numerical work, we can, of course, consider a finite number of particles. This number N may be several hundred. Our system consists of a container containing N particles. In order to minimize boundary effects we suppose the complete substance consists of many such squares, each of



In putting together this issue of *Computing in Science & Engineering*, we knew three things: it would be difficult to list just 10 algorithms; it would be fun to assemble the authors and read their papers; and, whatever we came up with in the end, it would be controversial. We tried to assemble the 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century.

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

Monte Carlo sampling nevertheless became one of the most widely used algorithms for its ability to sample from arbitrary distributions.

quantum many-body physics

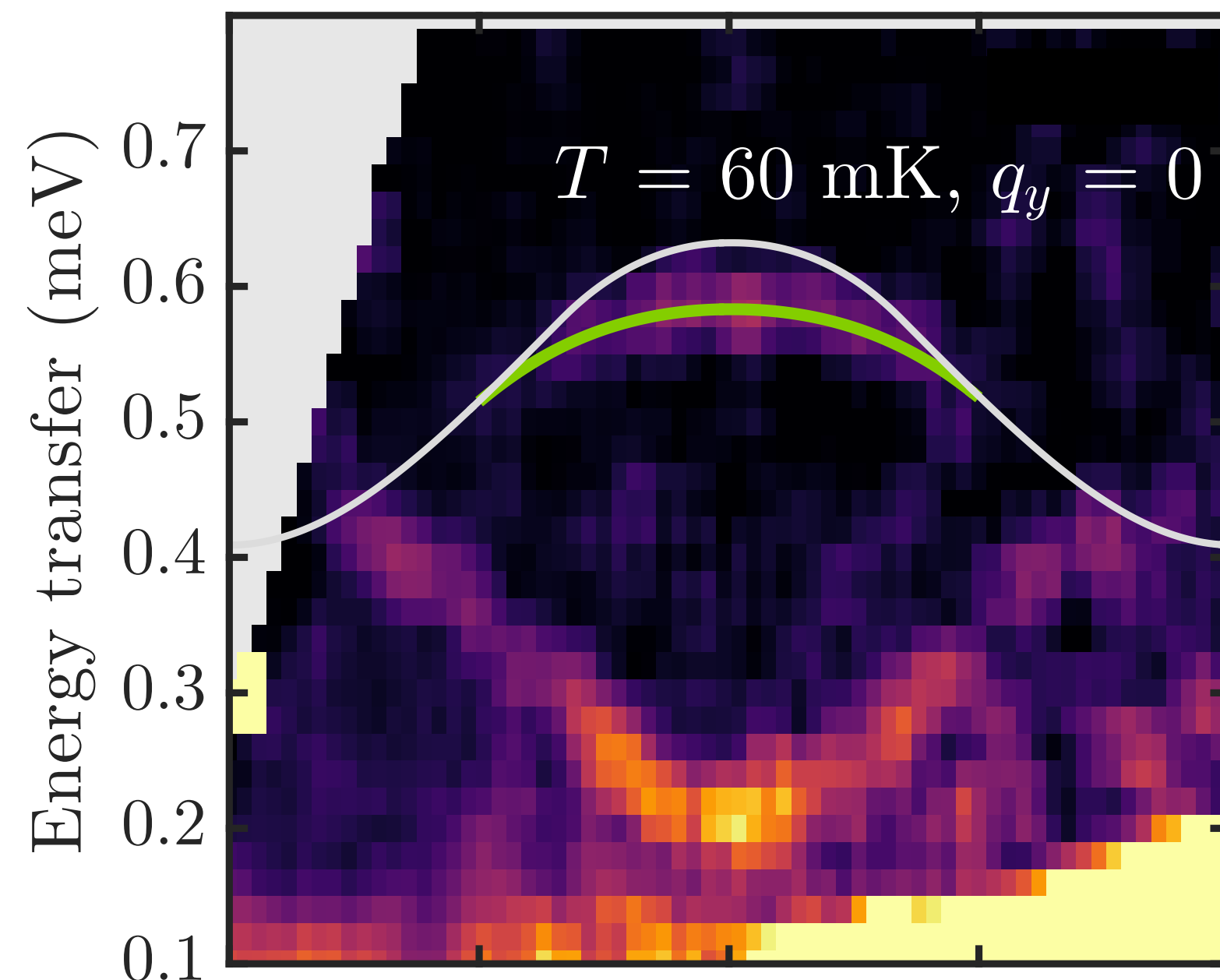
Cray @ ETH Zurich



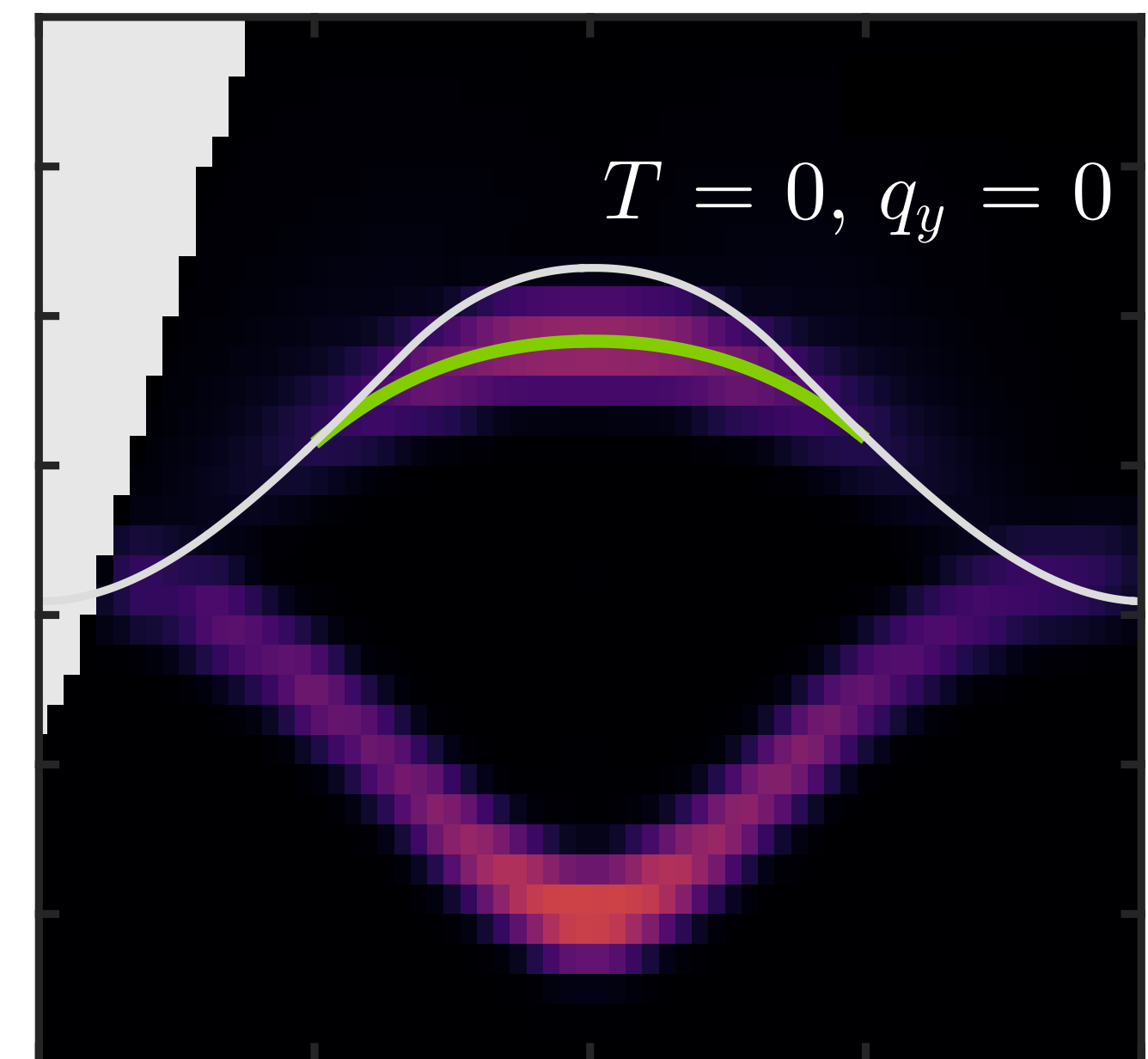
(1992)

quantum
many-body

Collective phenomena in quantum systems become accessible to **quantitative numerical simulations**, such as superconductivity, Bose-Einstein condensation & topological materials.



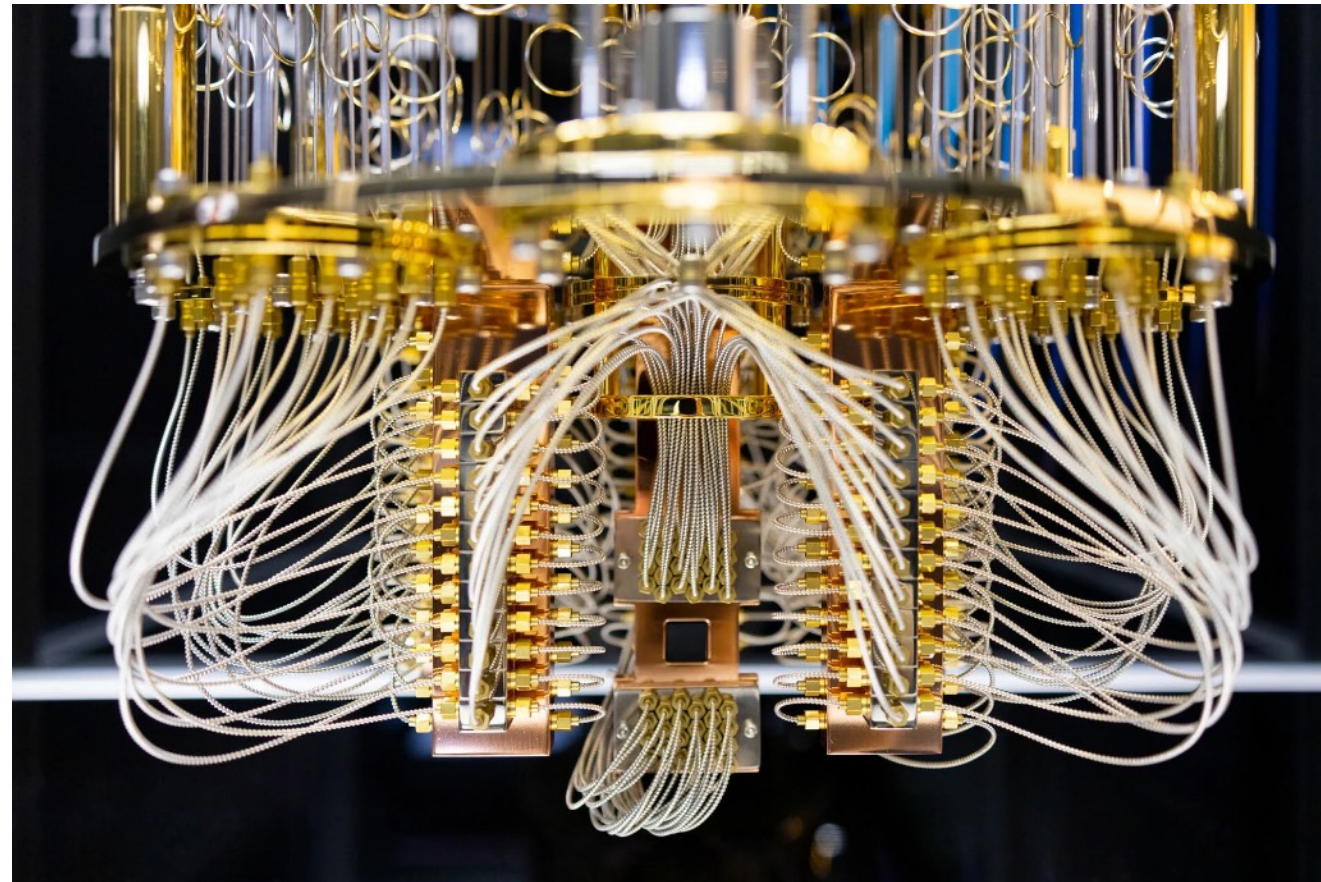
neutron scattering on
quantum material



density matrix renormalization group (DMRG)
numerical simulation

$(\text{quantum})^2$ many-body

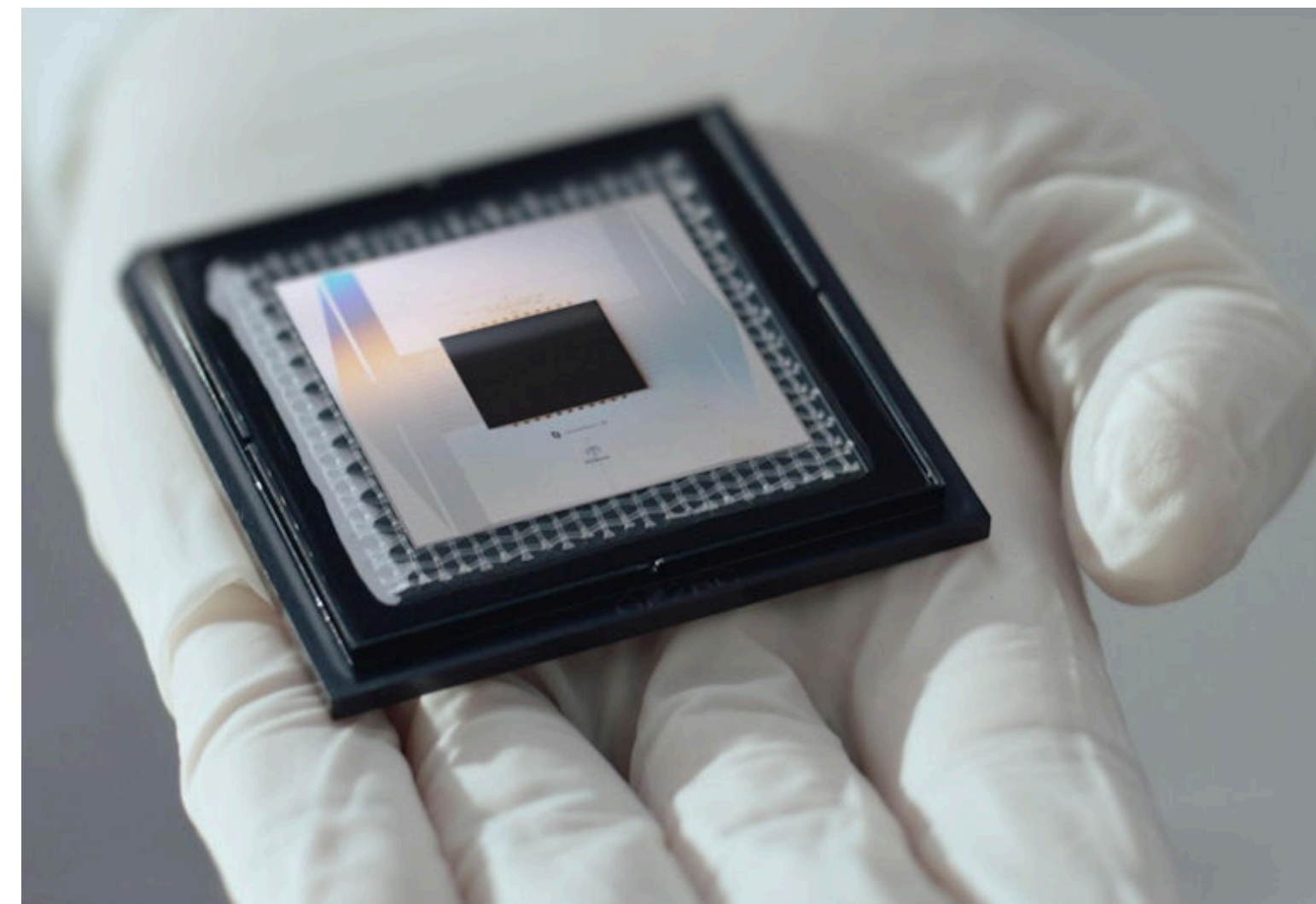
IBM Quantum (cloud)



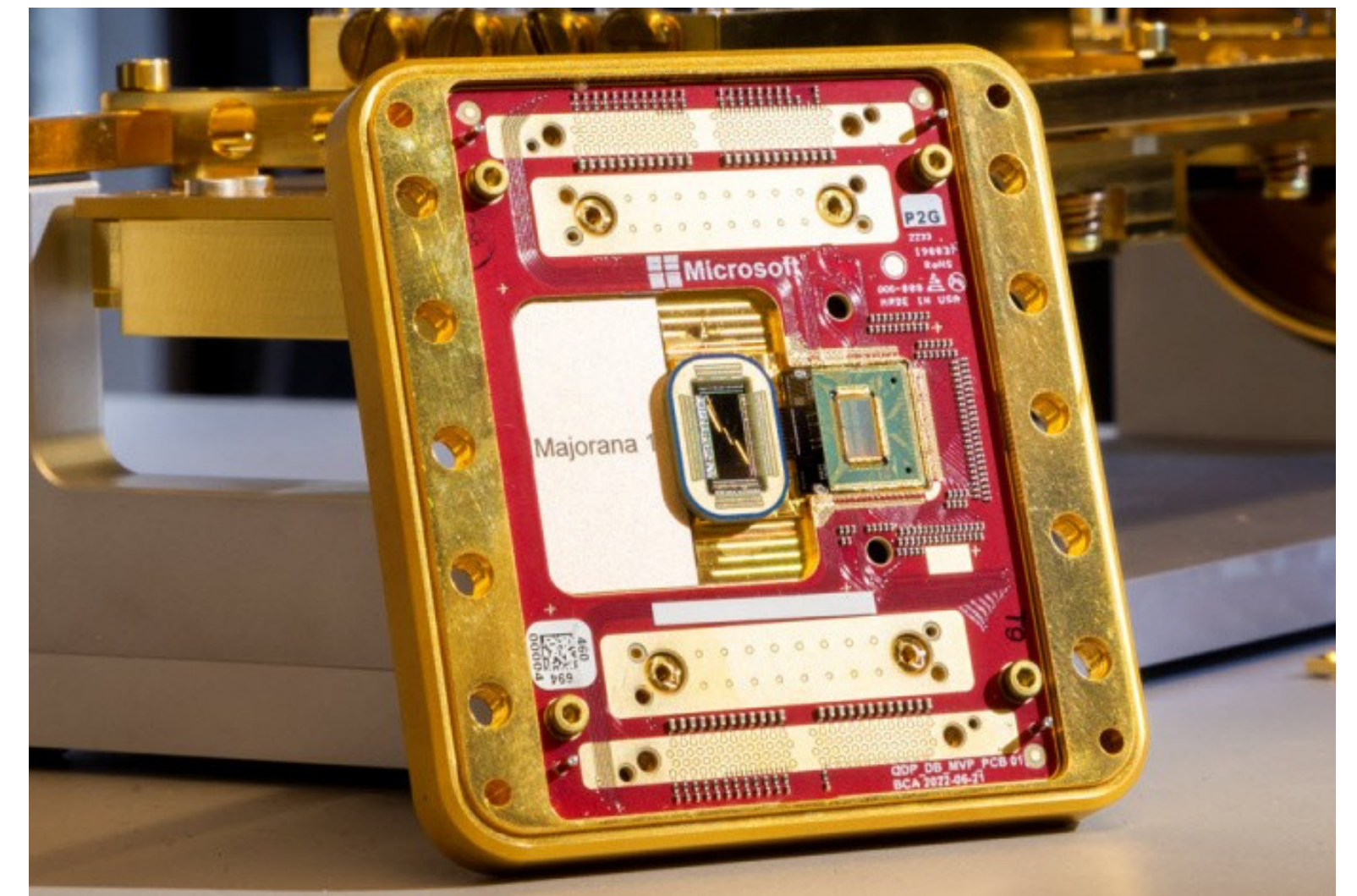
— (2019) —

$(\text{quantum})^2$
many-body

Quantum processors are emerging as a new experimental platform to simulate “**quantum on quantum**”, i.e. the non-linear **dynamics** of quantum systems with unprecedented control.



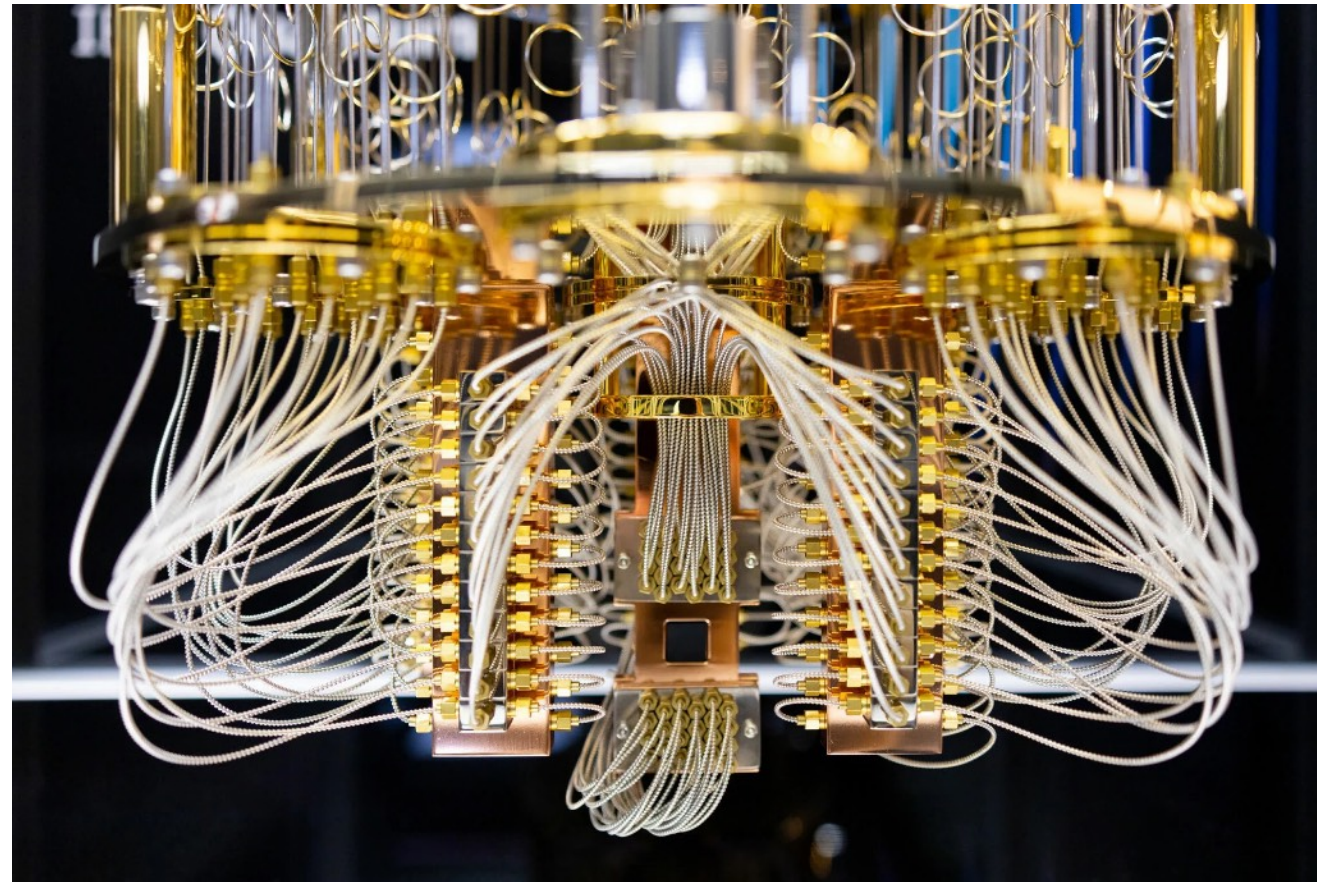
Google's Willow chip
(December 2024)



Microsoft's Majorana 1 chip
(February 2025)

(quantum)² many-body

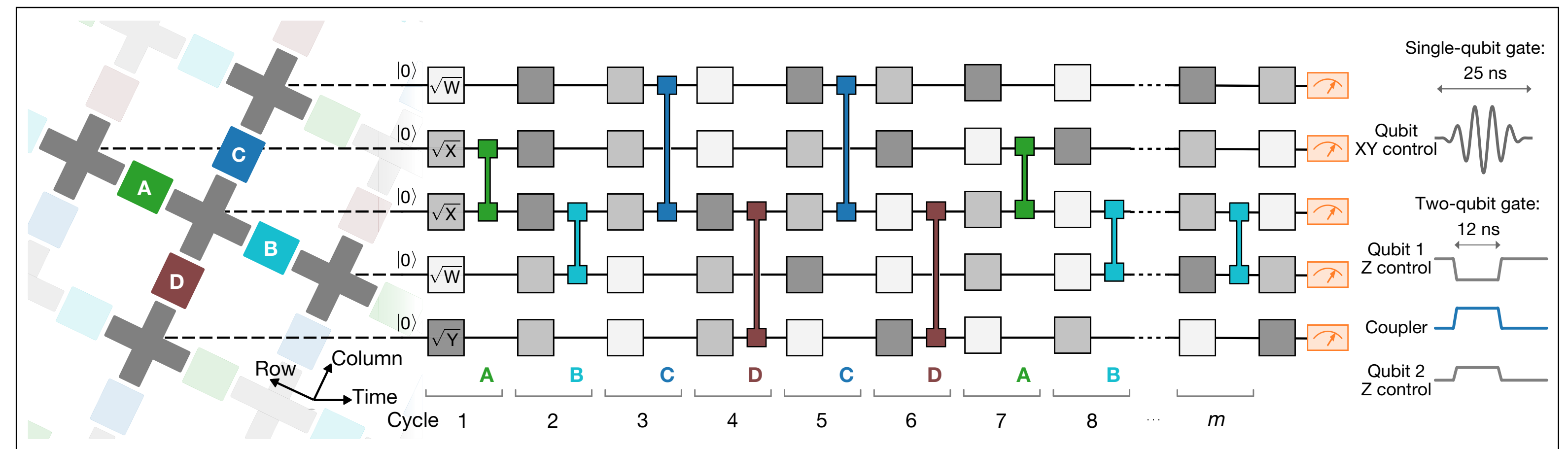
IBM Quantum (cloud)



(2019)

(quantum)²
many-body

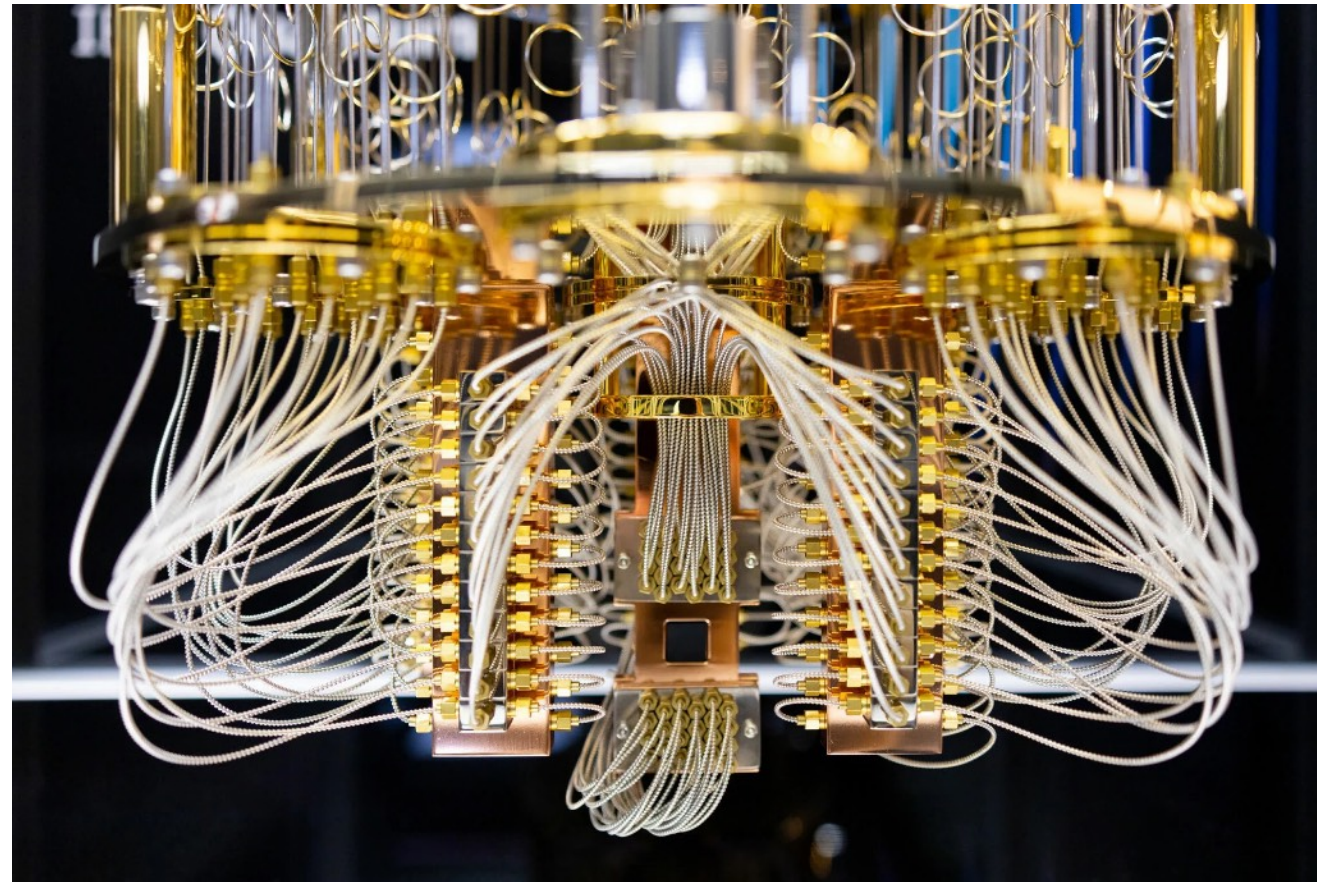
Quantum processors are emerging as a new experimental platform to simulate “**quantum on quantum**”, i.e. the non-linear **dynamics** of quantum systems with unprecedented control.



quantum circuits are the “assembler-level” of quantum computing

(quantum)² many-body

IBM Quantum (cloud)



Quantum processors are emerging as a new experimental platform to simulate “**quantum on quantum**”, i.e. the quantum **dynamics** of quantum systems with unprecedented control.

Article

Quantum supremacy using a programmable superconducting processor

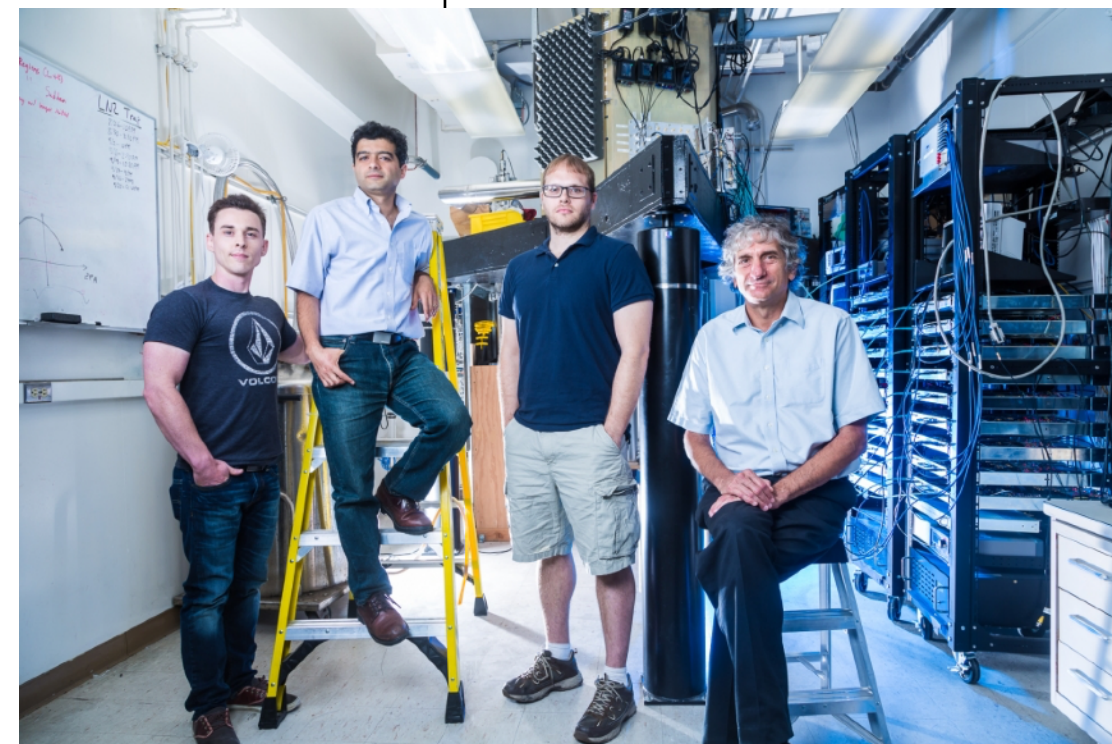
<https://doi.org/10.1038/s41586-019-1666-5>

Received: 22 July 2019

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Published online: 23 October 2019

Frank Arute¹, Kunal Arya¹, Ryan Babbush¹, Dave Bacon¹, Joseph C. Bardin^{1,2}, Rami Barends¹, Rupak Biswas³, Sergio Boixo¹, Fernando G. S. L. Brandao^{1,4}, David A. Buell¹, Brian Burkett¹, Yu Chen¹, Zijun Chen¹, Ben Chiaro⁵, Roberto Collins¹, William Courtney¹, Andrew Dunsworth¹, Edward Farhi¹, Brooks Foxen^{1,5}, Austin Fowler¹, Craig Gidney¹, Marissa Giustina¹, Rob Graff¹, Keith Guerin¹, Steve Habegger¹, Matthew P. Harrigan¹, Michael J. Hartmann^{1,6}, Alan Ho¹, Markus Hoffmann¹, Trent Huang¹, Travis S. Humble⁷, Sergei V. Isakov¹, Evan Jeffrey¹, Zhang Jiang¹, Dvir Kafri¹, Kostyantyn Kechedzhi¹, Julian Kelly¹, Paul V. Klimov¹, Sergey Knysh¹, Alexander Korotkov^{1,8}, Fedor Kostritsa¹, David Landhuis¹, Mike Lindmark¹, Erik Lucero¹, Dmitry Lyakh⁹, Salvatore Mandrà^{3,10}, Jarrod R. McClean¹, Matthew McEwen⁵, Anthony Megrant¹, Xiao Mi¹, Kristel Michielsen^{11,12}, Masoud Mohseni¹, Josh Mutus¹, Ofer Naaman¹, Matthew Neeley¹, Charles Neill¹, Murphy Yuezhen Niu¹, Eric Ostby¹, Andre Petukhov¹, John C. Platt¹, Chris Quintana¹, Eleanor G. Rieffel³, Pedram Roushan¹, Nicholas C. Rubin¹, Daniel Sank¹, Kevin J. Satzinger¹, Vadim Smelyanskiy¹, Kevin J. Sung^{1,13}, Matthew D. Trevithick¹, Amit Vainsencher¹, Benjamin Villalonga^{1,14}, Theodore White¹, Z. Jamie Yao¹, Ping Yeh¹, Adam Zalcman¹, Hartmut Neven¹ & John M. Martinis^{1,5*}



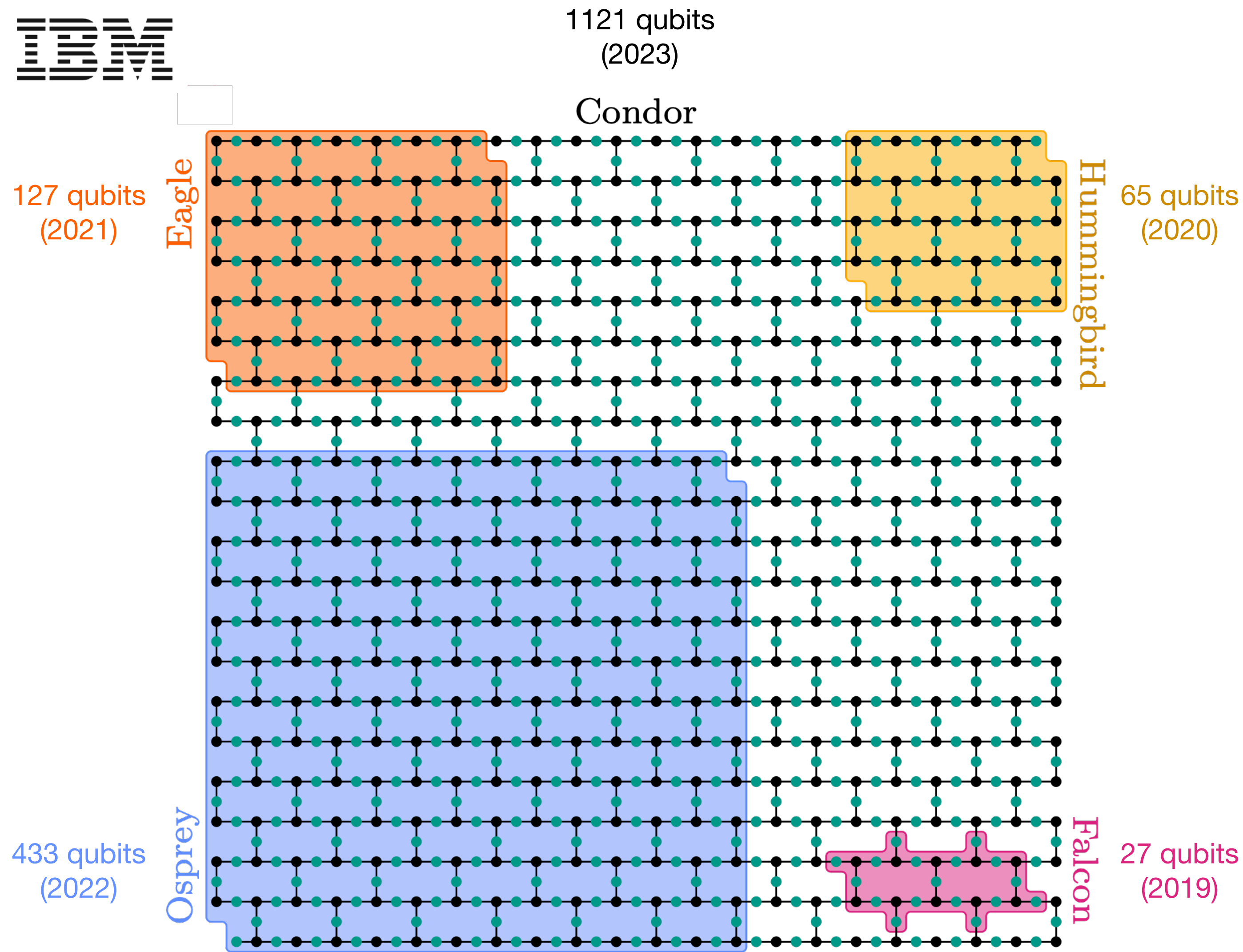
John Martinis (2019)

(2019)

(quantum)²
many-body

The promise of quantum computers is that **certain computational tasks** might be executed **exponentially faster** on a quantum processor than on a classical processor.

quantum hardware & simulations



Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G. S. L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, William Courtney, Andrew Dunsworth, Edward Farhi, Brooks Foxen, Austin Fowler, Craig Gidney, Marissa Giustina, Rob Graff, Keith Guerin, ... John M. Martinis + Show authors

Nature 574, 505–510 (2019) | [Cite this article](#)

RESEARCH ARTICLE | TOPOLOGICAL MATTER

Realizing topologically ordered states on a quantum processor

K. J. SATZINGER , Y.-J. LIU , A. SMITH , C. KNAPP , [...] AND P. ROUSHAN +93 authors [Authors Info & Affiliations](#)

SCIENCE • 2 Dec 2021 • Vol 374, Issue 6572 • pp. 1237–1241 • DOI: 10.1126/science.aba8378

Article | Open access | Published: 11 May 2023

Non-Abelian braiding of graph vertices in a superconducting processor

[Google Quantum AI and Collaborators](#)

Nature 618, 264–269 (2023) | [Cite th](#)

nature physics

Article

<https://doi.org/10.1038/s41567-024-02696-6>

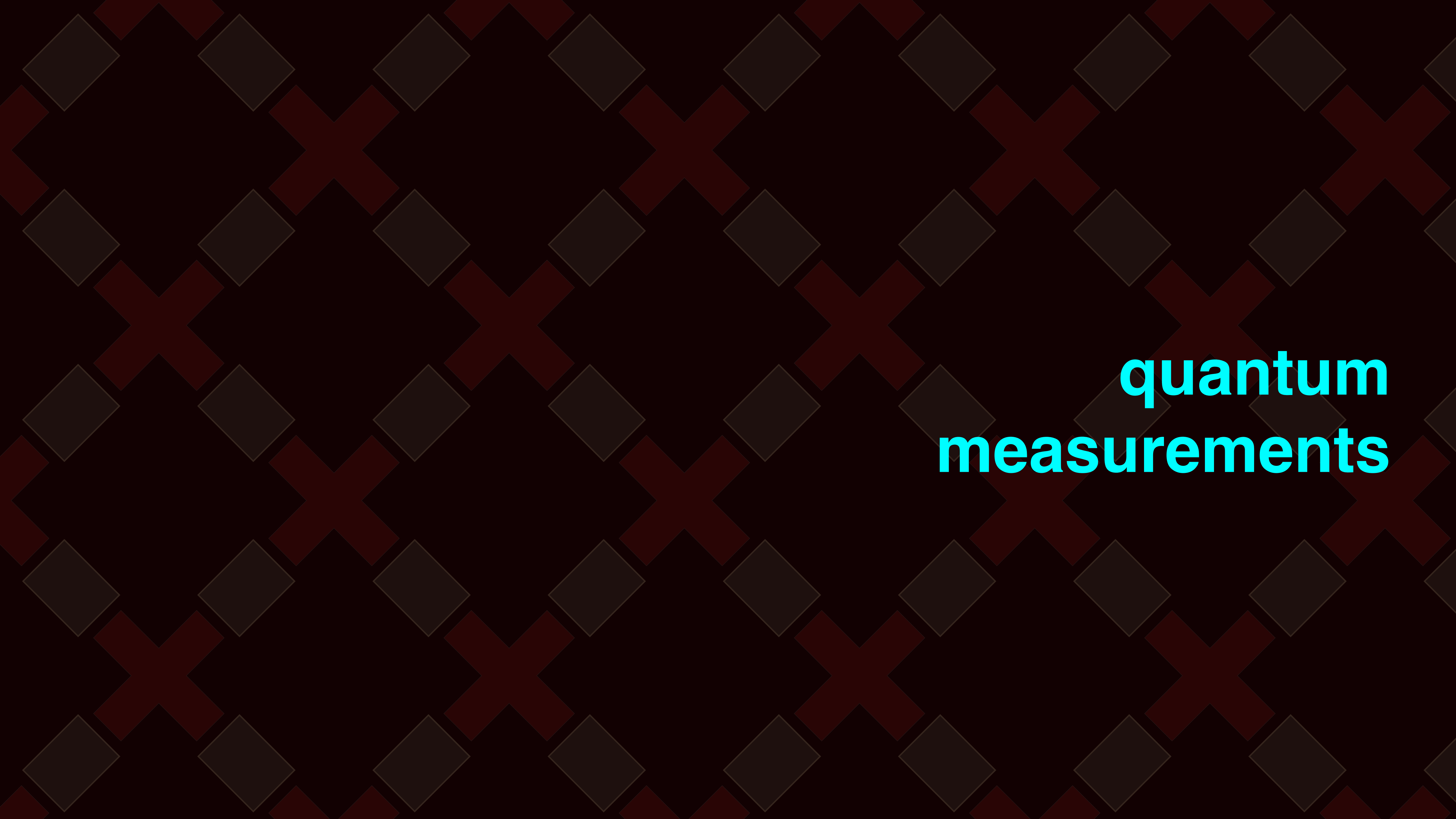
Nishimori transition across the error threshold for constant-depth quantum circuits

Received: 13 October 2023

Accepted: 7 October 2024

Published online: 16 December 2024

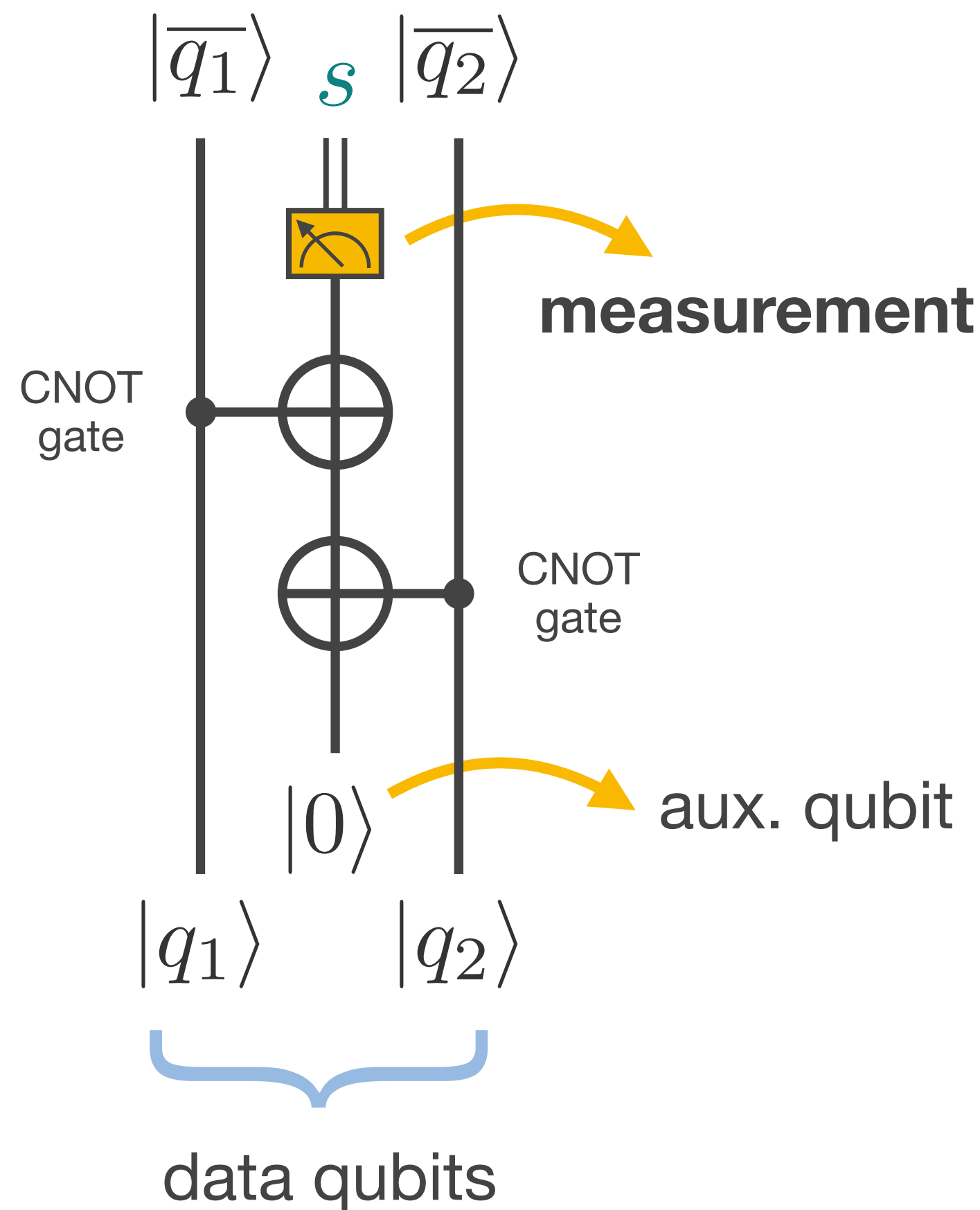
Edward H. Chen^{1,2} , Guo-Yi Zhu³ , Ruben Verresen⁴ , Alireza Seif⁵, Elisa Bäumer⁶, David Layden¹⁵, Nathanan Tantivasadakarn^{4,7}, Guanyu Zhu⁸, Sarah Sheldon⁹, Ashvin Vishwanath⁴, Simon Trebst³ & Abhinav Kandala⁹



**quantum
measurements**

joint measurements & entanglement

quantum circuit for 2-qubit
joint measurement



extract
information

shape
entanglement

product
state

$ q_1\rangle q_2\rangle$	$ \overline{q_1}\rangle \overline{q_2}\rangle$	s
$ 0\ 0\rangle$	$ 0\ 0\rangle$	0
$ 0\ 1\rangle$	$ 0\ 1\rangle$	1
$ 1\ 0\rangle$	$ 1\ 0\rangle$	1
$ 1\ 1\rangle$	$ 1\ 1\rangle$	0

syndrome measurement

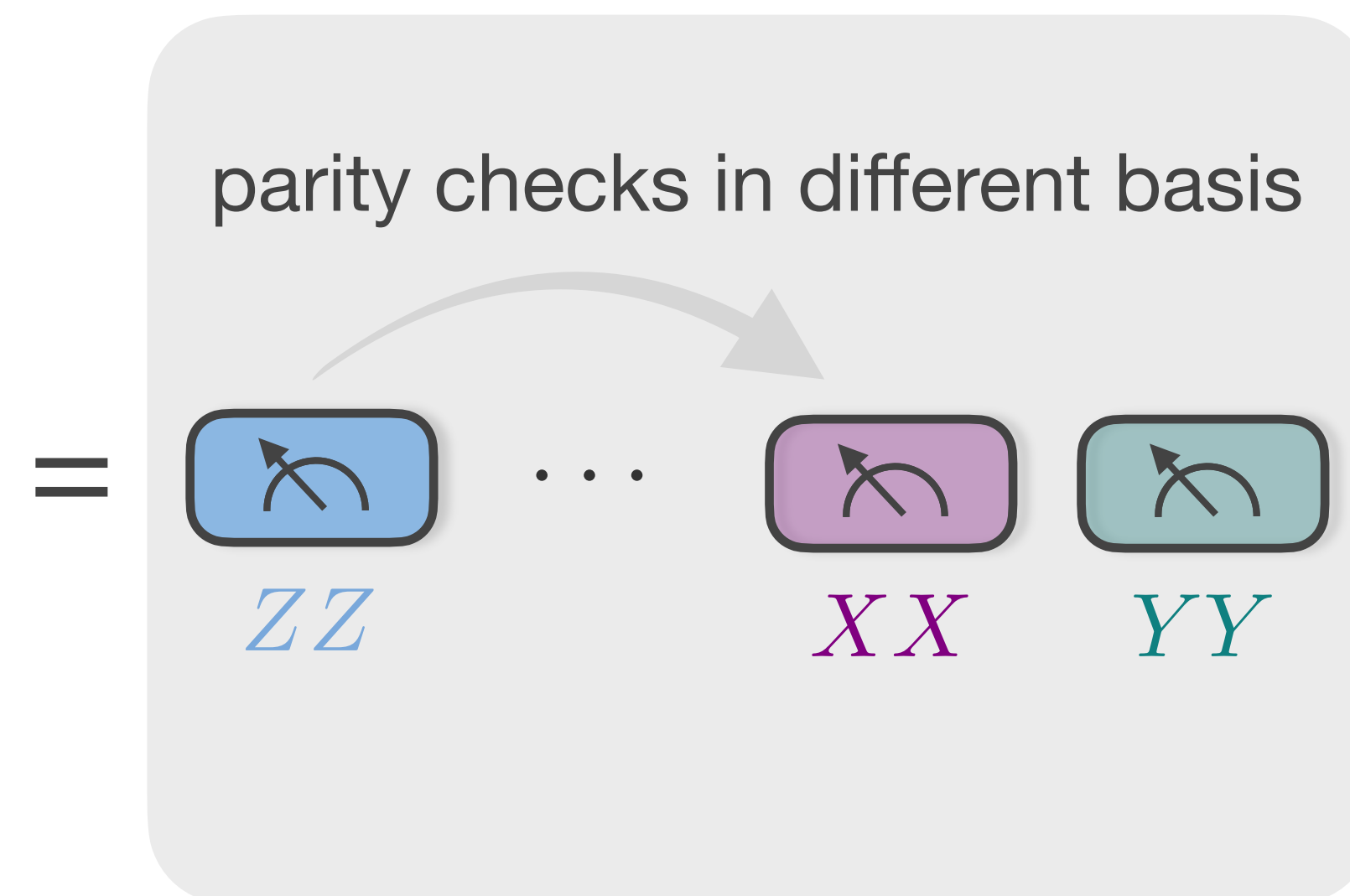
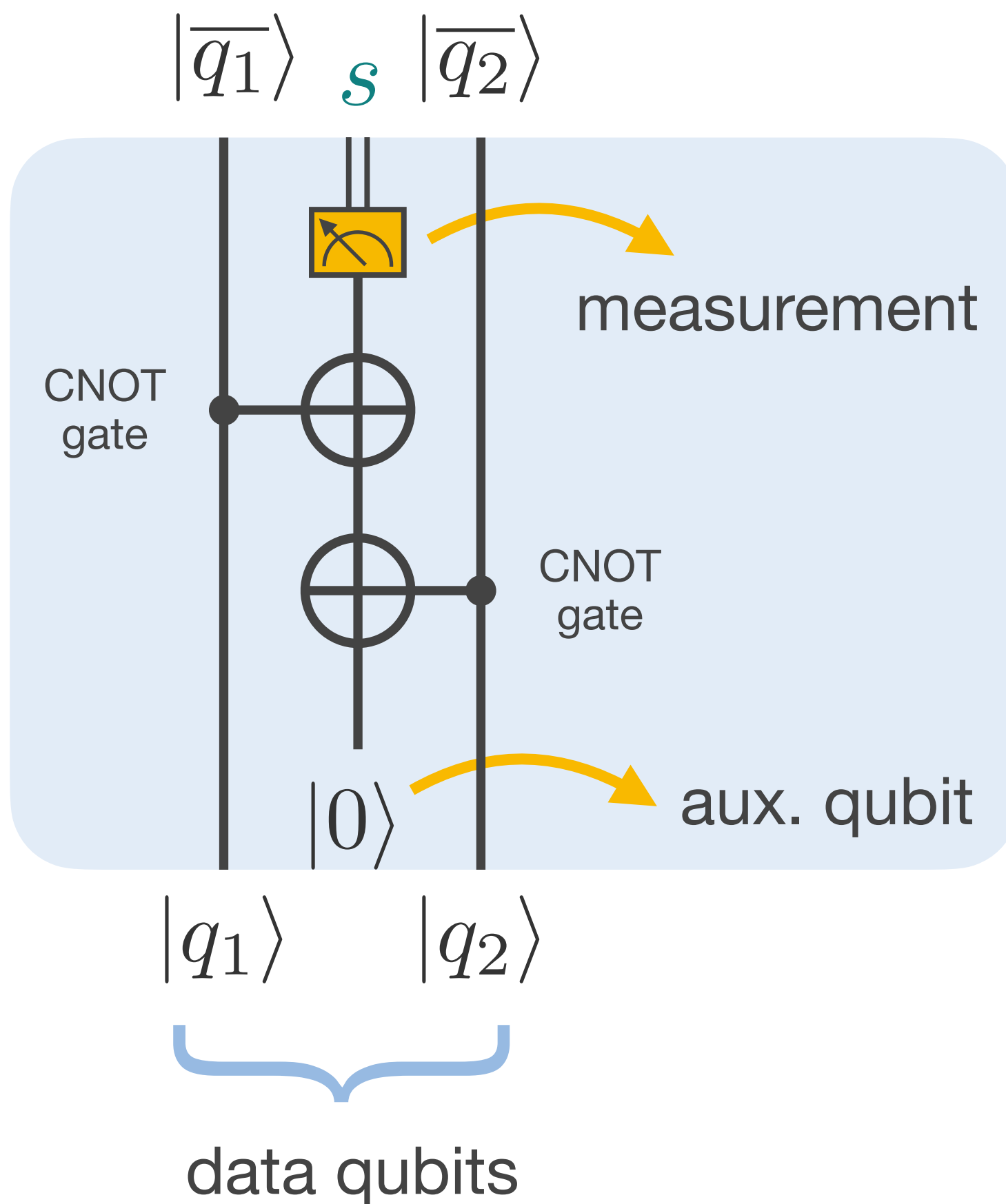
ZZ parity check

$ q_1\rangle q_2\rangle$	$ \overline{q_1}\rangle \overline{q_2}\rangle$	s
$ ++\rangle$	$(00\rangle + 11\rangle) / \sqrt{2}$	0
	$(01\rangle + 10\rangle) / \sqrt{2}$	1

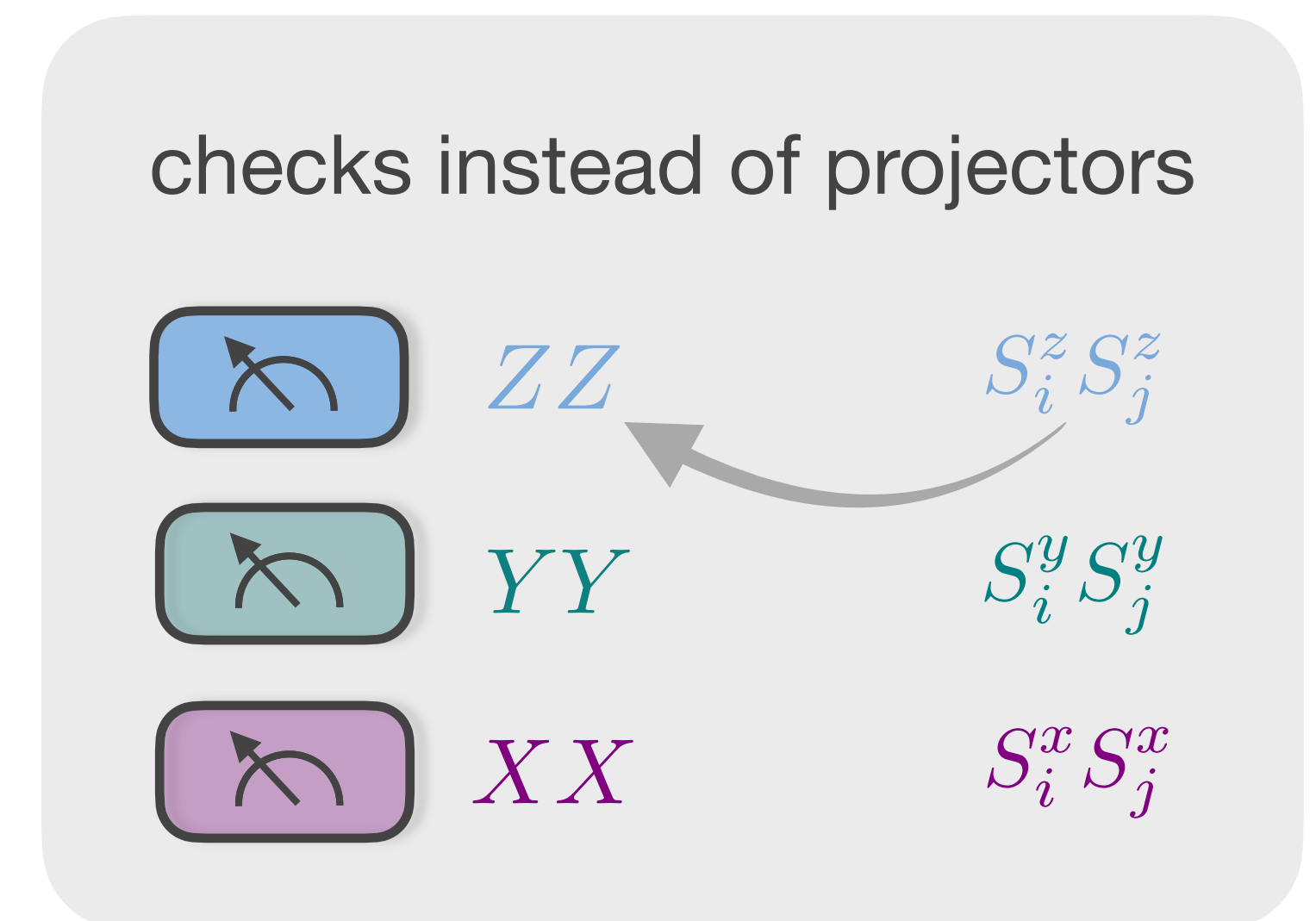
Bell
state

$$|++\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

joint measurements & entanglement



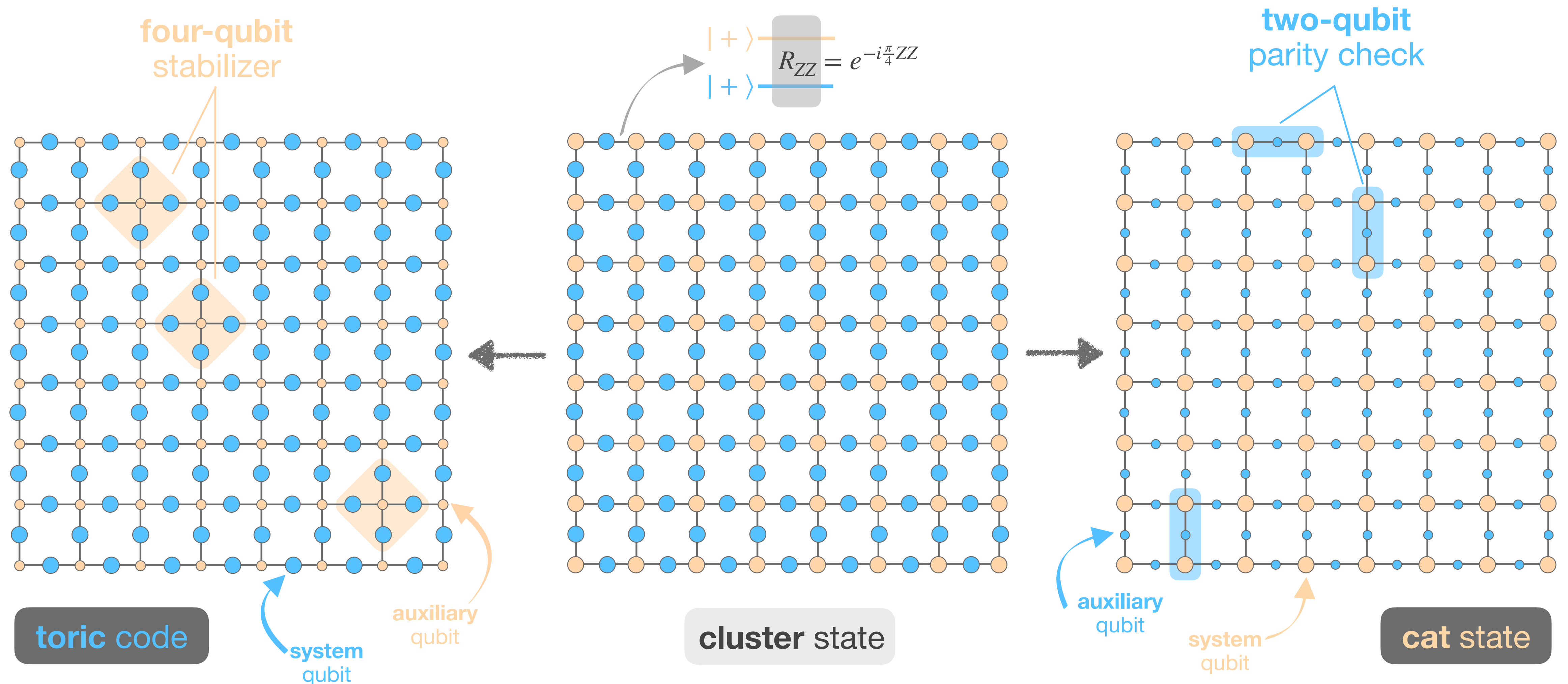
measurement dynamics



Hamiltonian dynamics

new toolbox for quantum many-body physics

joint measurements & entanglement



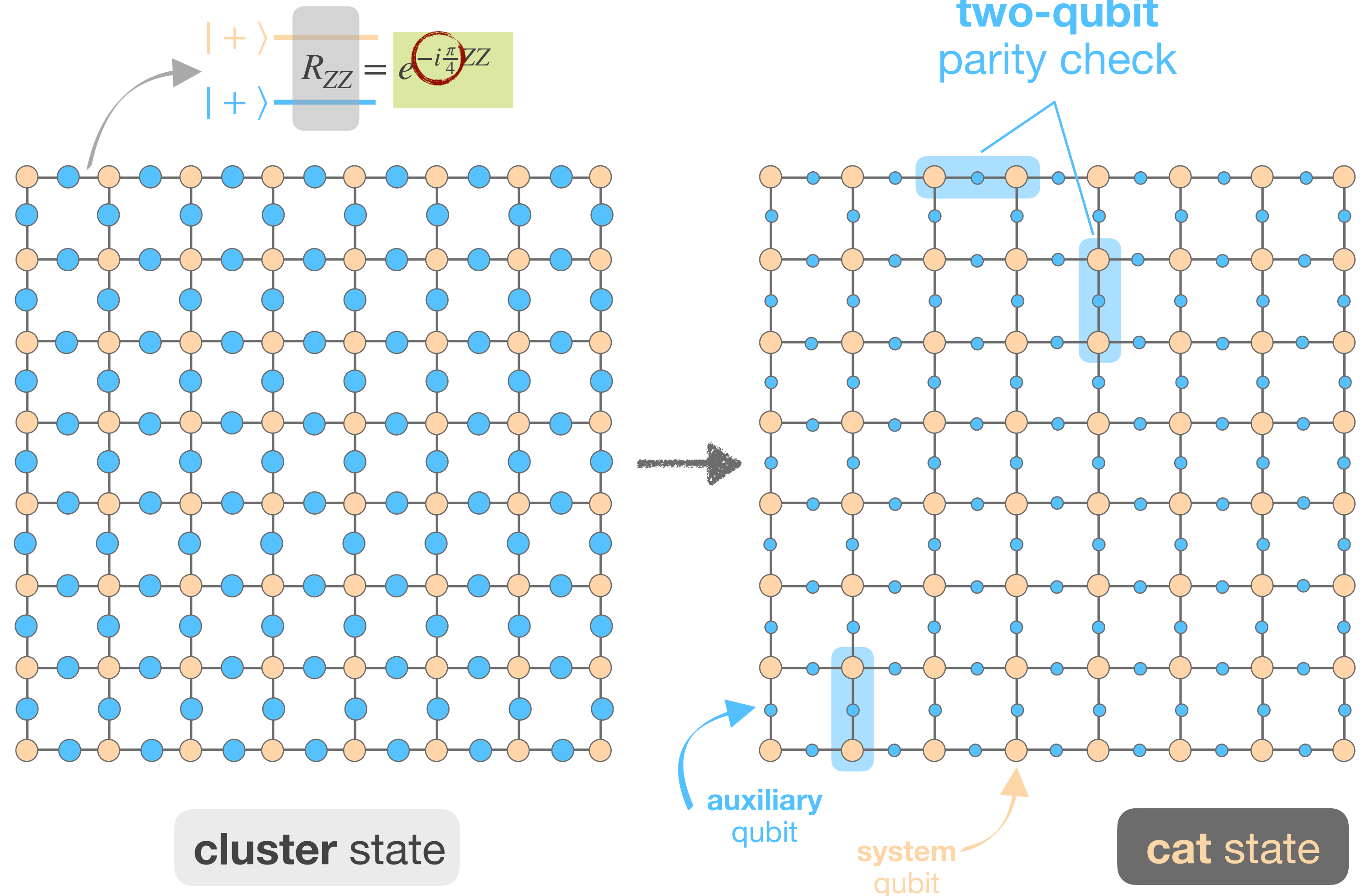
measurement-induced phase transitions

What happens if our **resource** — the cluster state — is **imperfectly prepared**?

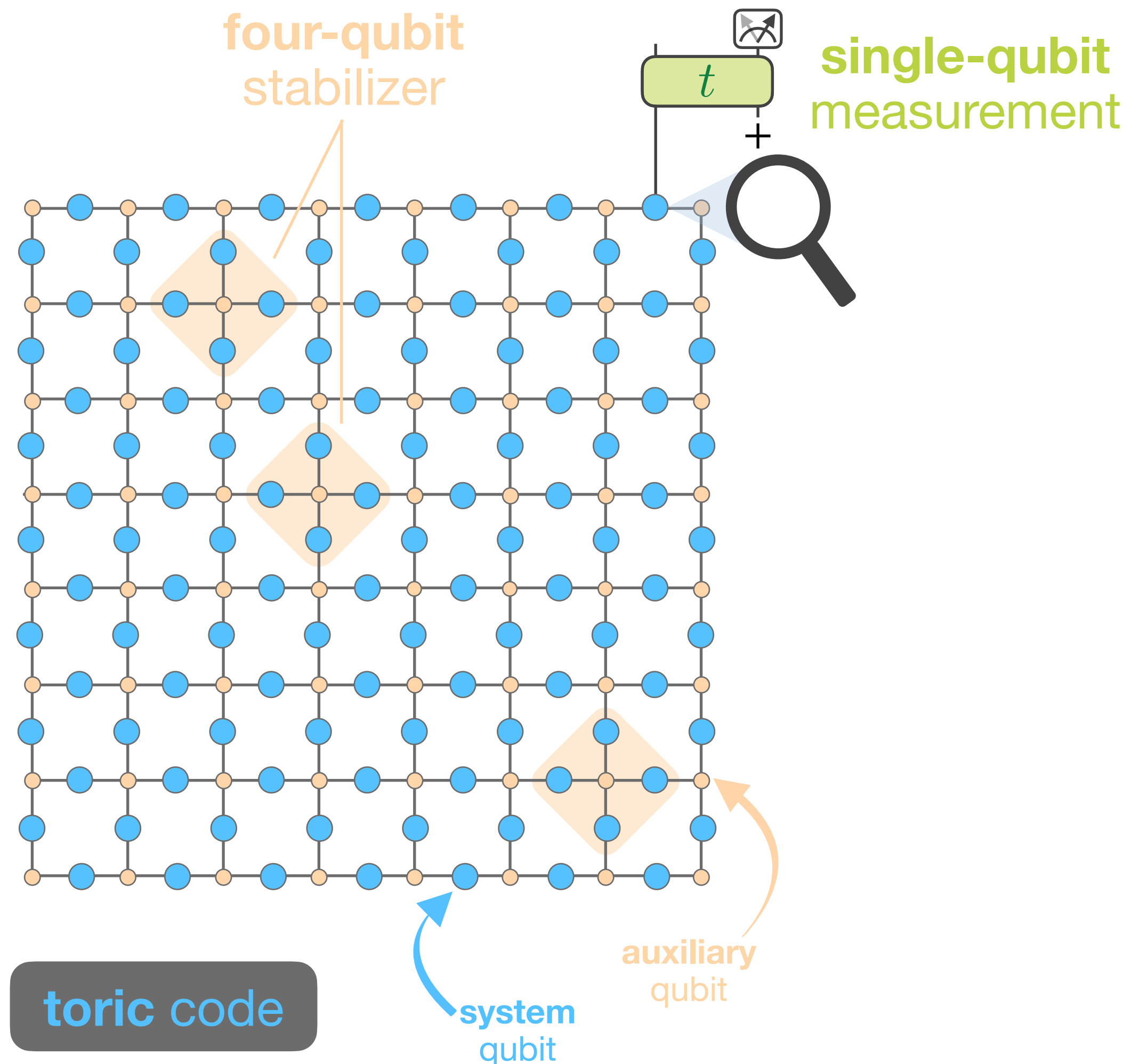
$$0 \leq t \leq \frac{\pi}{4}$$

Is the prepared **cat state** **stable** to this coherent deformation?
Is there a **threshold**?

Nishimori transition



measurement-induced phase transitions



What happens if, for the toric code, we weakly **monitor all system qubits**?

$$0 \leq t \leq \frac{\pi}{4}$$

Is the monitored **toric code stable** to this coherent deformation?
Is there a **threshold**?

Nishimori transition

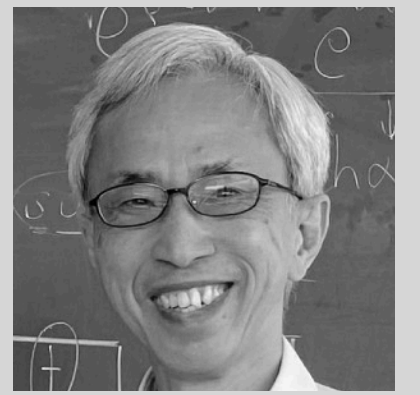
Nishimori physics

Phys. Rev. Lett. 131, 200201 (2023)

G-Y. Zhu, N. Tantivasadakarn, A. Vishwanath, R. Verresen



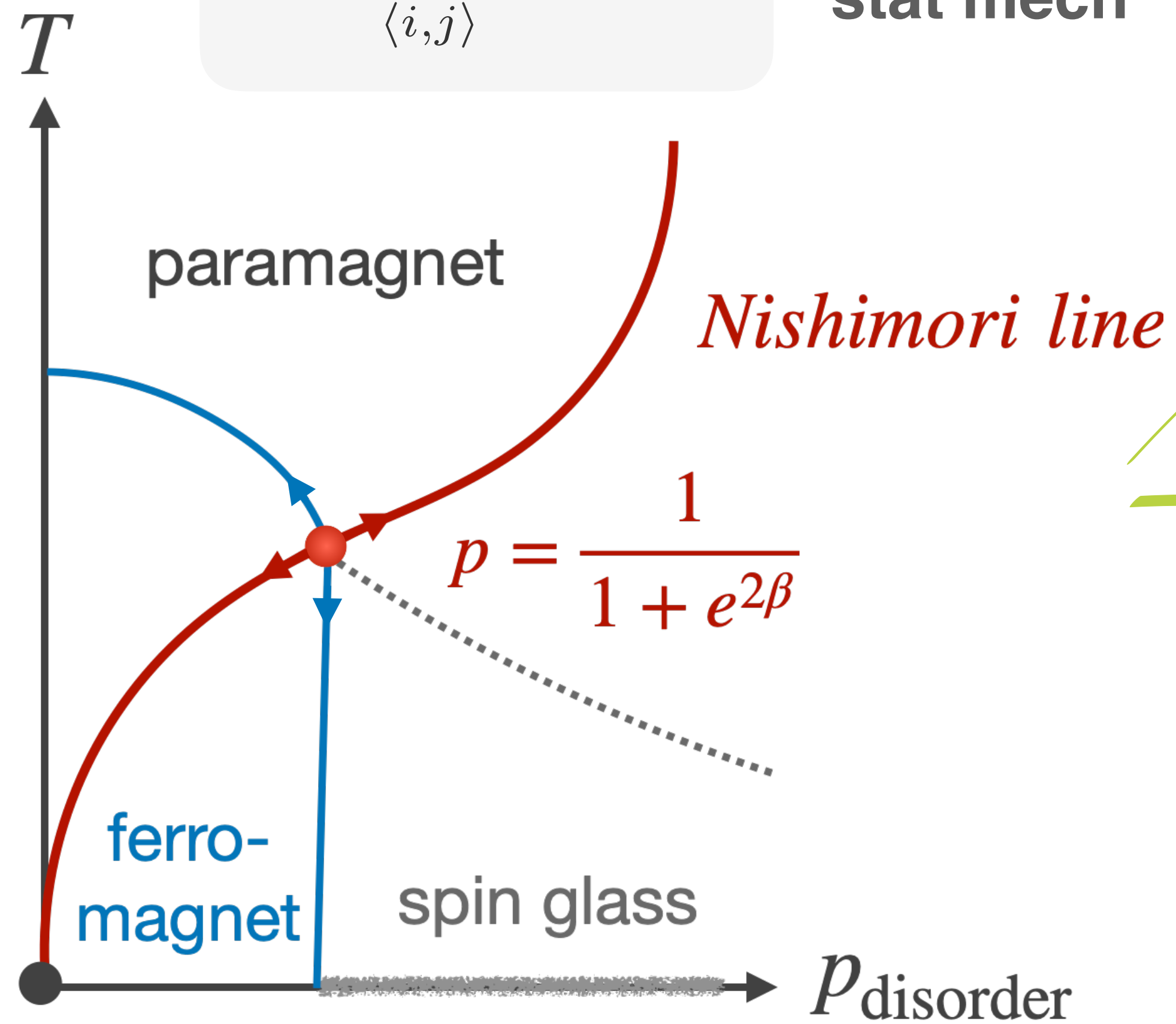
Nishimori physics



Nishimori (1981)

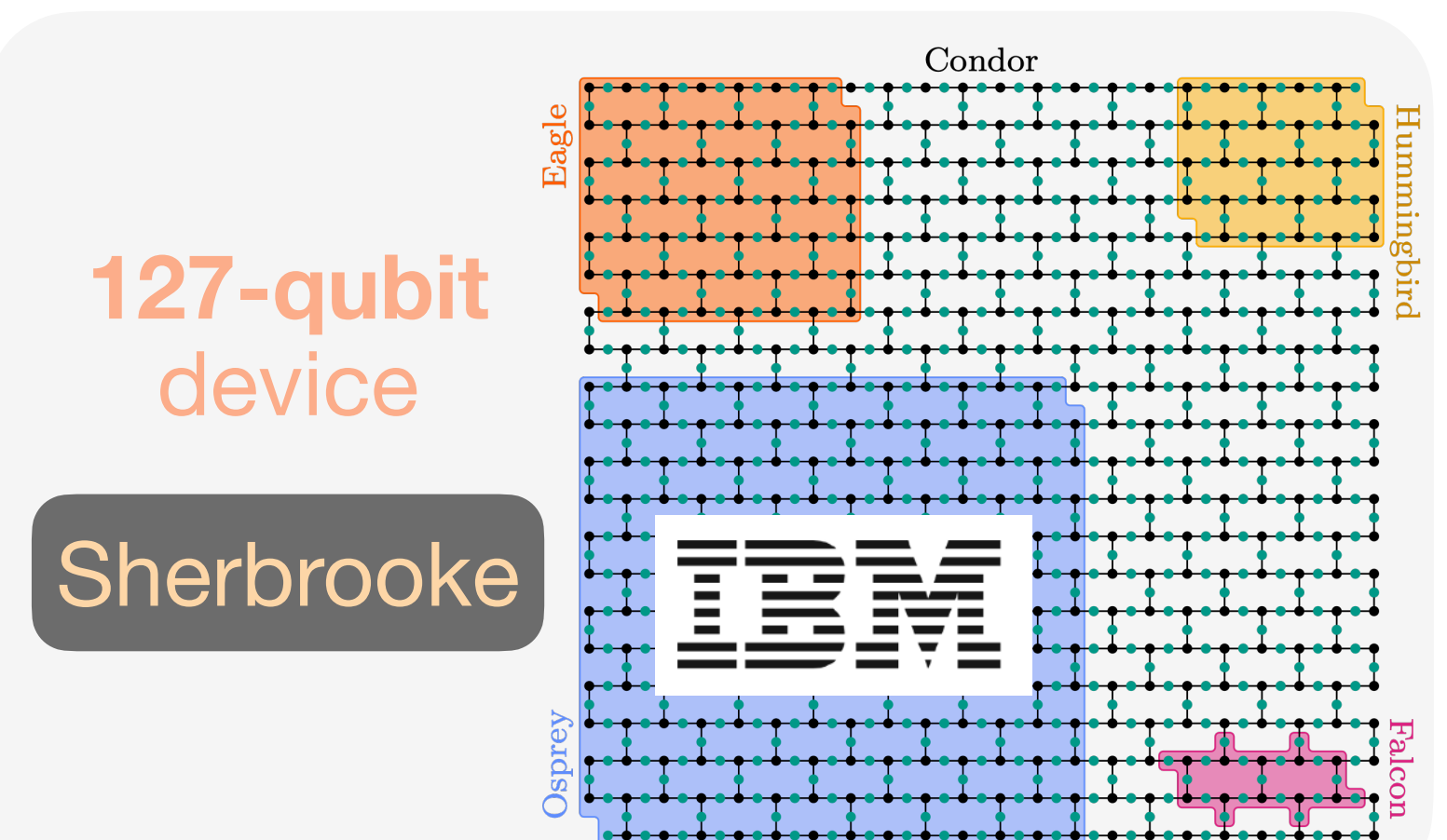
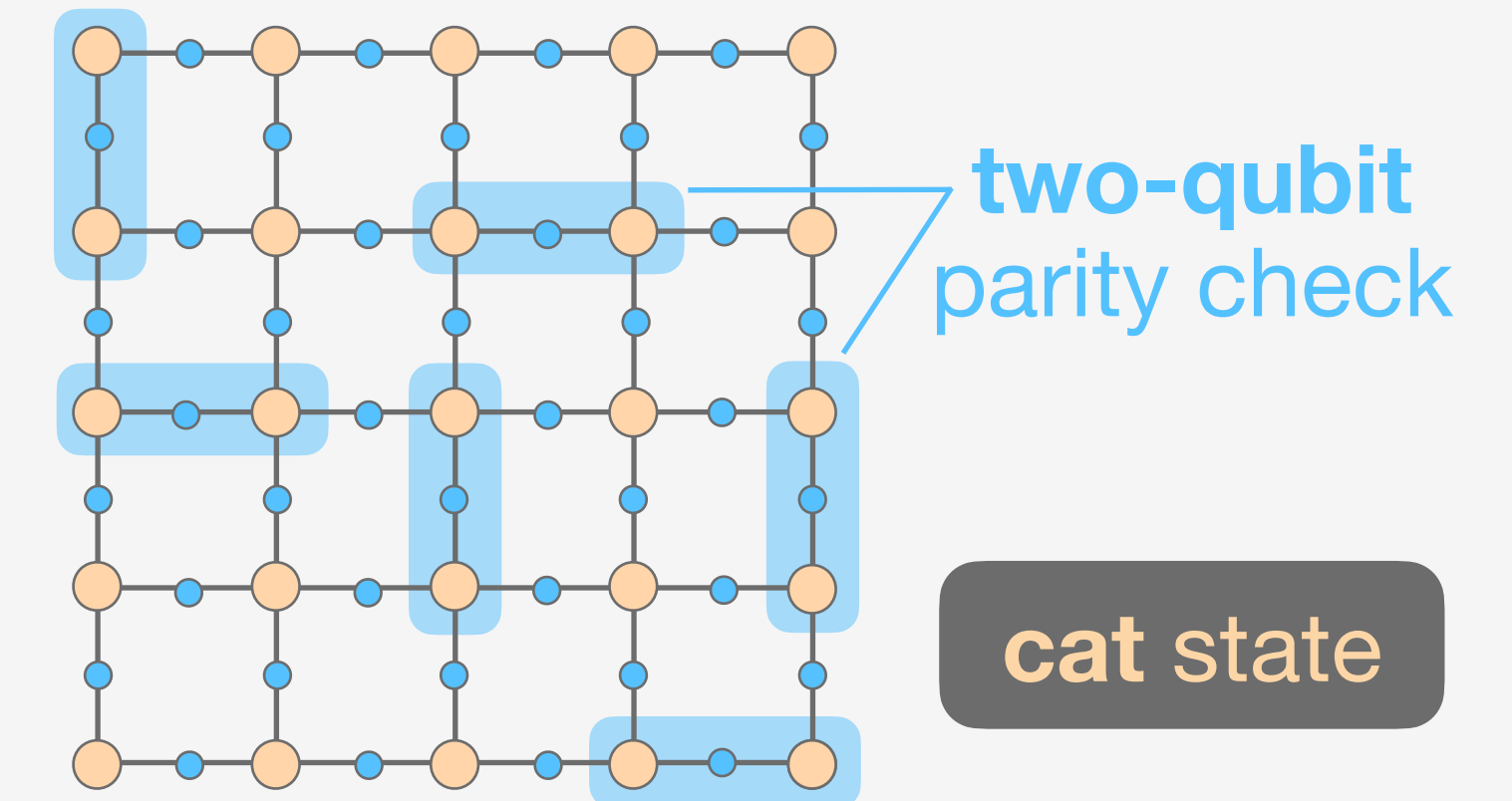
$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} S_i^z S_j^z$$

classical
stat mech

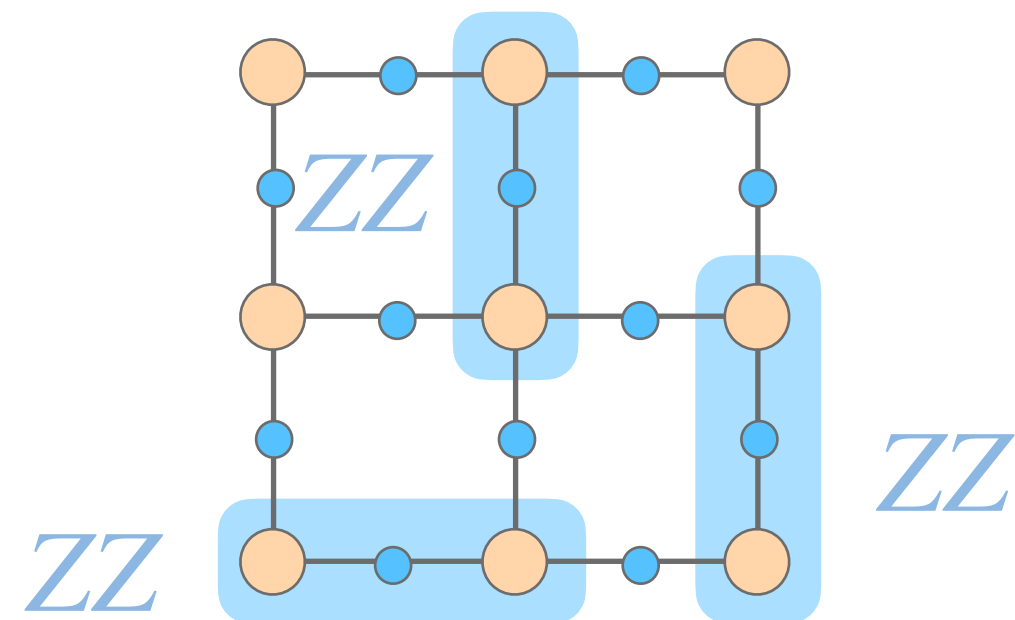


monitored
circuits

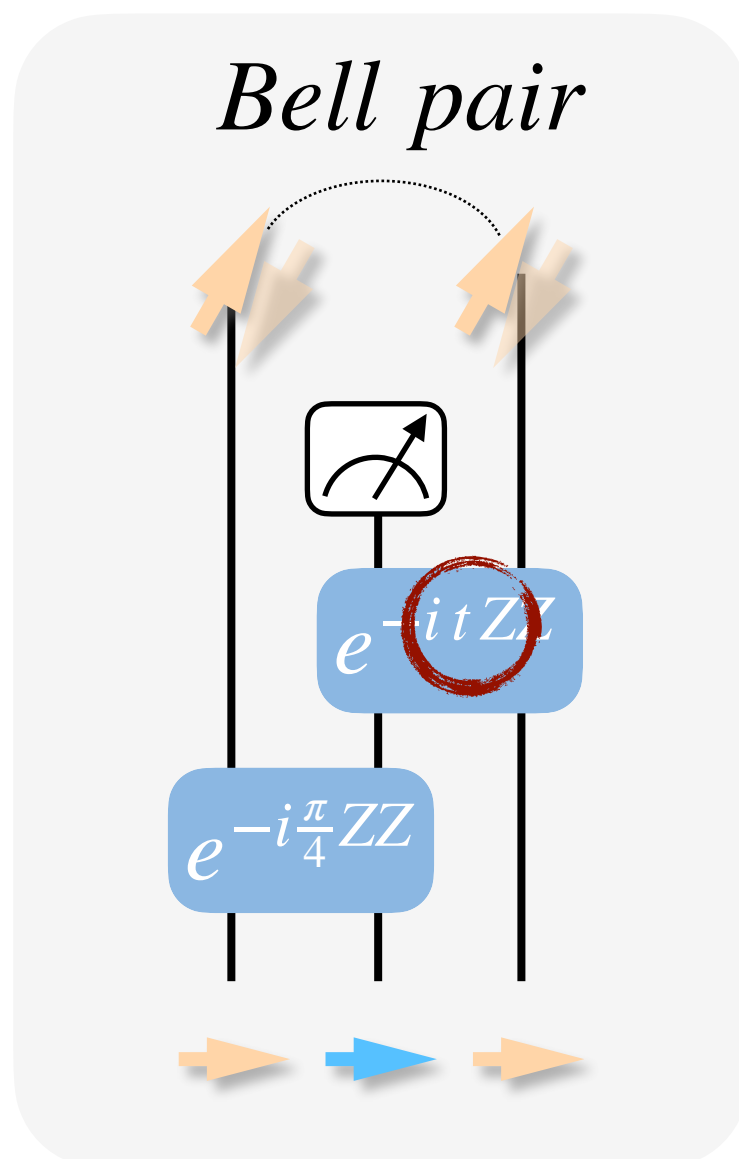
NISQ
devices



Nishimori's cat



Nishimori's cat



$$|\psi\{s\}\rangle = e^{-\frac{\beta}{2} \sum_{ij} J_{sij} \sigma_i^z \sigma_j^z} |+\rangle^{\otimes N}$$

Born's rule

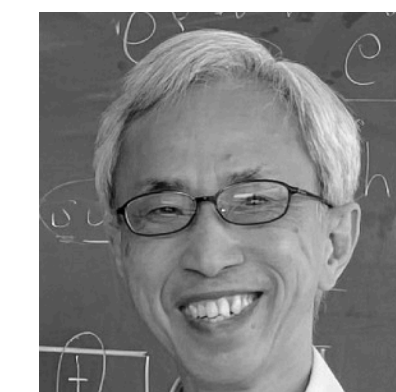
thermal **fluctuations** and **disorder** are **locked**

interpret as
classical
stat mech model

$$\tanh \frac{\beta J_{\pm}}{2} = \pm \tanh t$$

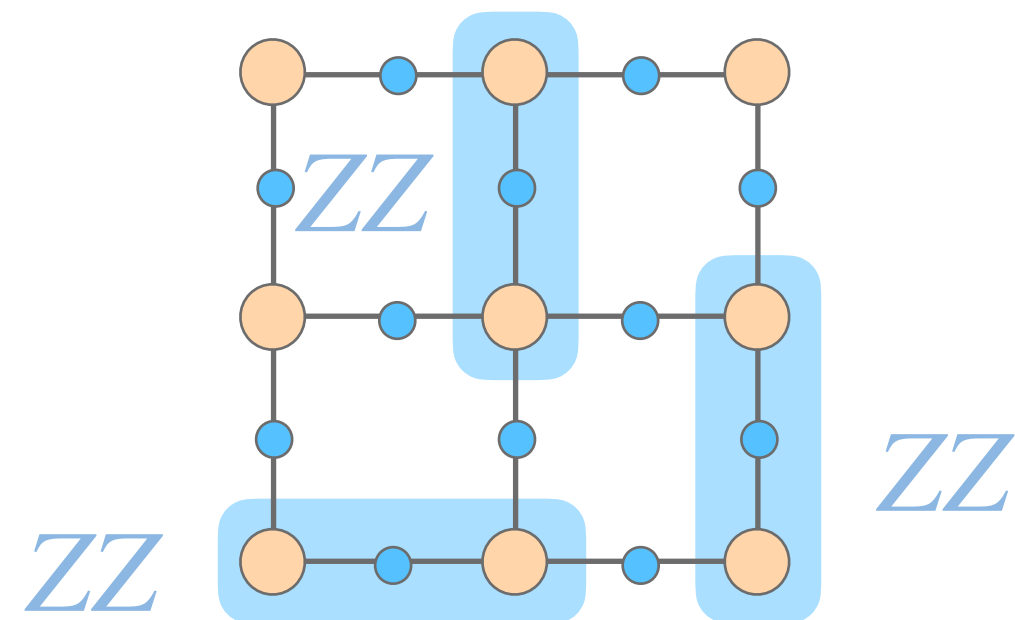
$$Z_{\{s\}} = \sum_{\{\sigma\}} e^{-\beta \sum_{ij} J_{sij} \sigma_i \sigma_j}$$

random bond Ising model

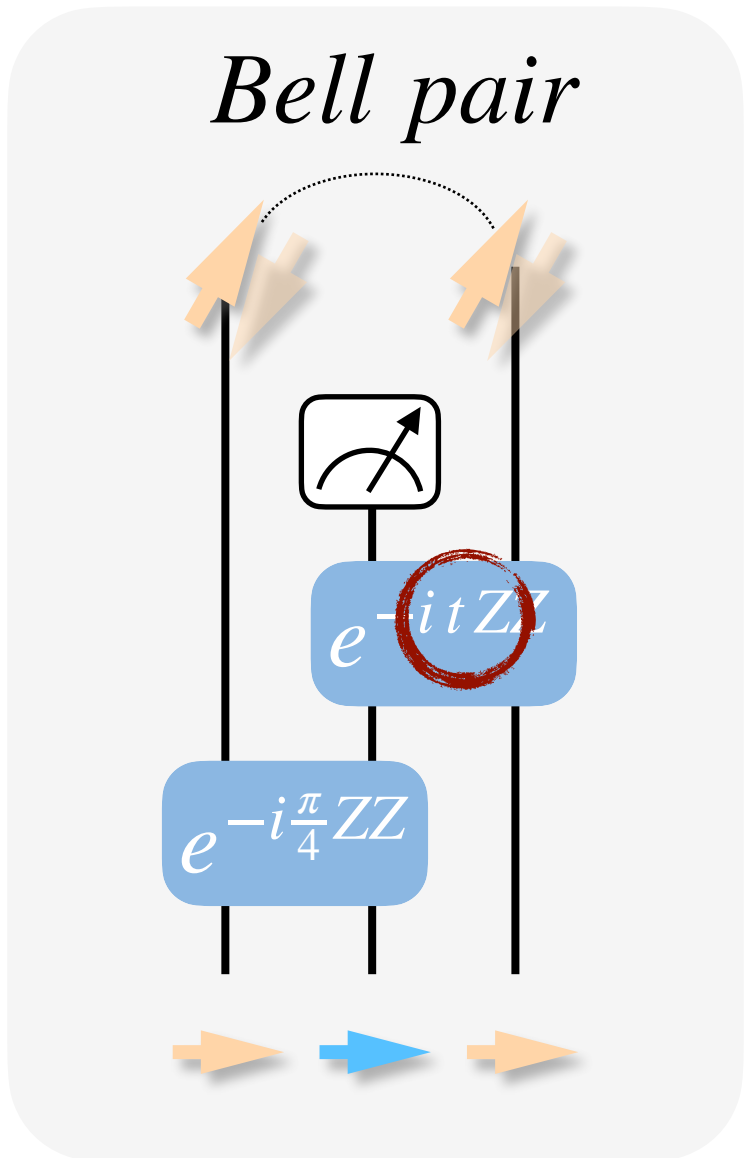


Nishimori (1981)

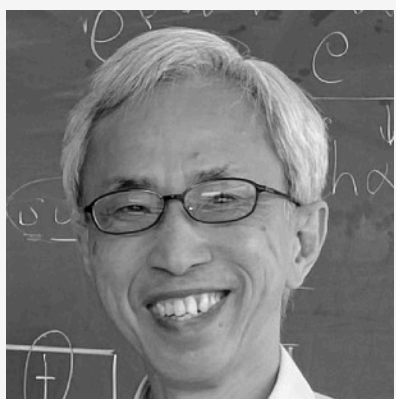
Nishimori's cat



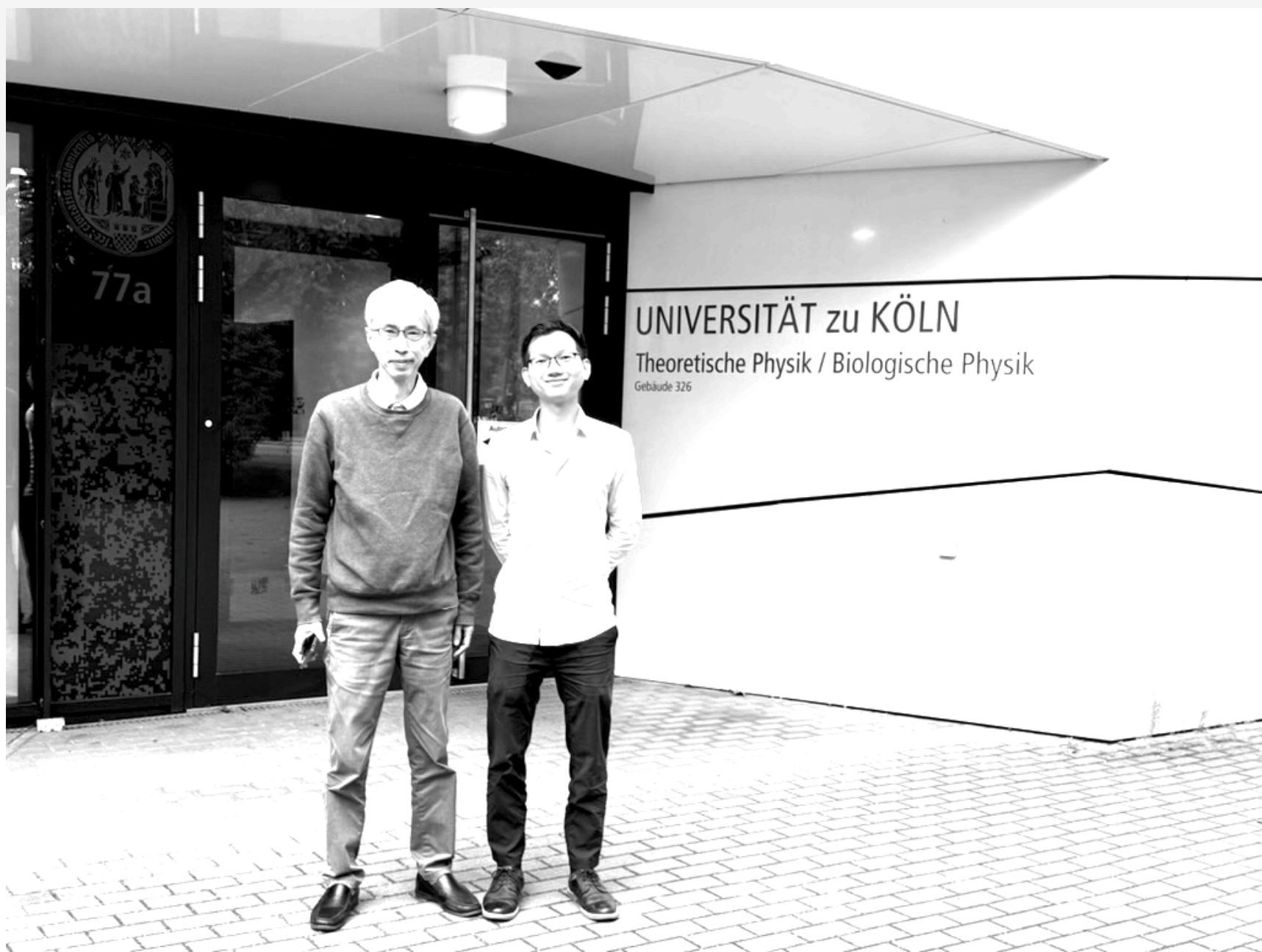
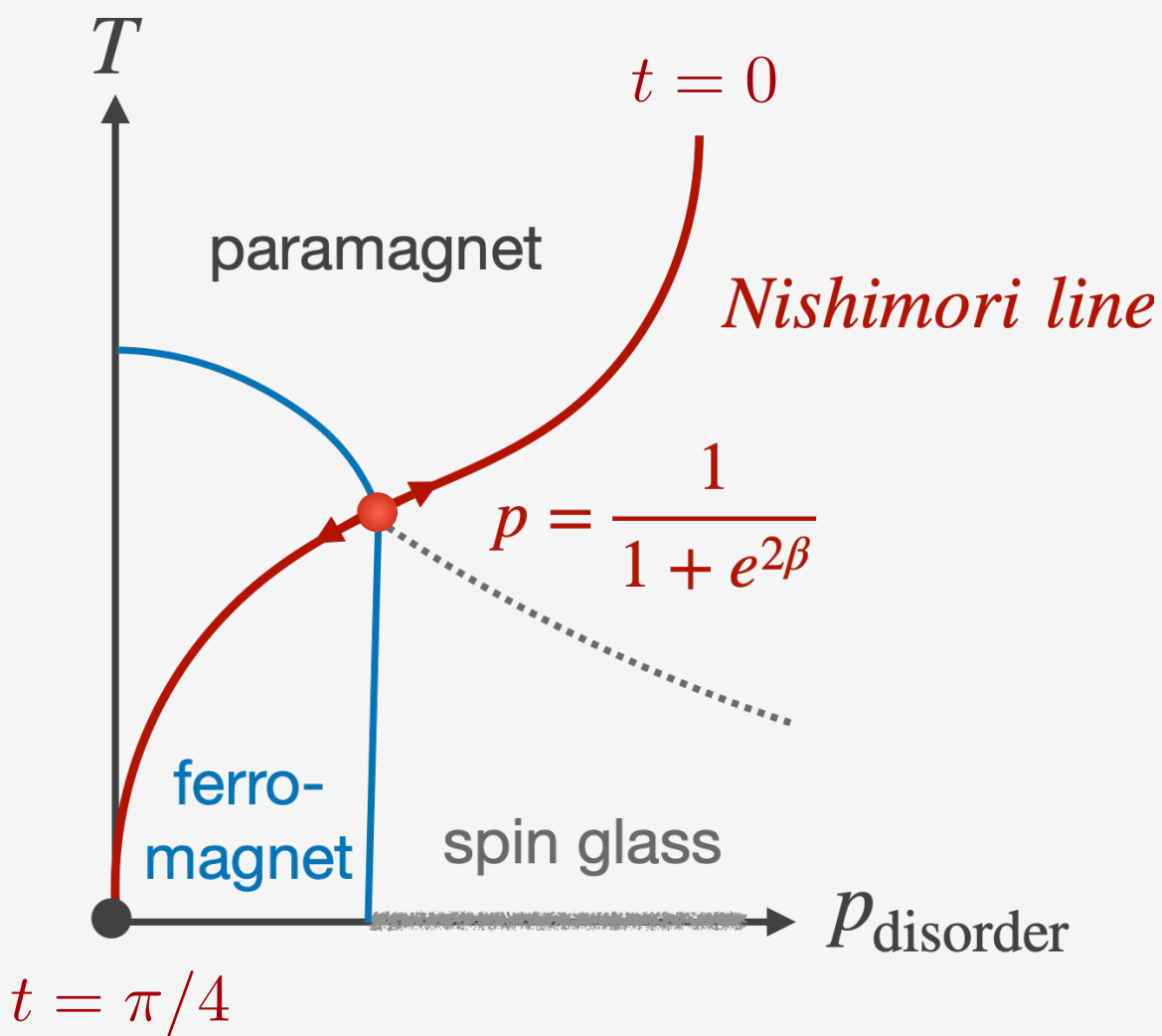
Nishimori's cat



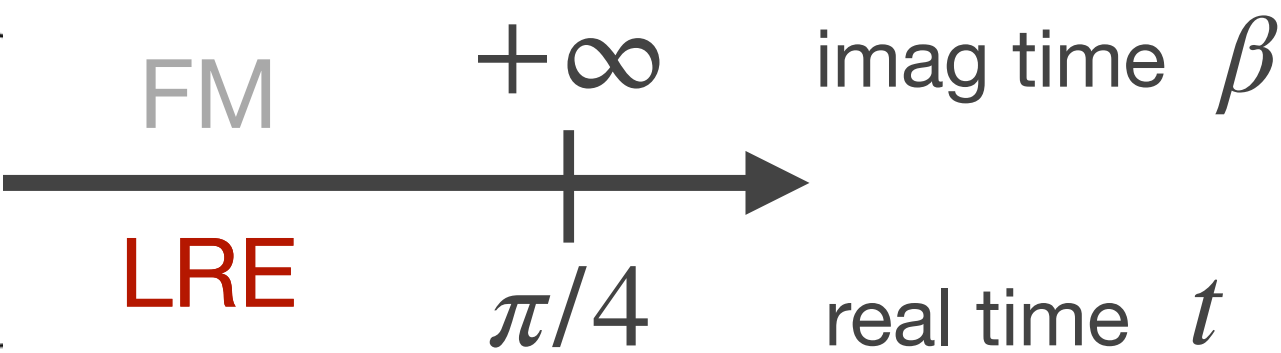
thermal fluctuations and disorder are **locked**



Nishimori (1981)



“low temperature”



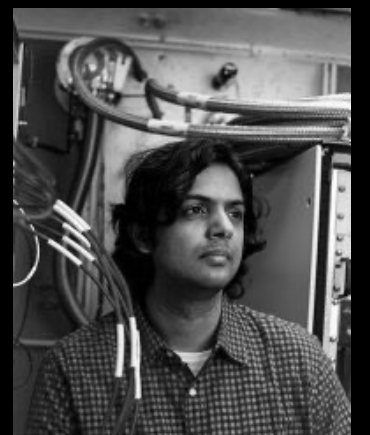
strong measurement

Eagle

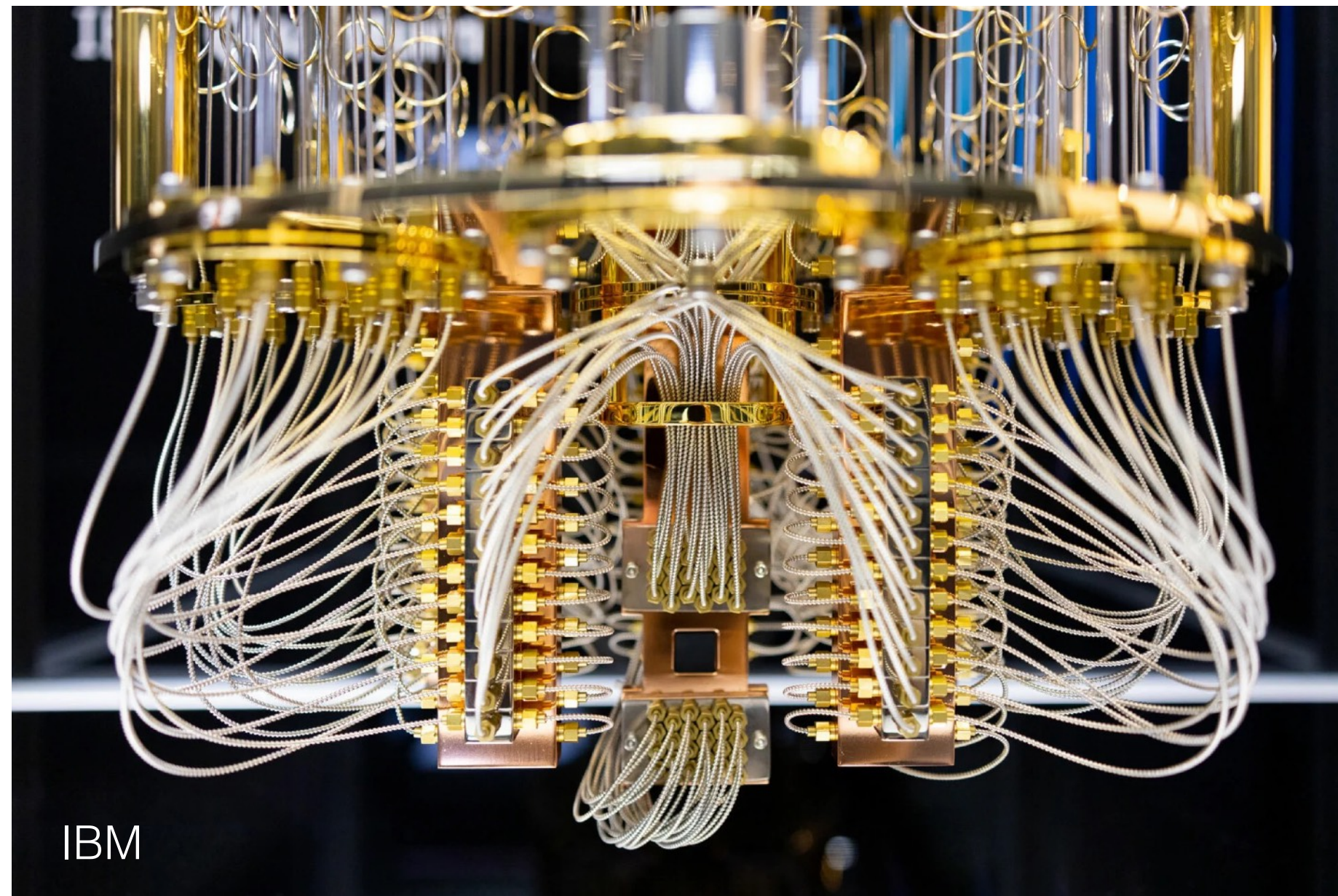
experiment

Nature Physics 21, 161 (2025)

E. Chen, G-Y. Zhu, R. Verresen, A. Seif, E. Bäumer, D. Layden, N. Tantivasadakarn, G. Zhu, S. Sheldon, A. Vishwanath, A. Kandala



IBM quantum cloud devices



NISQ devices built on transmon qubits

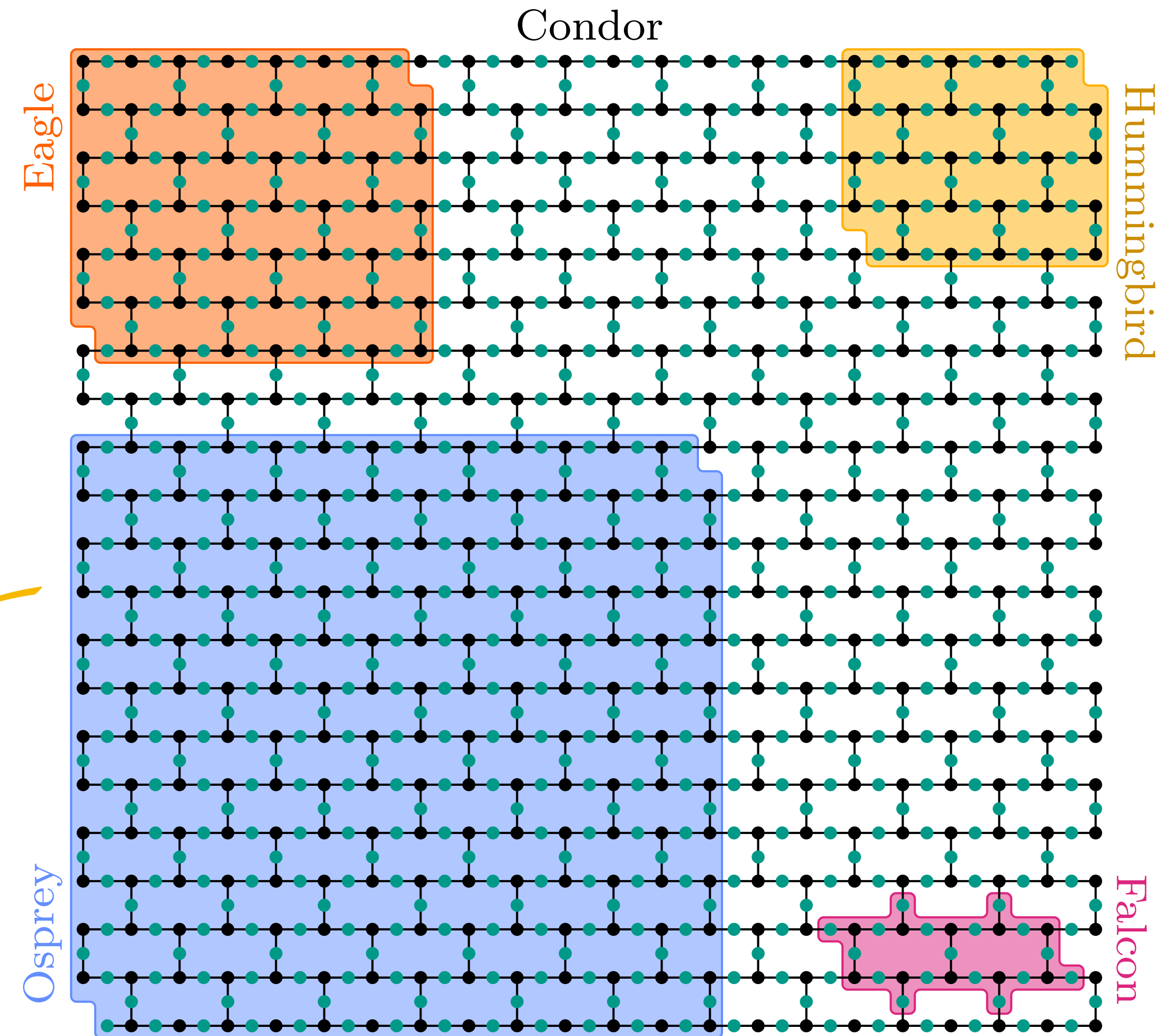
**noisy intermediate
scale quantum
devices**

heavy-hexagon

geometry

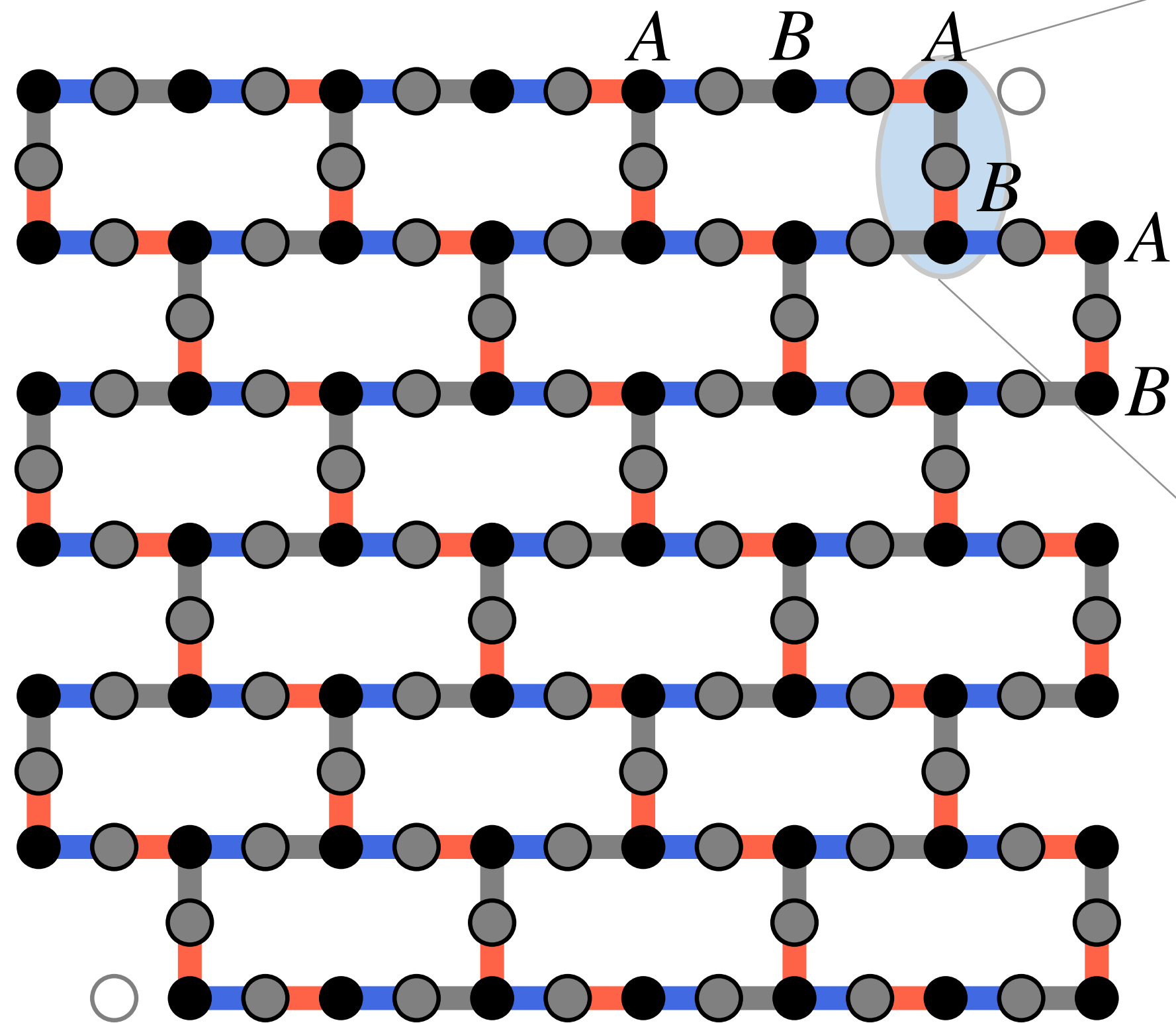
+

Ising evolution gates

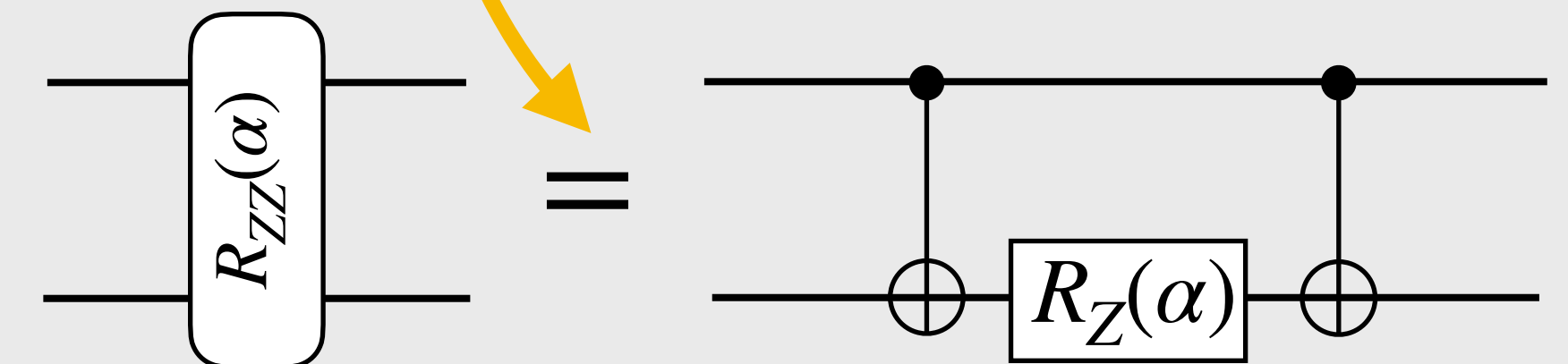
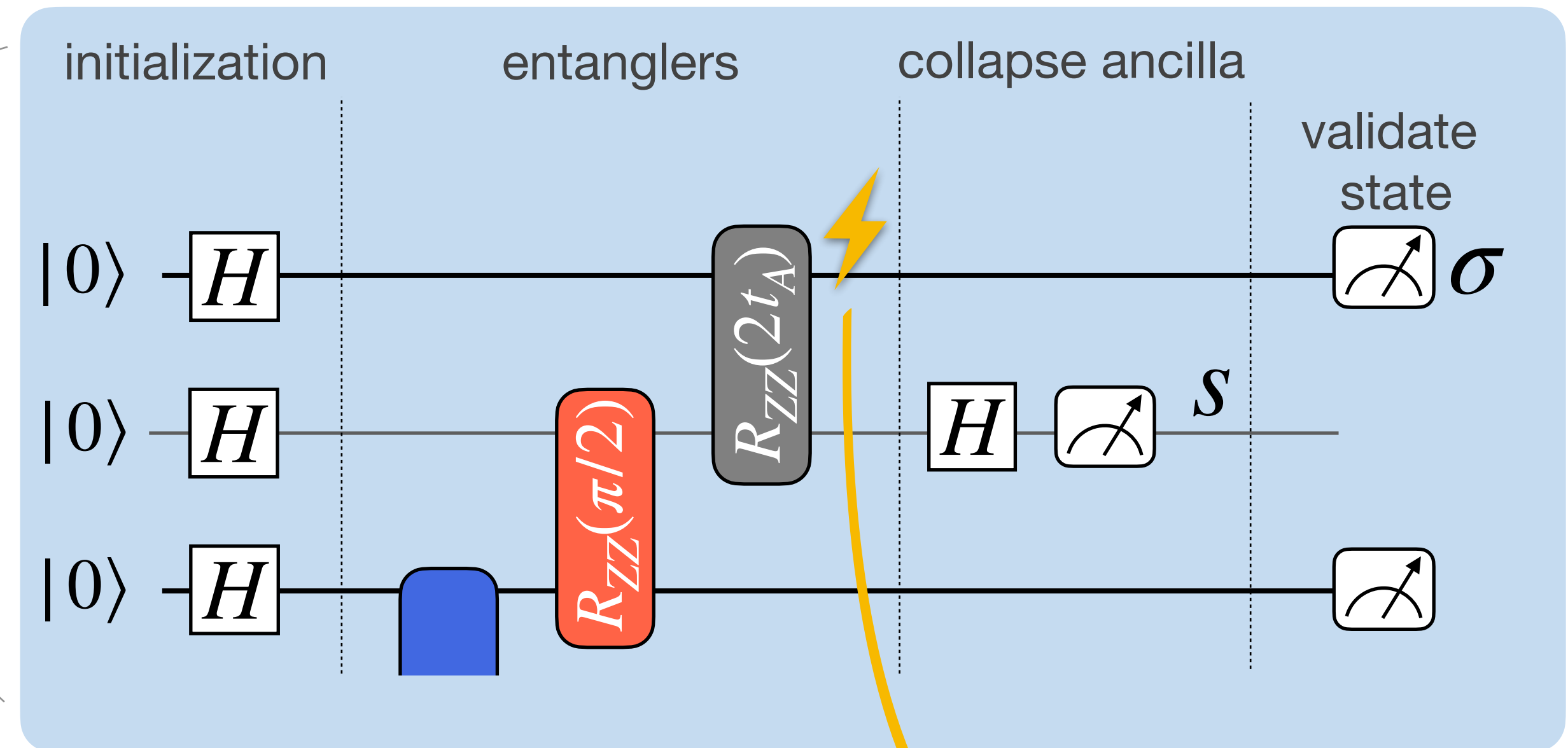


protocol on IBM heavy-hexagon lattice

IBM_Sherbrooke



- **system** qubit: site of honeycomb lattice
- **auxiliary** qubit: bond of honeycomb lattice

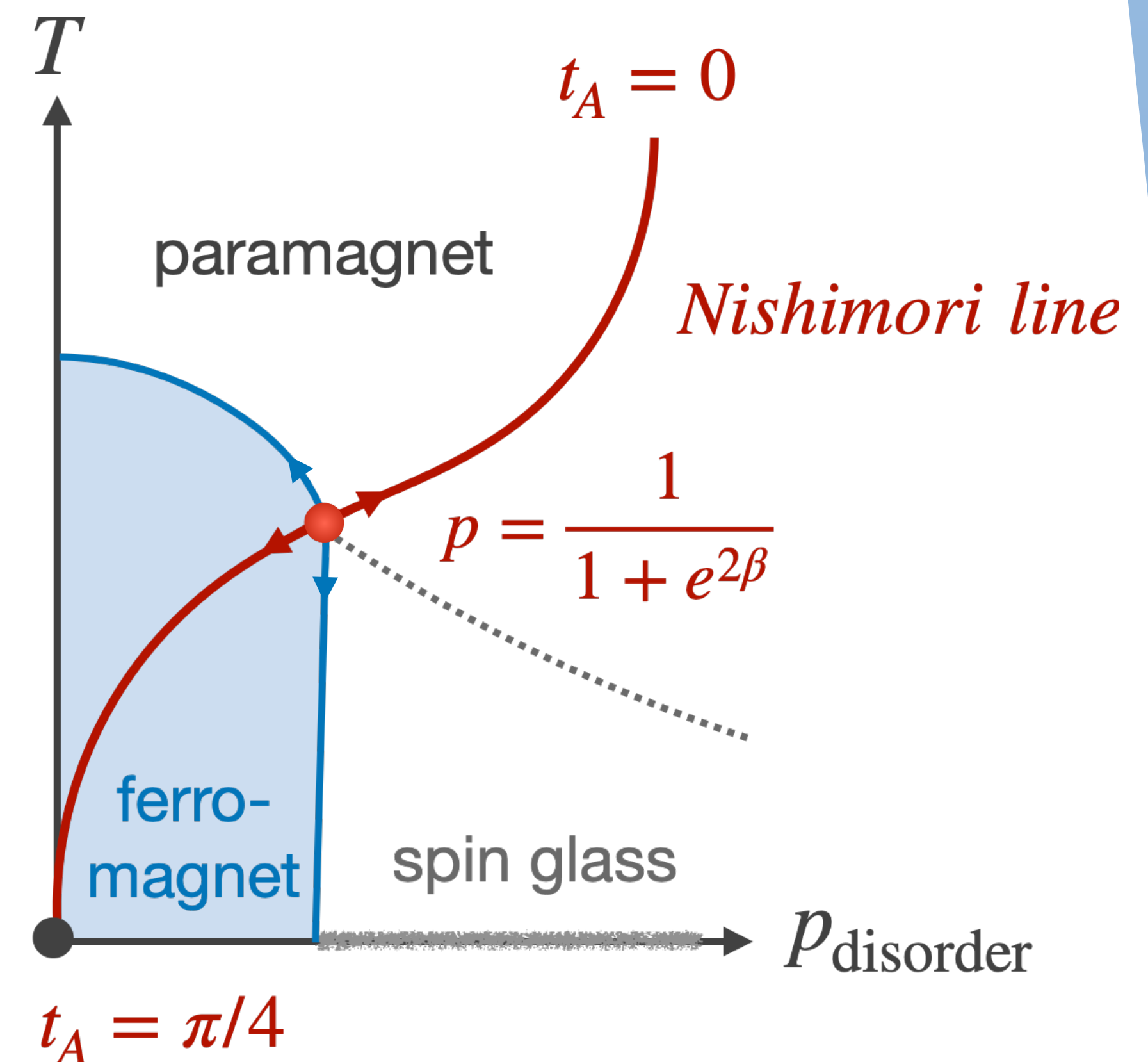
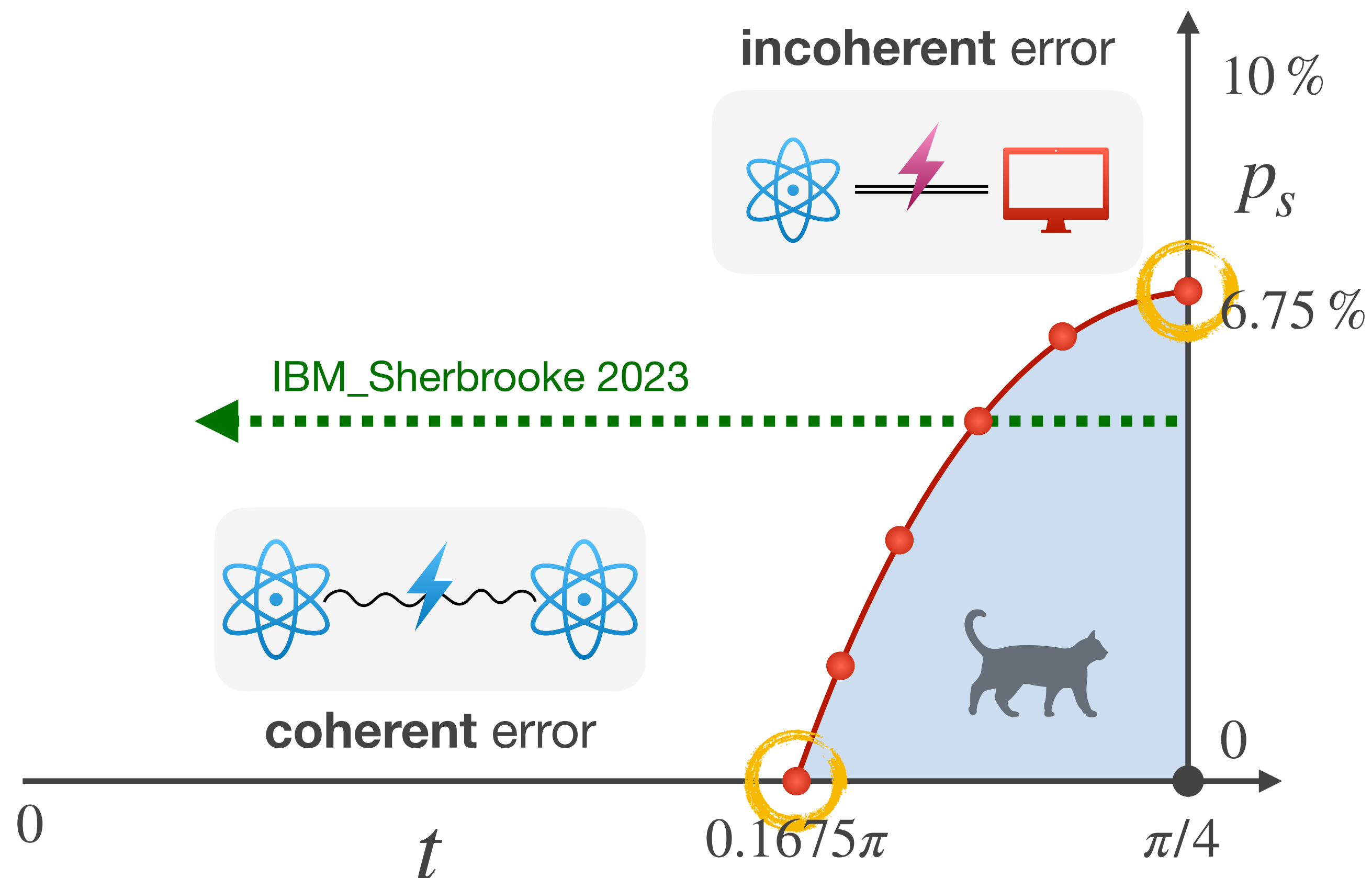


- $\alpha < \pi/4$ injects **tunable coherent error**
- generically a **non-Clifford** gate

incoherent & coherent errors



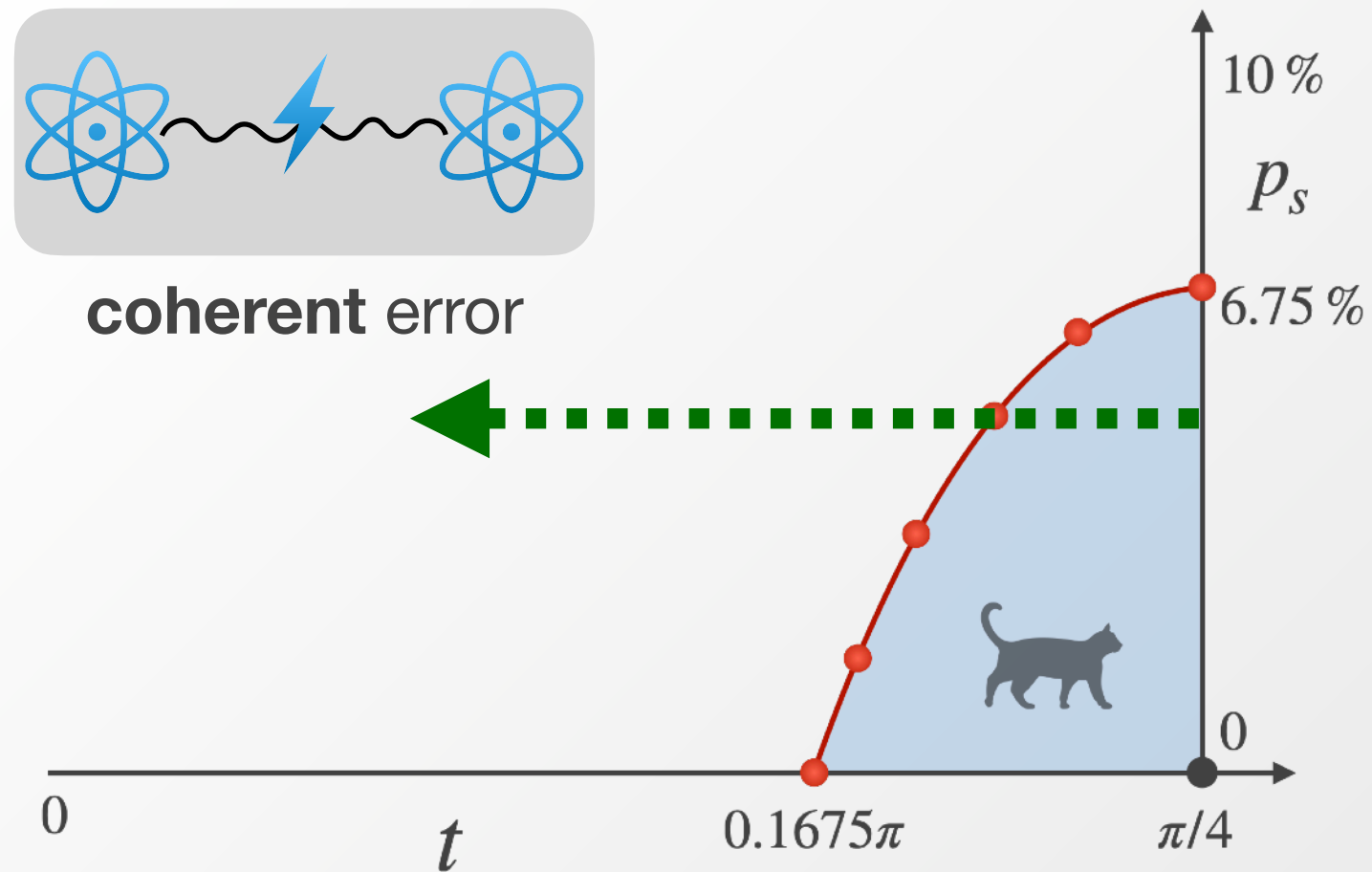
Dennis, Kitaev, Landahl, Preskill 2002



entire phase diagram is mapped to **Nishimori line**

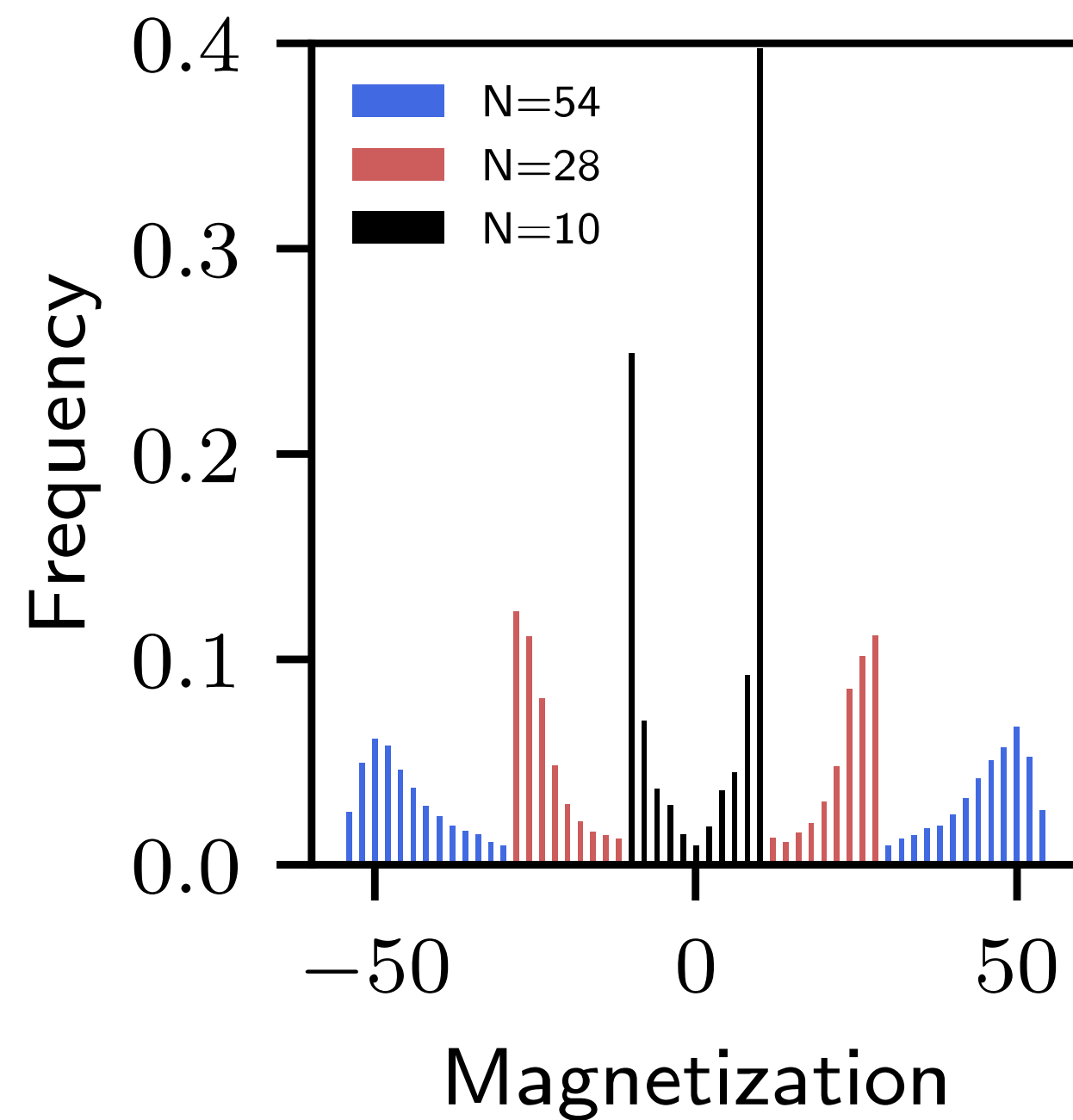
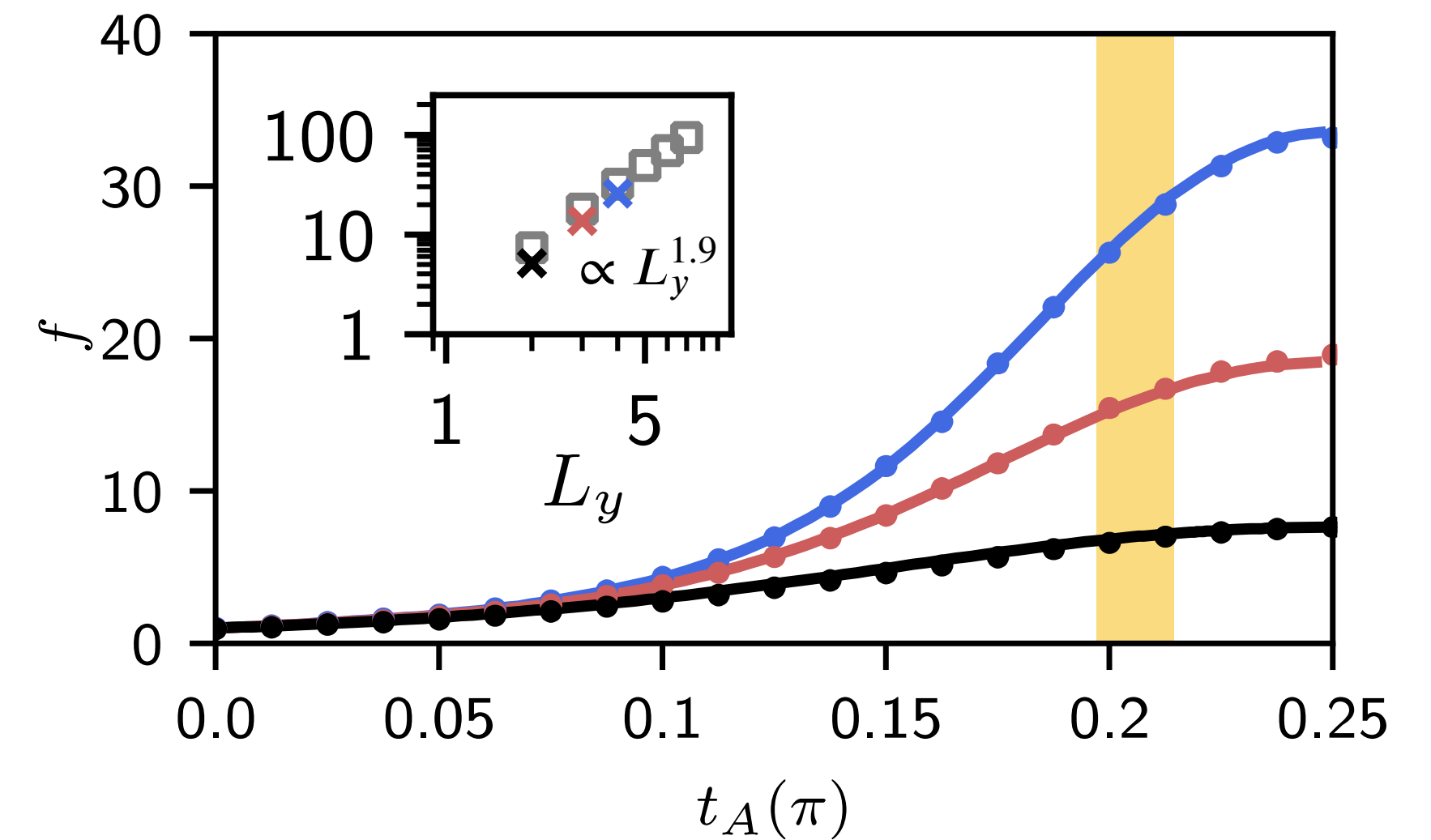
$$\tilde{p} = \frac{1 - (1 - 2p_s)\sin(2t_A)}{2}$$

coherent error transition



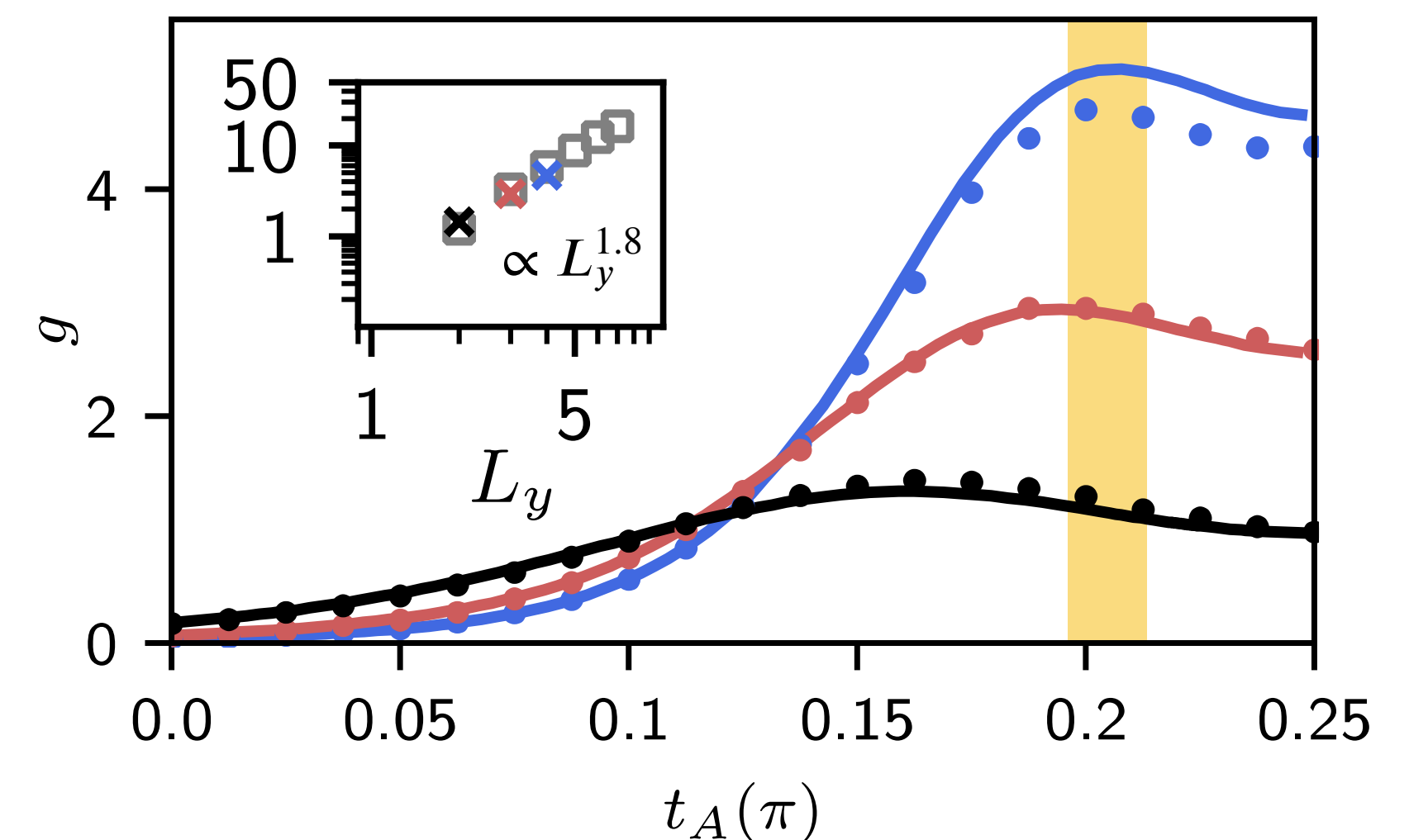
$$f = \frac{1}{N} (\langle M^2 \rangle - \langle M \rangle^2)$$

classical
correlations



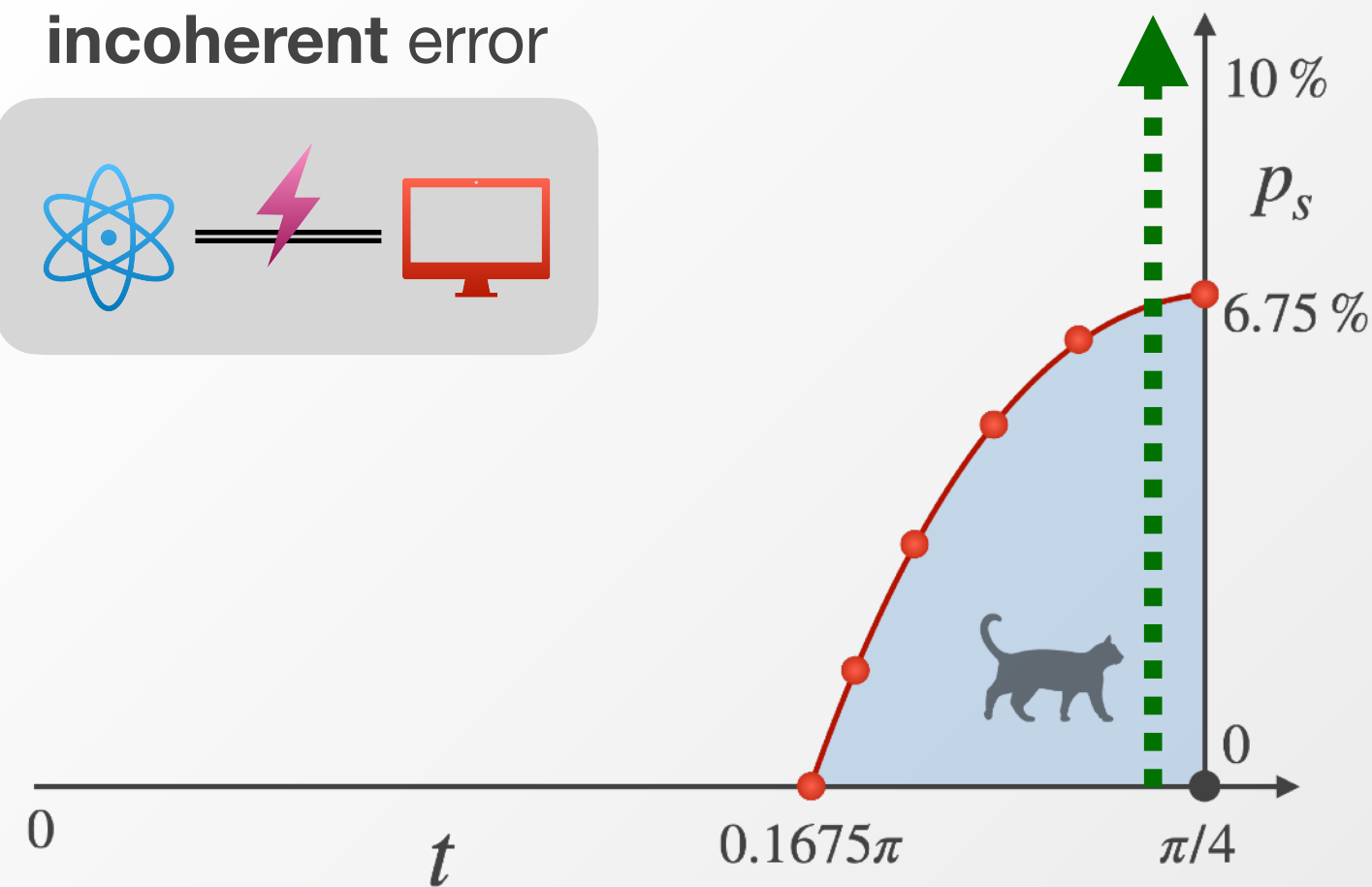
normalized
variance

$$g = \frac{1}{N^3} (\langle M^4 \rangle - \langle M^2 \rangle^2)$$

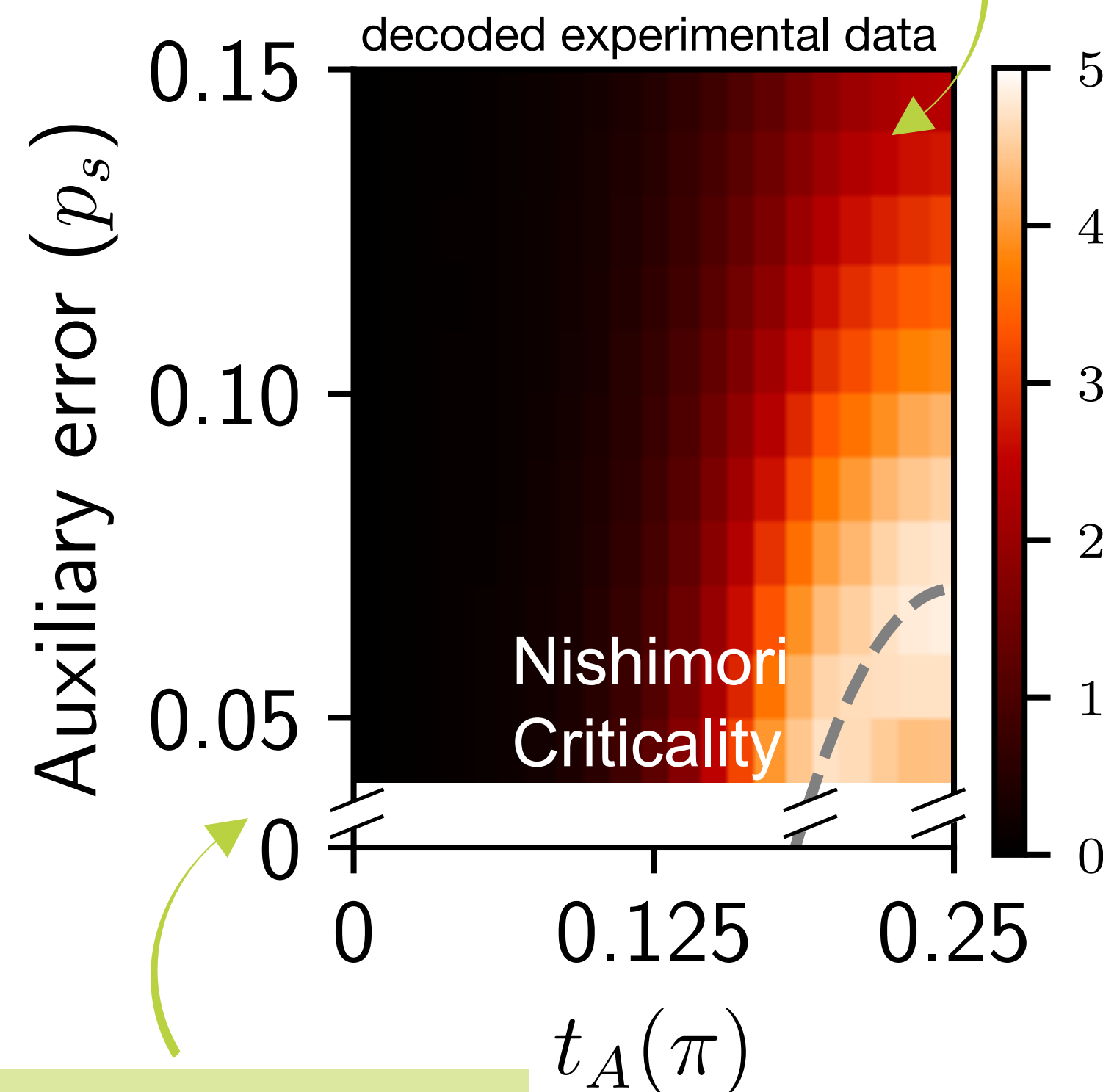


incoherent error transition

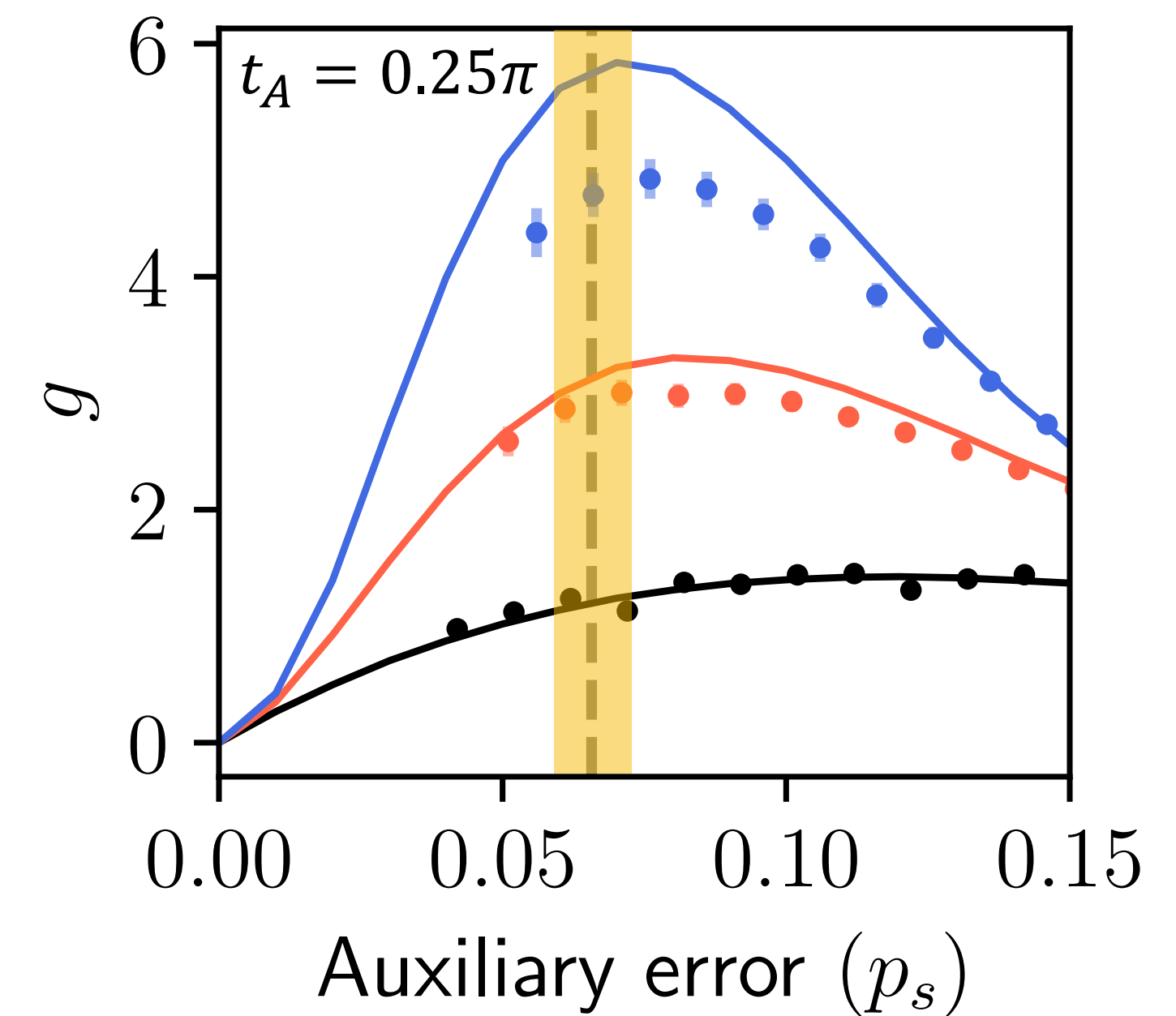
incoherent error



injected error
into classical computer



intrinsic error
of quantum device



... experimental data on IBM Sherbrooke
— theoretical benchmark with noise

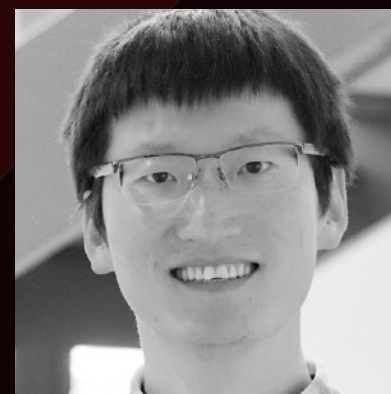
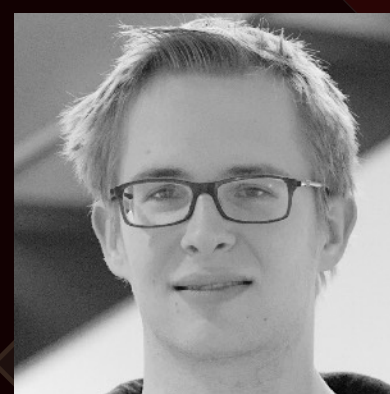
decoding transition

monitored toric codes

arXiv:2502.14034

PRX Quantum 5, 040313 (2024)

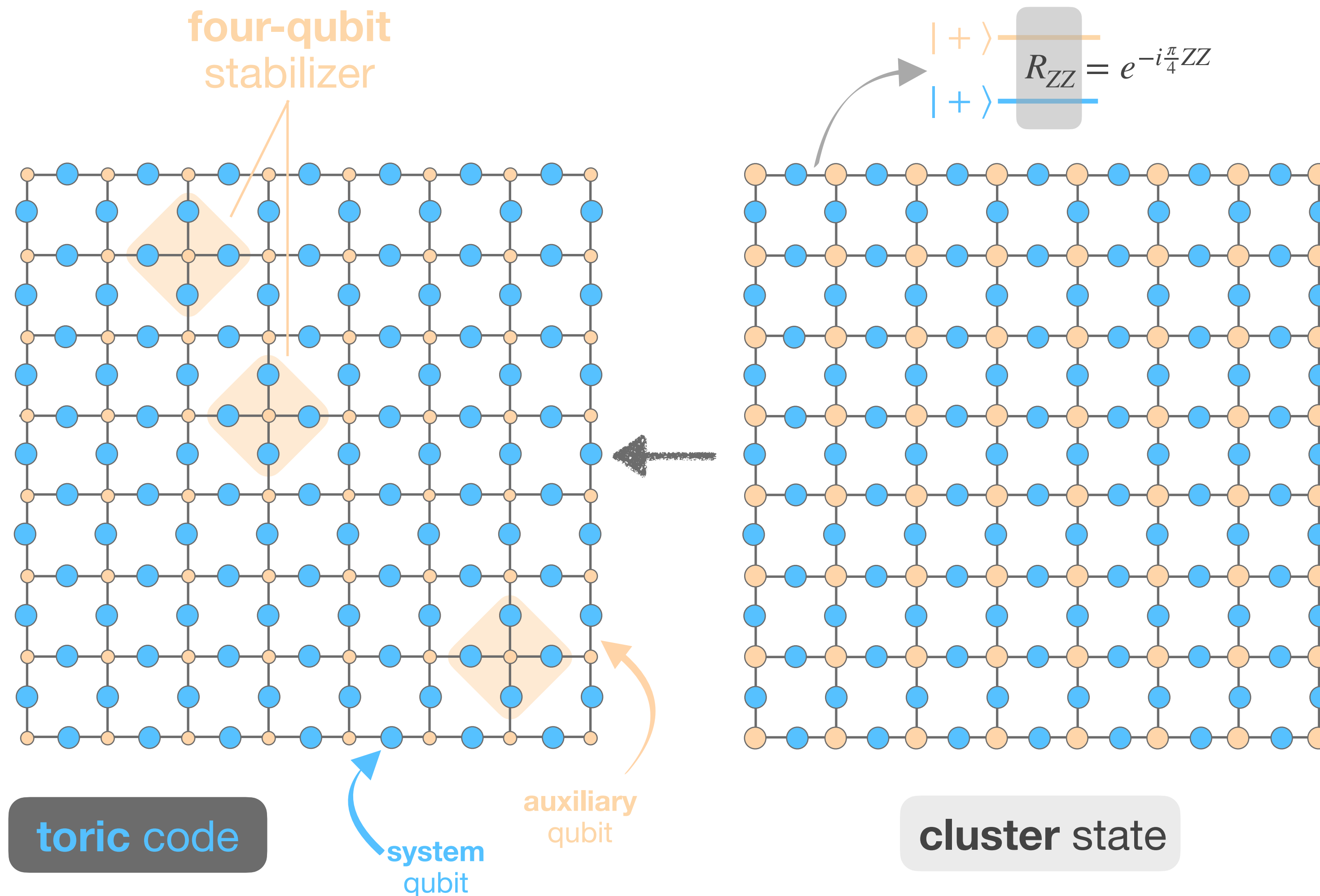
F. Eckstein, B. Han, Q. Wang, R. Vasseur, A. Ludwig, G-Y. Zhu



entanglement by measurement: **surface code**



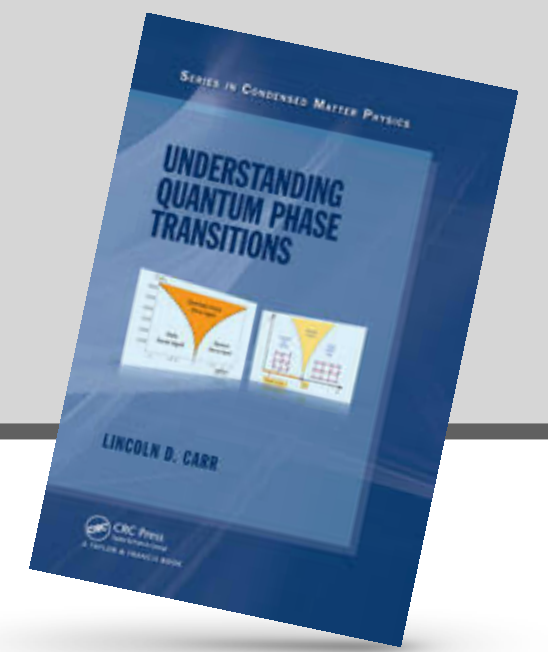
Kitaev (1997)



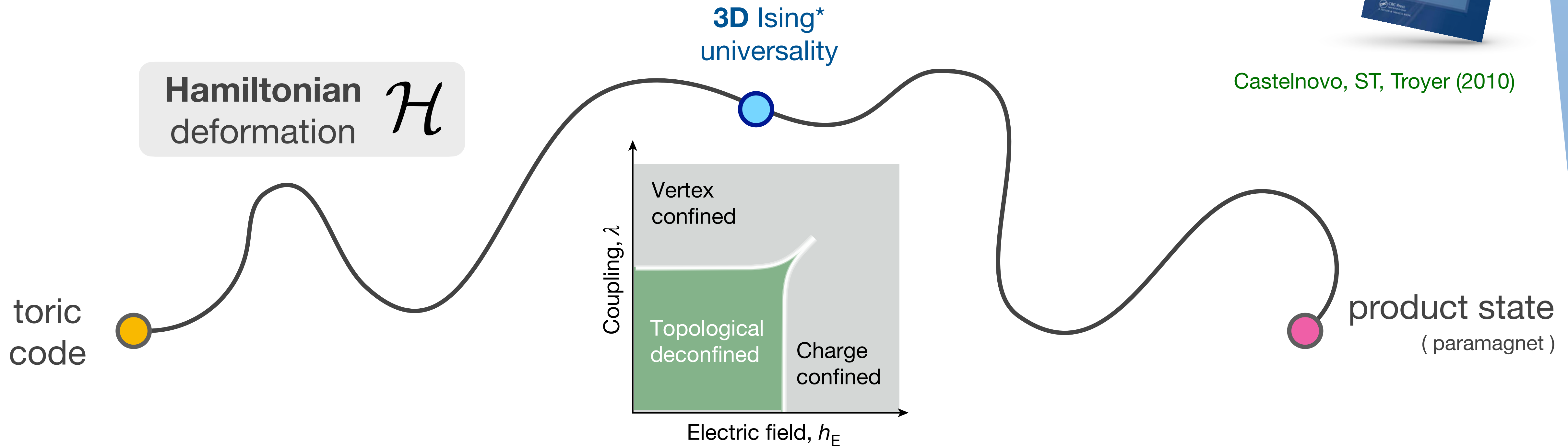
The toric/surface code
is an ingenious
measurement protocol.

The toric/surface code
is an exactly solvable
Hamiltonian model.

phase transitions & deformations



Castelnovo, ST, Troyer (2010)

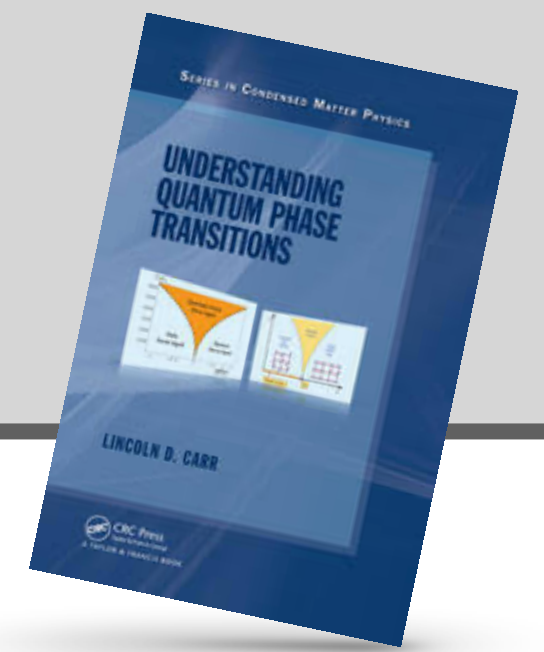


Article

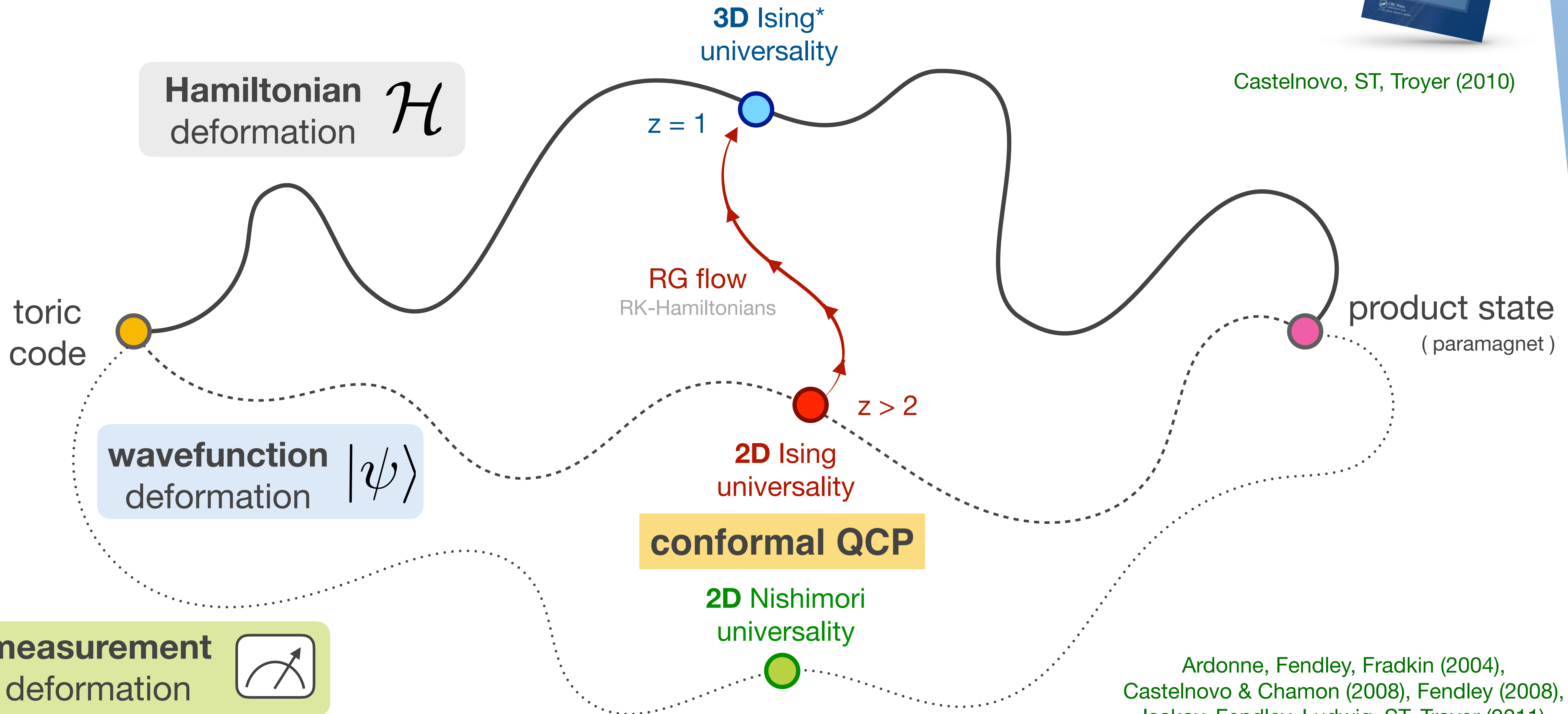
**Visualizing dynamics of charges and strings
in (2+1)D lattice gauge theories**

Tyler A. Cochran *et al.*, Nature **642**, 315 (2025)

phase transitions & deformations

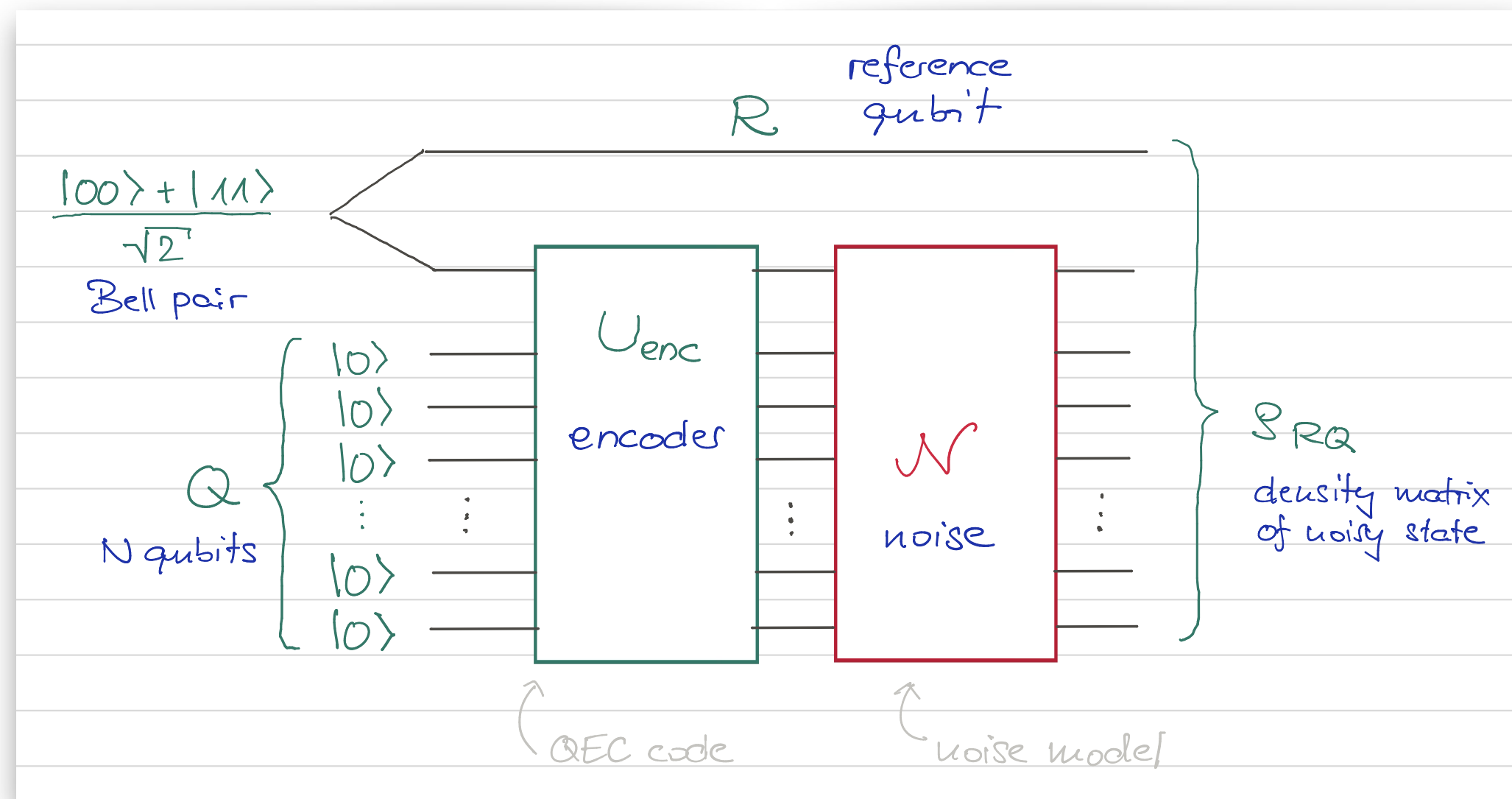


Castelnovo, ST, Troyer (2010)



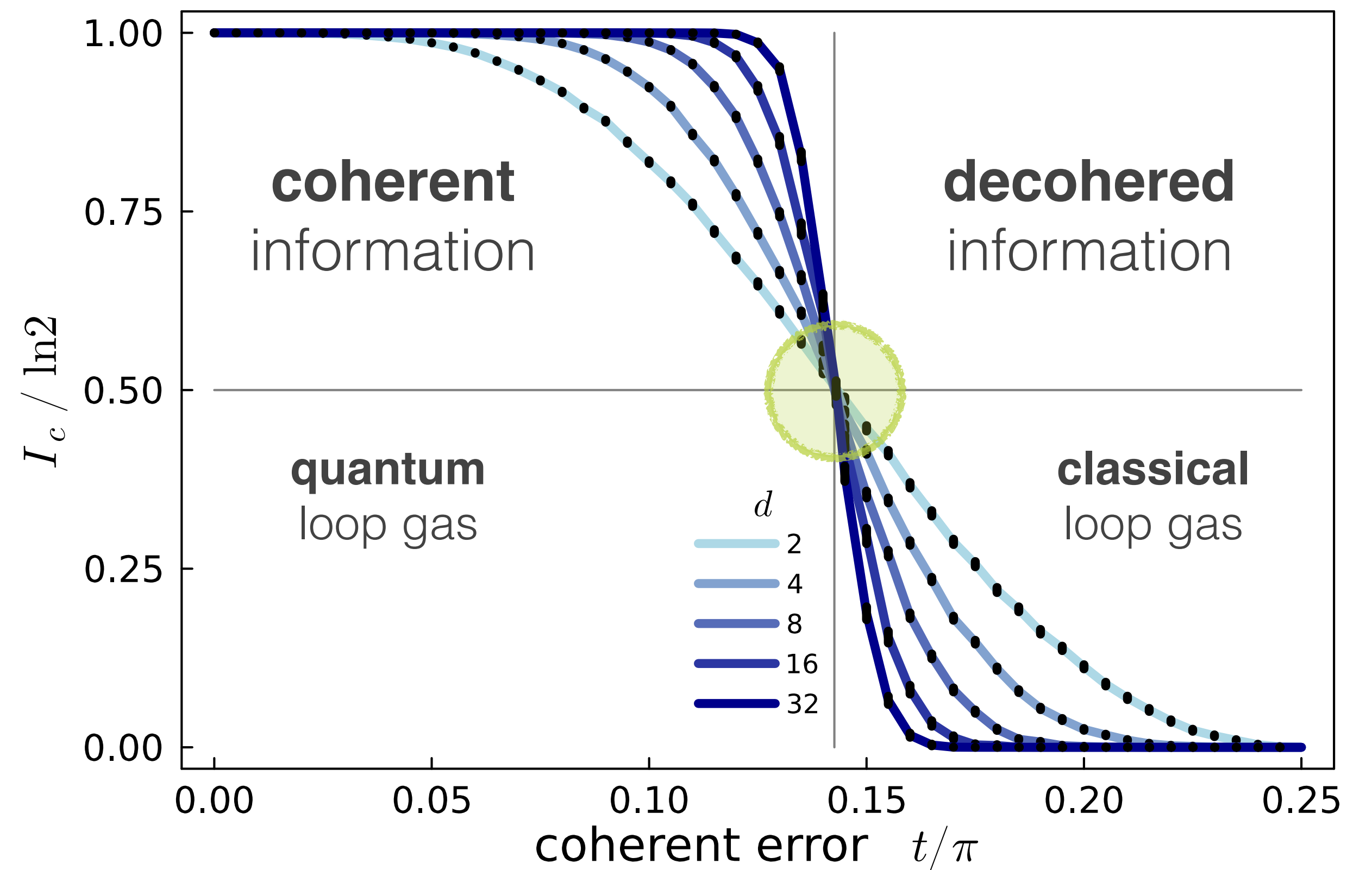
Ardonne, Fendley, Fradkin (2004),
Castelnovo & Chamon (2008), Fendley (2008),
Isakov, Fendley, Ludwig, ST, Troyer (2011)

toric code under **weak measurement**



$$I = S(\rho_Q) - S(\rho_{RQ})$$

$$|\psi\rangle \mapsto \begin{cases} \left(1 + \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle & \text{[Measurement 1]} \\ \left(1 - \tanh \frac{\beta}{2} Z_{ij}\right) |\psi\rangle & \text{[Measurement 2]} \end{cases}$$



Nishimori transition

F. Eckstein, B. Han, ST, G.Y. Zhu
PRX Quantum **5**, 040313 (2024)

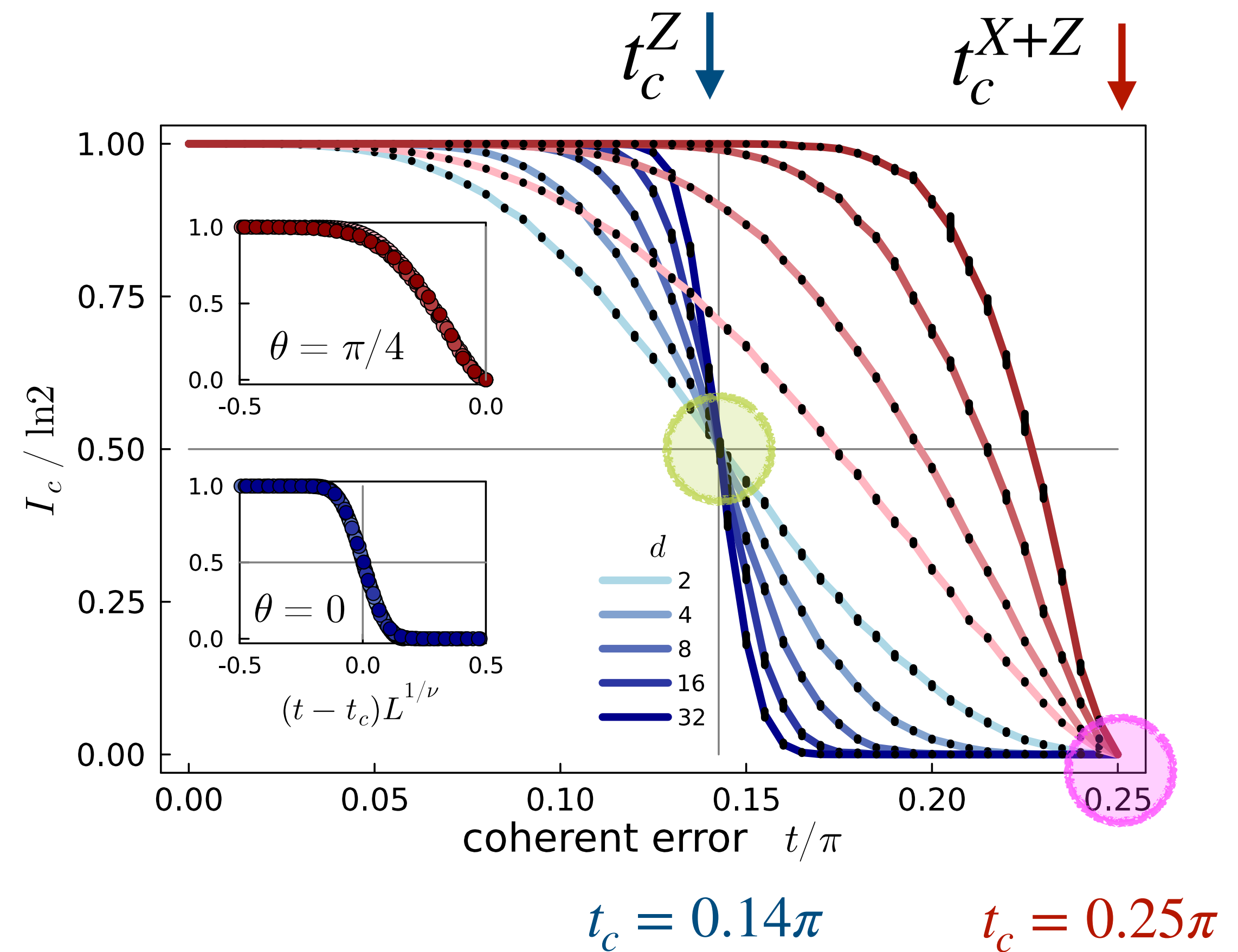
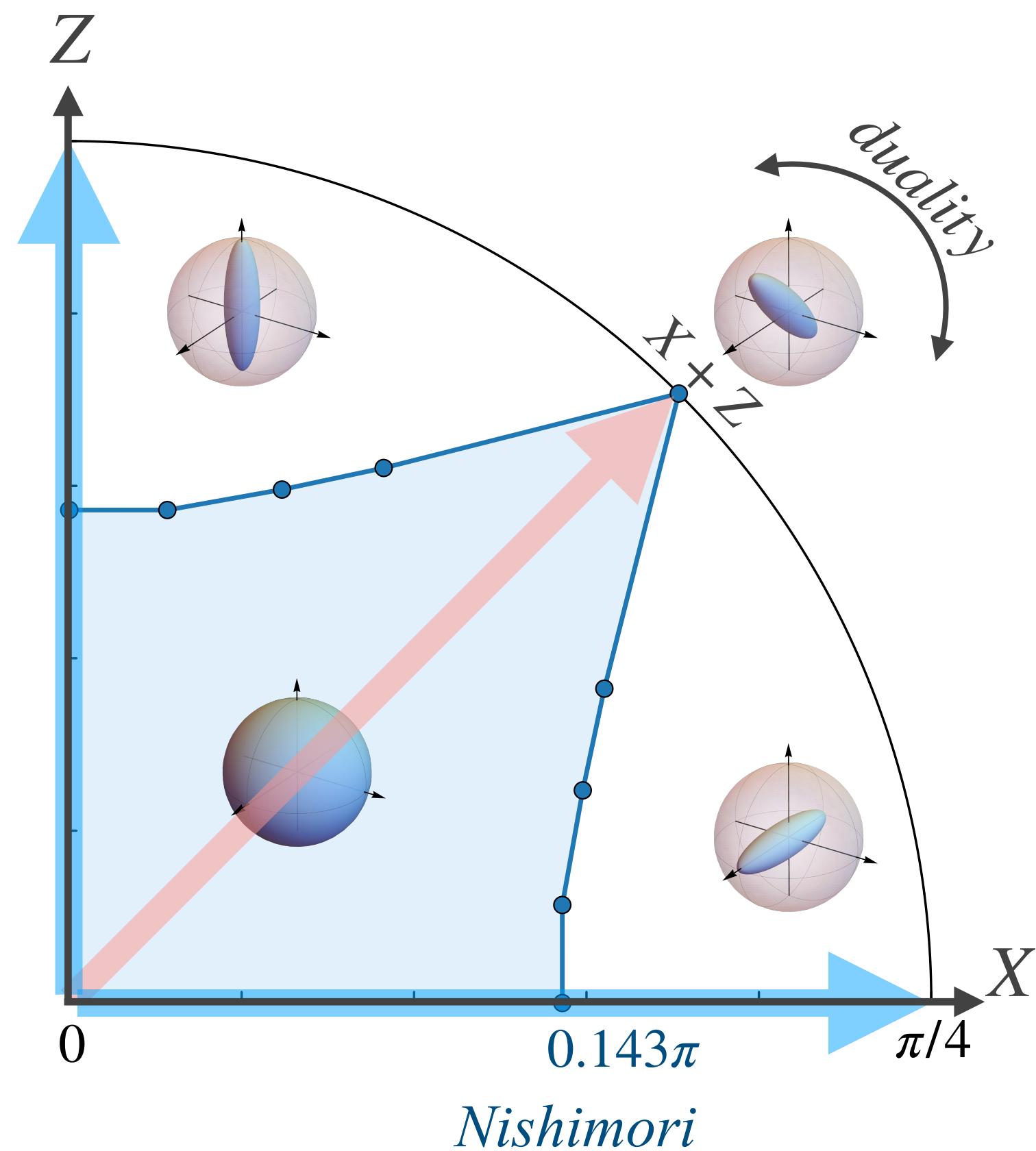
toric code under **weak measurement**



$$\Pi_{ij} \left[1 \pm \tanh \frac{\beta}{2} (\cos \theta Z_{ij} + \sin \theta X_{ij}) \right] |\psi\rangle$$

measurement **angle**

Self-duality makes the toric code absolutely **stable** against decoherence.

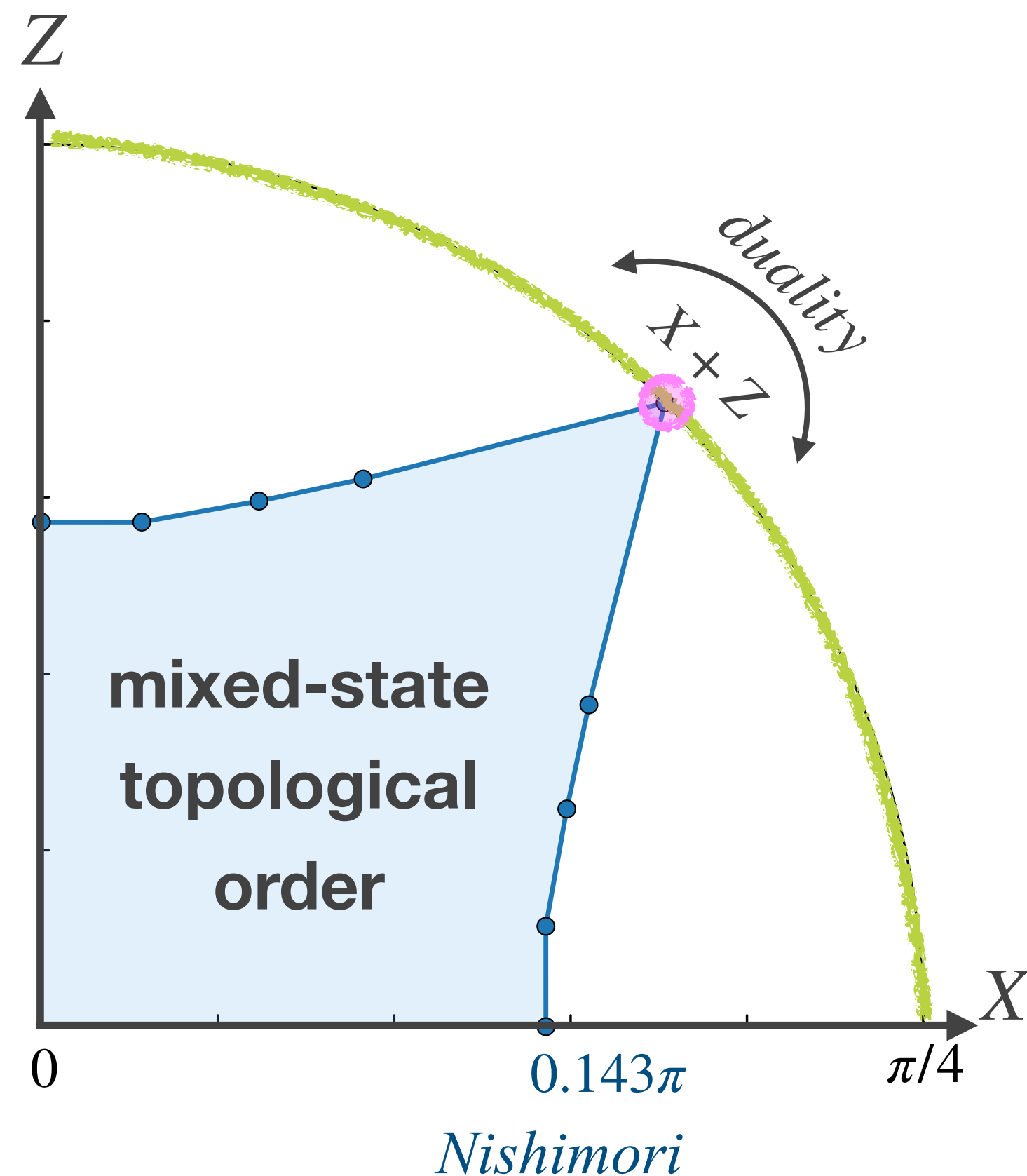


toric code & strong measurement

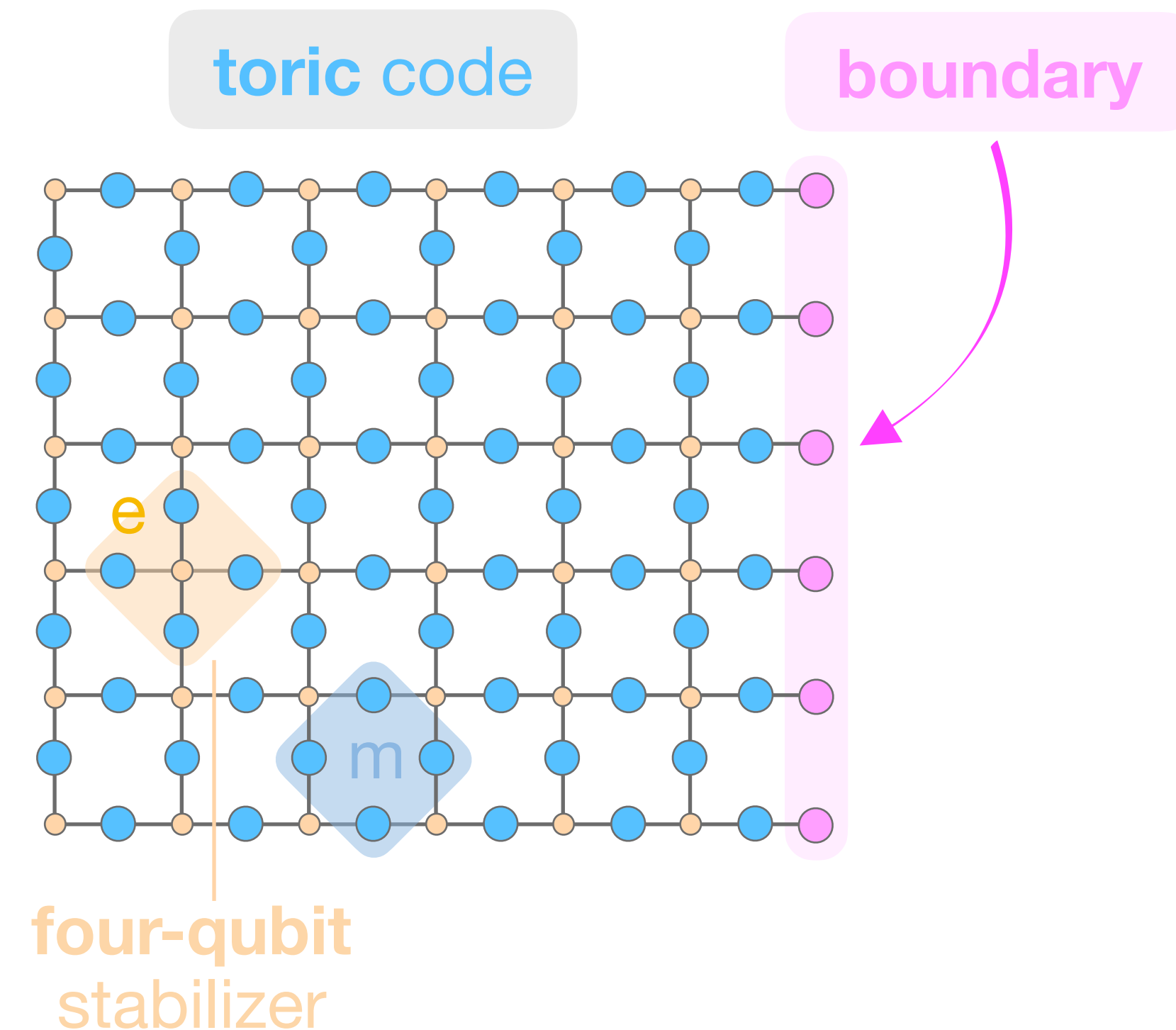


$$\Pi_{ij} \left[1 \pm \tanh \frac{\beta}{2} (\cos \theta Z_{ij} + \sin \theta X_{ij}) \right] |\psi\rangle$$

measurement angle



Why is there a **critical point** when all the bulk qubits **collapse**?

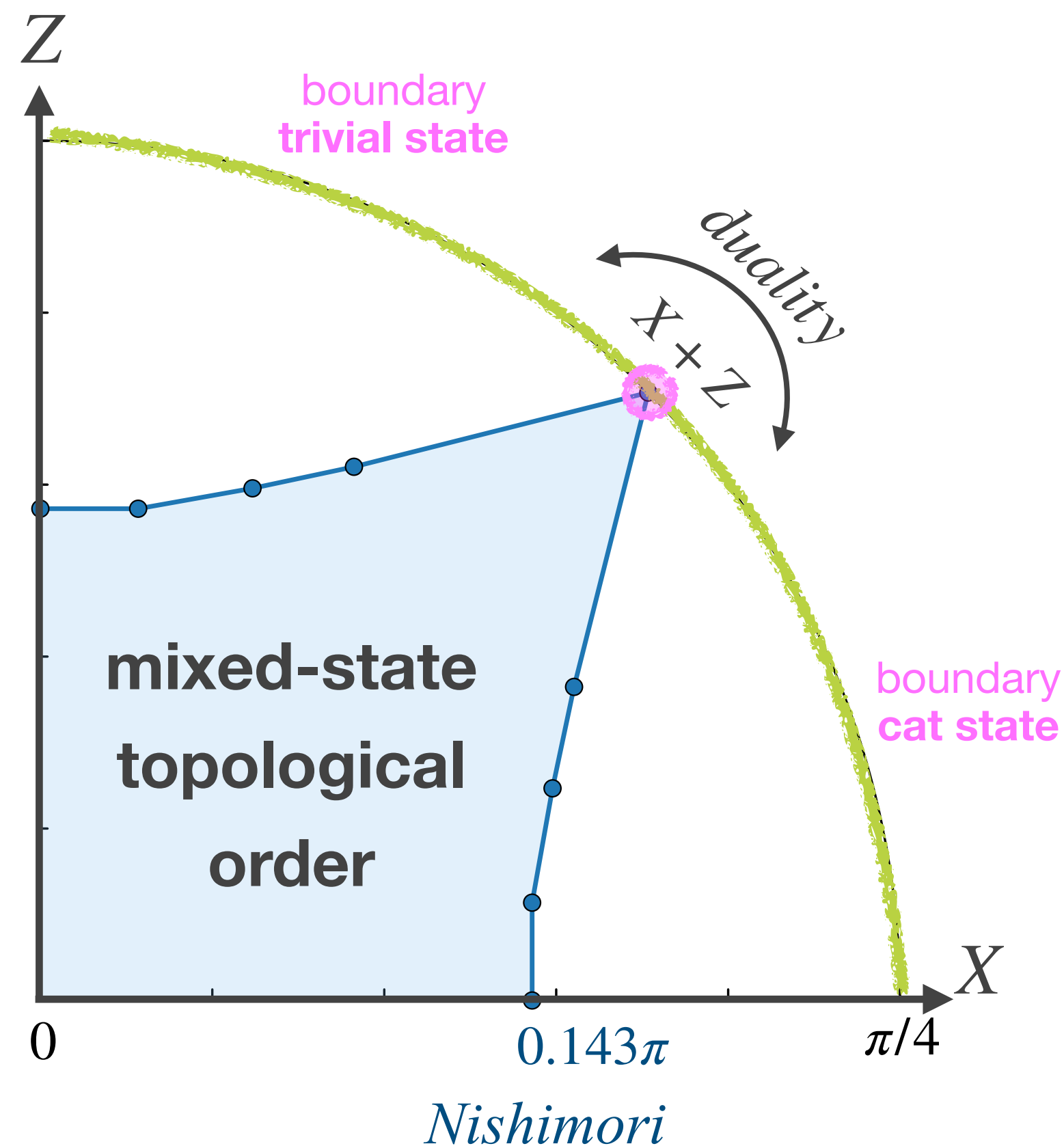


bulk-boundary correspondence



$$\Pi_{ij} \left[1 \pm \tanh \frac{\beta}{2} (\cos \theta Z_{ij} + \sin \theta X_{ij}) \right] |\psi\rangle$$

measurement **angle**



Why is there a **critical point** when all the bulk qubits **collapse**?

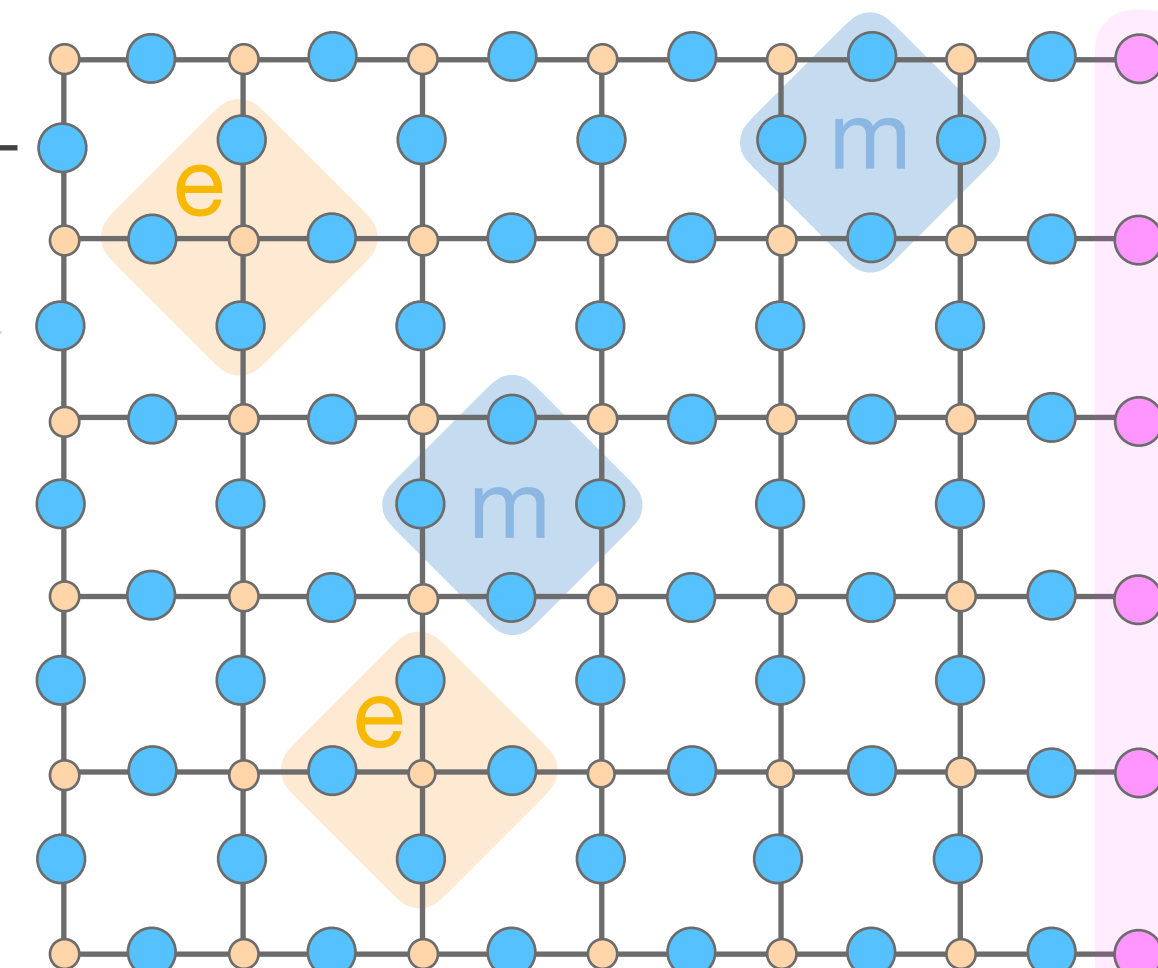
measure bulk qubits

bulk

mixed-state phase transition

toric code

boundary



measurement-induced phase transition

boundary

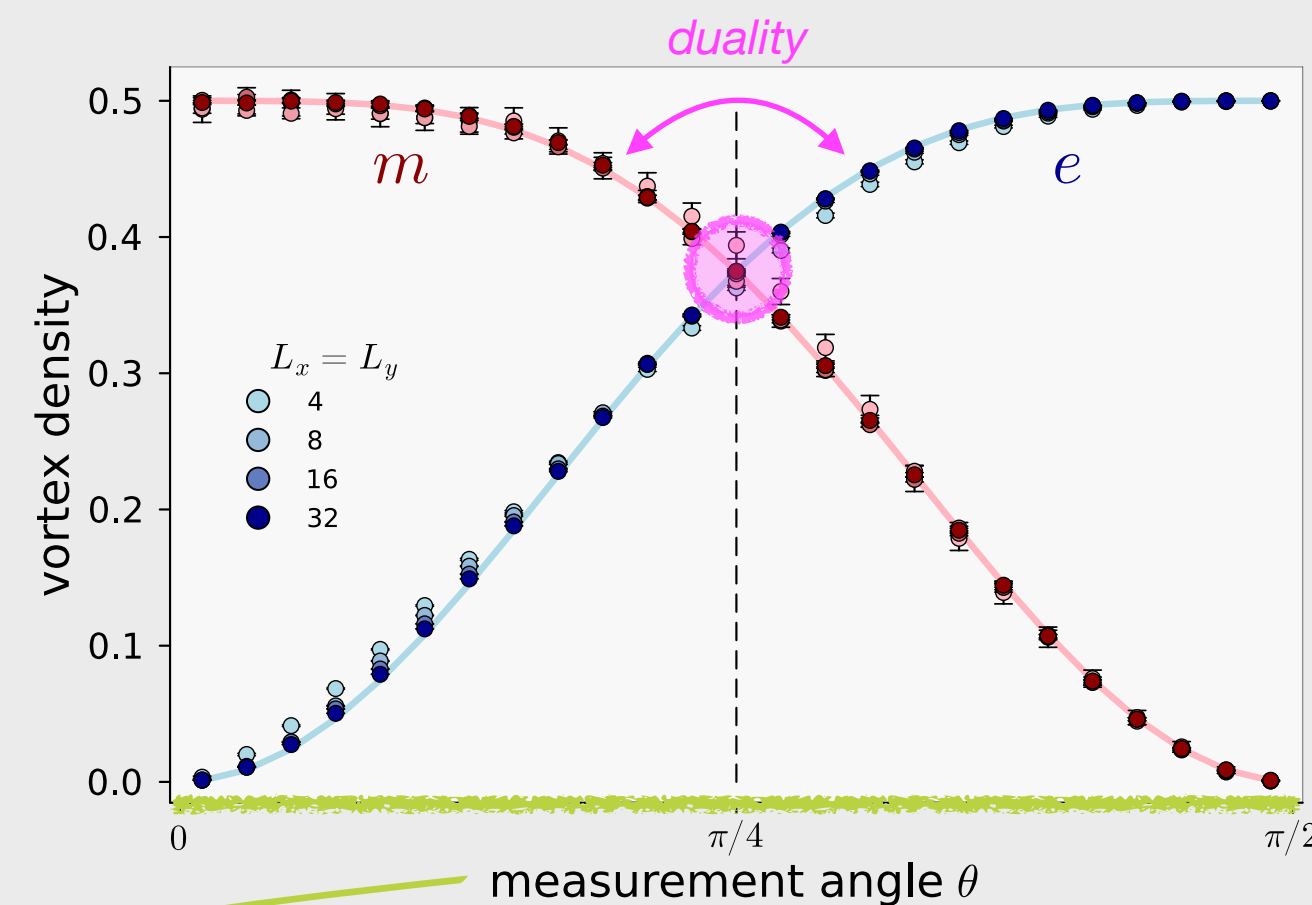
"time"

$$\rho = \sum_{em} \overset{\text{classical}}{|em\rangle\langle em|_C} \otimes \overset{\text{quantum}}{|\psi(em)\rangle\langle\psi(em)|_Q}$$

bulk-boundary correspondence



criticality is dictated by self-duality

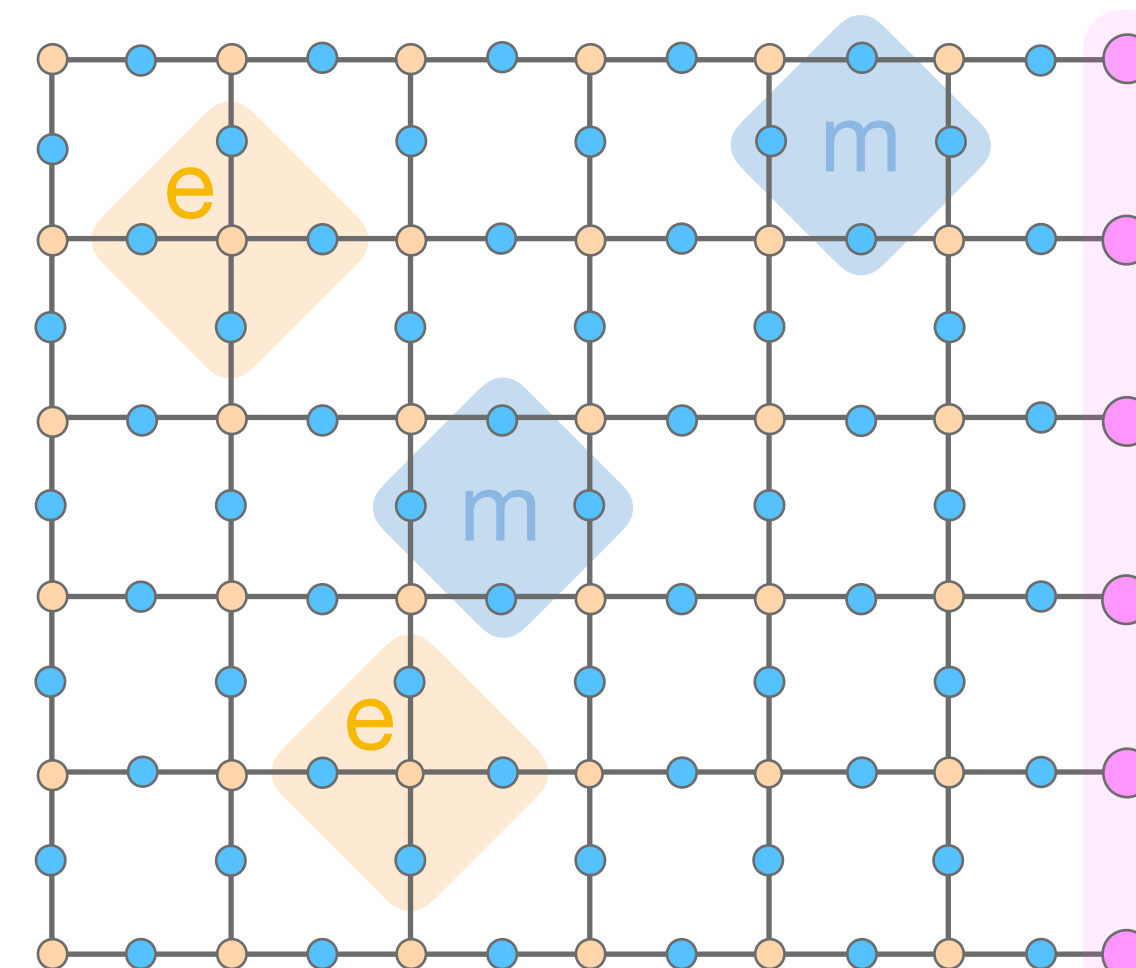


weak Kramers-Wannier duality at self-dual point

bulk

mixed-state
phase transition

toric code



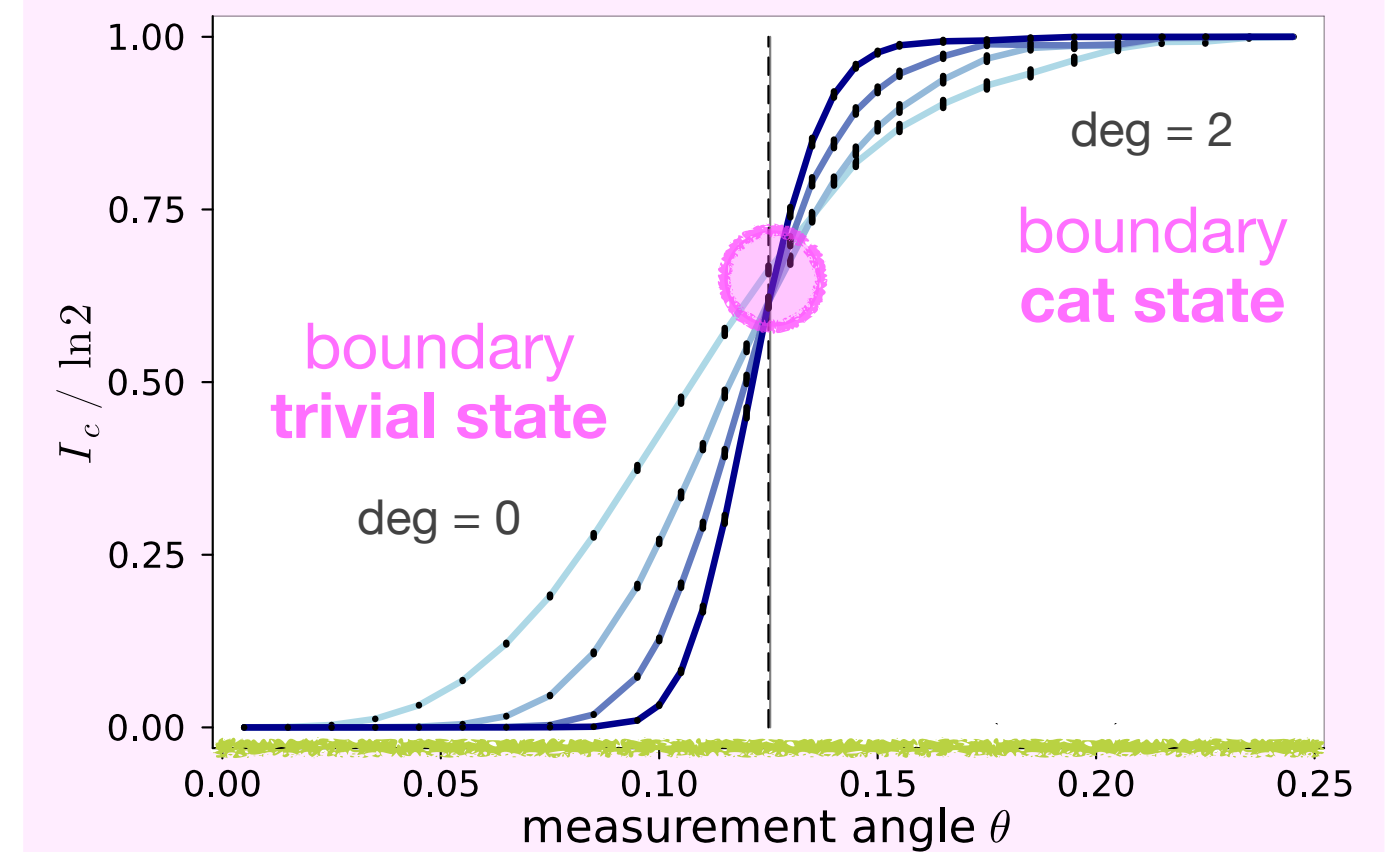
"time"

boundary

measurement-induced
phase transition

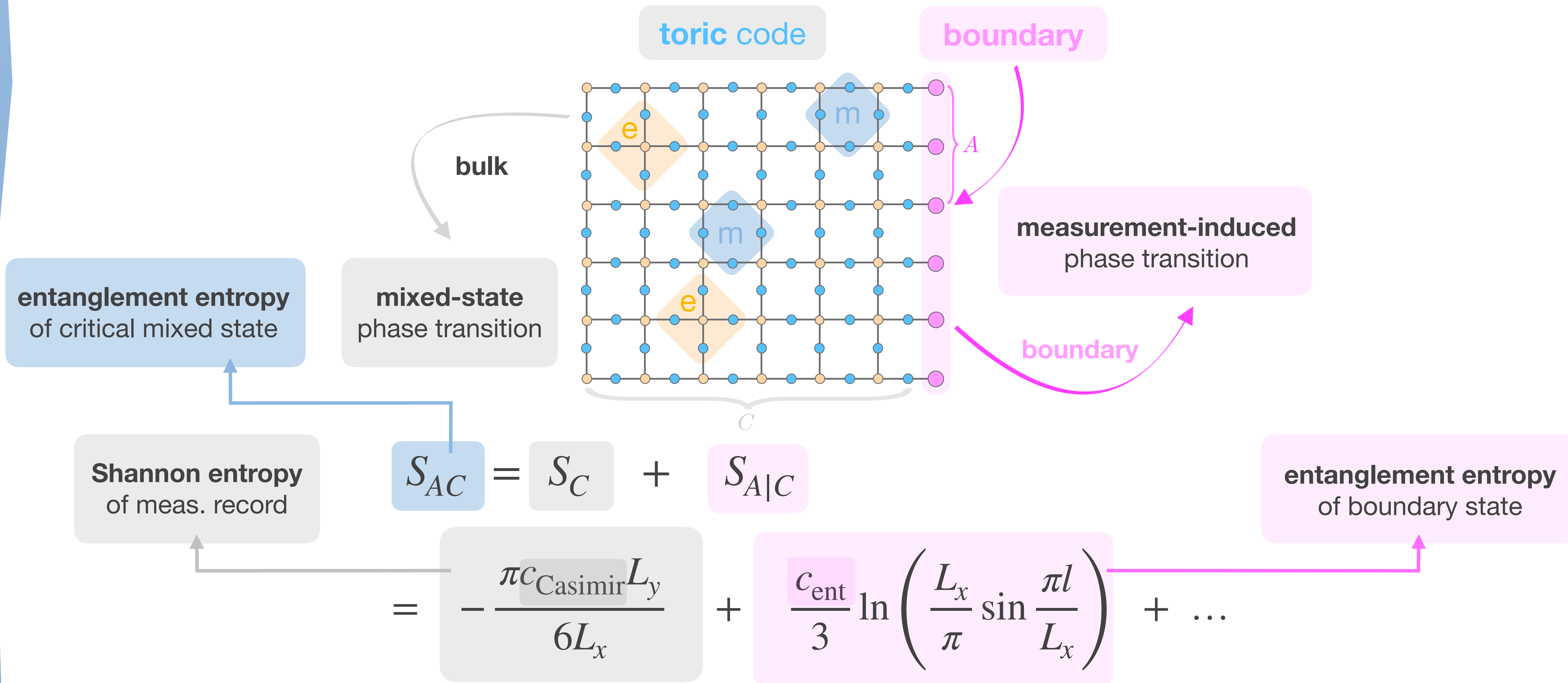
boundary

coherent information evaluates the size of the
degenerate space of the boundary state

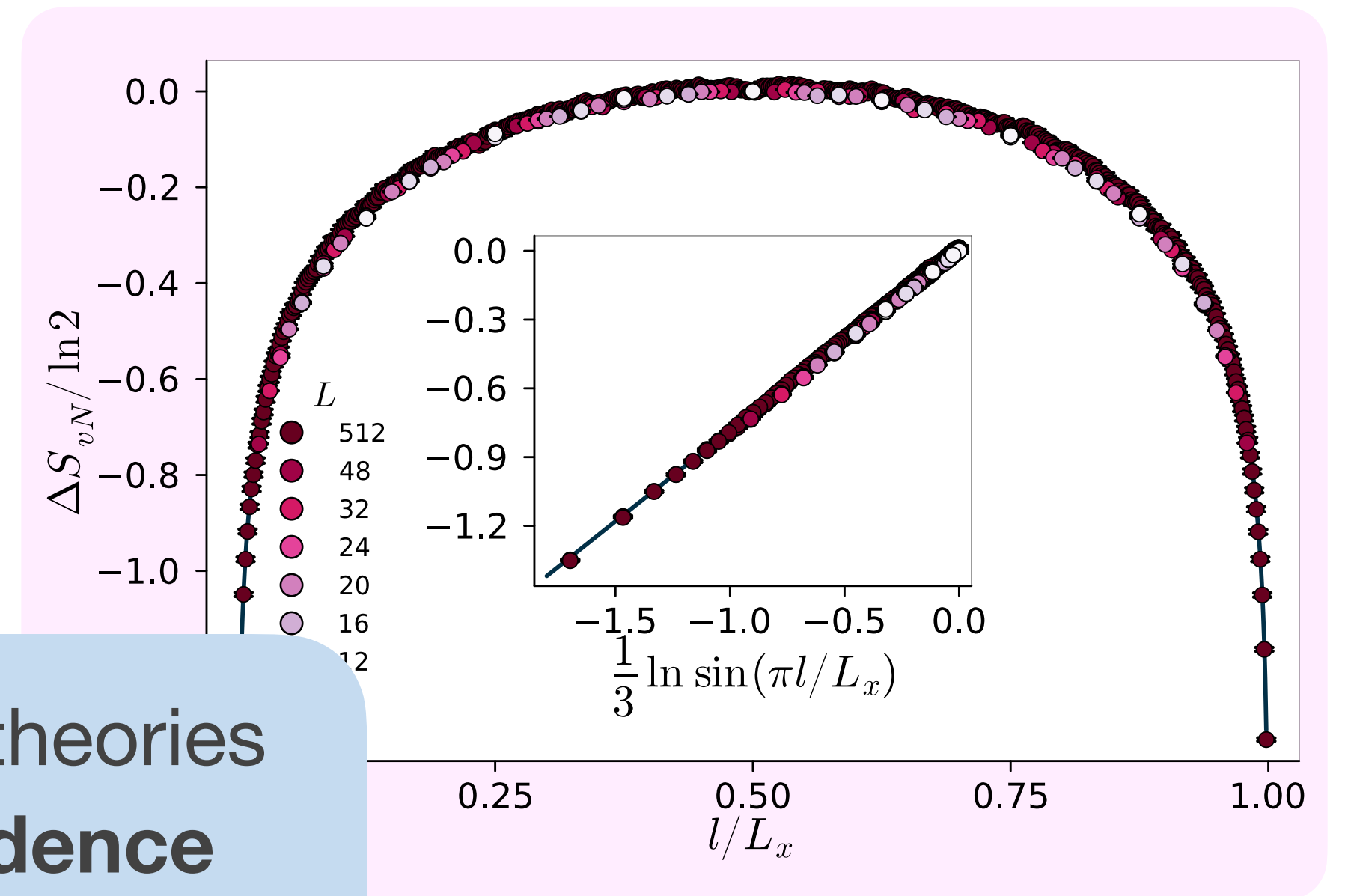
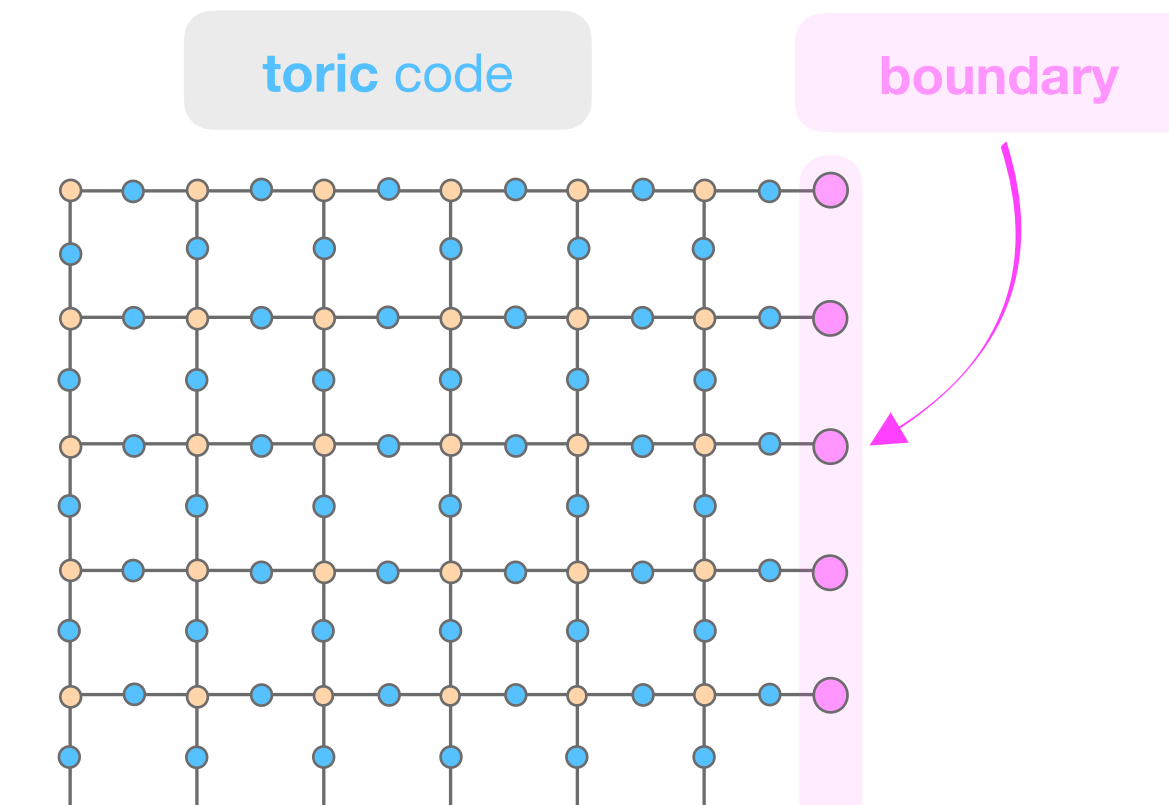
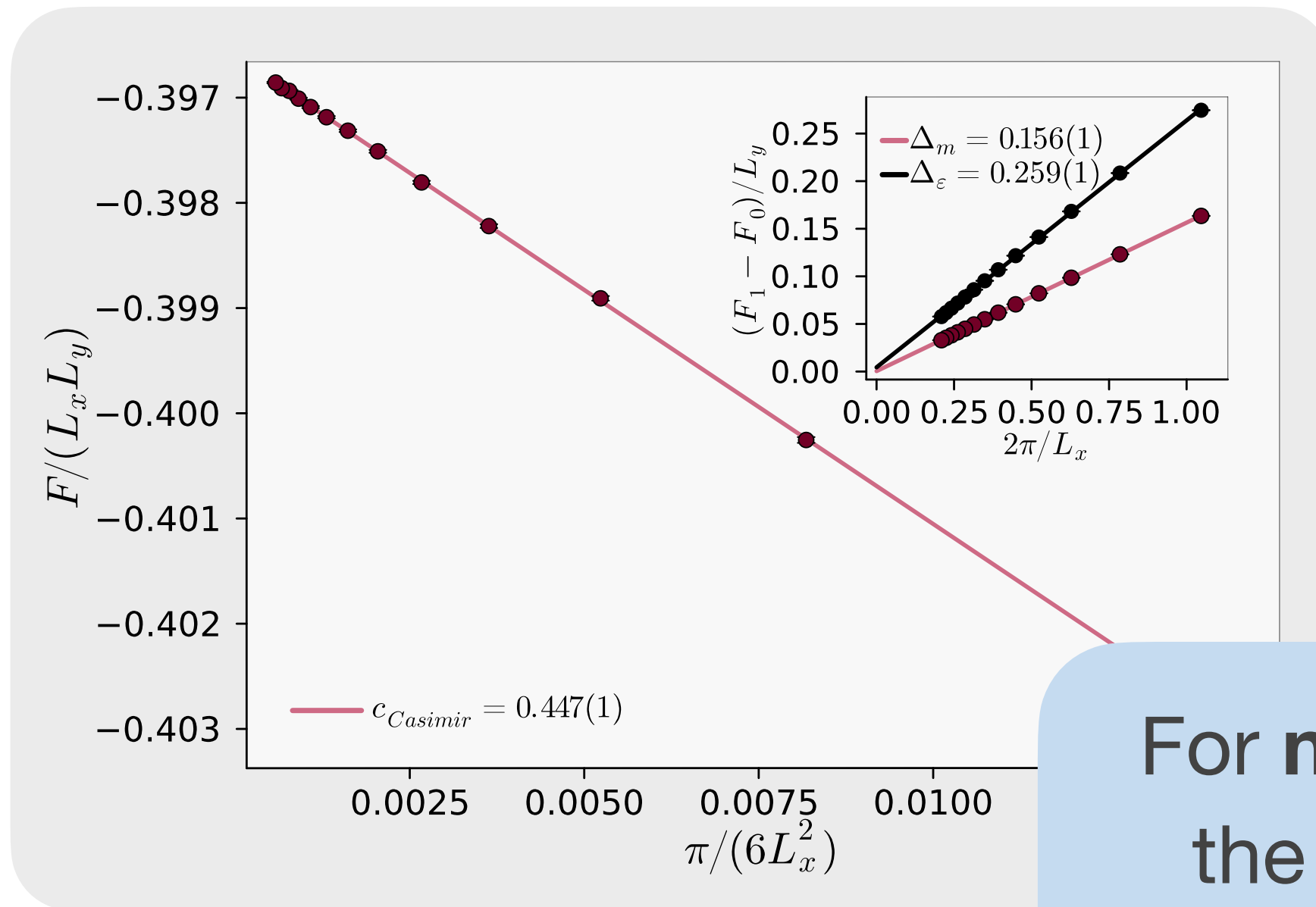


$$\rho = \sum_{em} \overset{\text{classical}}{|em\rangle\langle em|_C} \otimes \overset{\text{quantum}}{|\psi(em)\rangle\langle\psi(em)|_Q}$$

bulk-boundary correspondence



self-dual criticality



For non-unitary conformal field theories
the **bulk-boundary correspondence**
generally allows c_{Casimir} and c_{ent} to differ.

$$c_{\text{Casimir}} = 0.447(1)$$

$$c_{\text{ent}}^{\text{vN}} = 0.795(1)$$

Shannon entropy
of meas. record

$$S_{AC} = S_C + S_{A|C}$$

entanglement entropy
of boundary state

$$= -\frac{\pi c_{\text{Casimir}} L_y}{6L_x} + \frac{c_{\text{ent}}}{3} \ln \left(\frac{L_x}{\pi} \sin \frac{\pi l}{L_x} \right) + \dots$$

universality classes

Ising

no disorder

Z_2 SSB

spontaneous
symmetry breaking

**Kramers-Wannier
duality**

$$KW_\rho \propto \rho$$

Nishimori

Born-rule disorder
(1-replica limit)

Z_2 SW-SSB

strong-to-weak spontaneous
symmetry breaking

**broken KW
duality**

$$c_{\text{Casimir}} = 0.464(4) \quad c_{\text{ent}}^{\text{vN}} = 0.41956(3)$$

weak self-dual

Born-rule disorder
(1-replica limit)

Z_2 SW-SSB

strong-to-weak spontaneous
symmetry breaking

**weak KW
duality**

$$KW_\rho KW = \rho$$

$$c_{\text{Casimir}} = 0.447(1) \quad c_{\text{ent}}^{\text{vN}} = 0.795(1)$$

universality classes

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no disorder

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Z_2 SW-SSB

strong-to-weak spontaneous
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weak KW
duality

tricritical

Born-rule disorder
(1-replica limit)

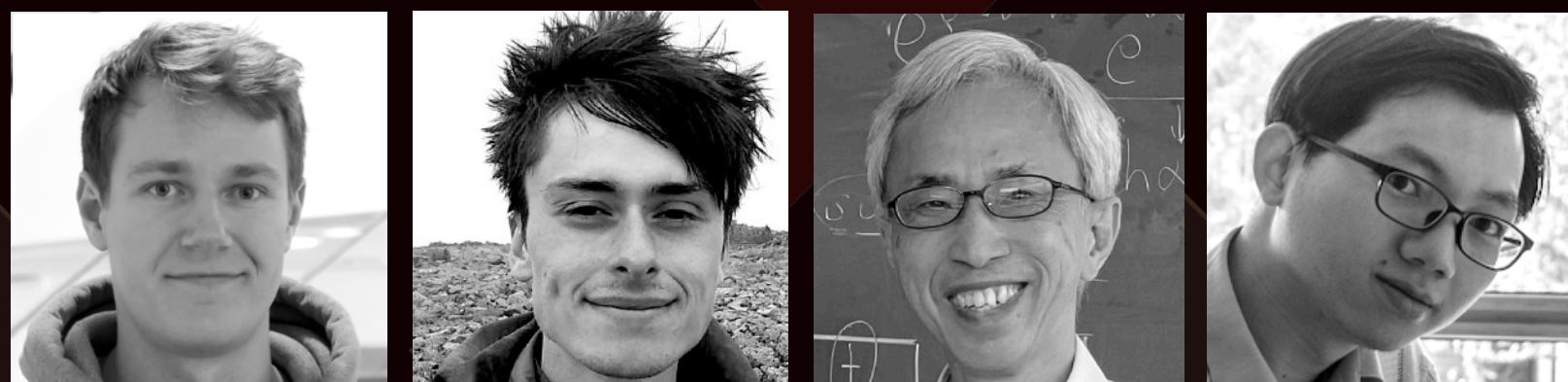
strong/weak/broken
 Z_2 symmetry phases meet

learning transitions

learning transitions

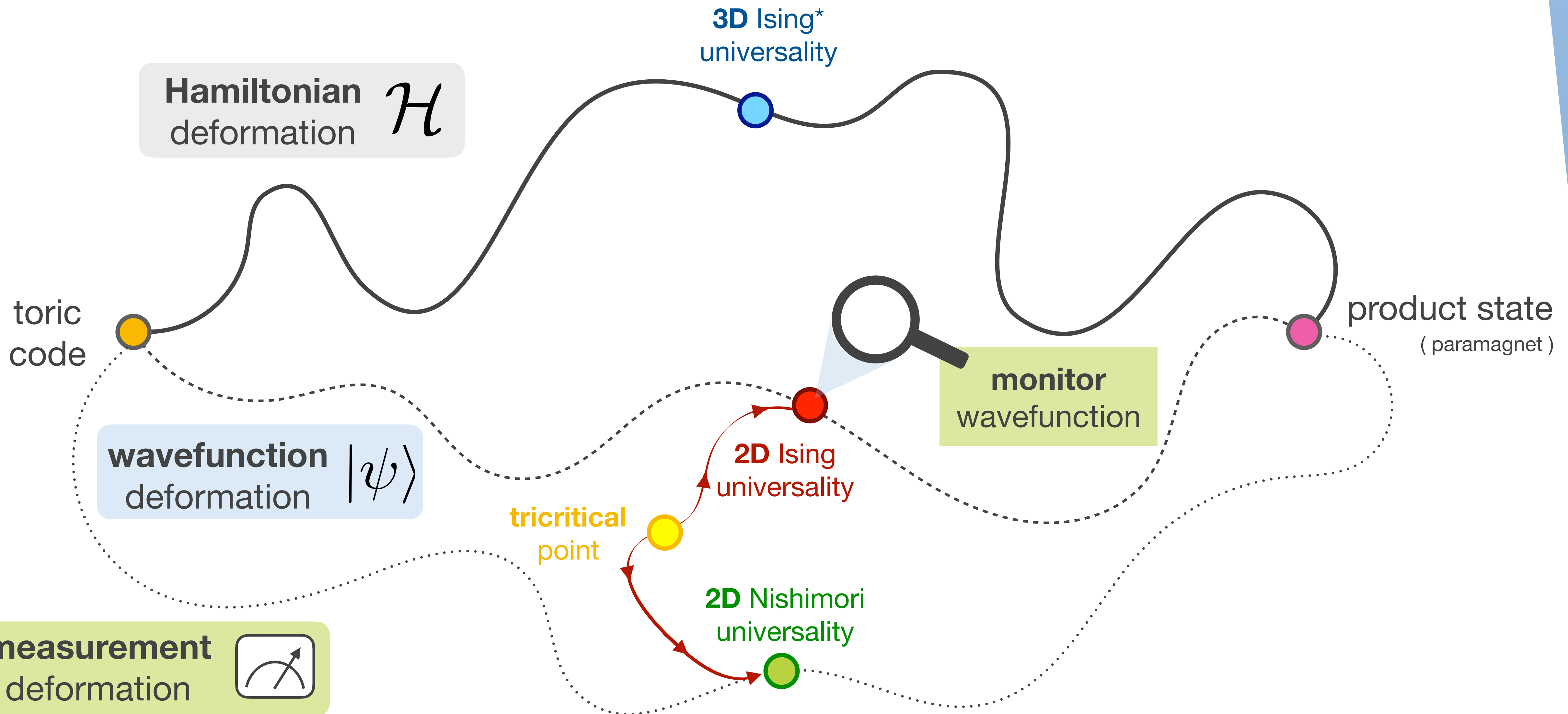
arXiv:2504.12385

M. Pütz, S. Garratt, H. Nishimori, G-Y. Zhu



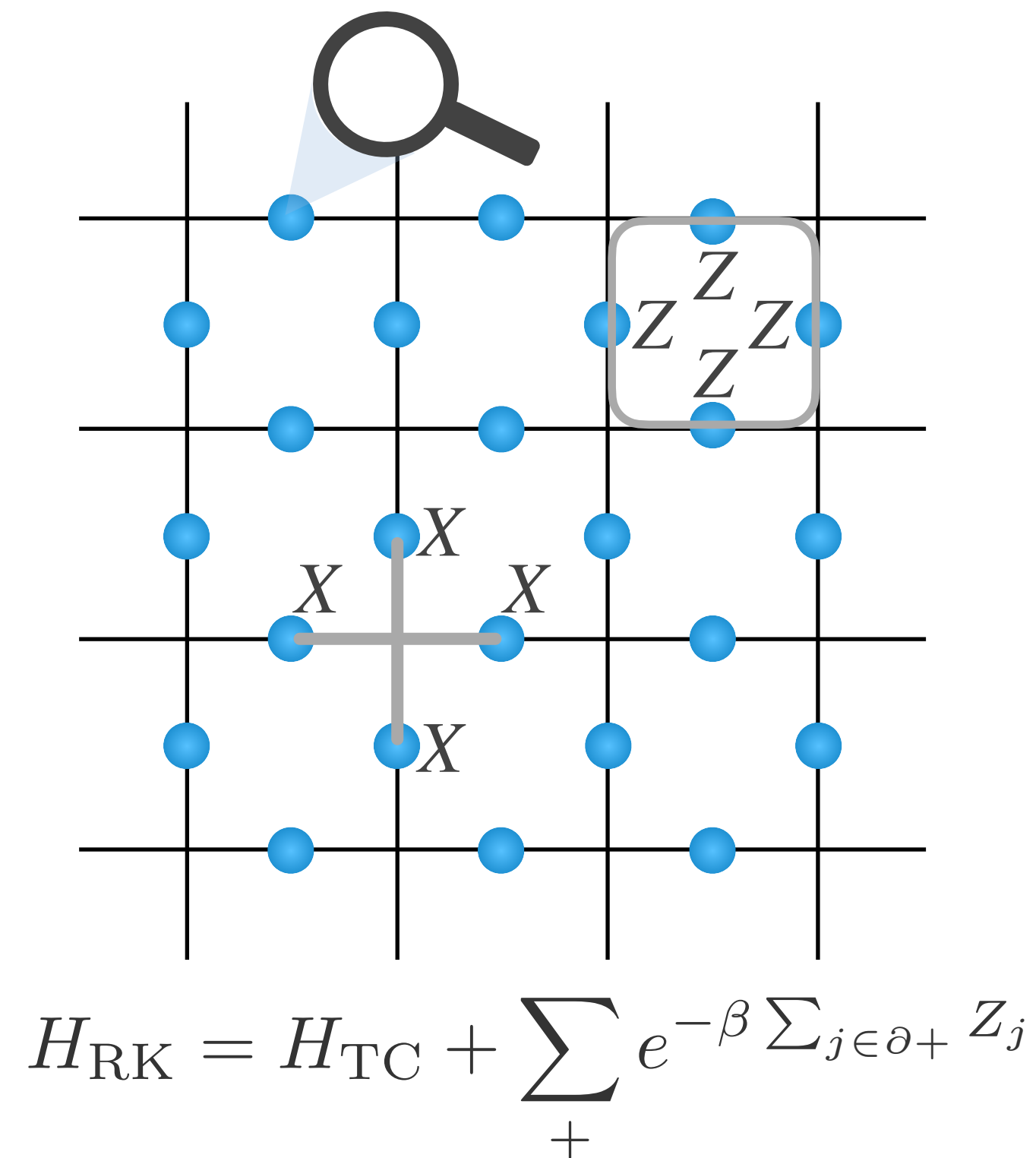
See also
Nahum & Jacobsen, arXiv:2504.01264
Kim, von Keyserlink & Lamacraft, arXiv:2504.08888

Monitored quantum criticality

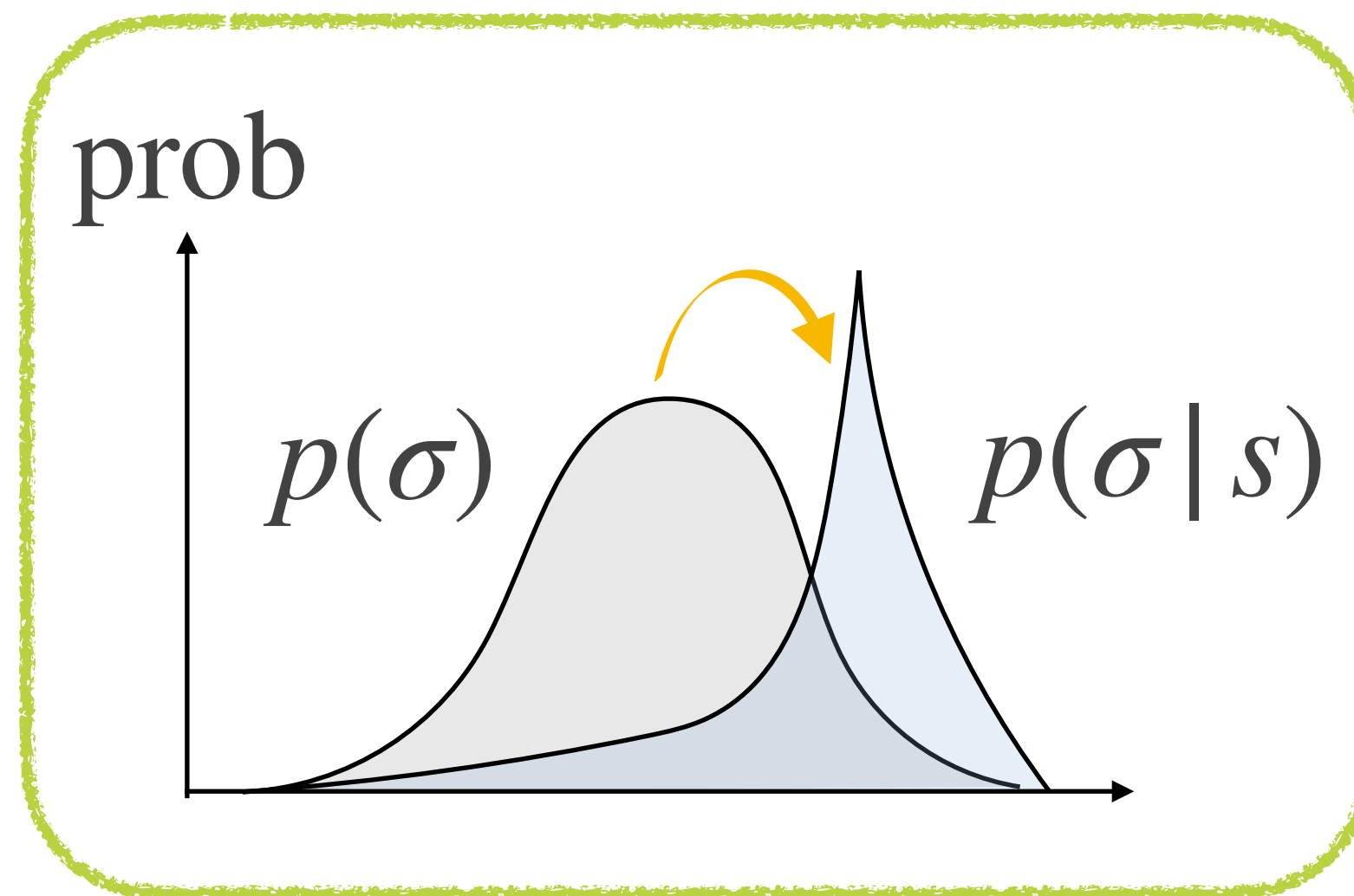


quantum monitoring & classical inference

deformed toric code

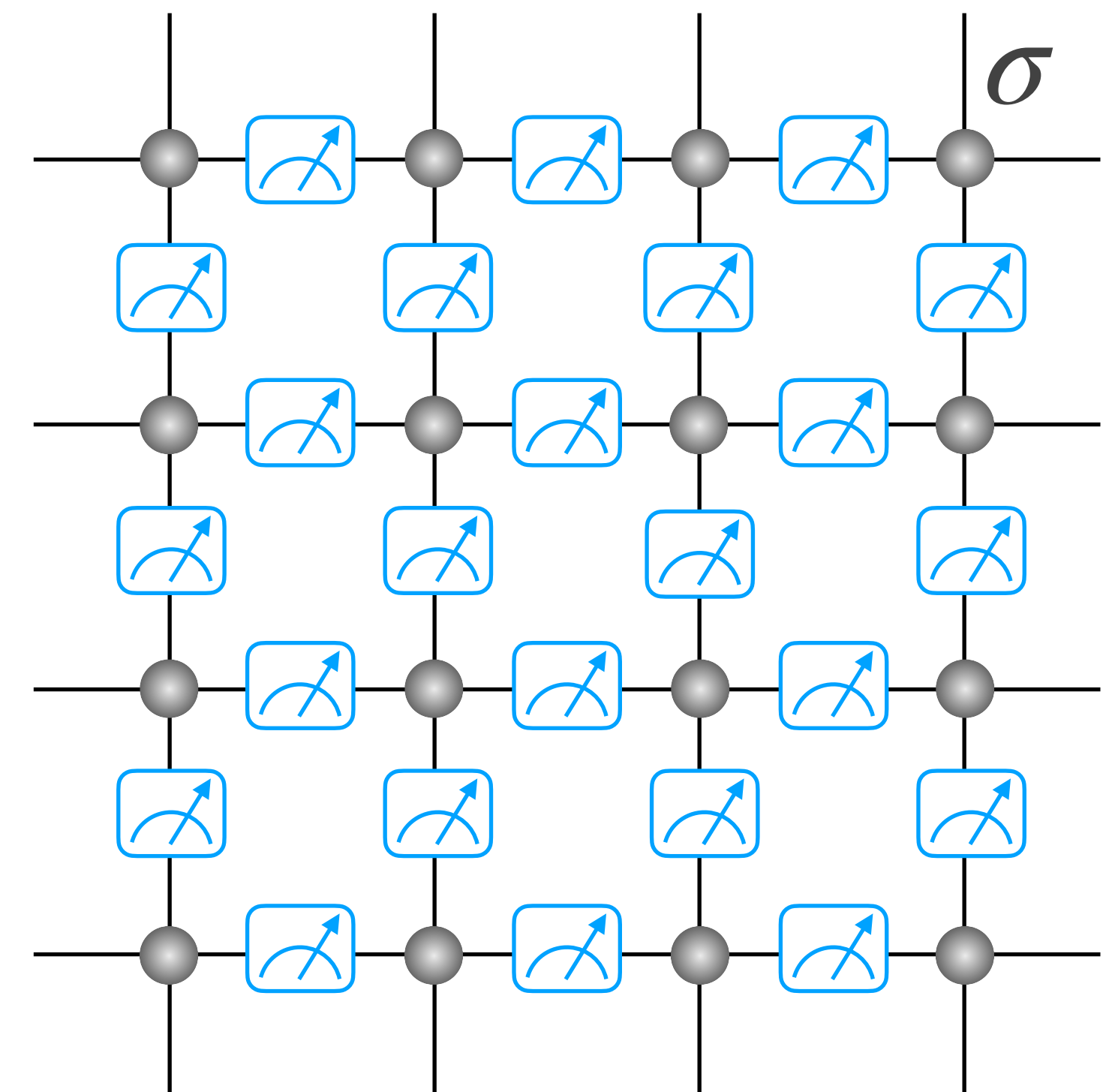


Born's rule



conditional distribution

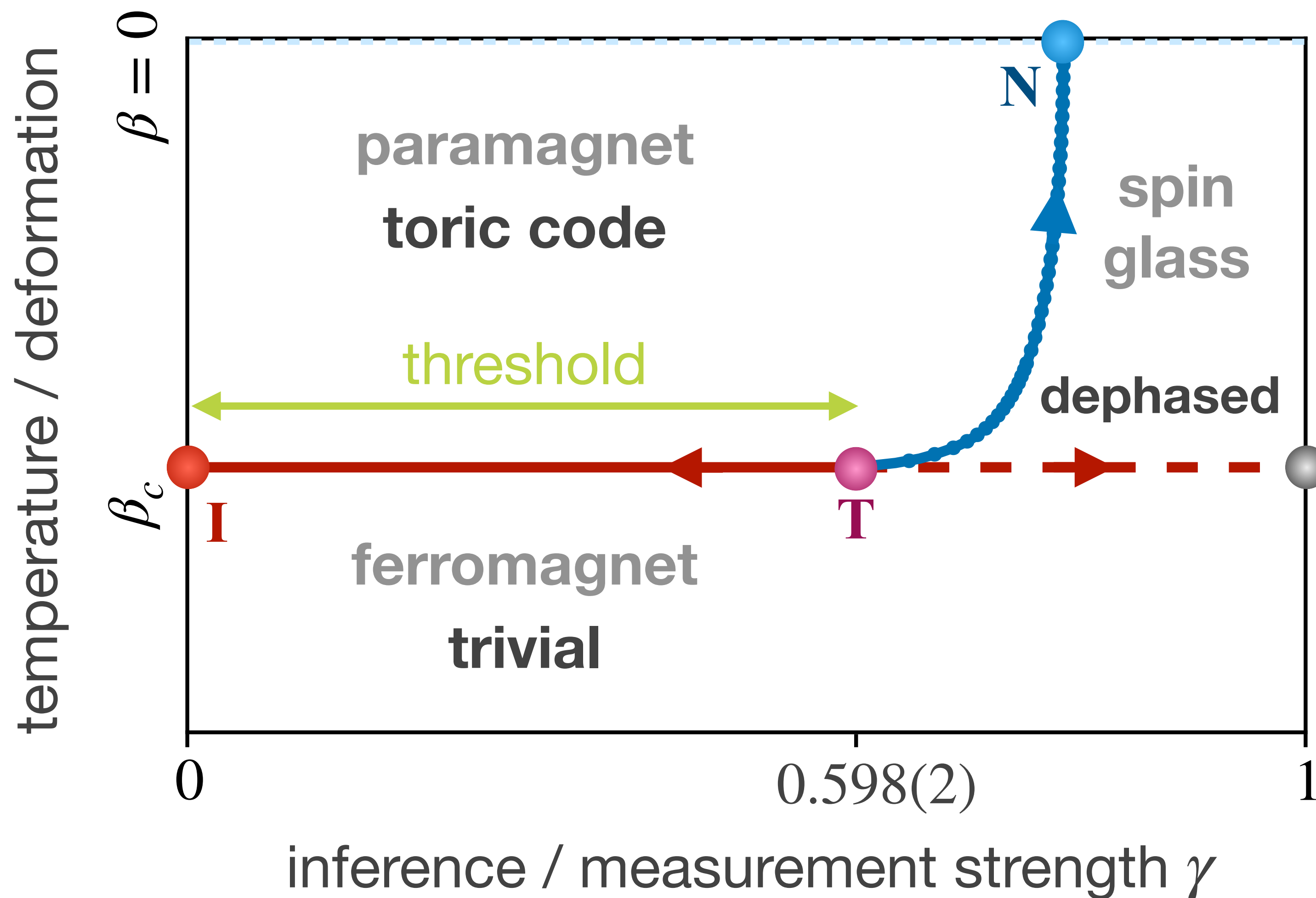
critical Ising model



Bayesian inference

learning transitions

Bayesian / Born phase diagram

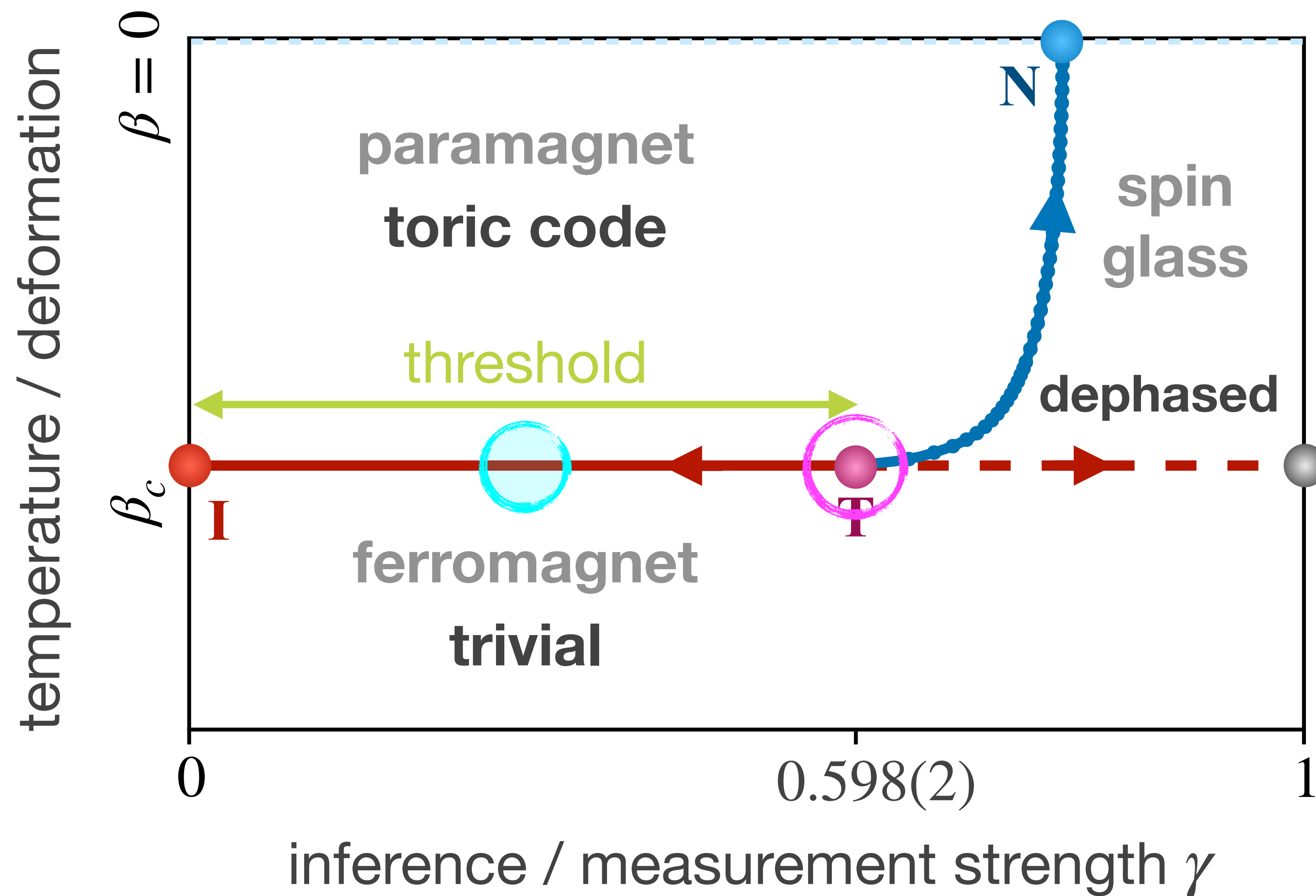


dictionary of phases

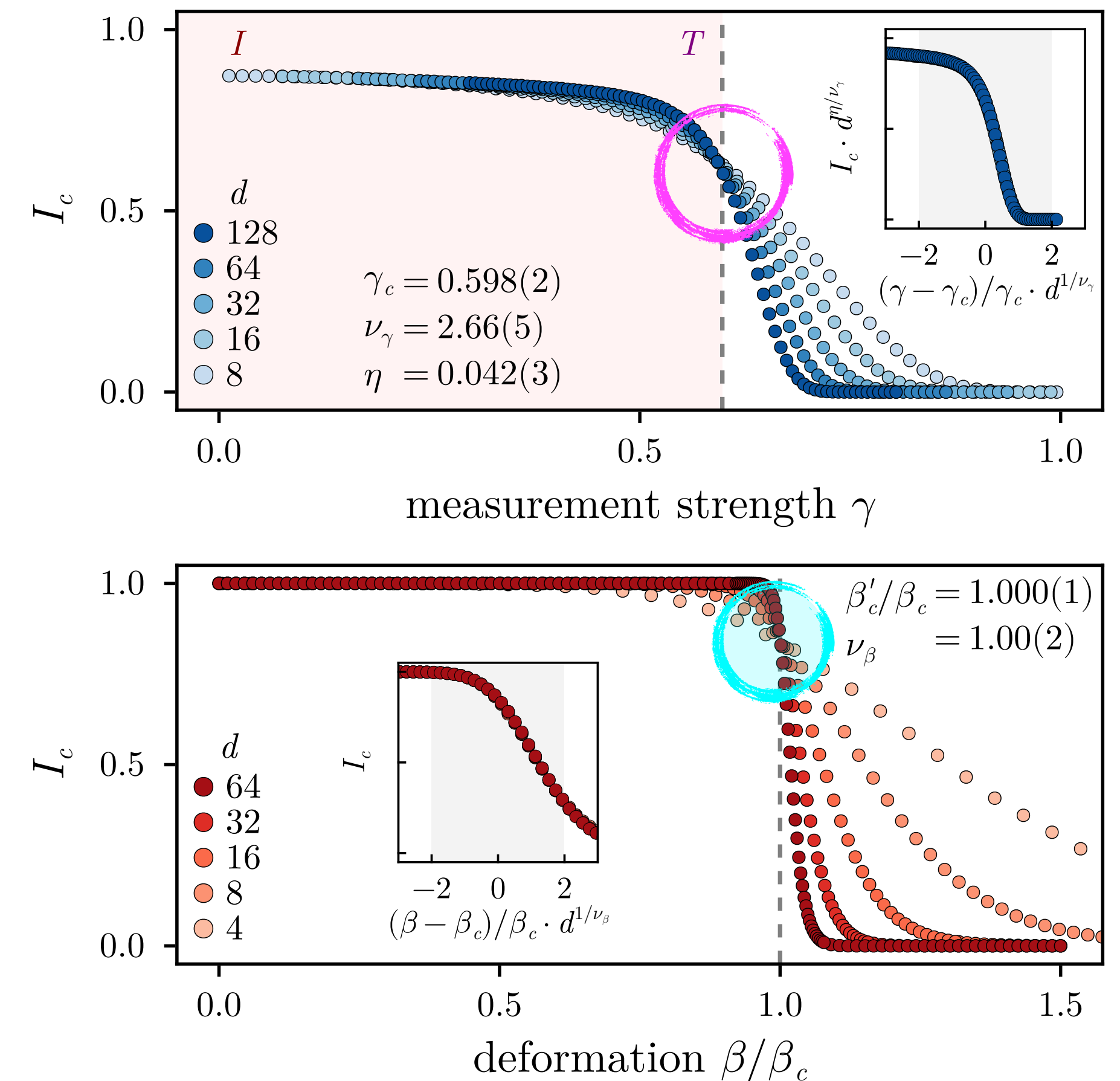
classical Ising	measured toric code	$\mathbb{Z}_2$ symmetry
paramagnet	quantum memory	strong
spin glass	classical memory	weak
ferromagnet	lost memory	broken

learning transitions

Bayesian / Born phase diagram

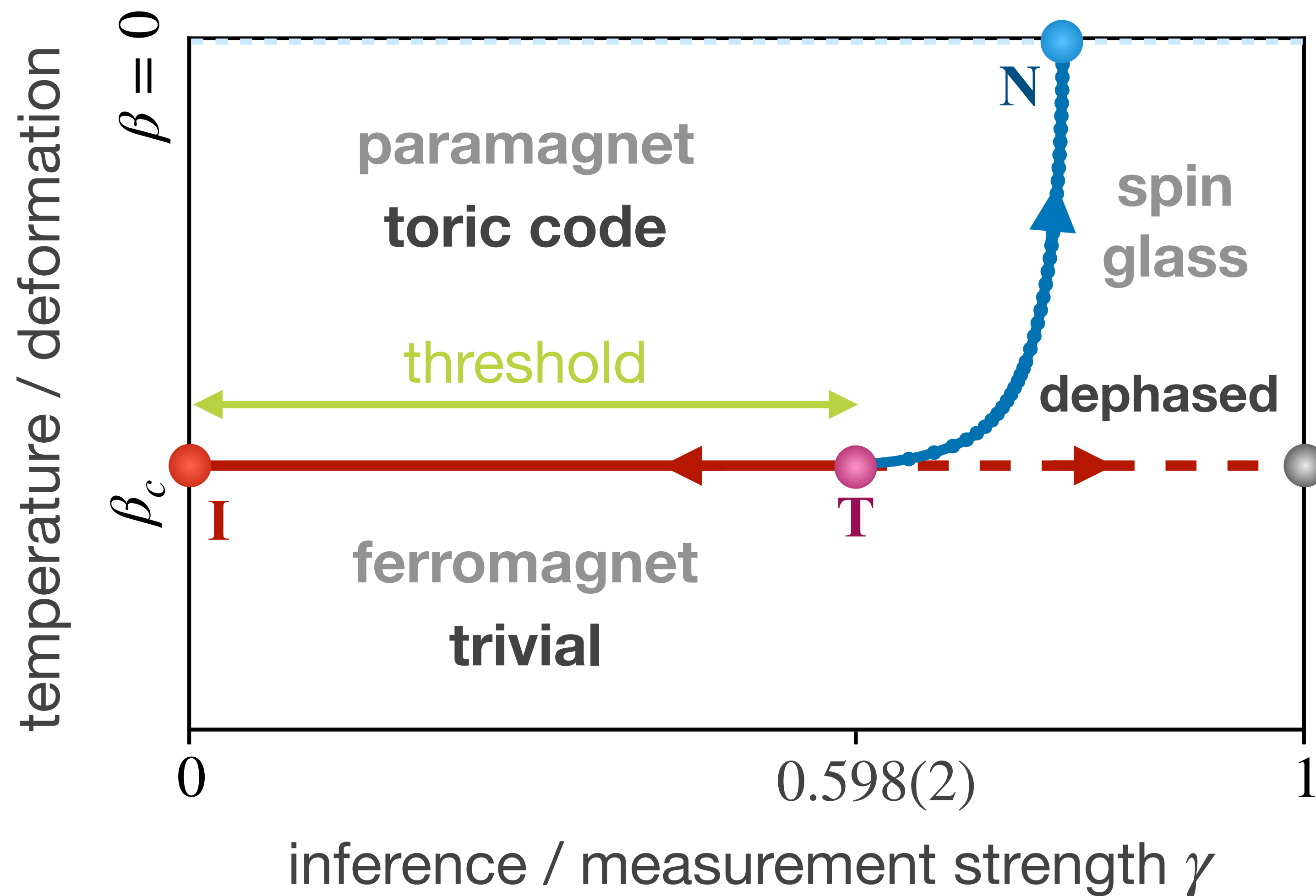


coherent information cuts



learning transitions

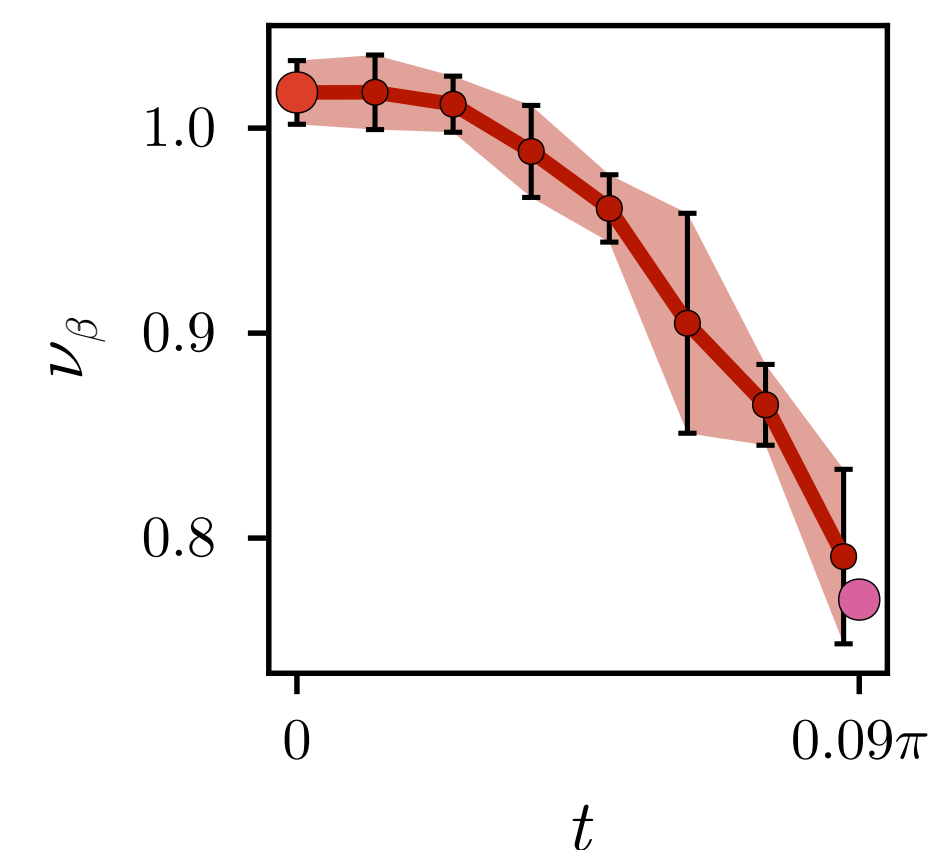
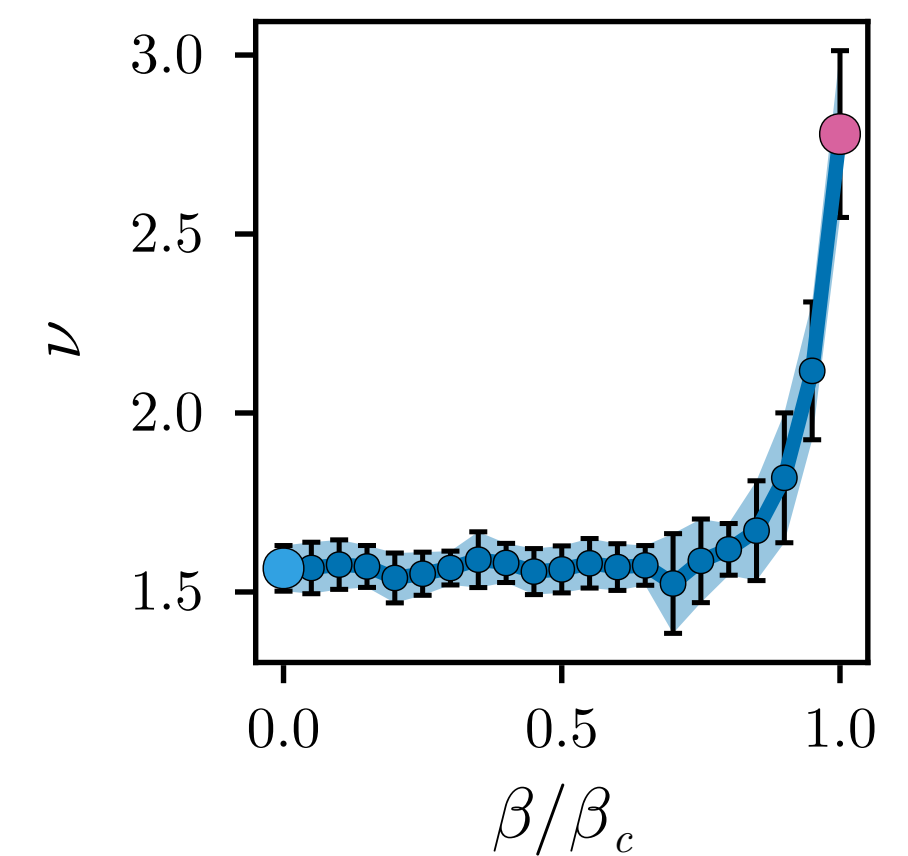
Bayesian / Born phase diagram



RG flows

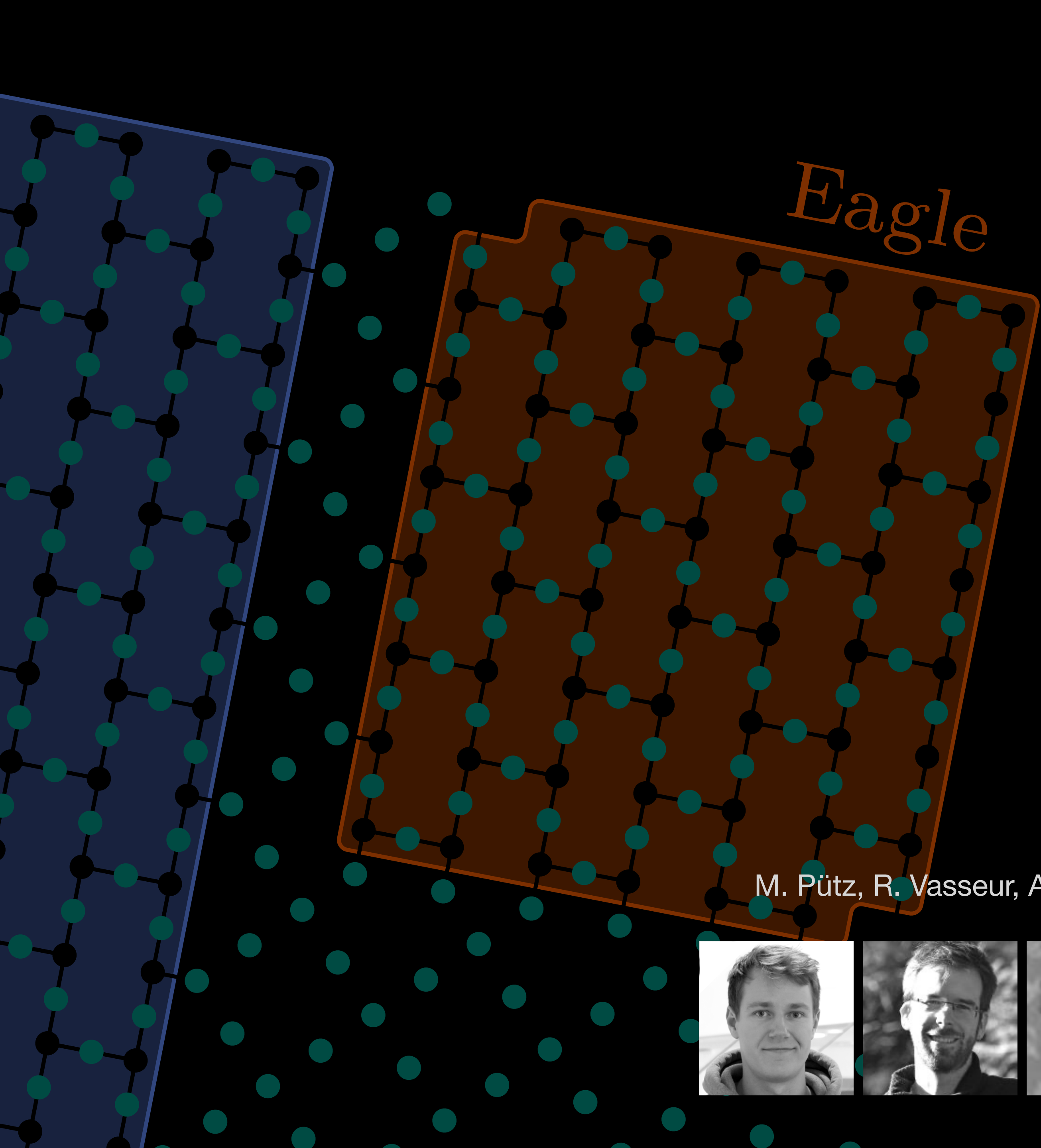
Nishimori

tricritical



tricritical

Ising



Eagle

qubit loss

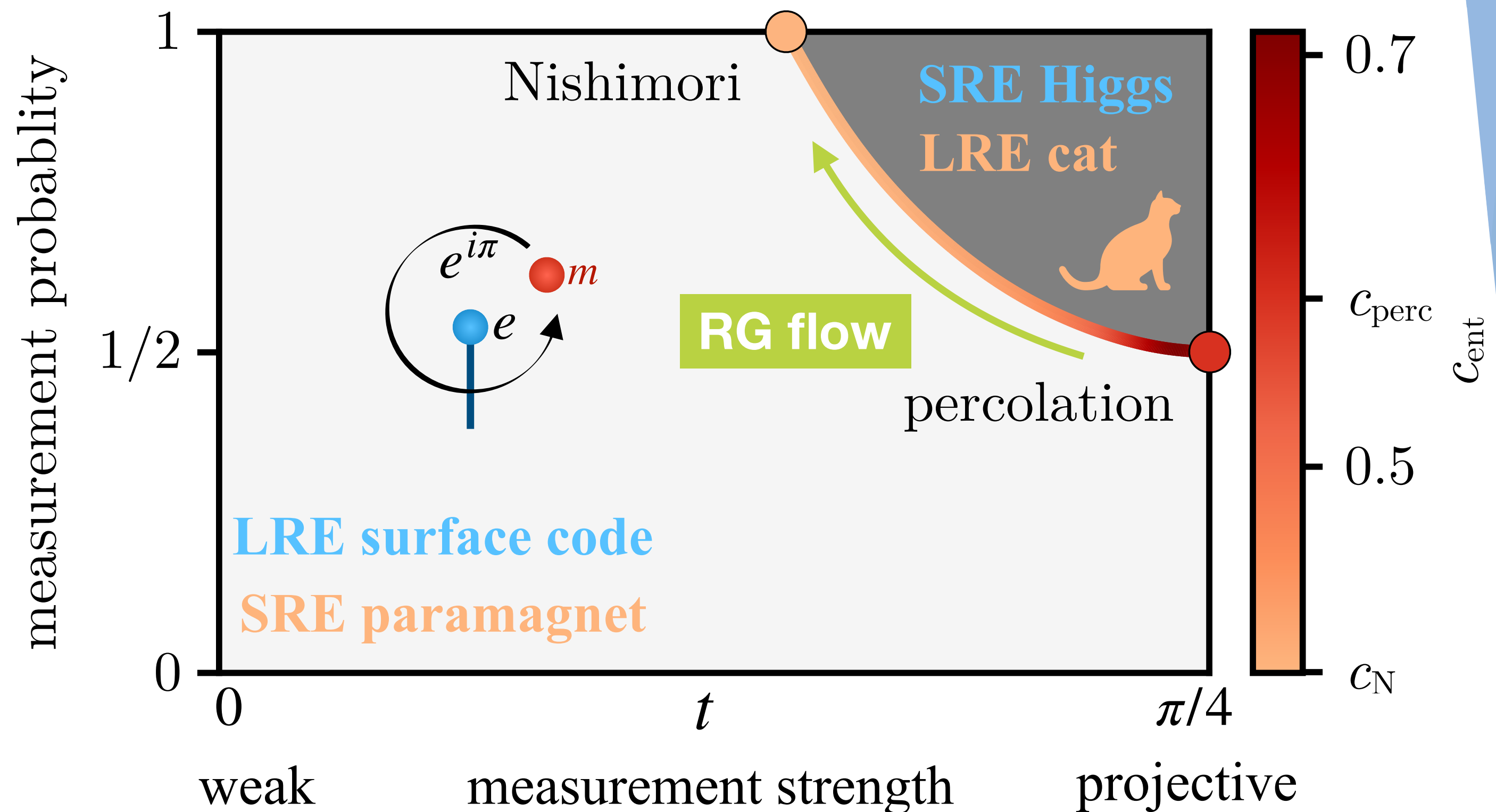
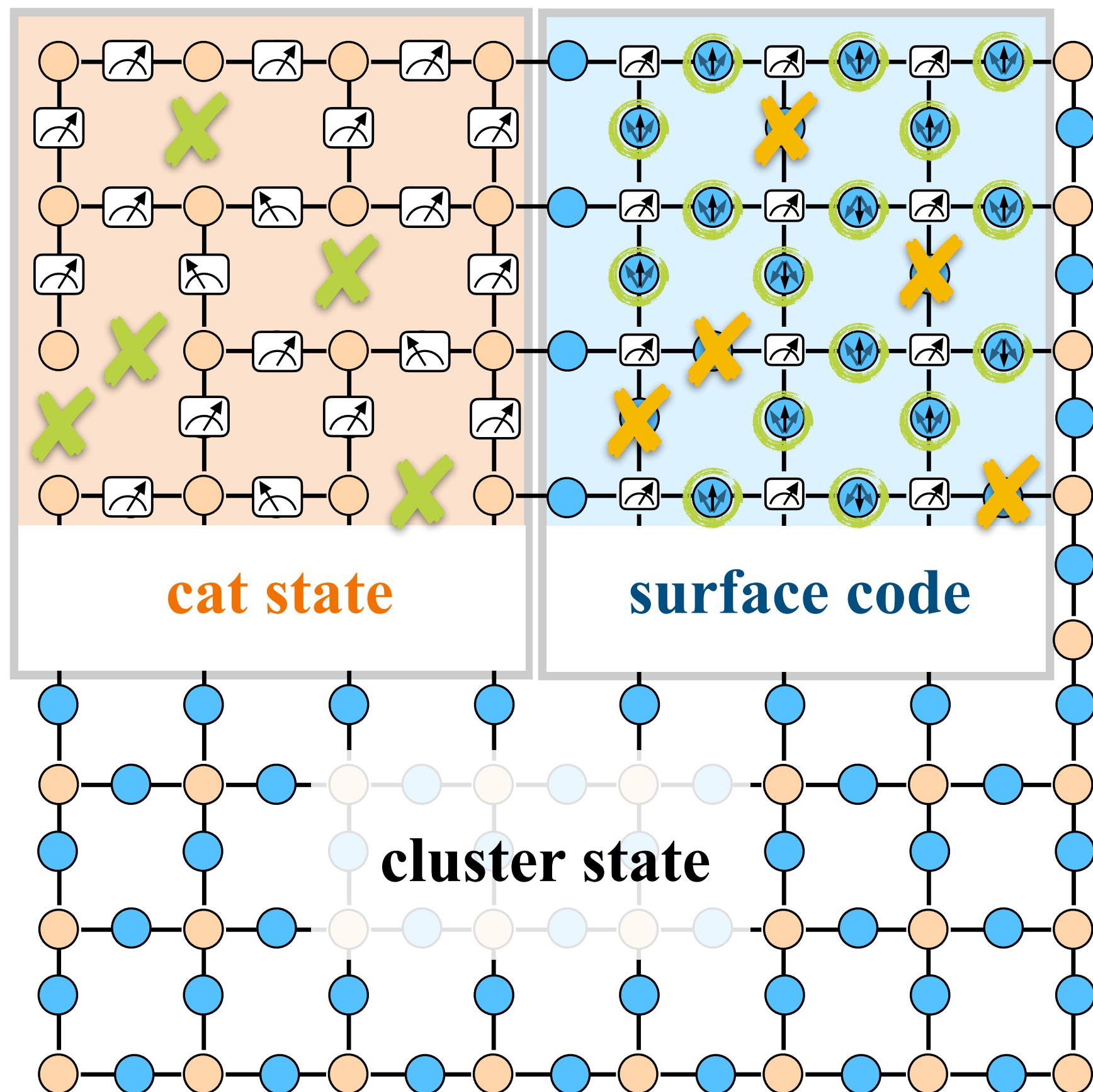
arXiv:2505.22720
PRX Quantum (2025)

M. Pütz, R. Vasseur, A. Ludwig, G-Y. Zhu



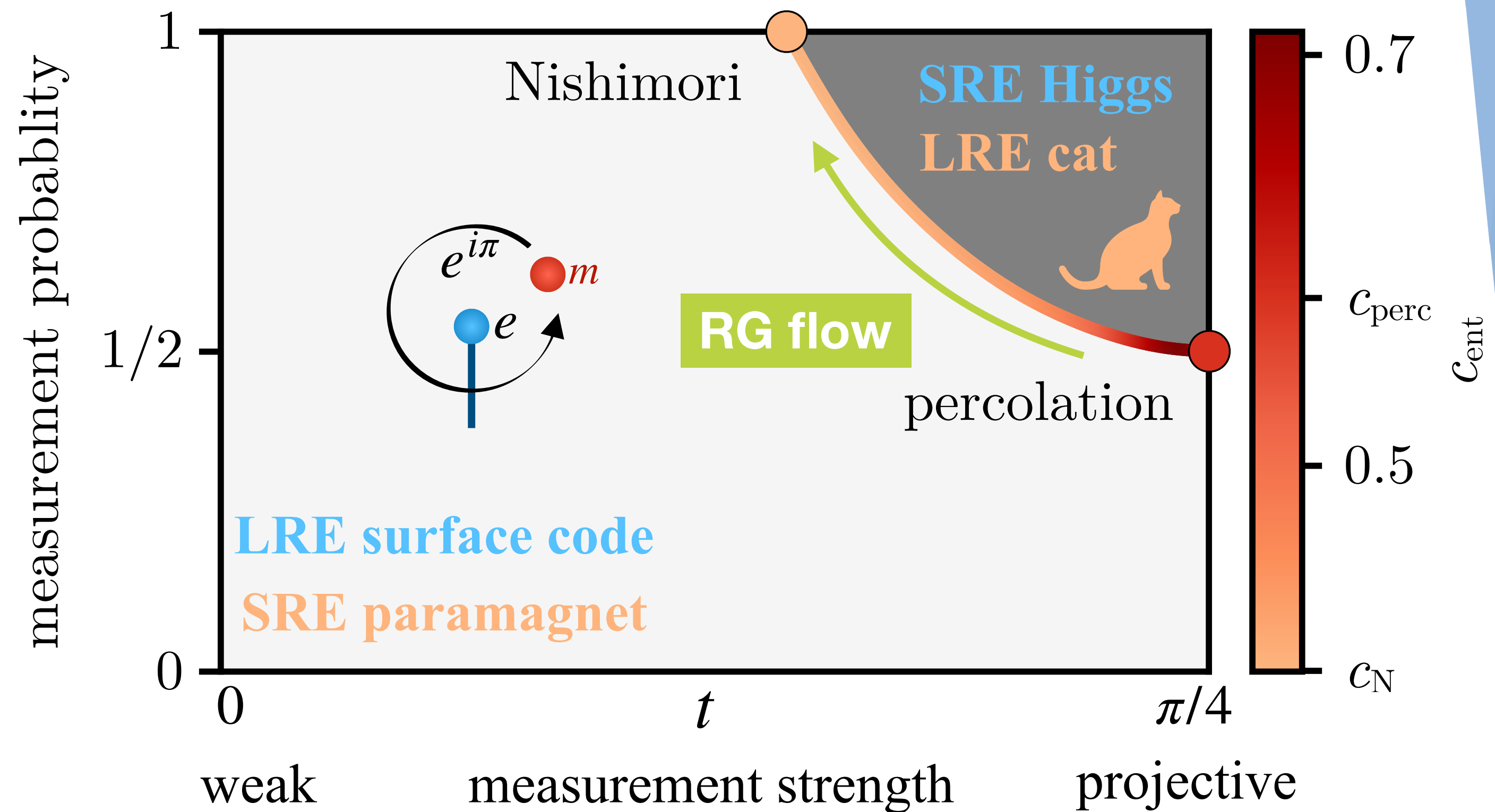
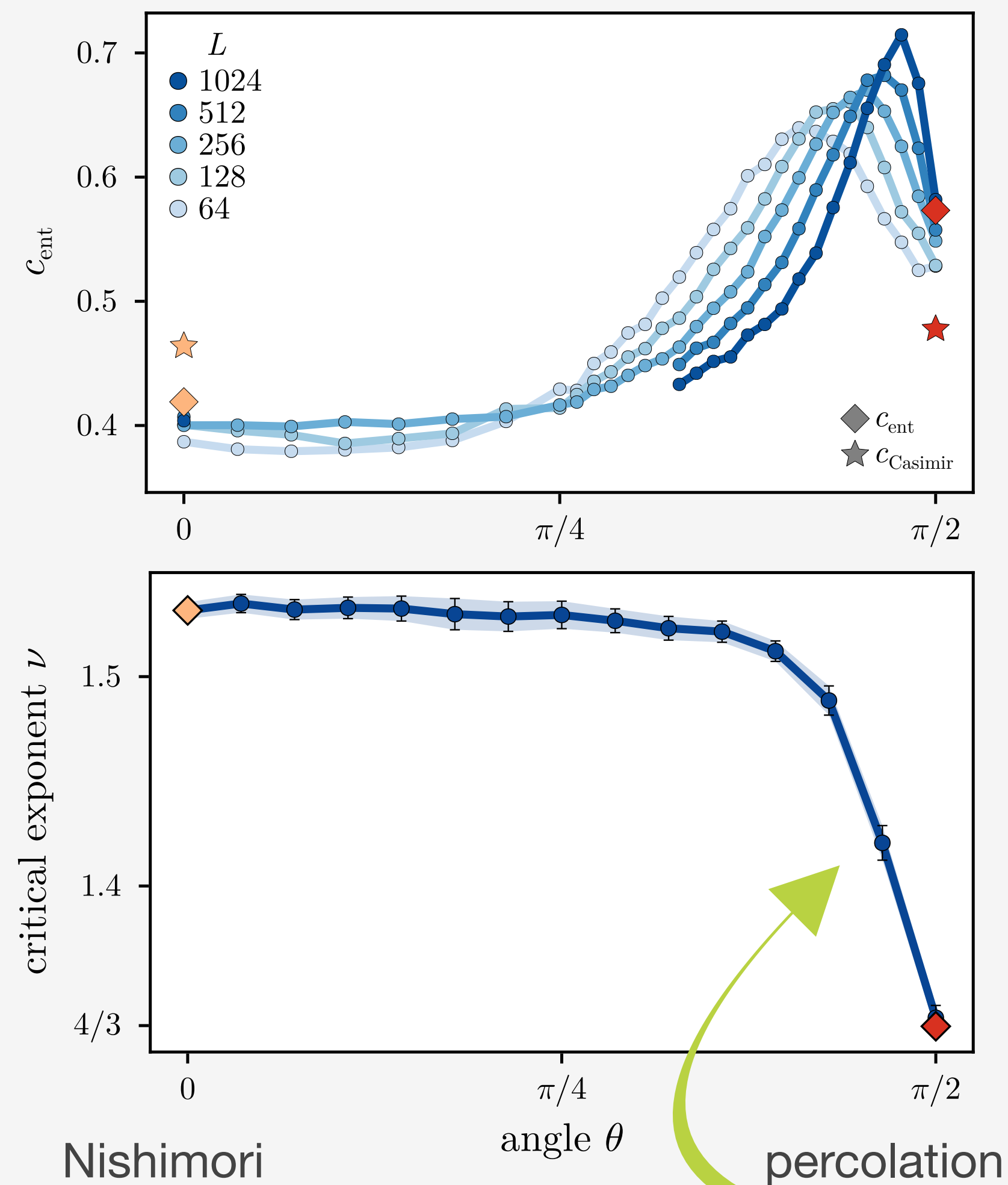
Nishimori / percolation & **RG flows**

qubit loss



**RG flow from percolation
to Nishimori criticality**

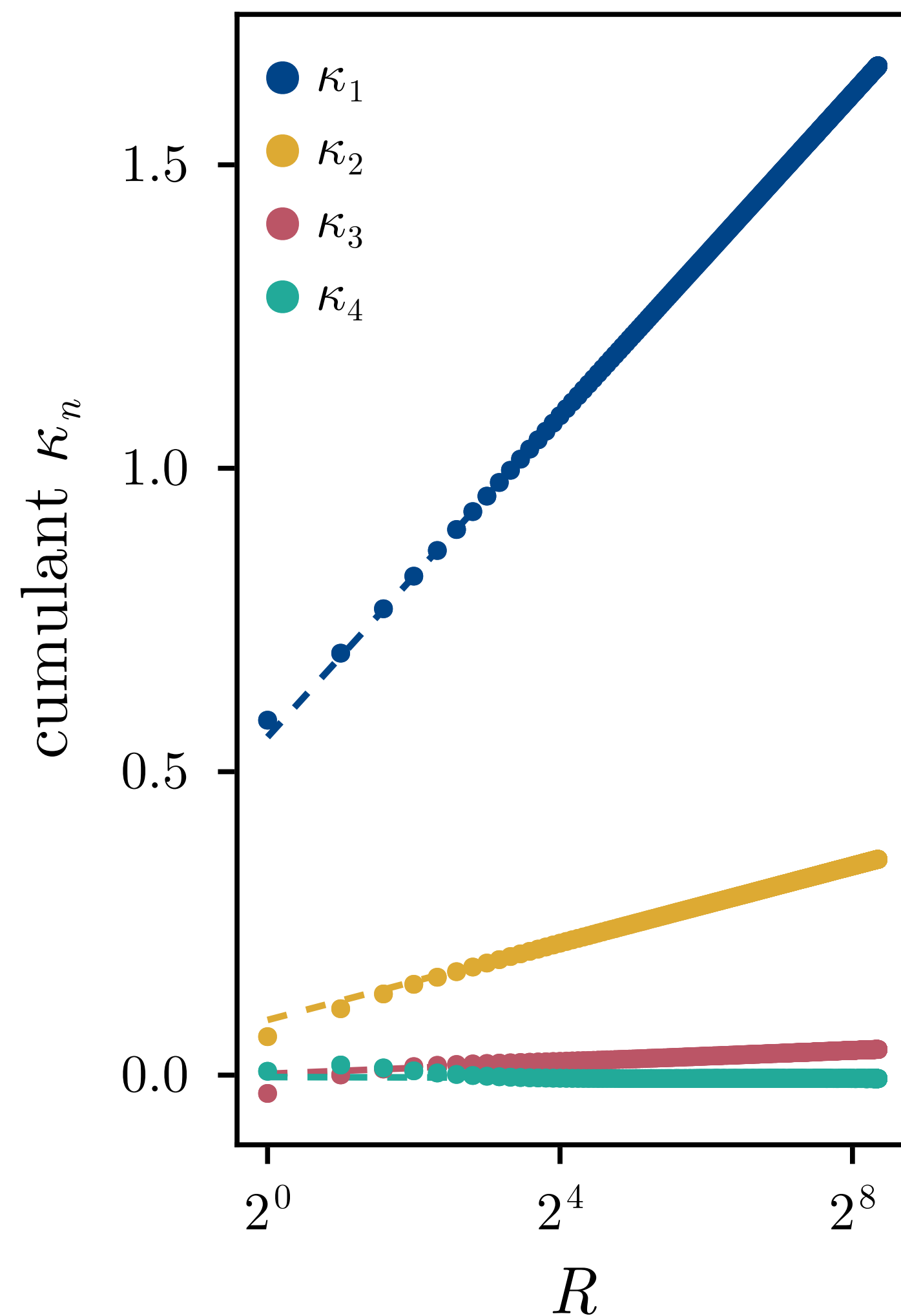
Nishimori / percolation & **RG flows**



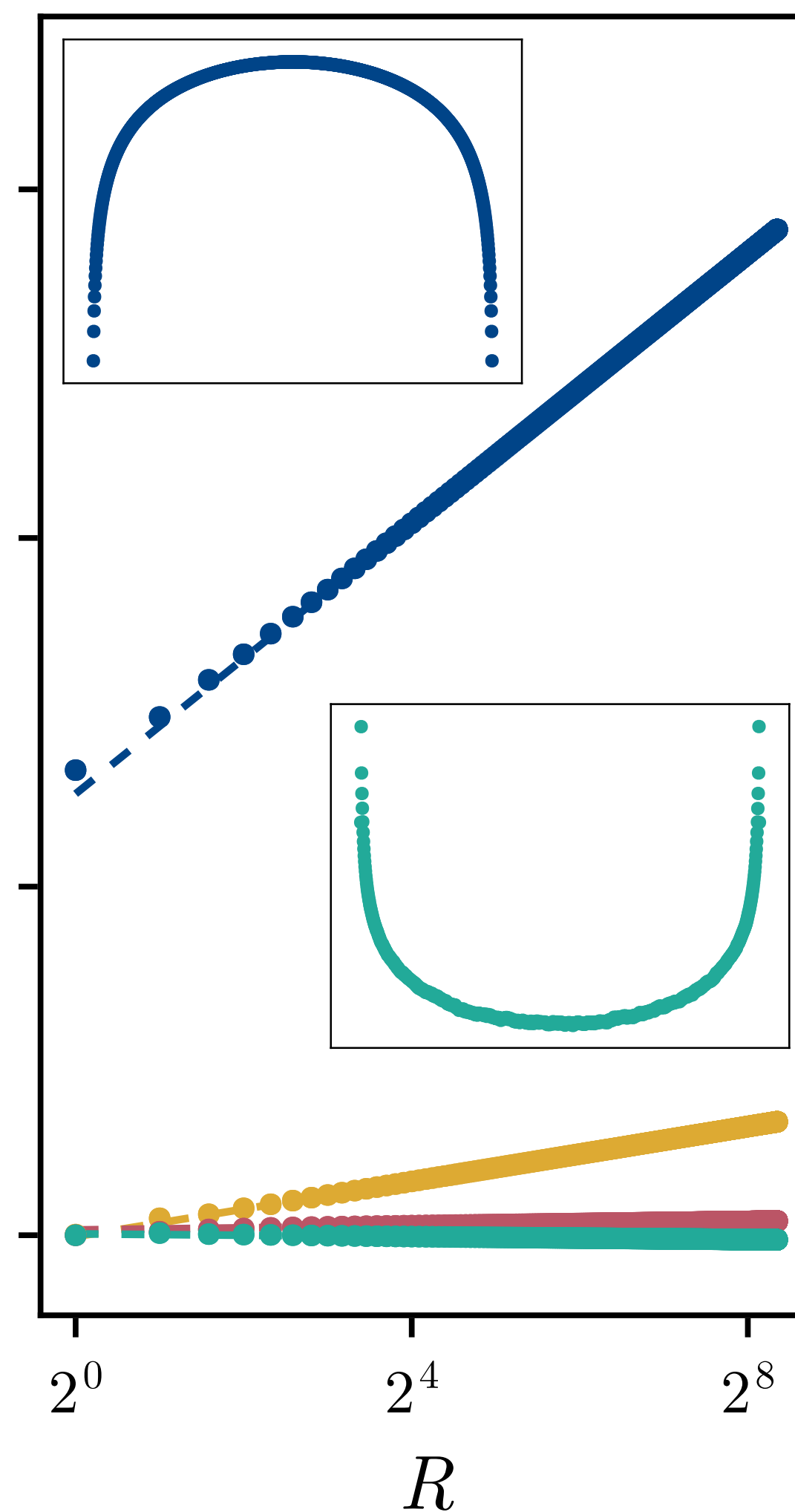
**RG flow from percolation
to Nishimori criticality**

Nishimori / percolation & multifractality

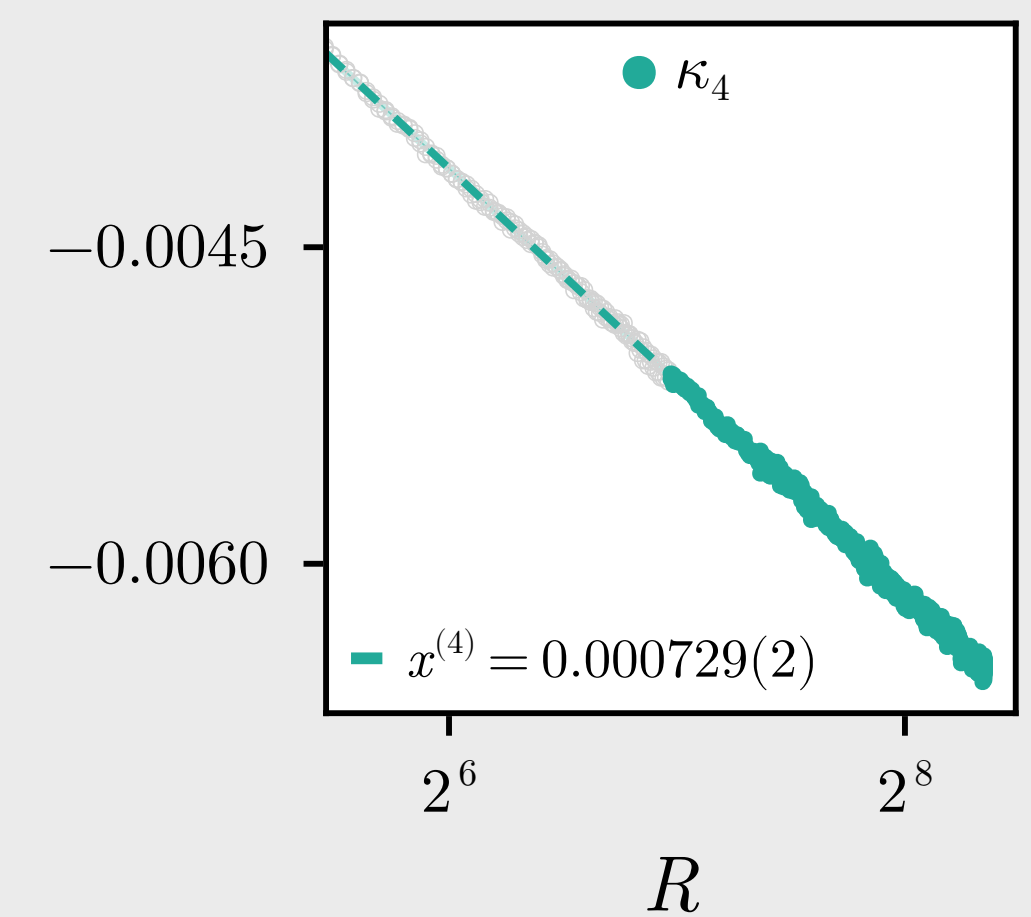
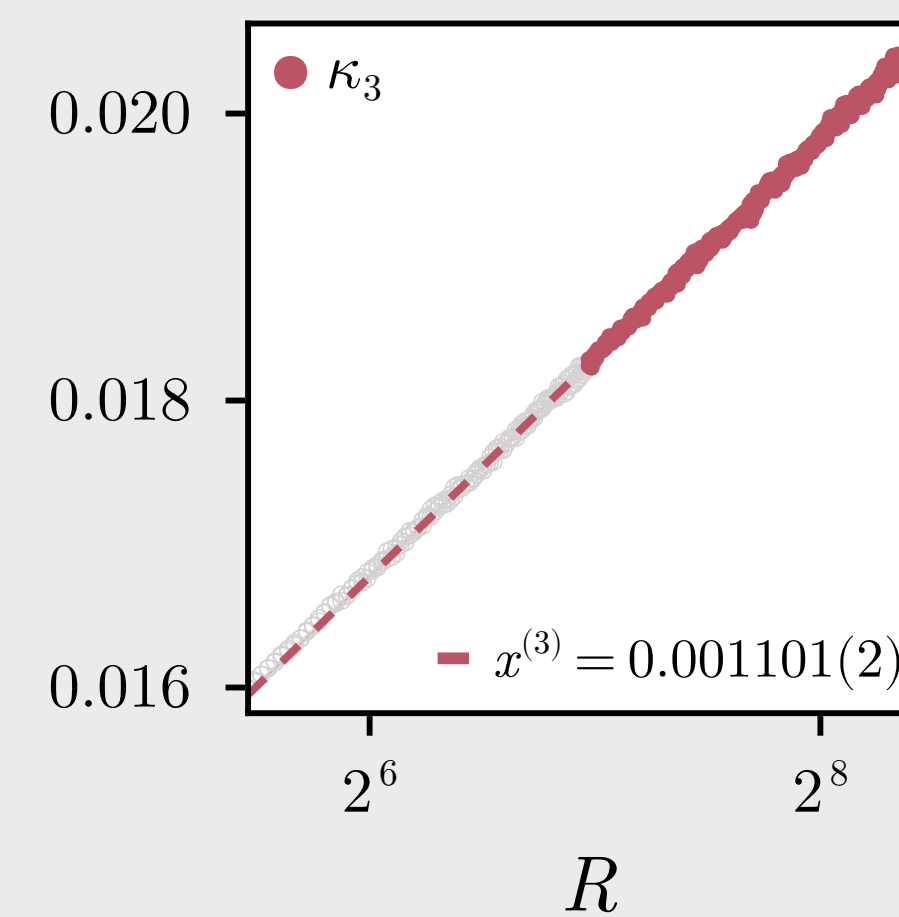
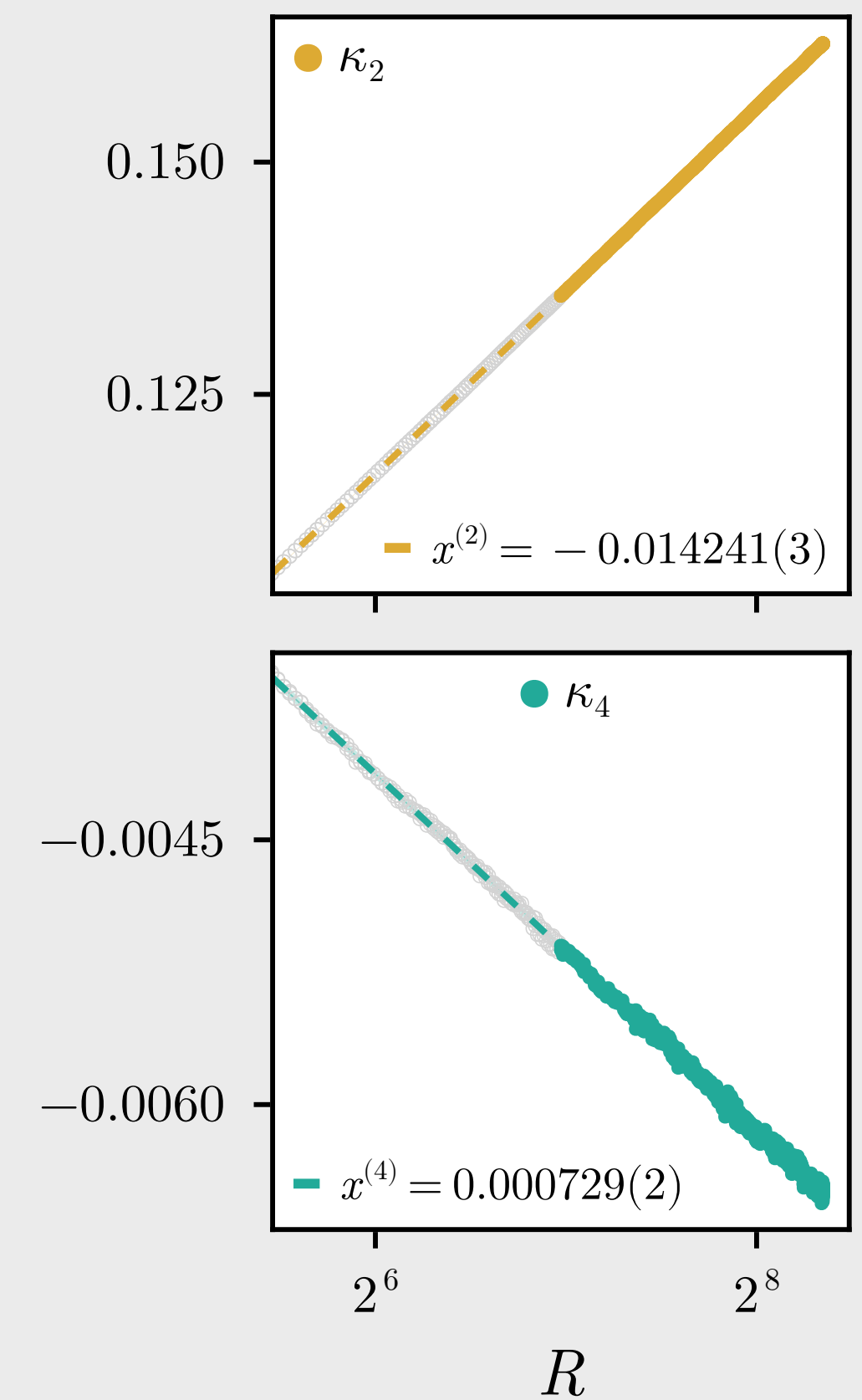
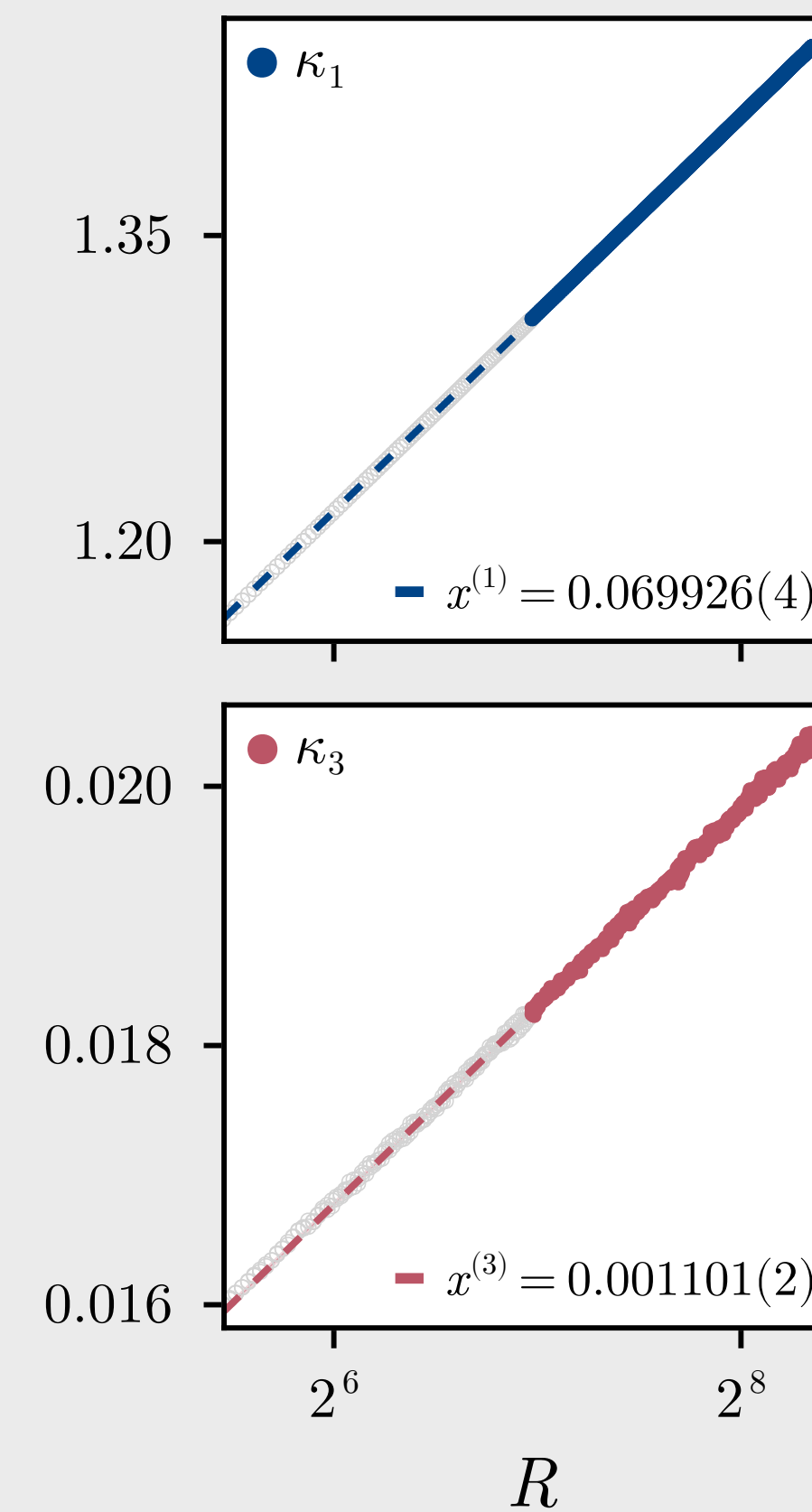
percolation



Nishimori



Both percolation and Nishimori criticality exhibit **multifractal spectra** of scaling dimensions.



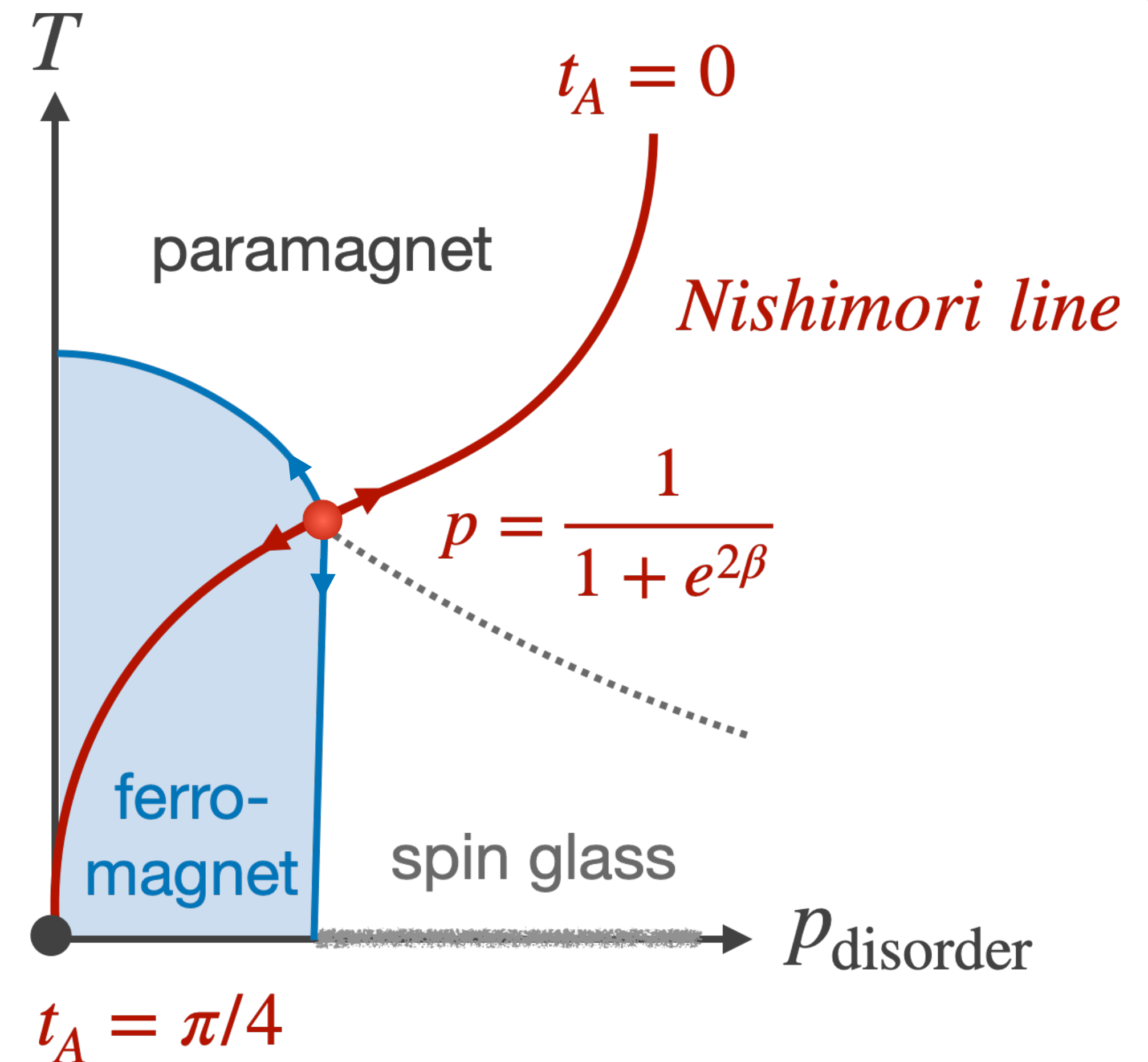


perspectives

Nishimori physics

- a staple of classical **statistical physics**
- but **ubiquitous in quantum** physics
 - enforced by **Born's rule**
 - induced by **coherent** and **incoherent** errors
 - **RG flow** form percolation, self-dual, tricriticality
 - an emerging **fixed point universality class** for **non-unitary** conformal QCPs?

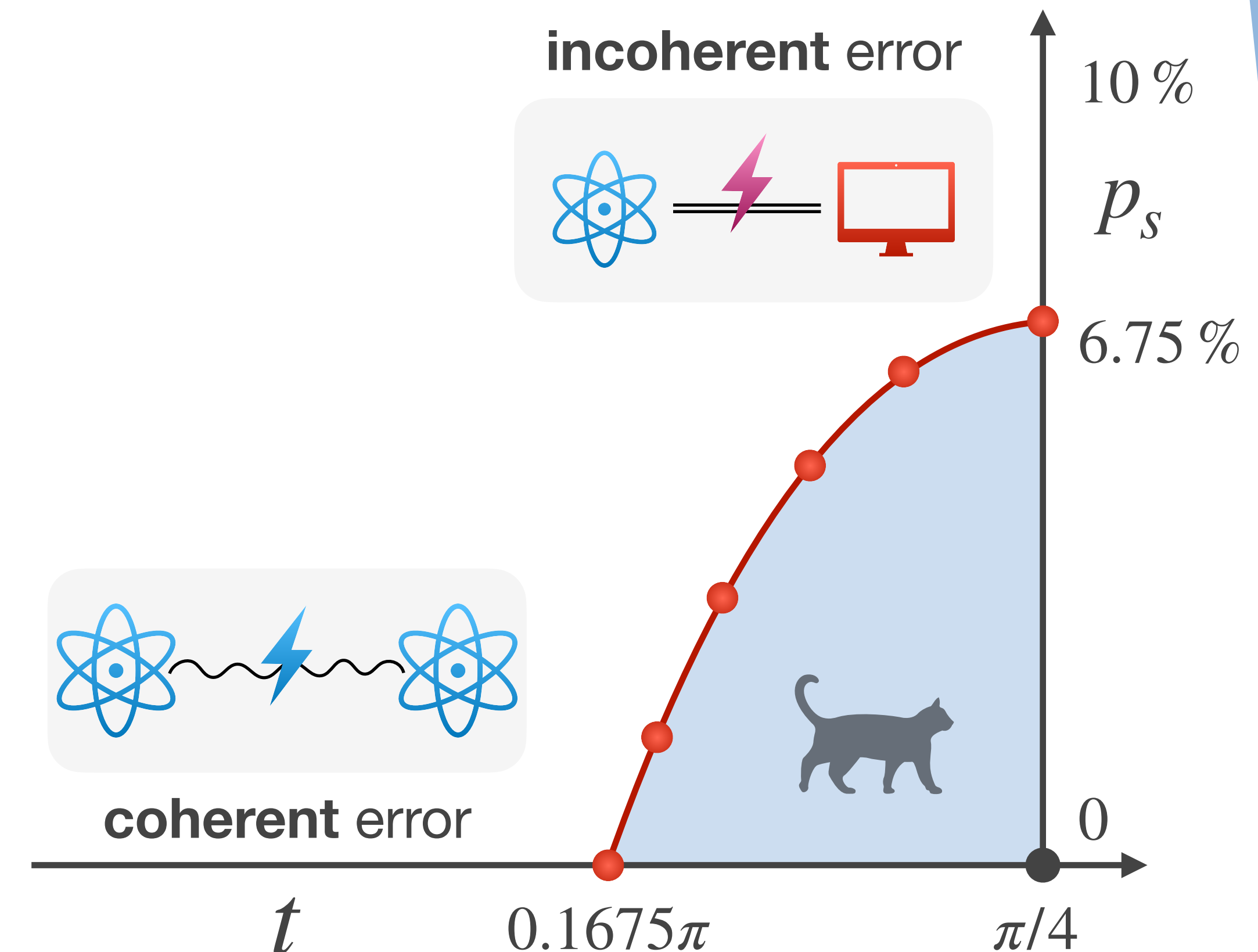
theory – Phys. Rev. Lett. **131**, 200201 (2023)
experiment (IBM) – Nature Physics **21**, 161 (2025)



Nishimori physics

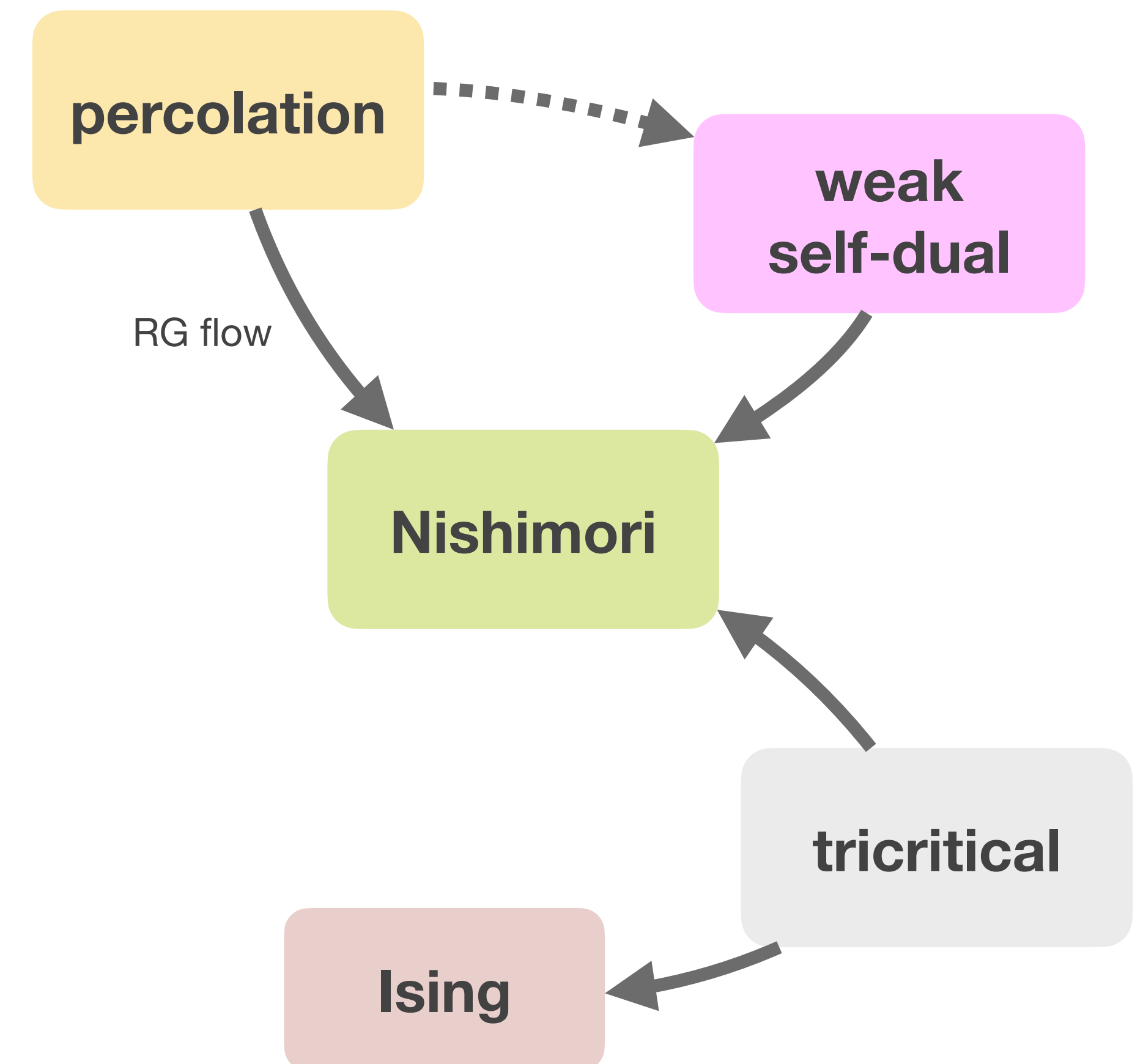
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theory – Phys. Rev. Lett. **131**, 200201 (2023)
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percolation – arXiv:2505.22720
self-dual – arXiv:2502.14034, PRX Quantum 5, 040313 (2024)
tricritical – arXiv:2504.12385

Thanks!