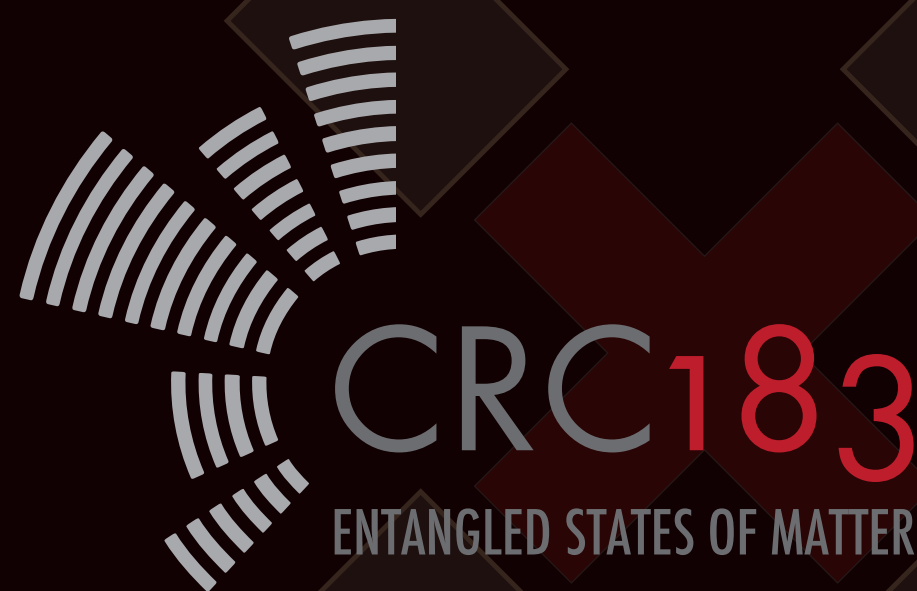


Qubit Fractionalization and Emergent Majorana Liquids in Quantum Circuits

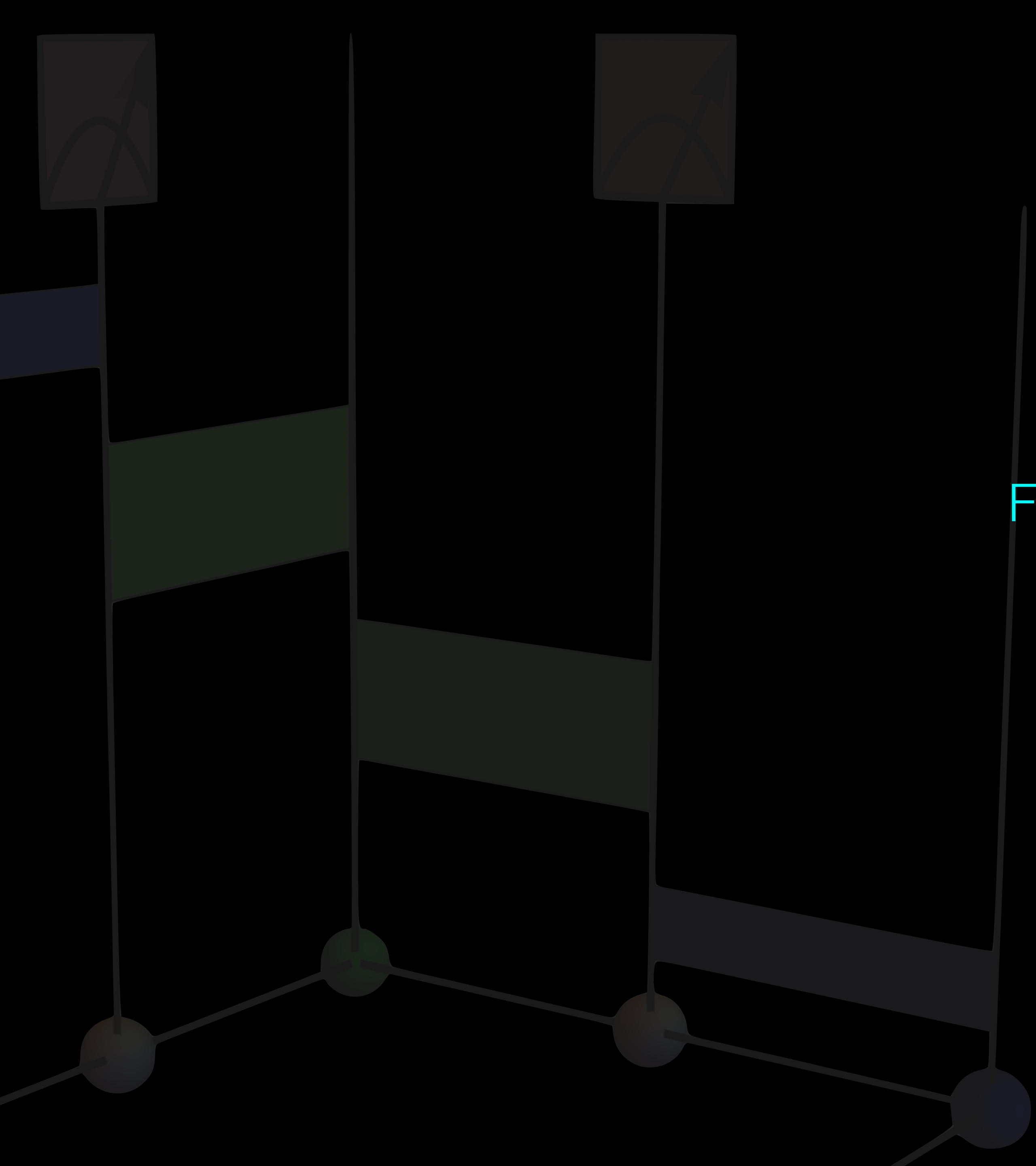


Simon Trebst
University of Cologne



Fractionalization and Emergent Gauge Fields in Quantum Matter

ICTP Trieste, December 2023



Fractionalization & Emergent Gauge Fields
in **Quantum Matter**



finite-temperature Kitaev spin liquids

PRL **113**, 197205 (2014)

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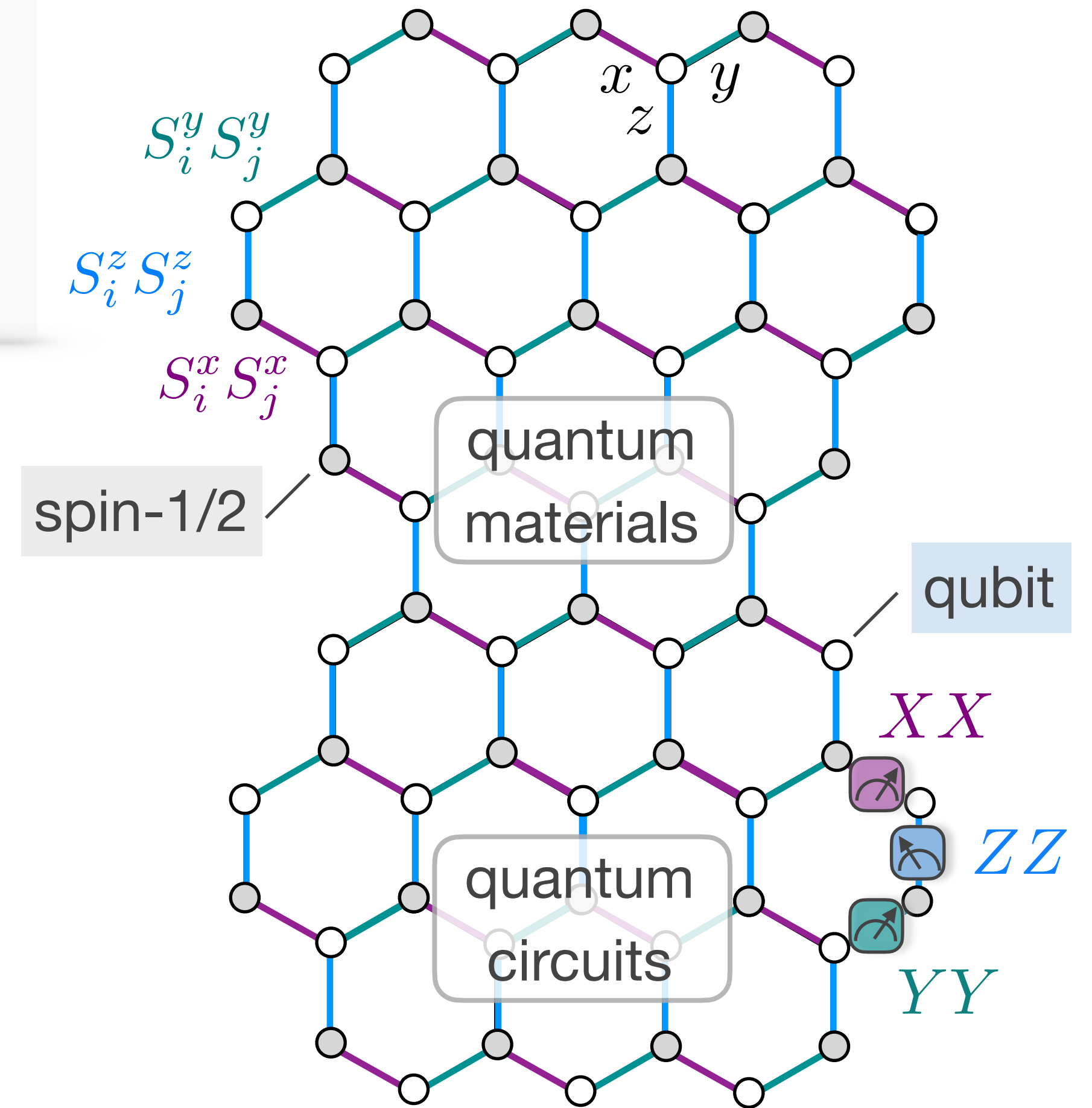
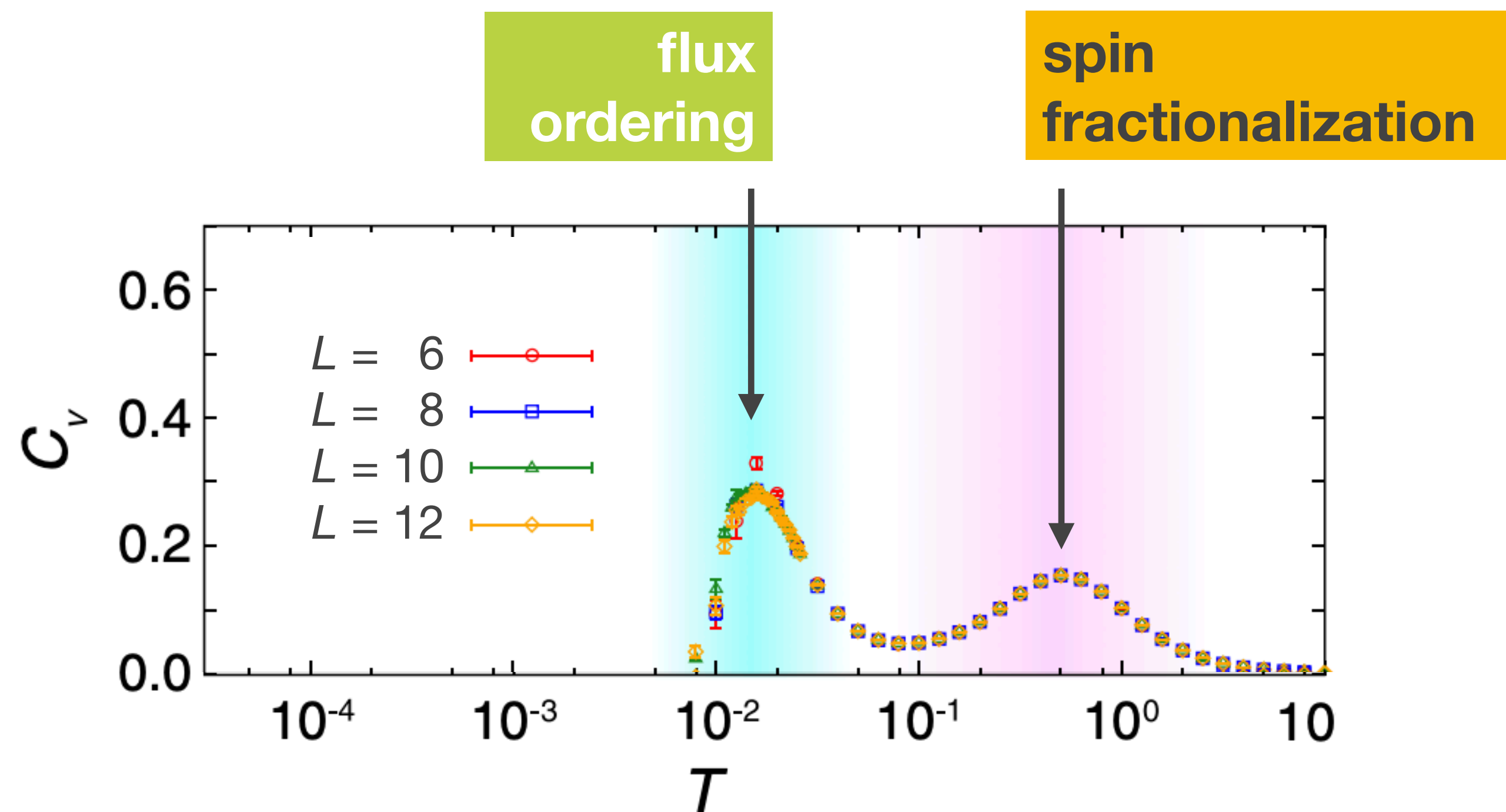
Vaporization of Kitaev Spin Liquids

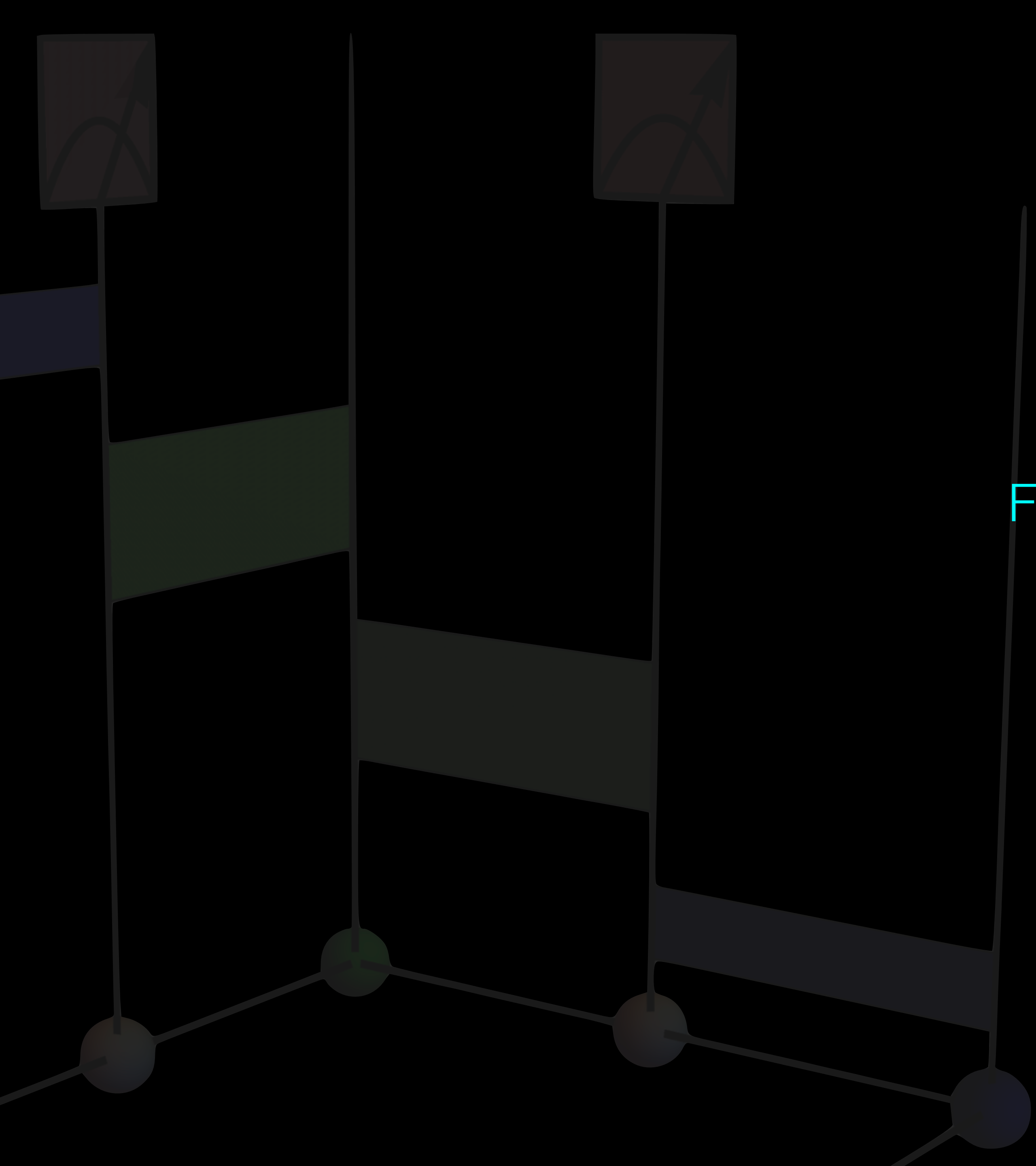
Joji Nasu,¹ Masafumi Udagawa,² and Yukitoshi Motome²

¹Department of Physics, Tokyo Institute of Technology, Ookayama, 2-12-1, Meguro, Tokyo 152-8551, Japan

²Department of Applied Physics, University of Tokyo, Hongo, 7-3-1, Bunkyo, Tokyo 113-8656, Japan

(Received 24 July 2014; revised manuscript received 9 October 2014; published 7 November 2014)

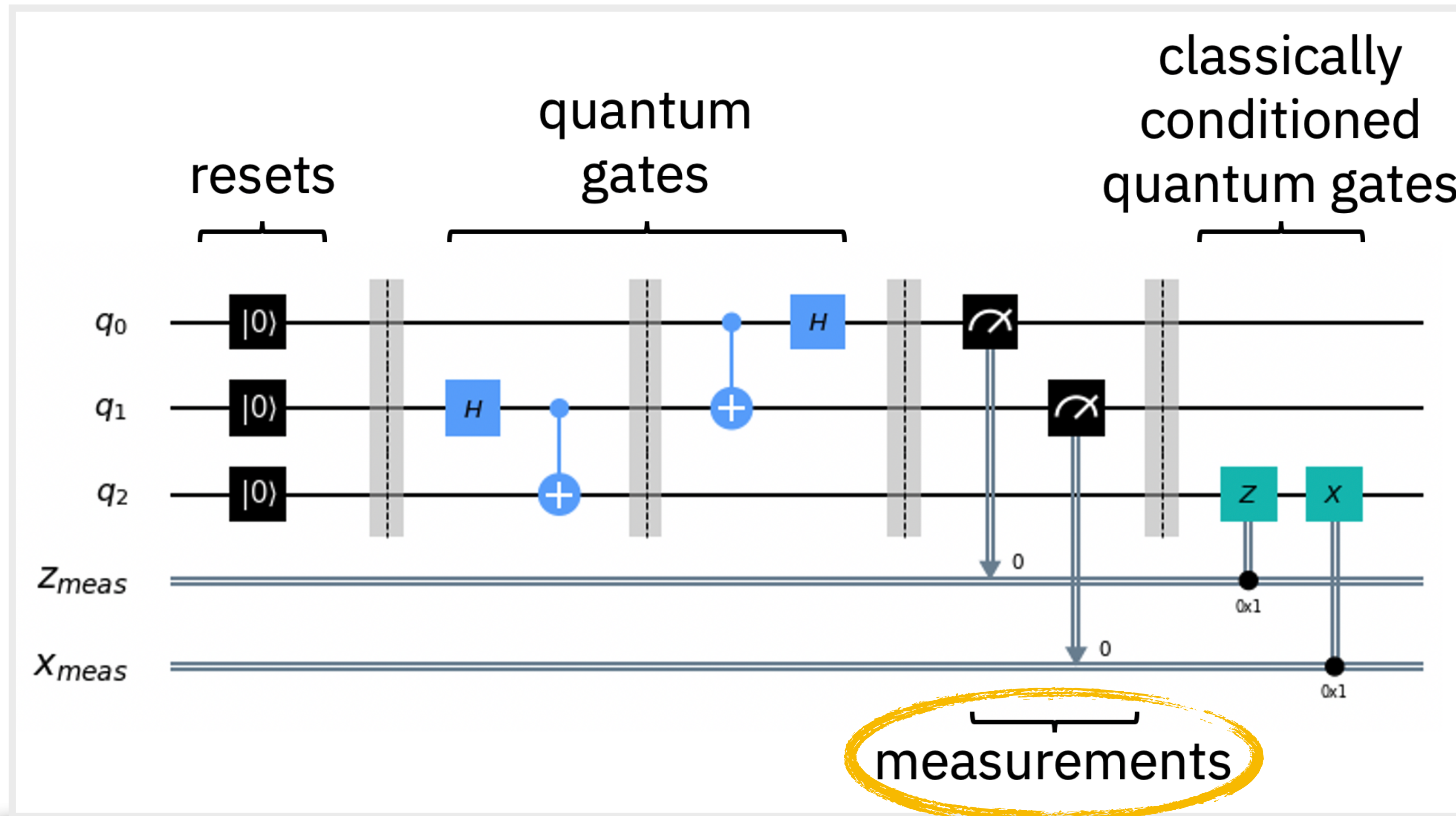




Fractionalization & Emergent Gauge Fields
in **Quantum Circuits**



quantum circuits in a nutshell



Quantum computing in a nutshell, Qiskit documentation / IBM Quantum

quantum measurements

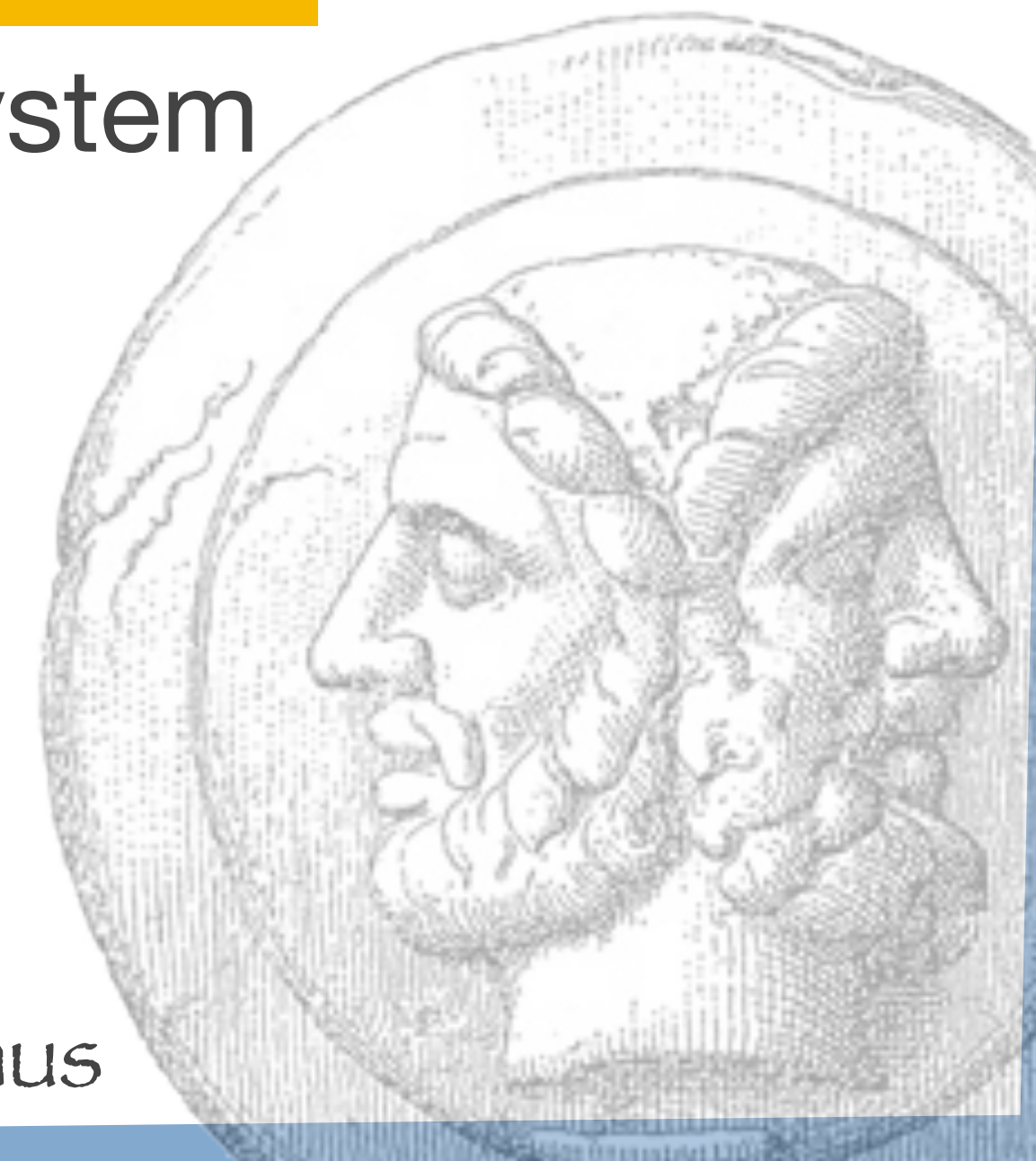


“About your cat, Mr. Schrödinger — I have good news and bad news.”

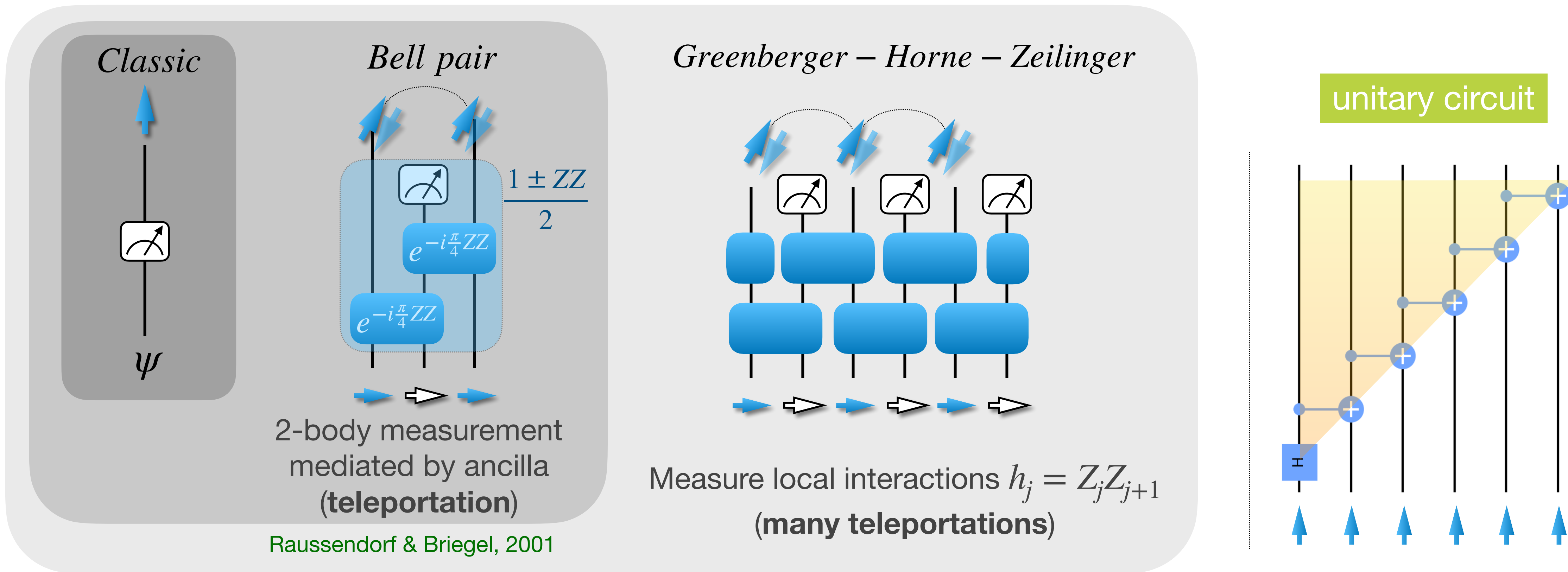
Quantum measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system

double-faced Janus



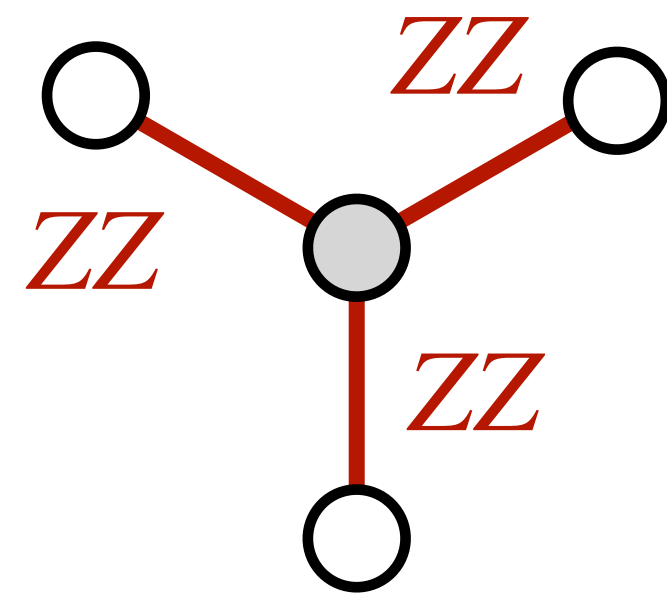
quantum states from measurements



$$t \propto O(1)$$

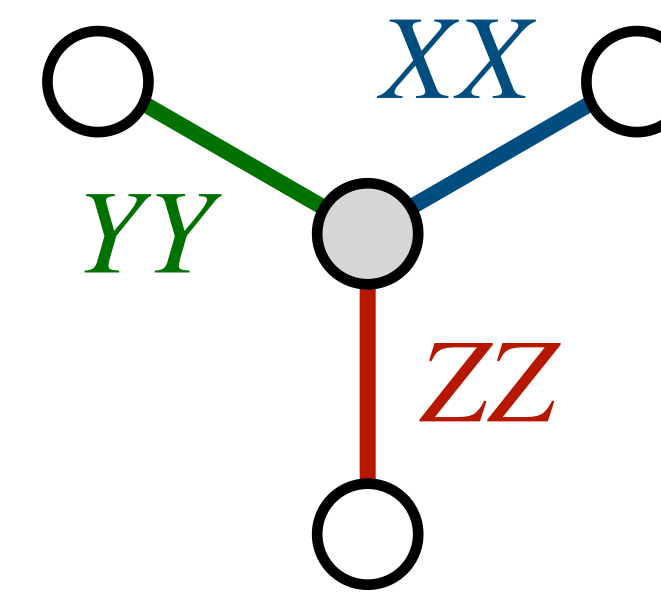
$$t \propto L$$

commuting vs non-commuting measurements



Nishimori's cat

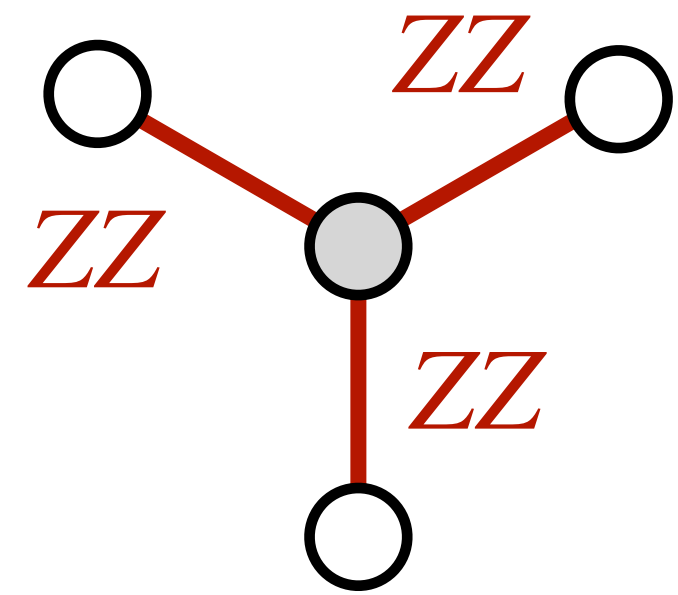
- commuting
- parallelized
- no dynamics



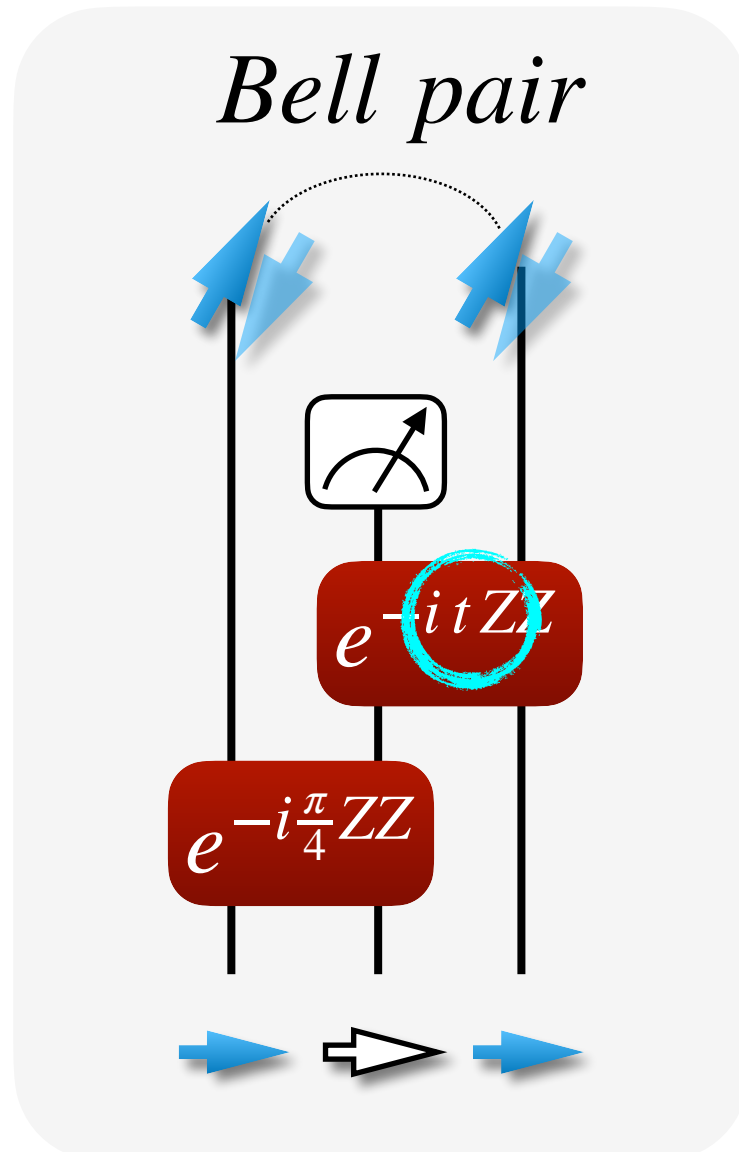
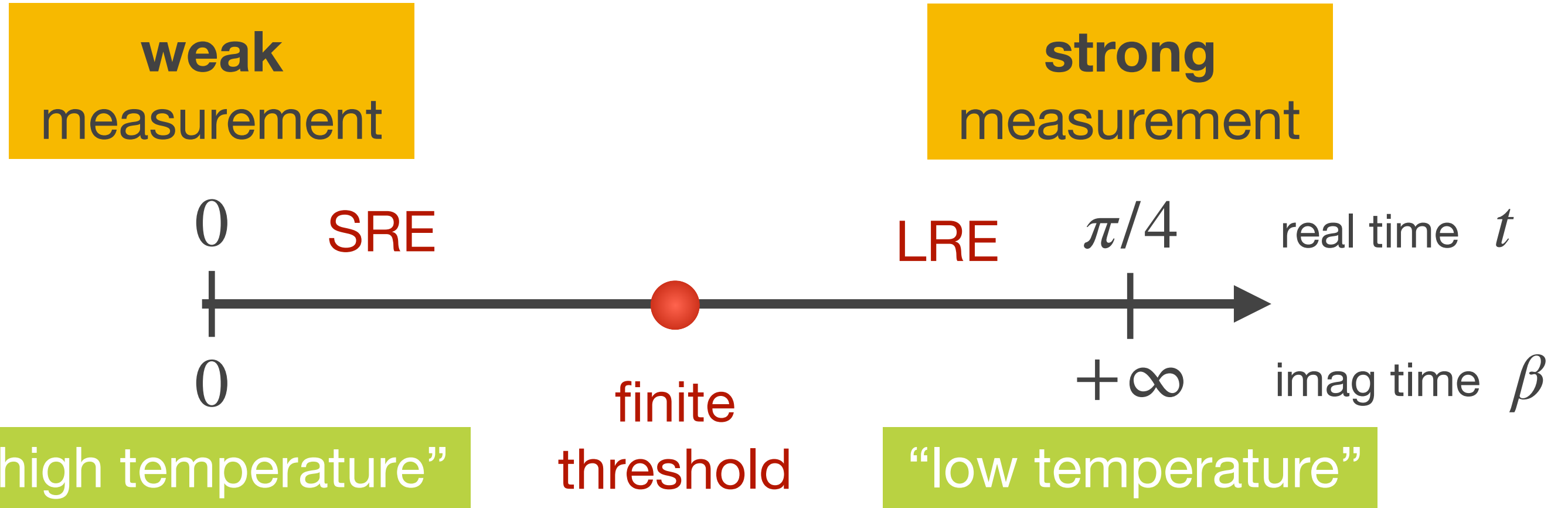
Kitaev spin liquid

- non-commuting
- sequential
- dynamics

Nishimori's cat



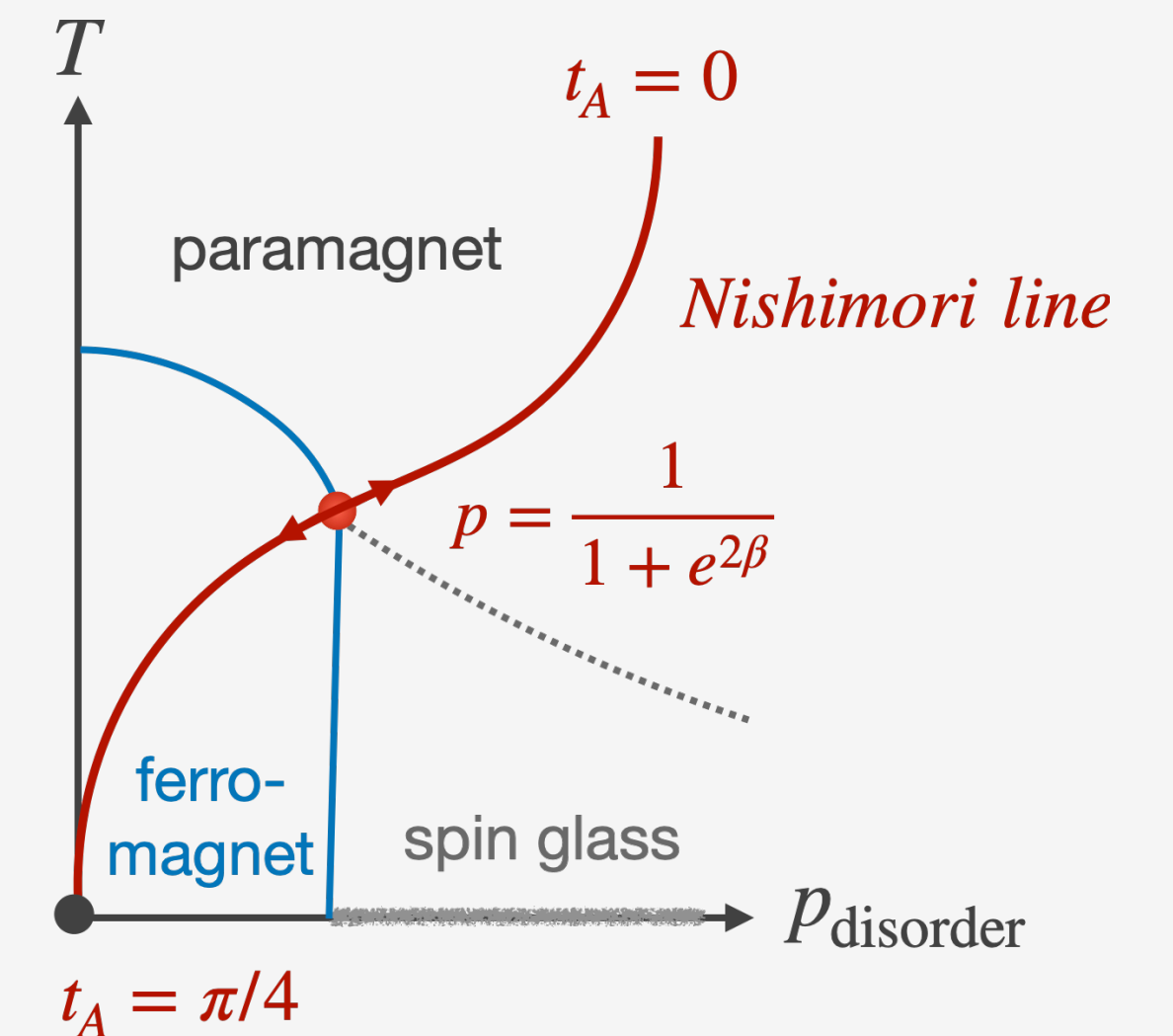
Nishimori's cat



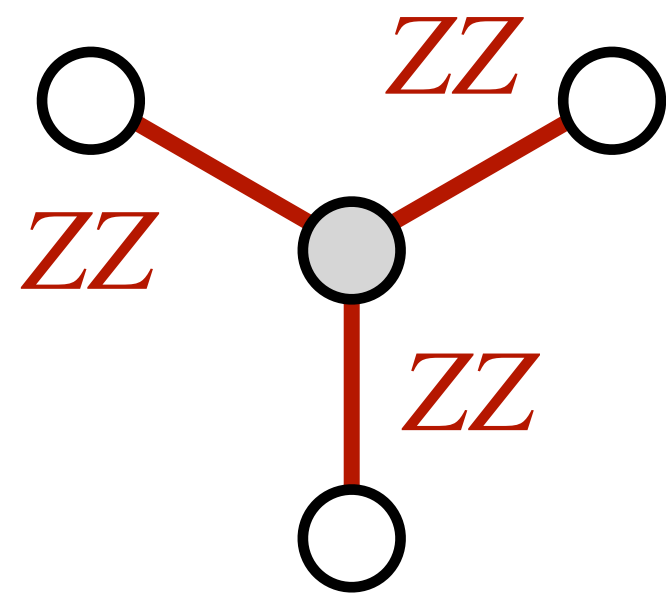
thermal fluctuations and disorder are **locked**

Nishimori (1981)

theory – Phys. Rev. Lett. 131, 200201 (2023)
 experiment (IBM) – arXiv:2309.02863 (2023)

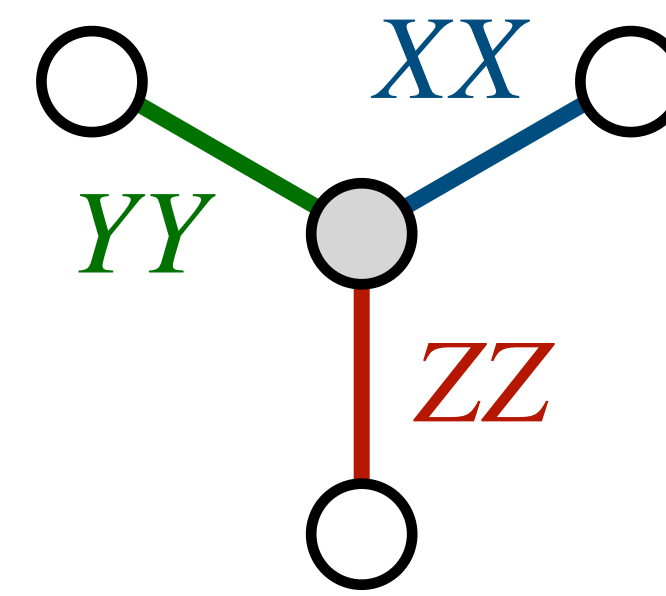


commuting vs non-commuting measurements



Nishimori's cat

- commuting
- parallelized
- no dynamics



Kitaev spin liquid

- non-commuting
- sequential
- dynamics

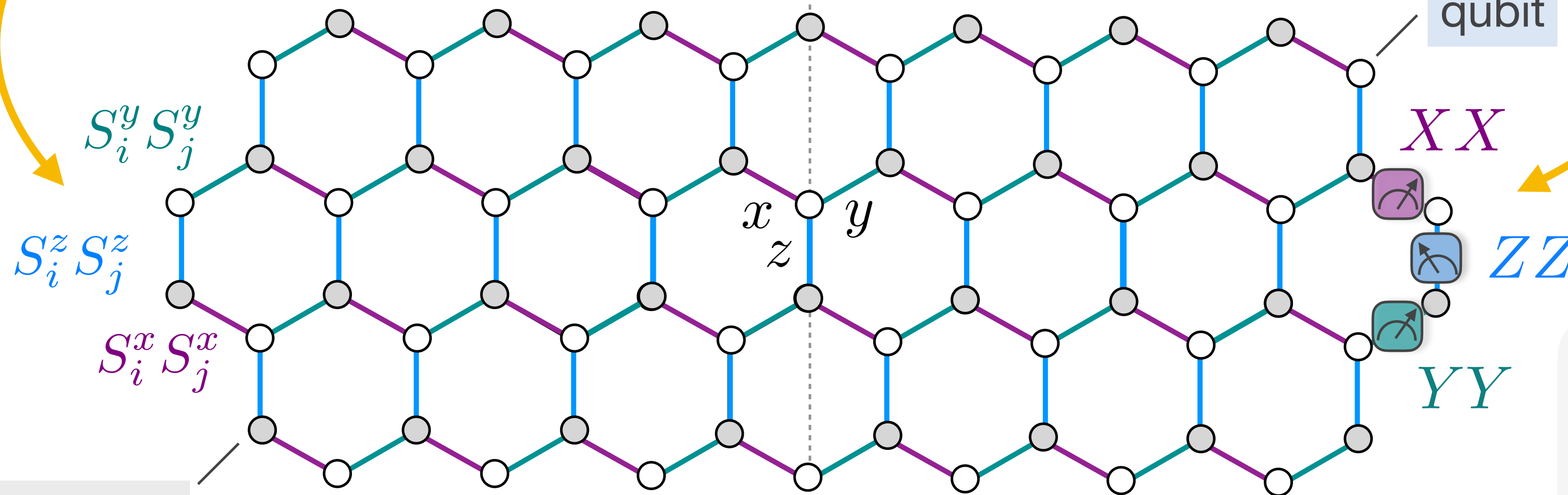
frustration and entanglement



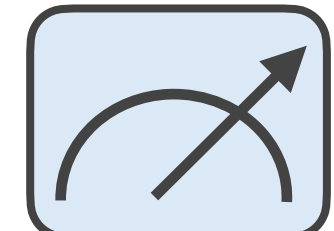
Kitaev (2006)

\mathcal{H}

magnetic exchange



spin-1/2



Clifford gates

Hamiltonian

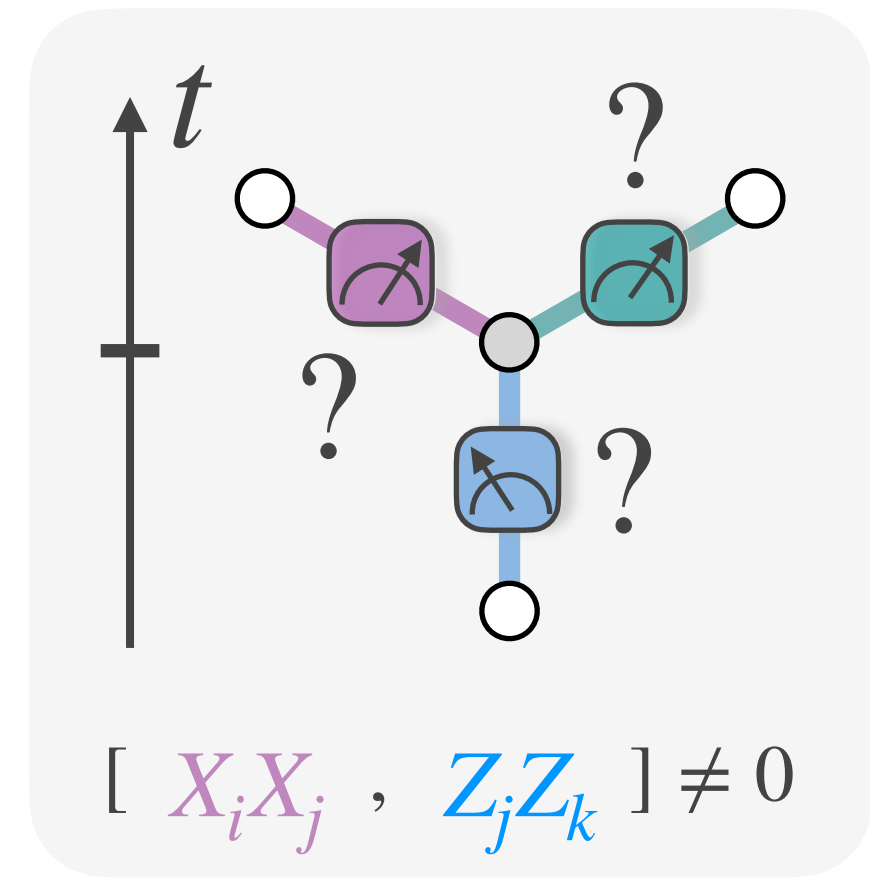
measurement

ground state

minimizes global energy

dynamical state

eigenstate of measurements



no state can satisfy every local interaction

frustration

non-commuting operators

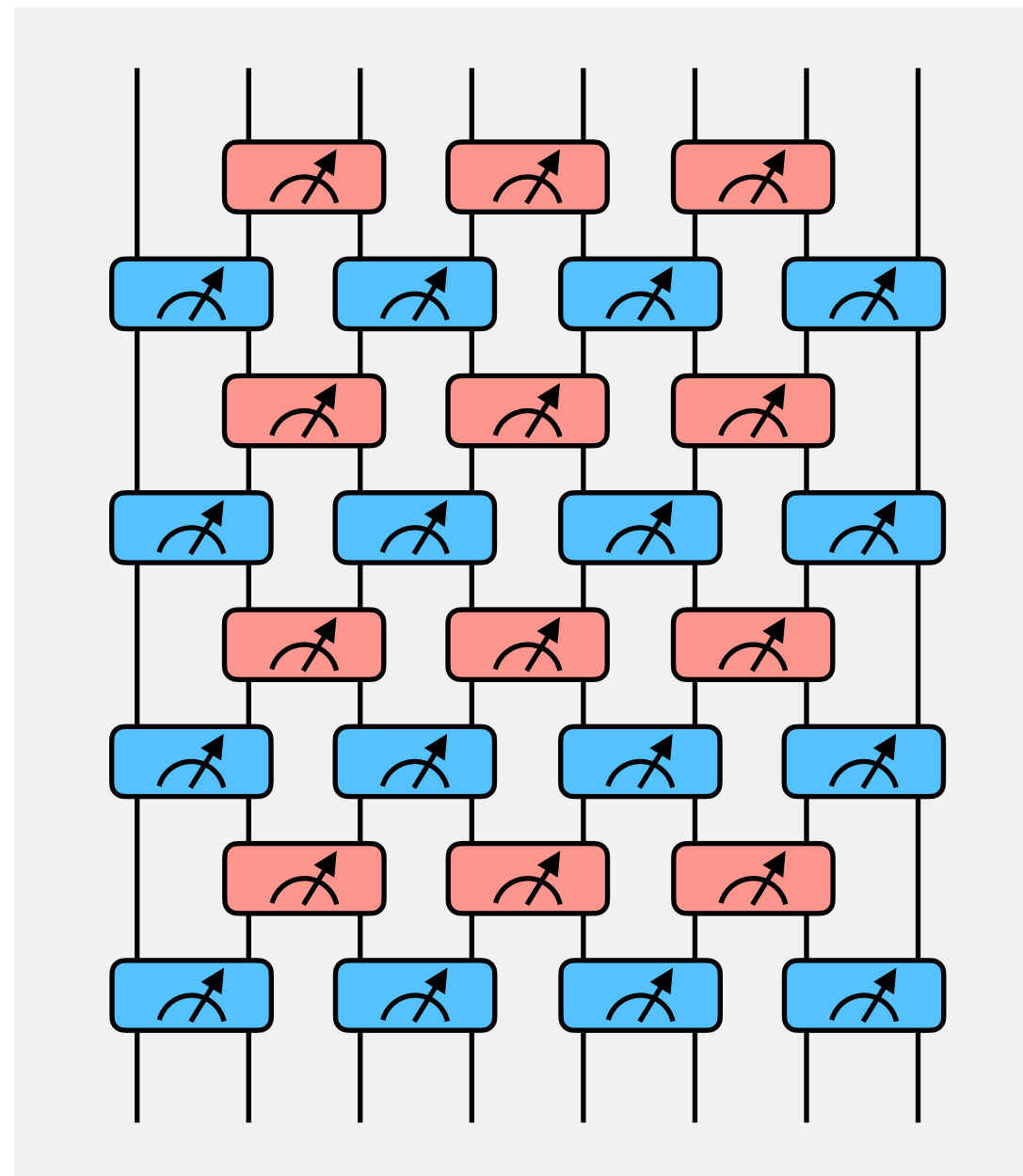
- cannot be measured simultaneously
- will be over-written

imaginary time vs. measurement-only

Hamiltonian ground state

$$e^{-\beta H} |\psi_0\rangle$$

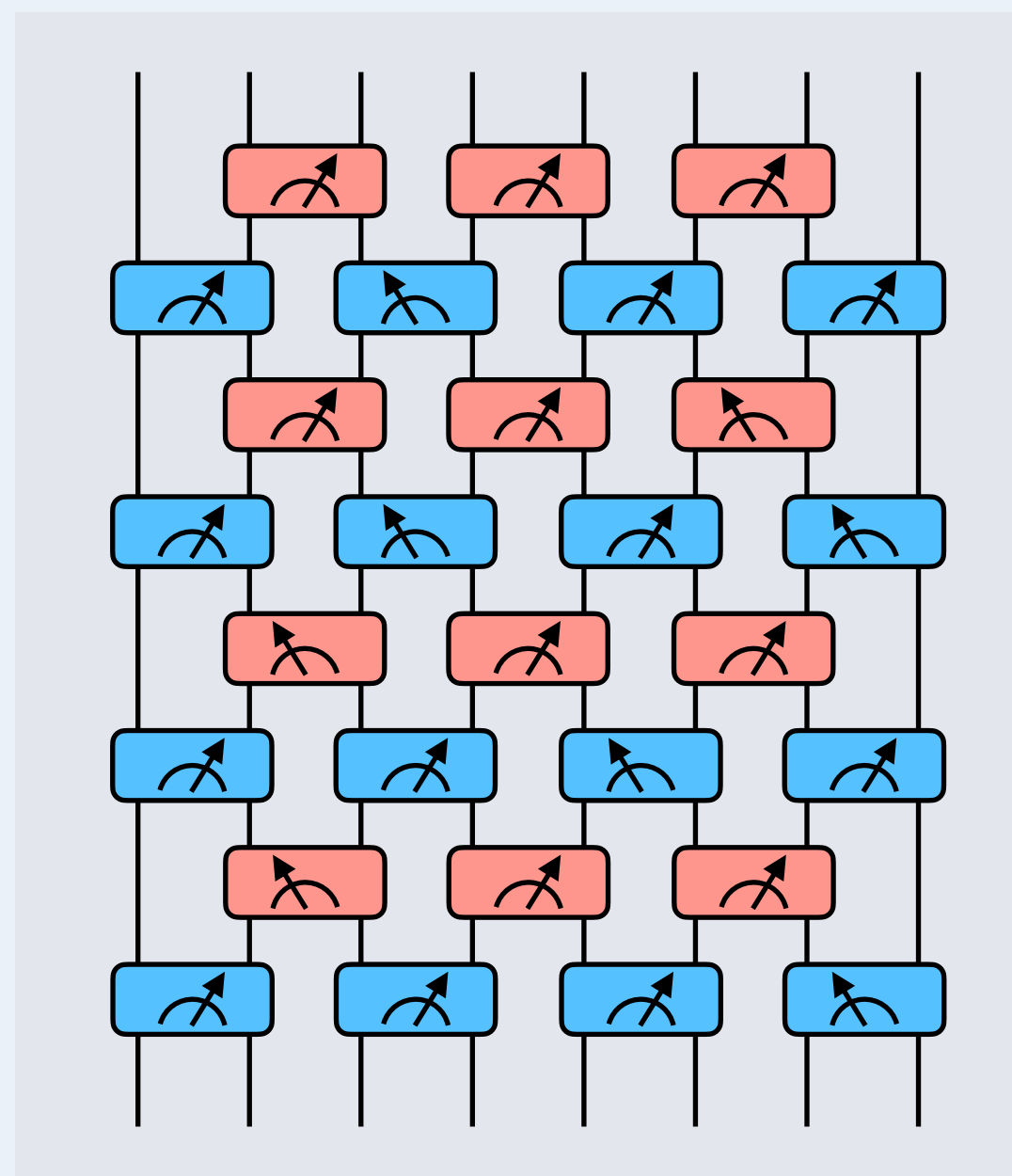
$e^{-\tau H_r}$



- brickwall circuit
- no disorder
- $\tau \ll 1$

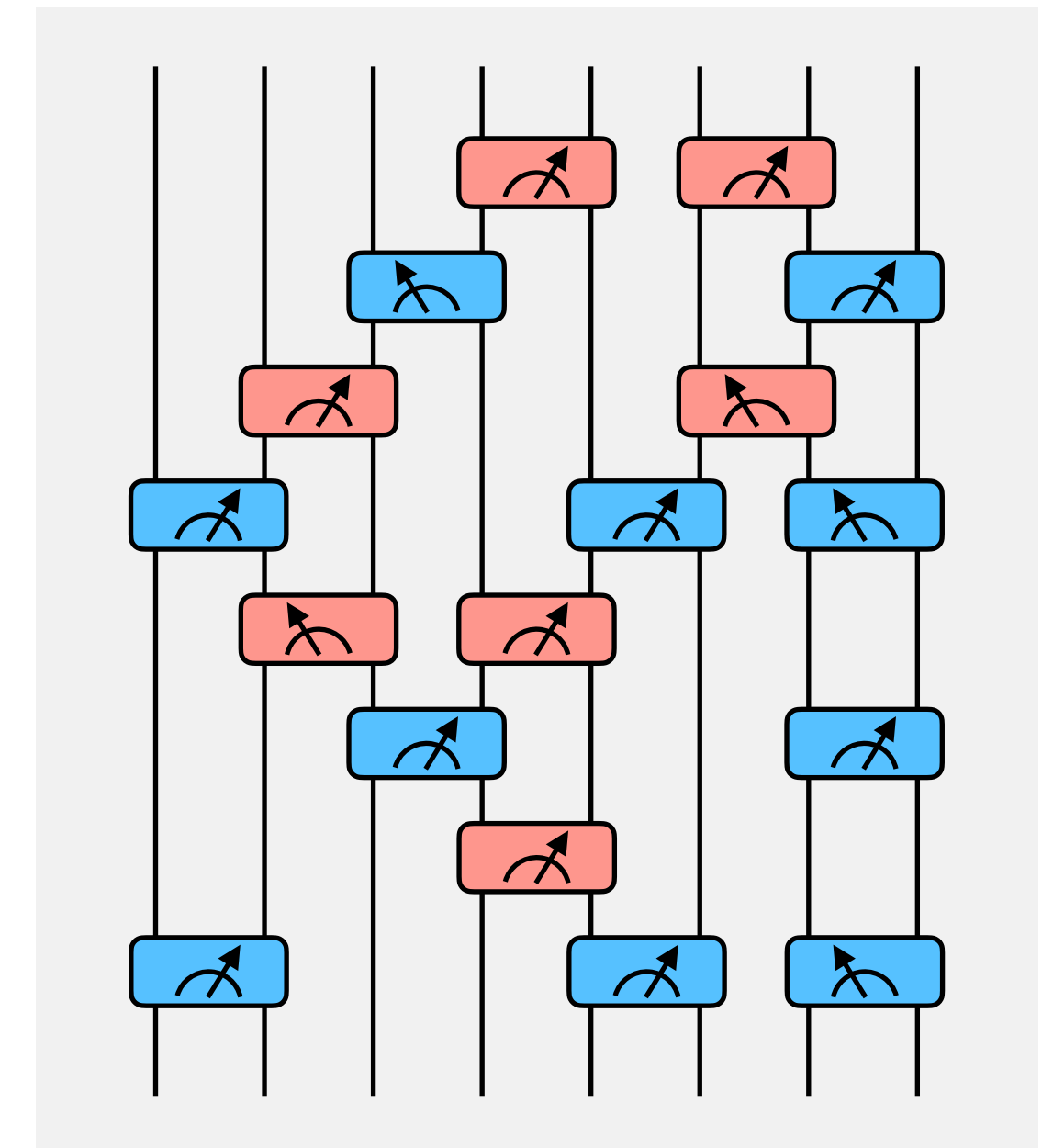
Floquet weak measurement

$$(e^{\mp\tau H_r} \dots e^{\mp\tau H_0}) |\psi_0\rangle$$



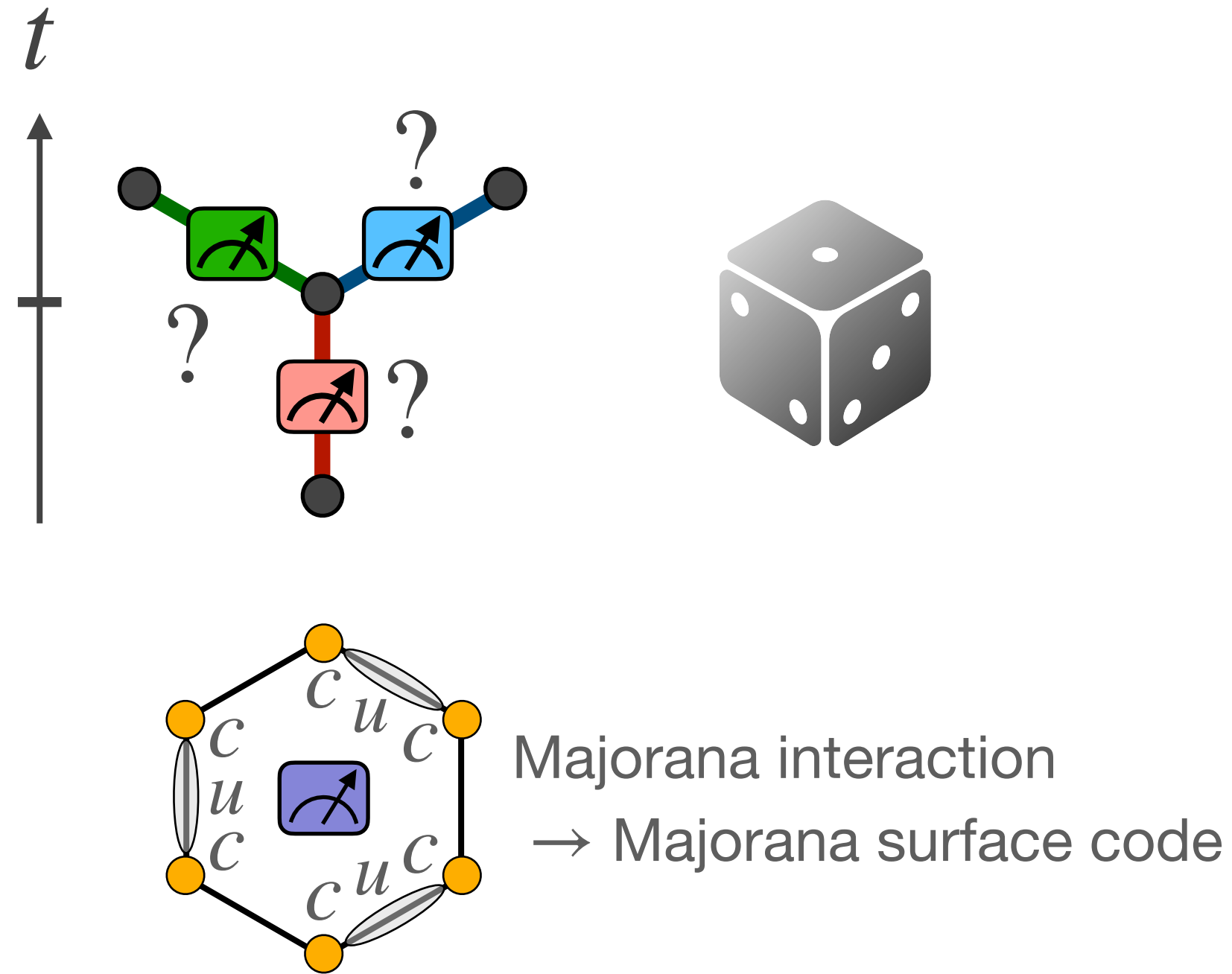
- brickwall circuit
- **Born** disorder
- $\tau \in [0, +\infty)$

random weak/strong measurement



- **stochastic** circuit
- **Born** disorder
- $\tau \in [0, +\infty)$

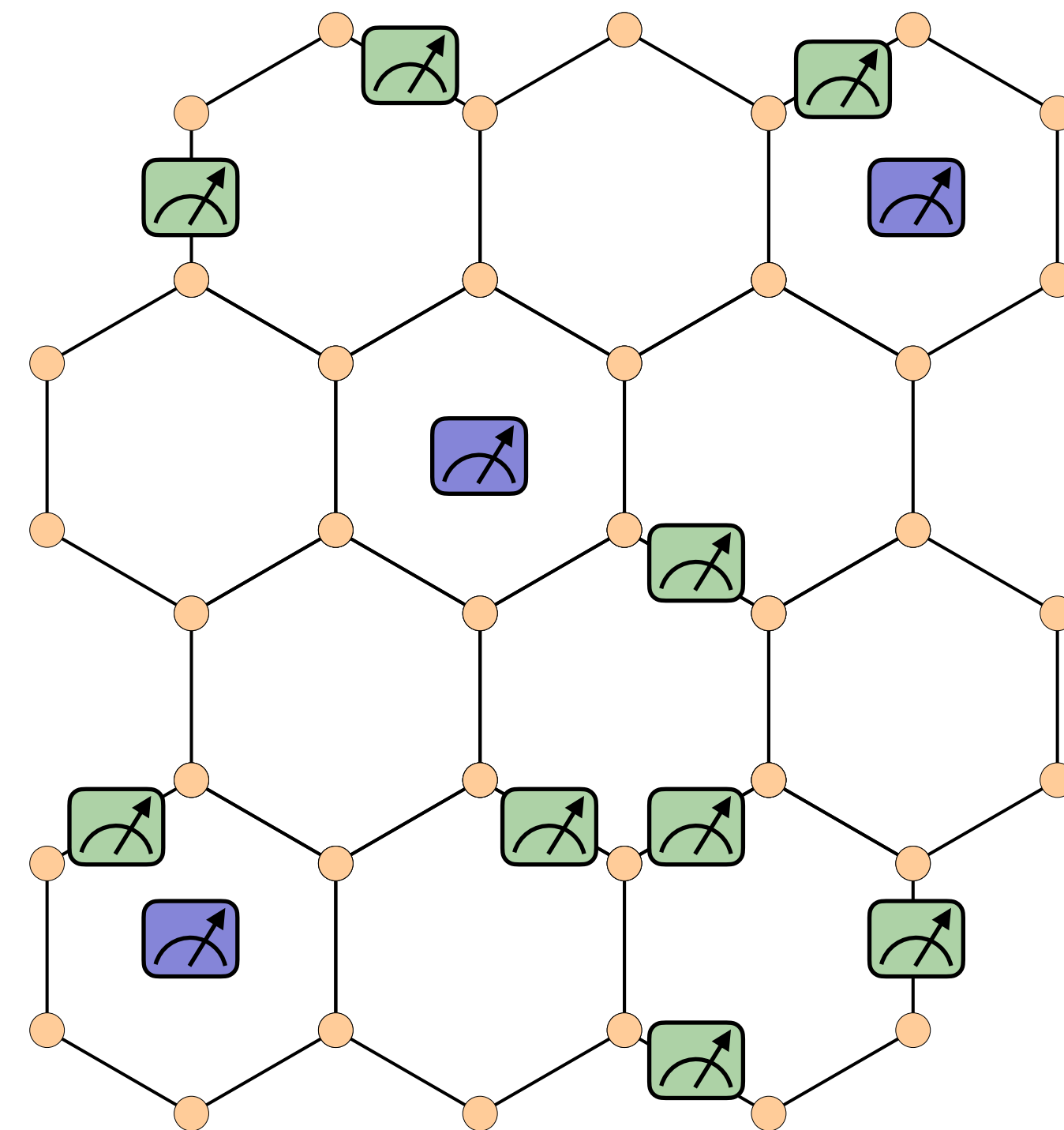
random projective Kitaev measurements



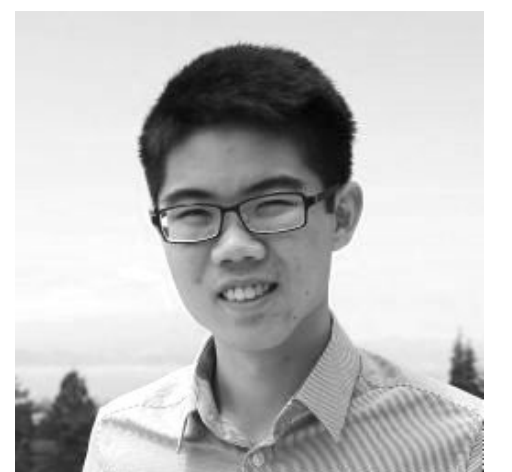
Clifford circuit

even **interacting** problem can be simulated in polynomial time (in Heisenberg picture)

a snapshot: randomly chosen measurements

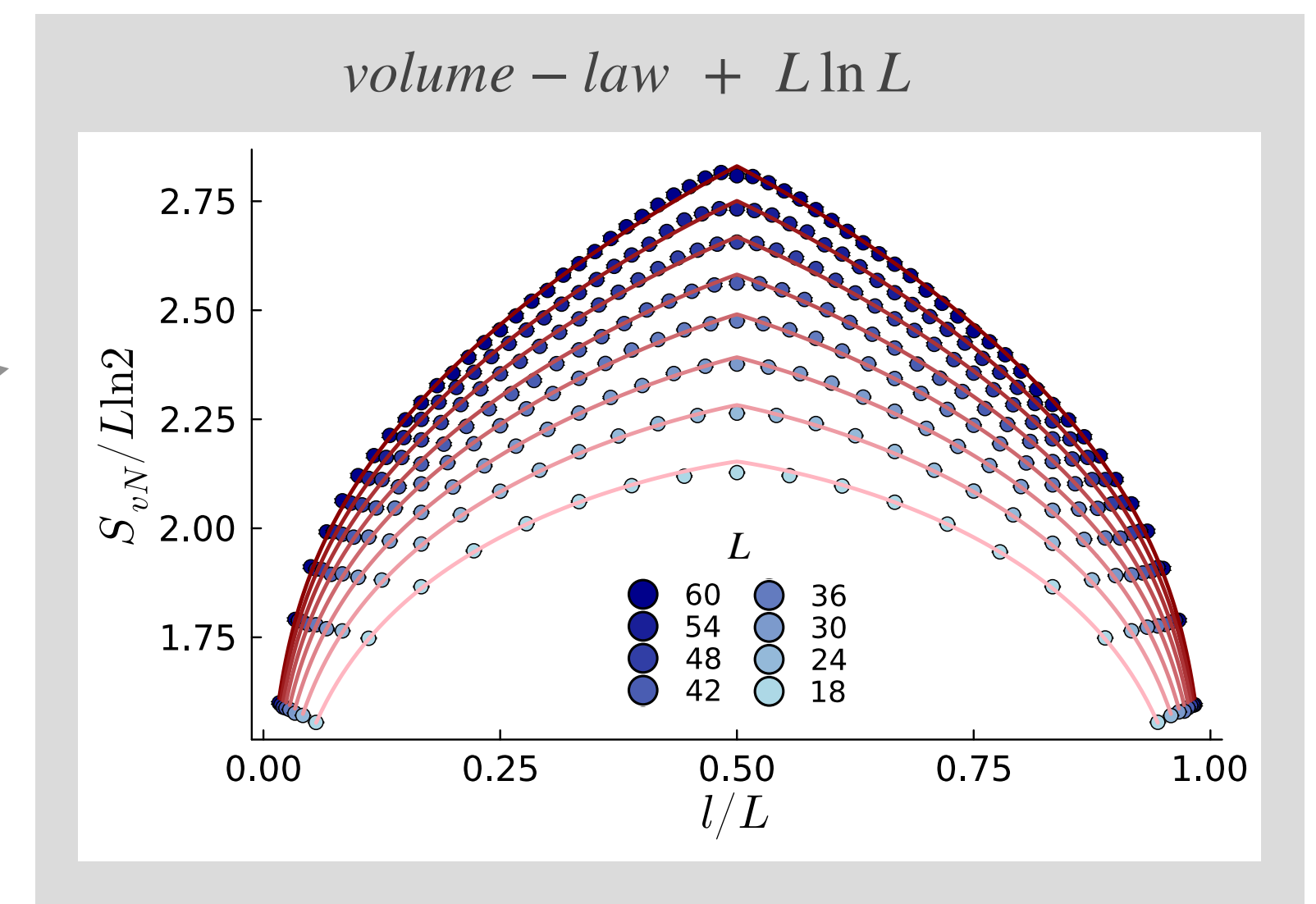
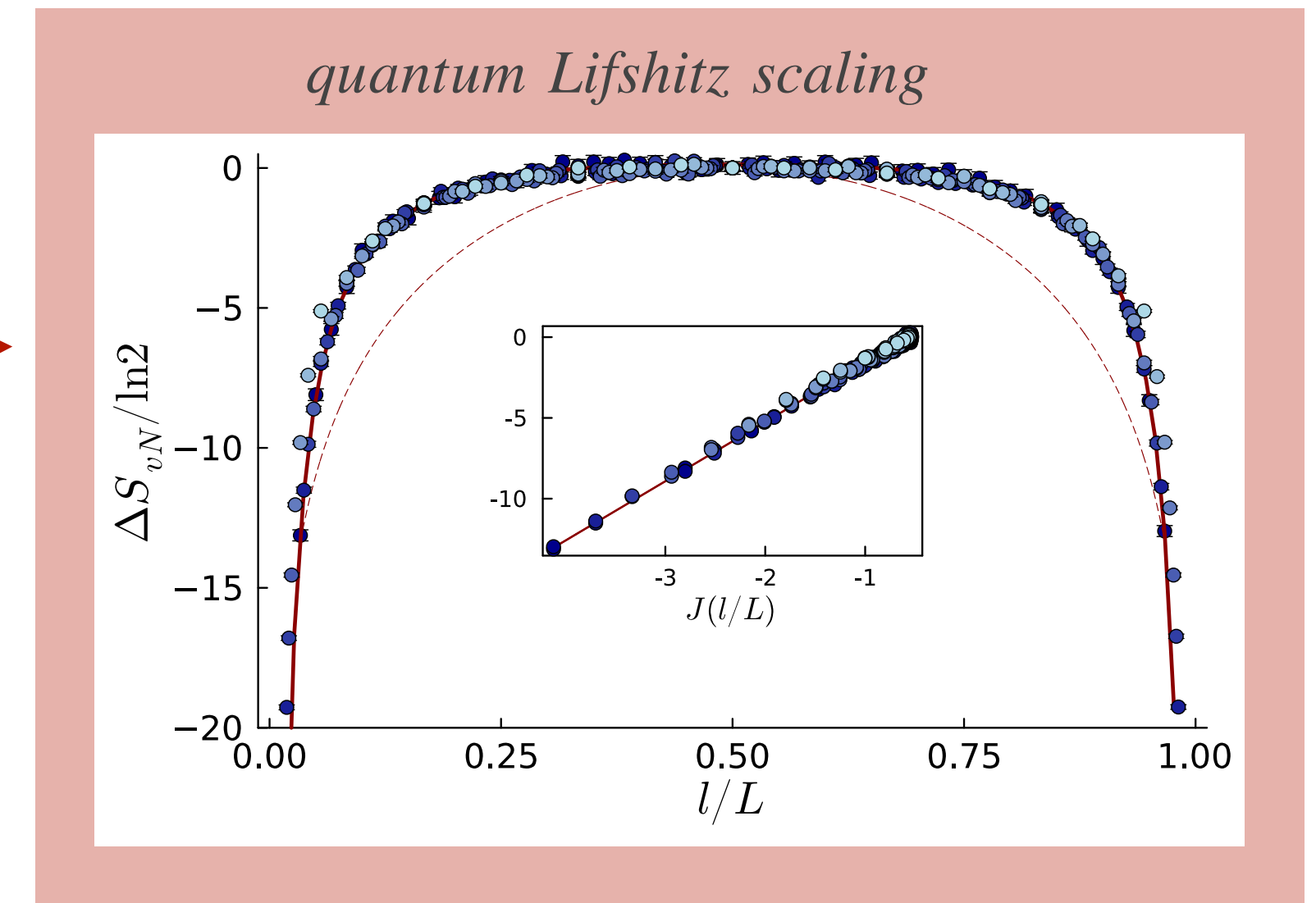
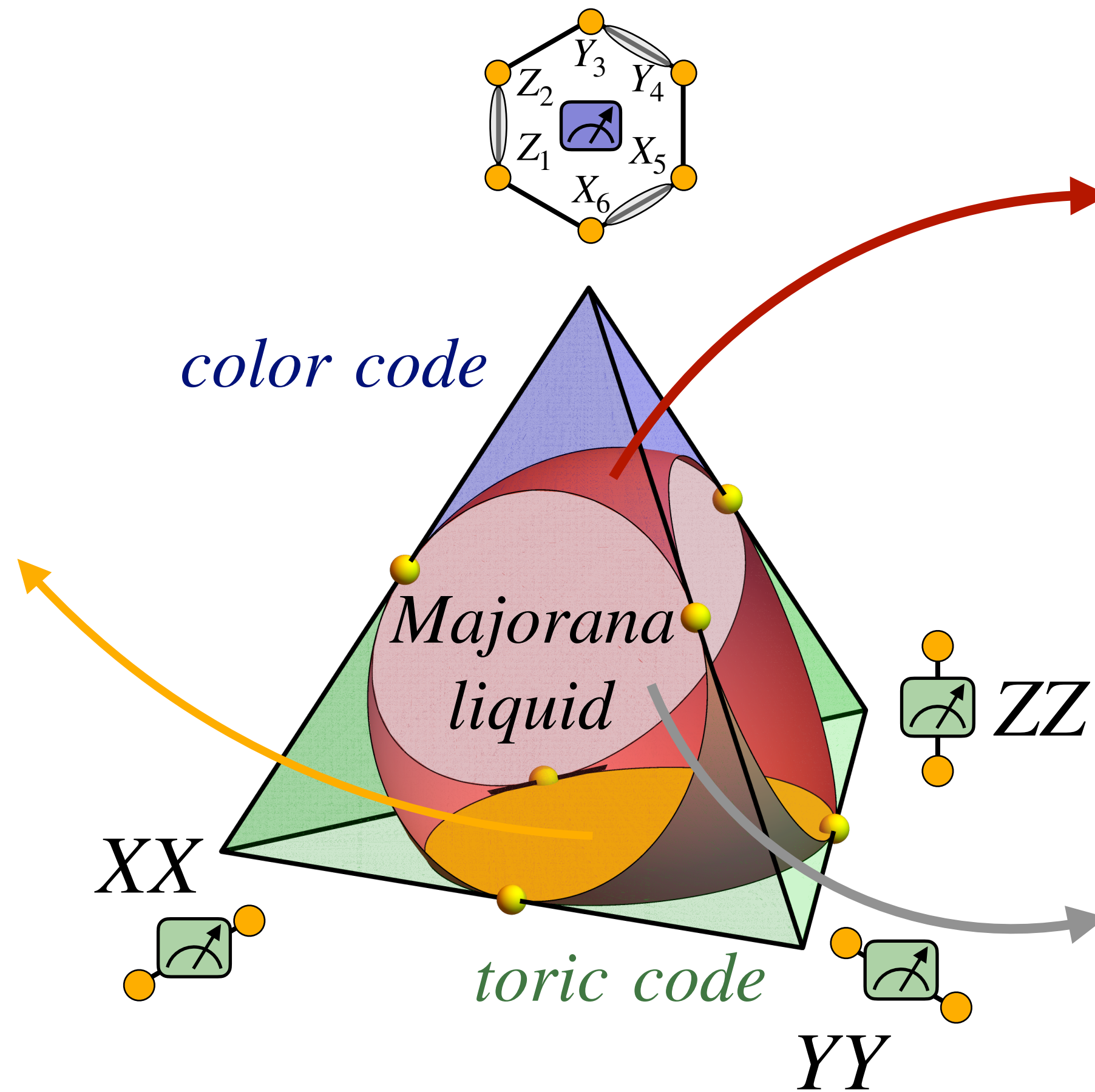
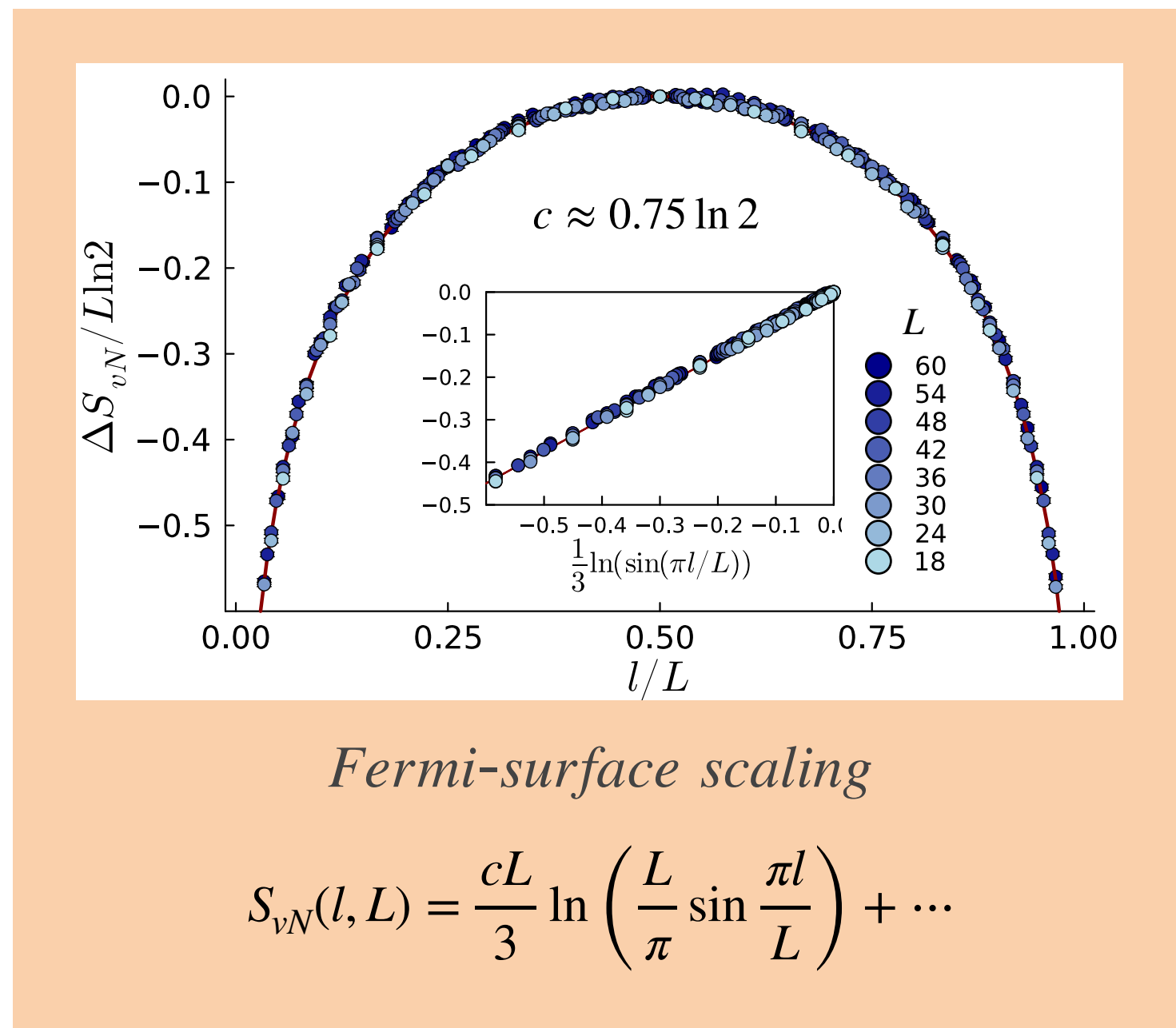


Guo-Yi Zhu



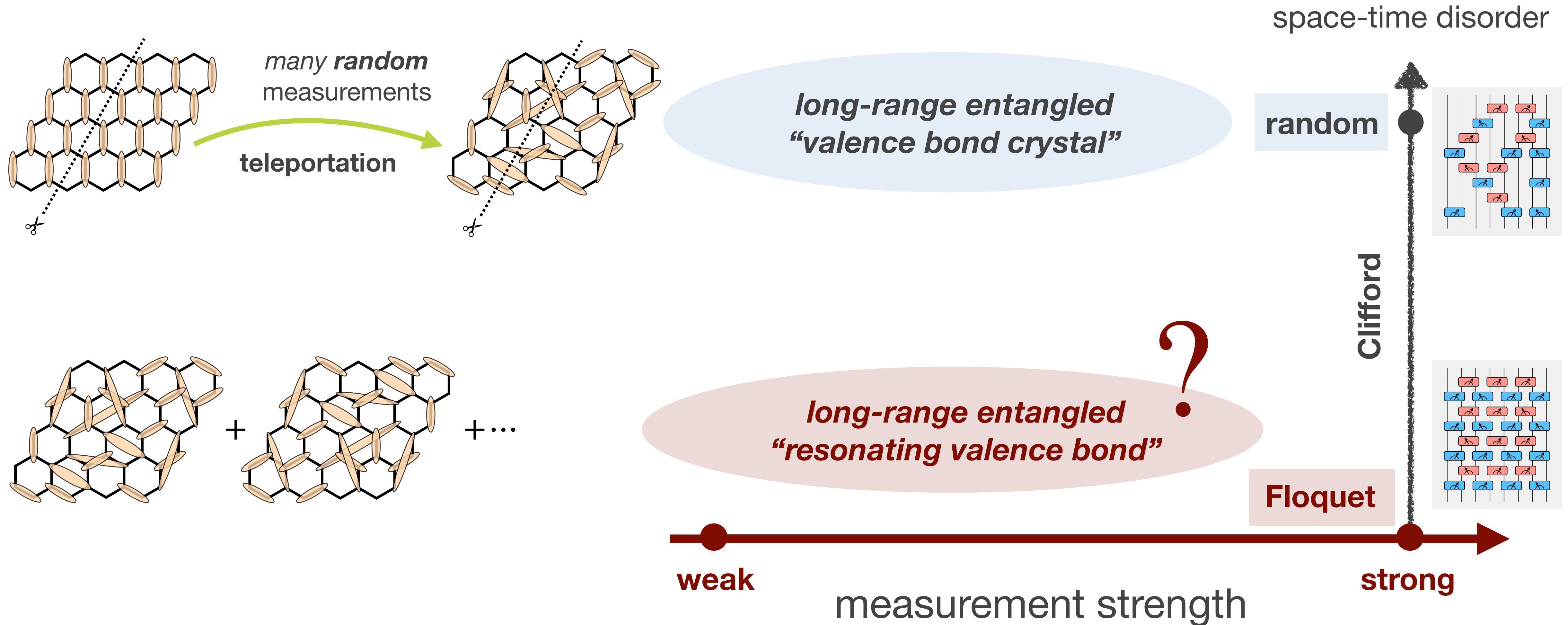
Nathanan
Tantivasadakarn

entanglement phase diagram



Nahum, Skinner 2020; Lavasani, Luo, Vijay 2023; Sriram, Rakovszky, Khemani, Ippoliti 2023
 Zhu, Tantivasadakarn, ST 2023: + Majorana interaction

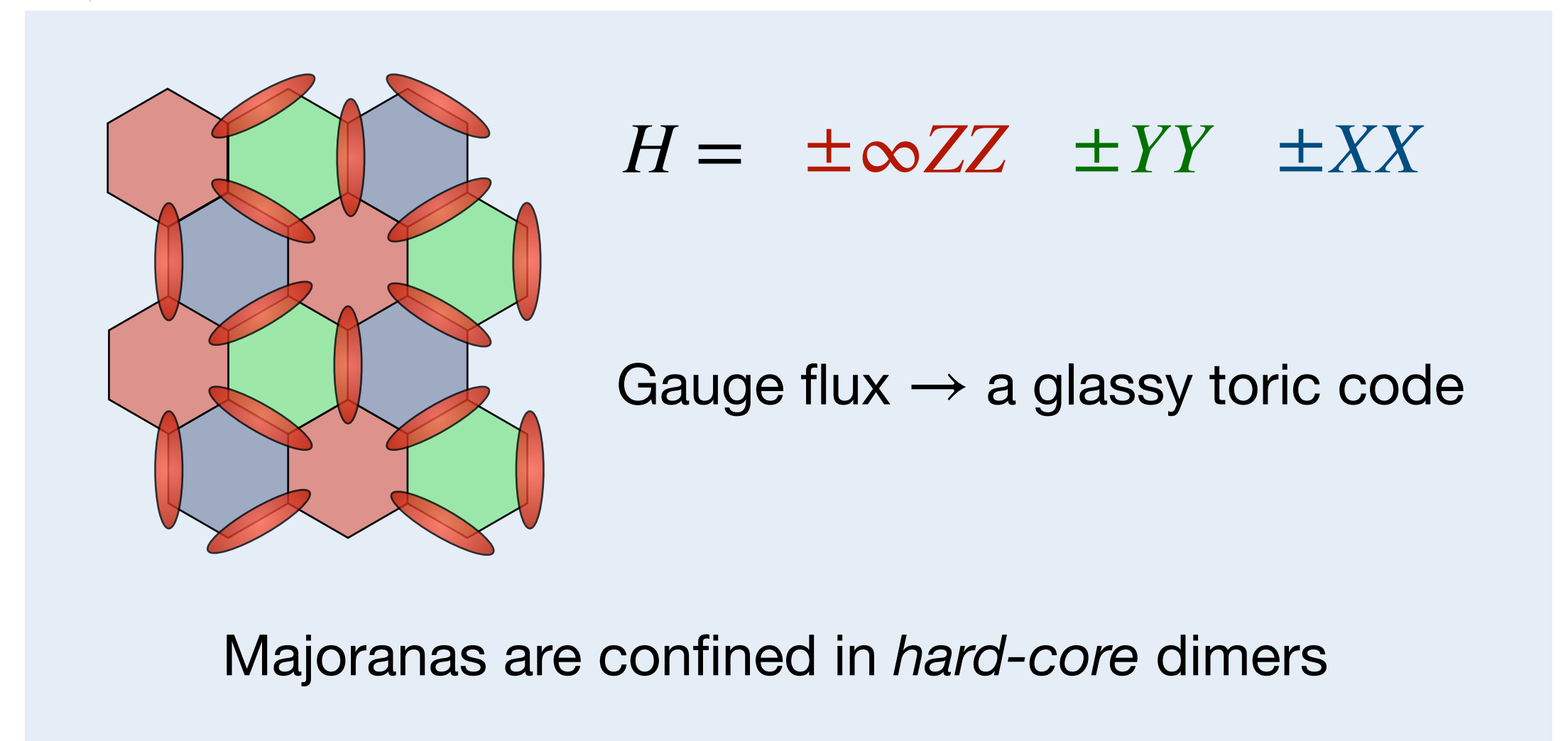
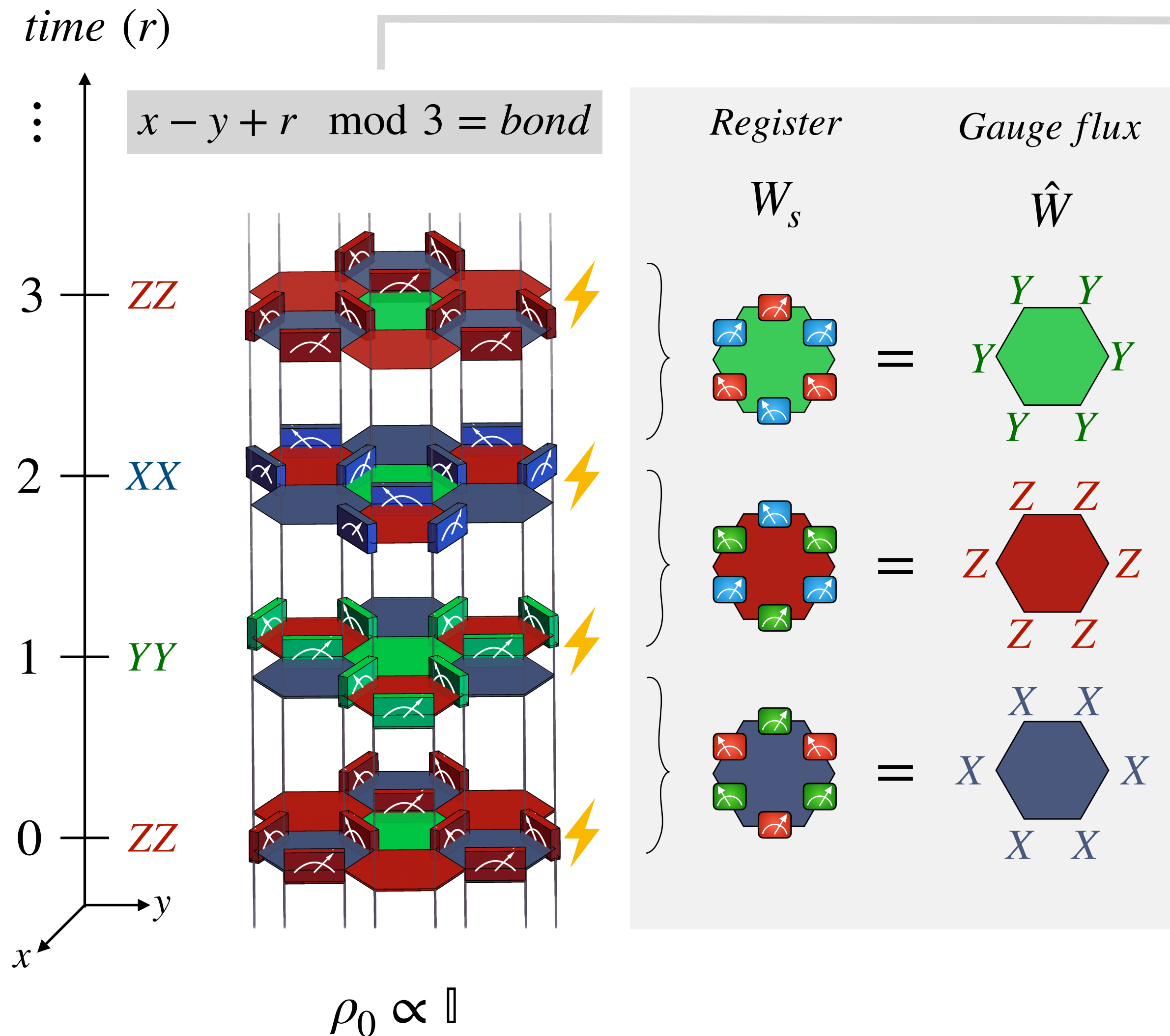
measurement, teleportation, and beyond



dynamical protocol

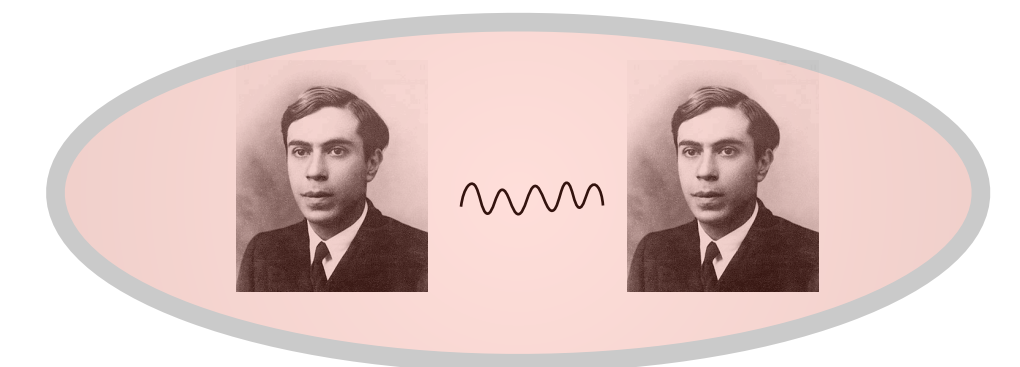


Hastings, Haah (2021)



Questions:

- How to liberate Majorana?
- Stability of the code?



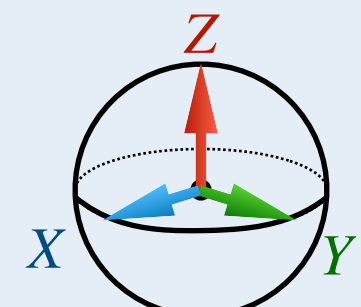
coherent error / weak measurement \rightarrow

soften dimers – a channel for Majorana to escape !

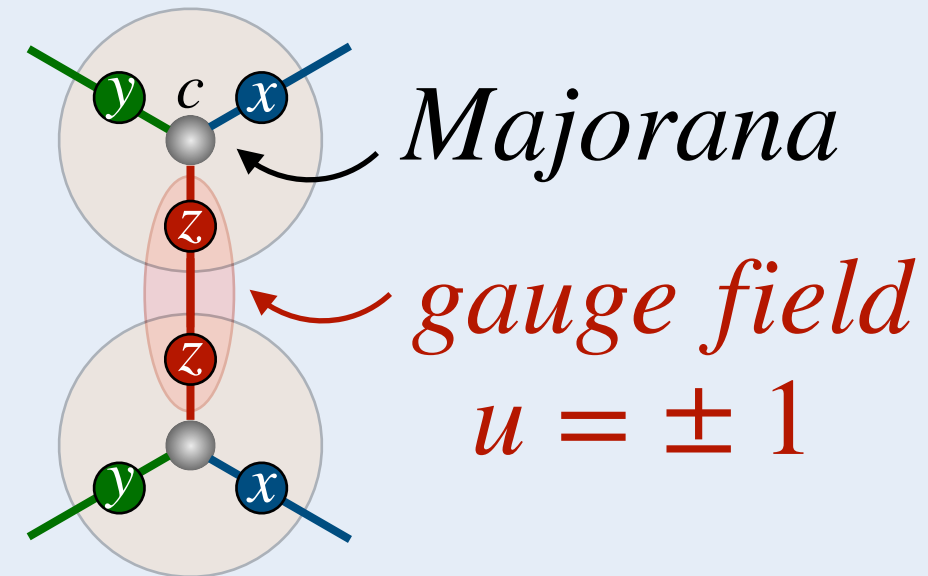
Majorana, flux pillars, loops



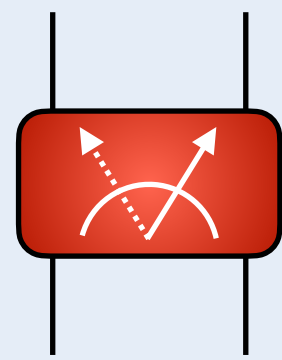
Guo-Yi Zhu



$$\begin{aligned} Z &= ib^z c \\ X &= ib^x c \\ Y &= ib^y c \end{aligned}$$



$$\exp(-\tau s Z Z) \quad (s = \pm 1)$$

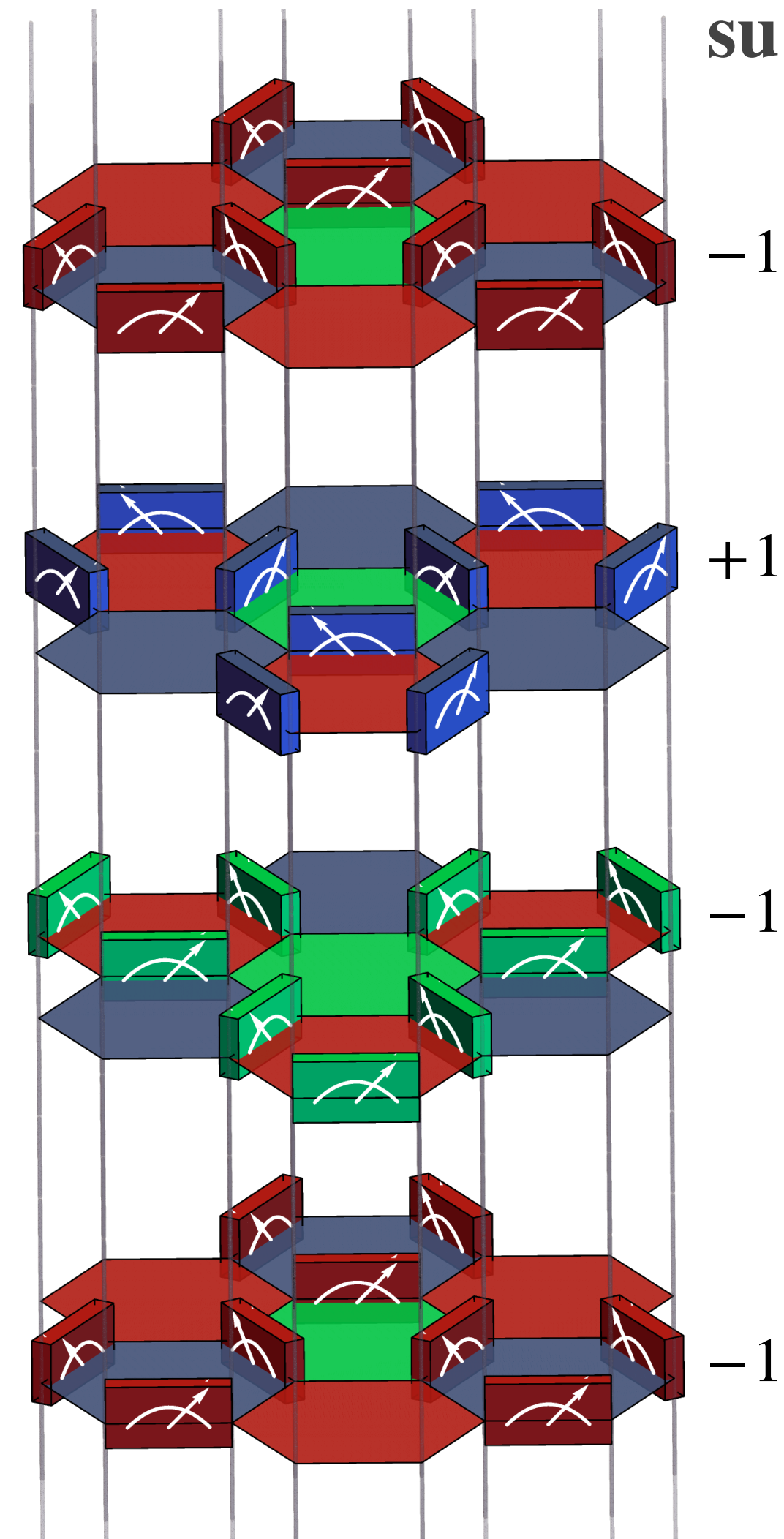


$$\exp(-\tau \quad (su) \quad ic_A c_B)$$

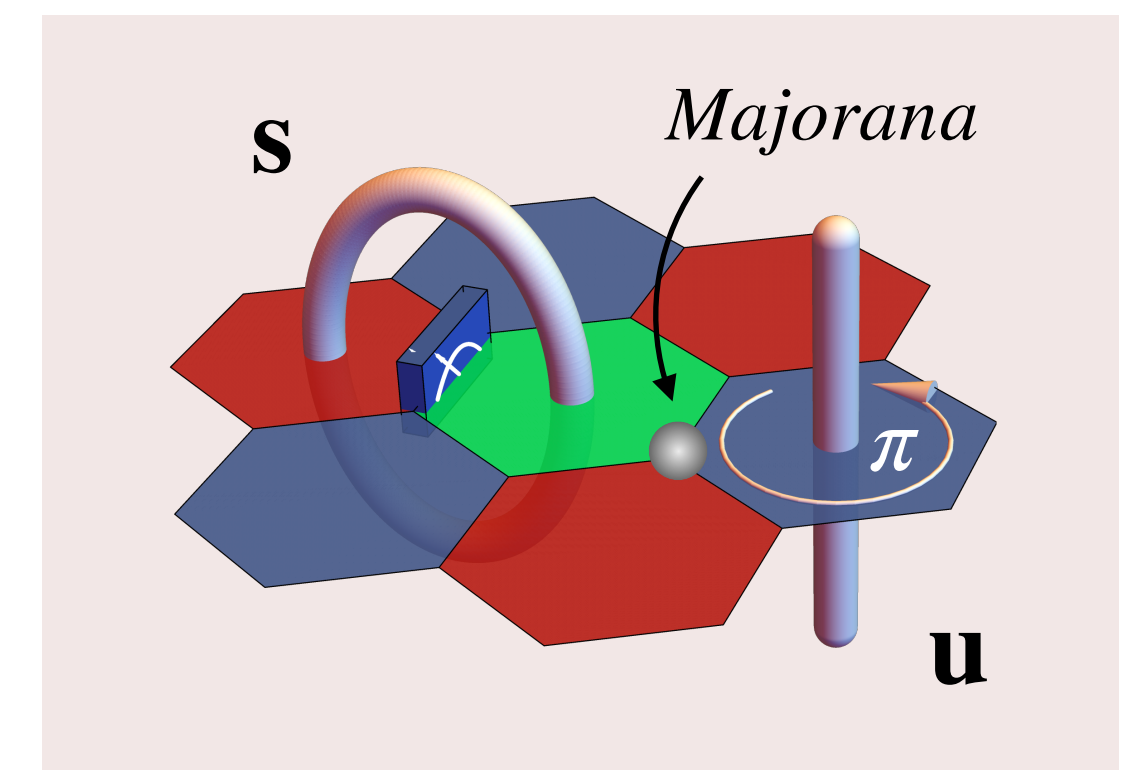
measurement strength
 $\tau = \tanh^{-1}(\sin(2t))$

net gauge field

Majorana bilinear



random Gaussian fermion circuit
conditioned on
gauge trajectory su

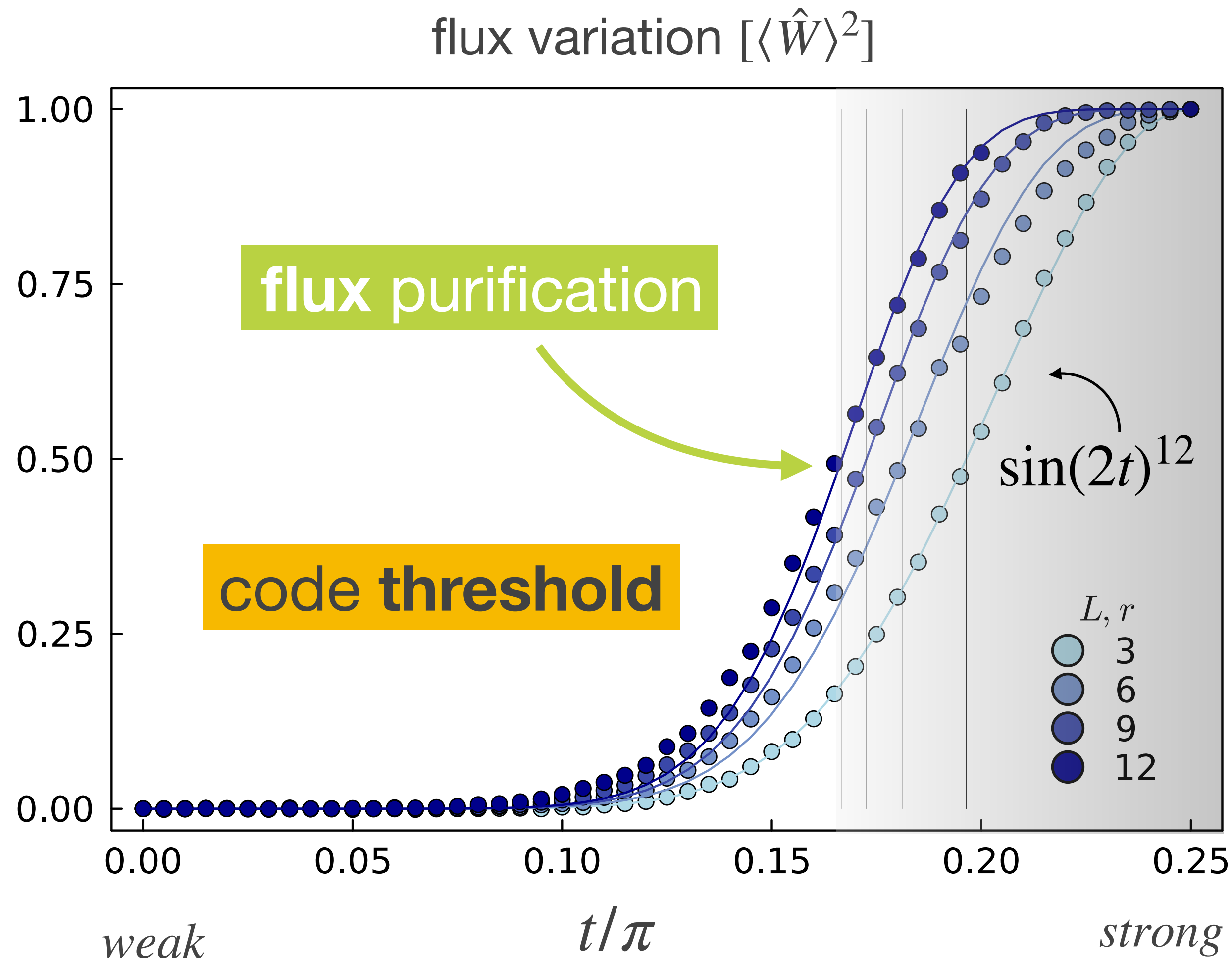


Born probability

=

Majorana partition function

purification of fluxes



- **flux** expectation value

$$[\langle \hat{W} \rangle] = 0$$

quantum average
 ┌
 └
 measurement average

- **Edwards-Anderson** order parameter

$$[\langle \hat{W} \rangle^2] = \sum_{\mathbf{s}} P(\mathbf{s}) \langle \hat{W} \rangle_{\mathbf{s}}^2 = \sum_q \sum_{\mathbf{s}, \mathbf{u}} \frac{P_{\mathbf{s}} \cdot P_{\mathbf{s}\mathbf{u}}}{P(\mathbf{s})} \left(\prod_{l \in q} u_l \right)$$

- **exponential** purification

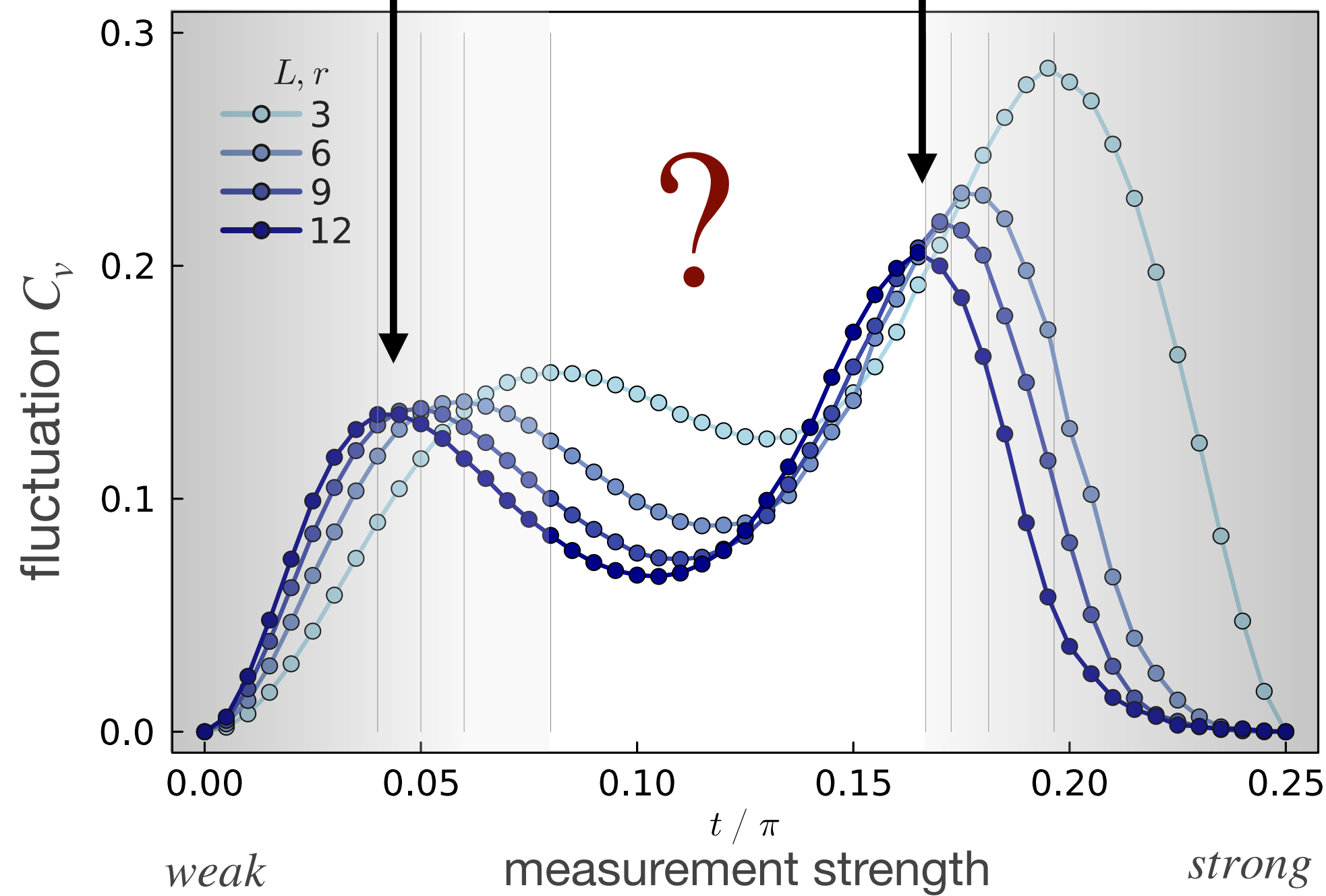
$$S_u := -\log_2 \frac{1 + [\langle \hat{W} \rangle^2]}{2} \approx \left(-\log_2 \frac{1 + \sin(2t)^{12}}{2} \right)^{\frac{r+1}{4}}$$

but there is more – double-peaks

weak measurement-only circuit

qubit fractionalization

flux purification



circuit depth

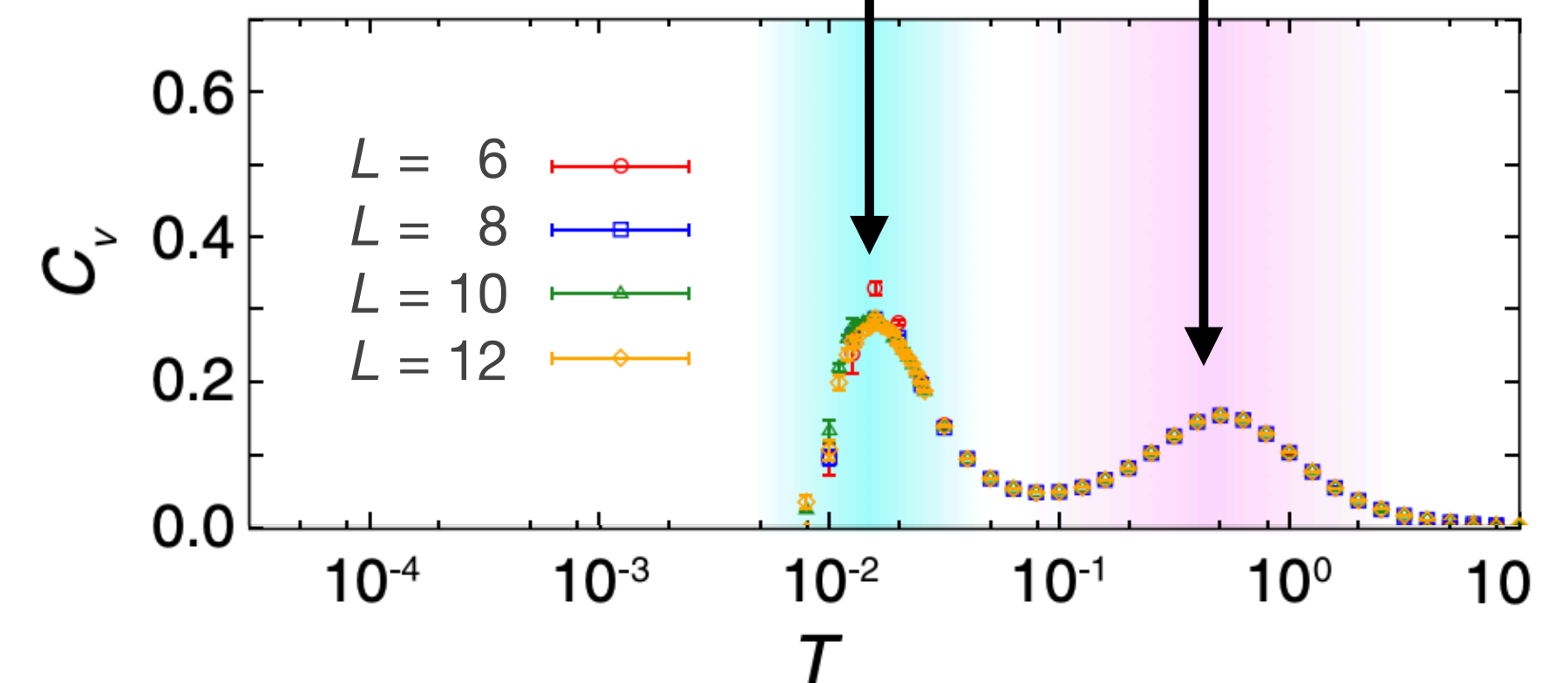
$$\rho_{\text{su}} \propto \exp\left(-\frac{\beta}{4} \mathbf{c} H_{\text{su}} \mathbf{c}\right)$$

effective Hamiltonian

Hamiltonian at finite temperature

flux purification

spin fractionalization



Nasu, Udagawa, Motome, 2014

PRL 113, 197205 (2014)

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7 NOVEMBER 2014

Vaporization of Kitaev Spin Liquids

Joji Nasu,¹ Masafumi Udagawa,² and Yukitoshi Motome²

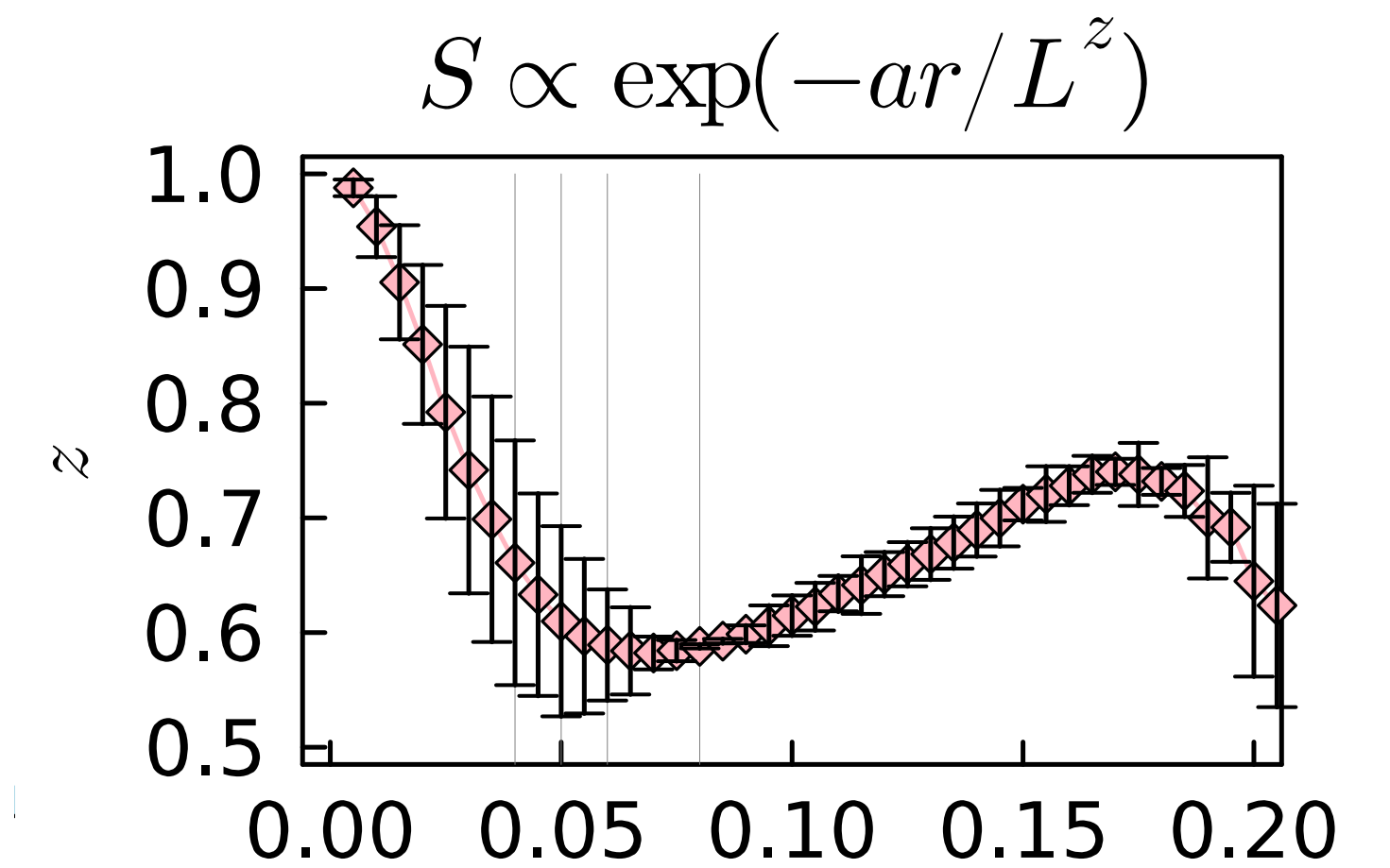
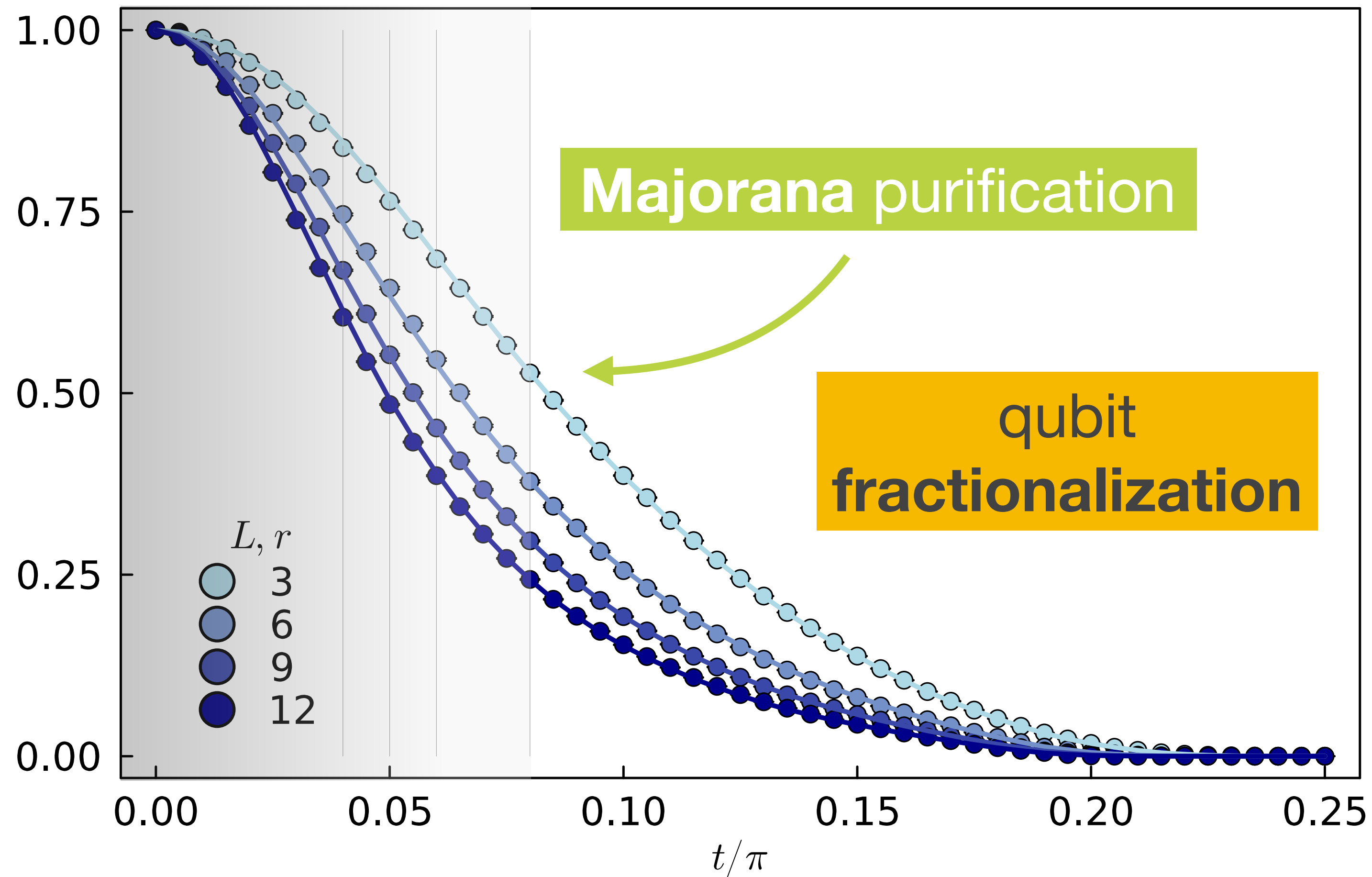
¹Department of Physics, Tokyo Institute of Technology, Ookayama, 2-12-1, Meguro, Tokyo 152-8551, Japan

²Department of Applied Physics, University of Tokyo, Hongo, 7-3-1, Bunkyo, Tokyo 113-8656, Japan
(Received 24 July 2014; revised manuscript received 9 October 2014; published 7 November 2014)

purification of Majoranas

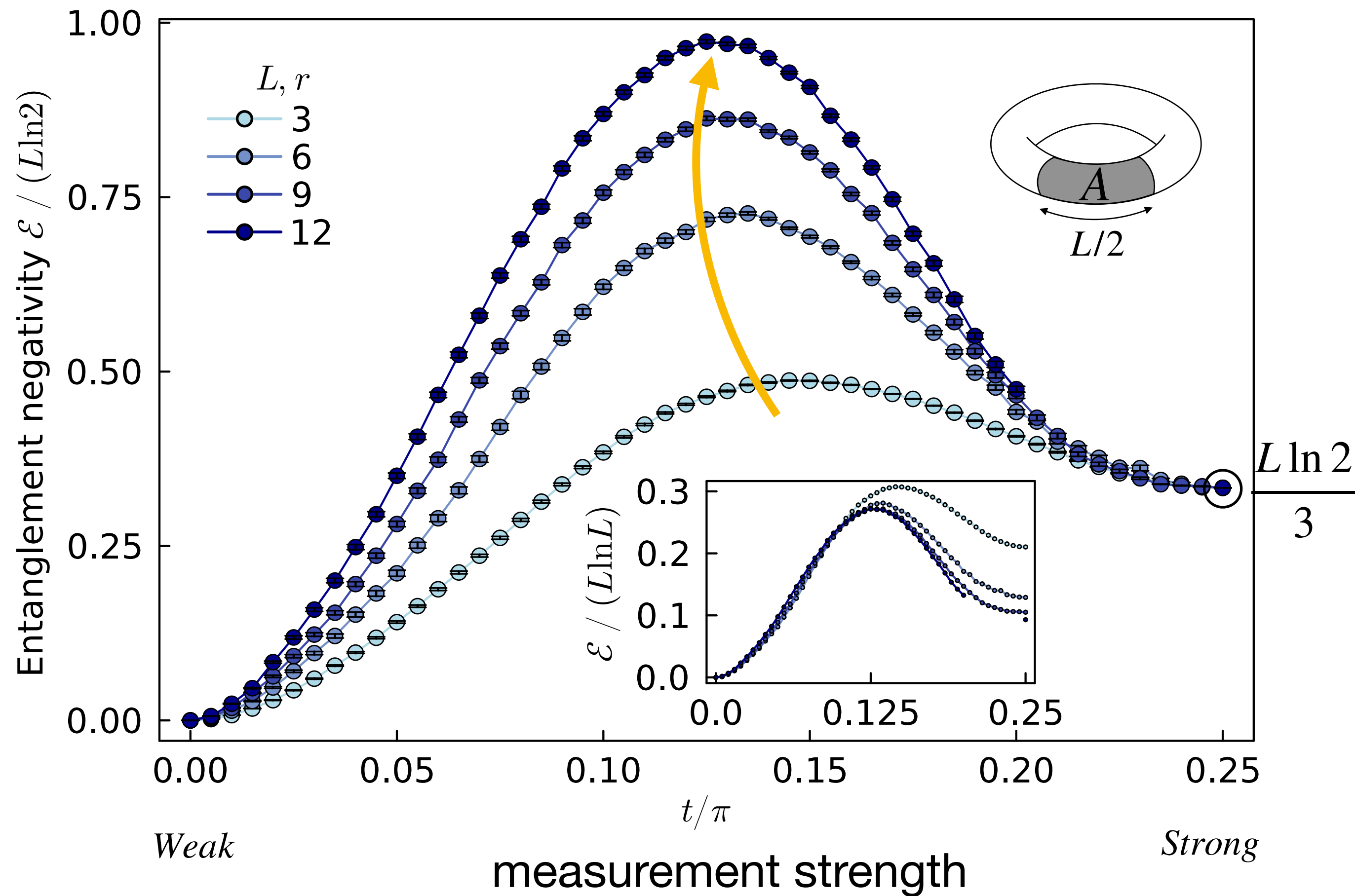
Majorana entropy density [$\ln 2$]

$$S = \beta(E - F)$$



dynamical
critical exponent

Majorana liquid



- fermionic **entanglement negativity**
 - definition: response under *partial* time reversal
 - property: **distill out thermal entropy**
 - diagnose: **mixed state entanglement**

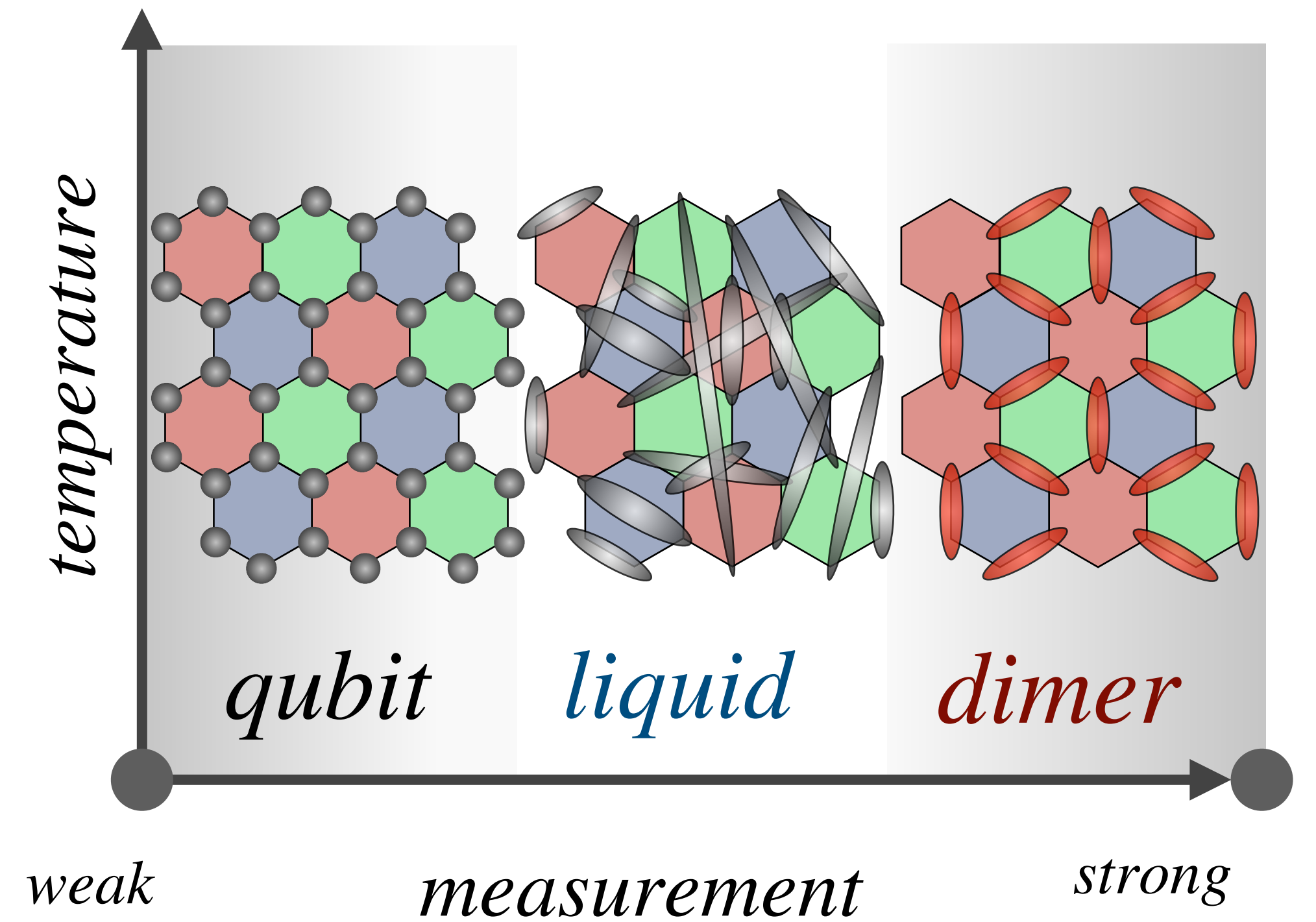
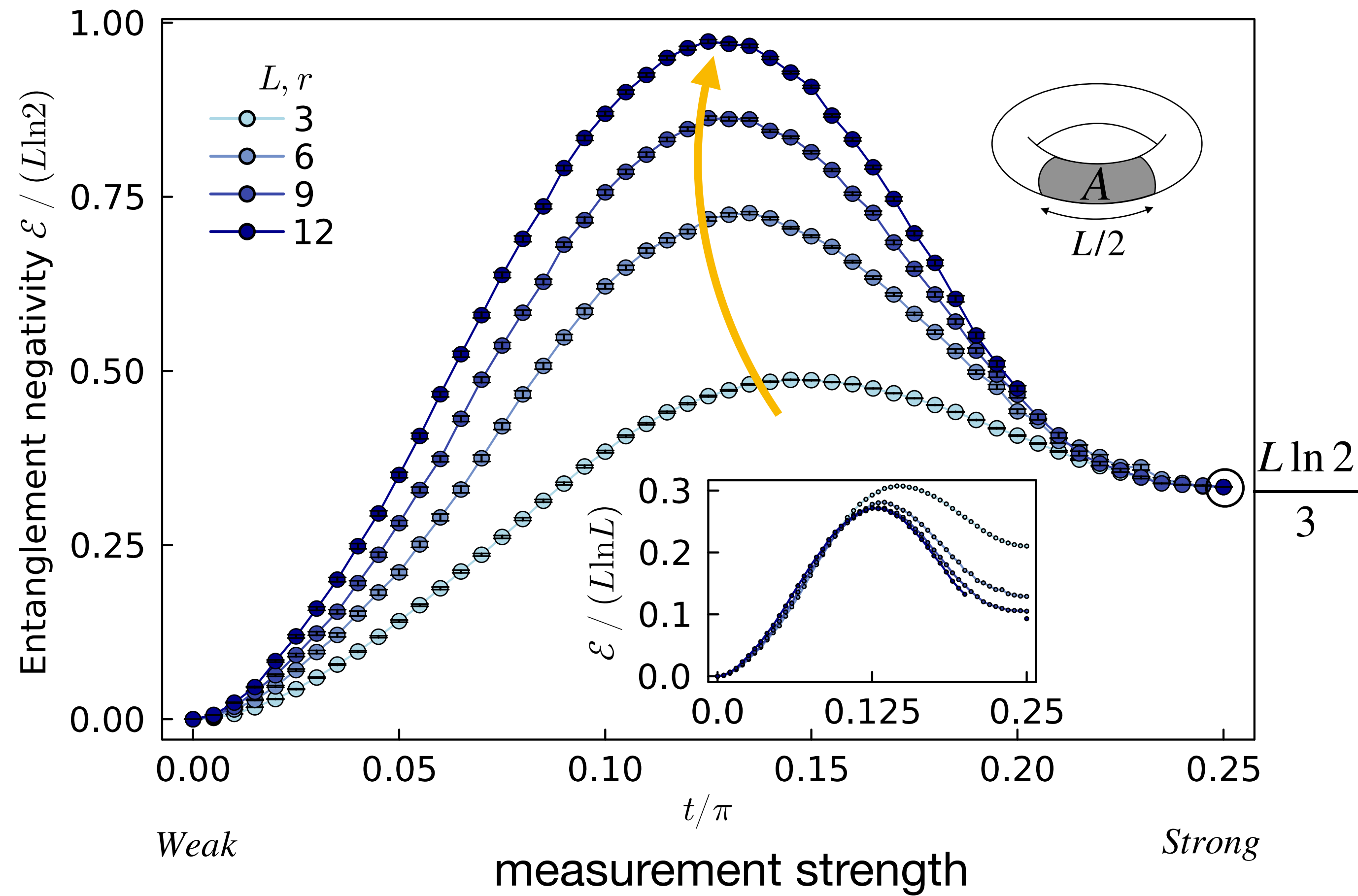
Shapourian, Shiozaki, Ryu, 2017

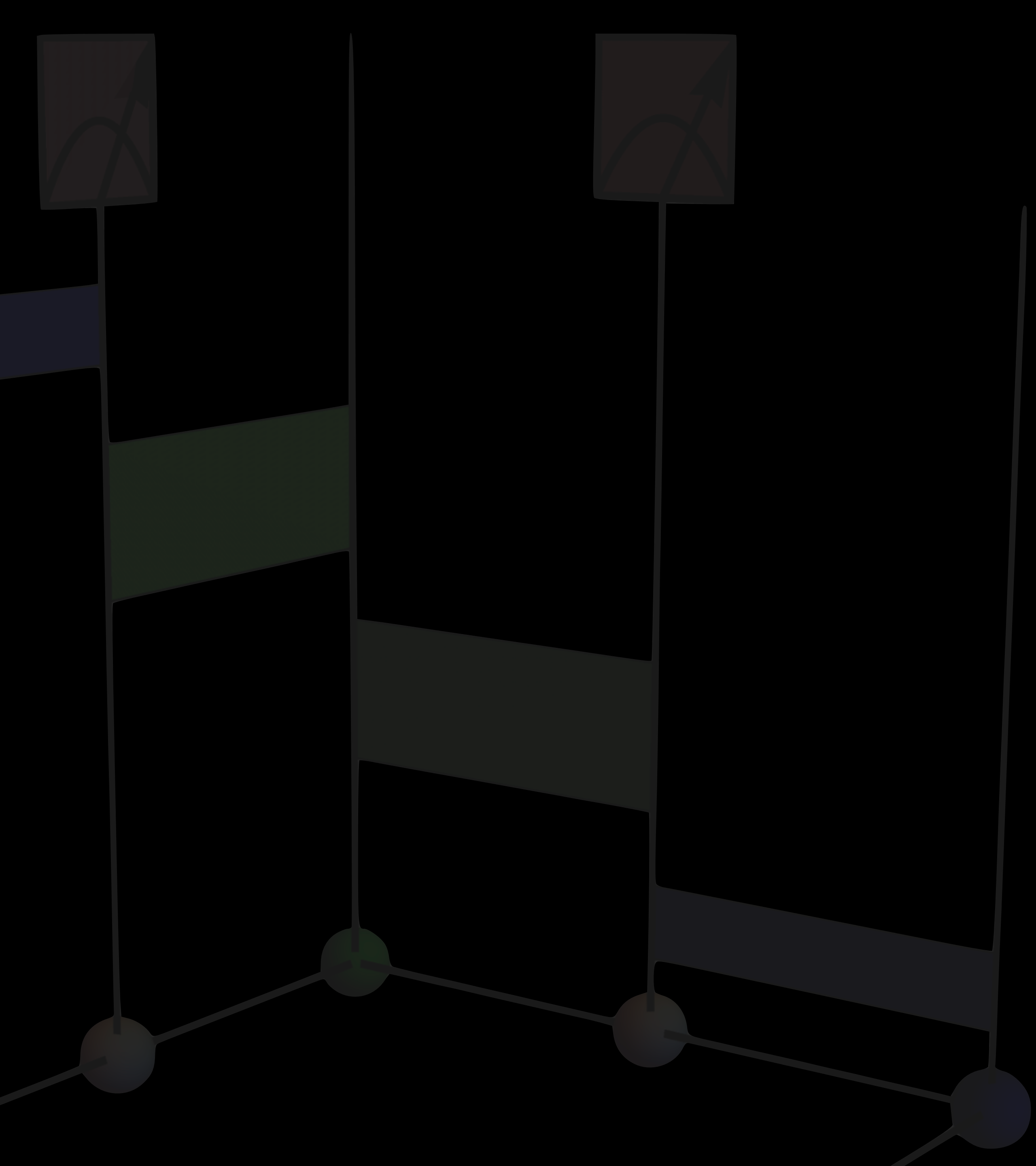
$$\mathcal{E} = \sum_{\text{su}} p_{\text{su}} \cdot \ln || \rho_{\text{su}}^{R_A} ||_1$$

- **entanglement phase transition**
stable fermion phase with $L \ln L$ entanglement

Fava, Piroli, Swann, Bernard, Nahum, NLM, 2023

Majorana liquid





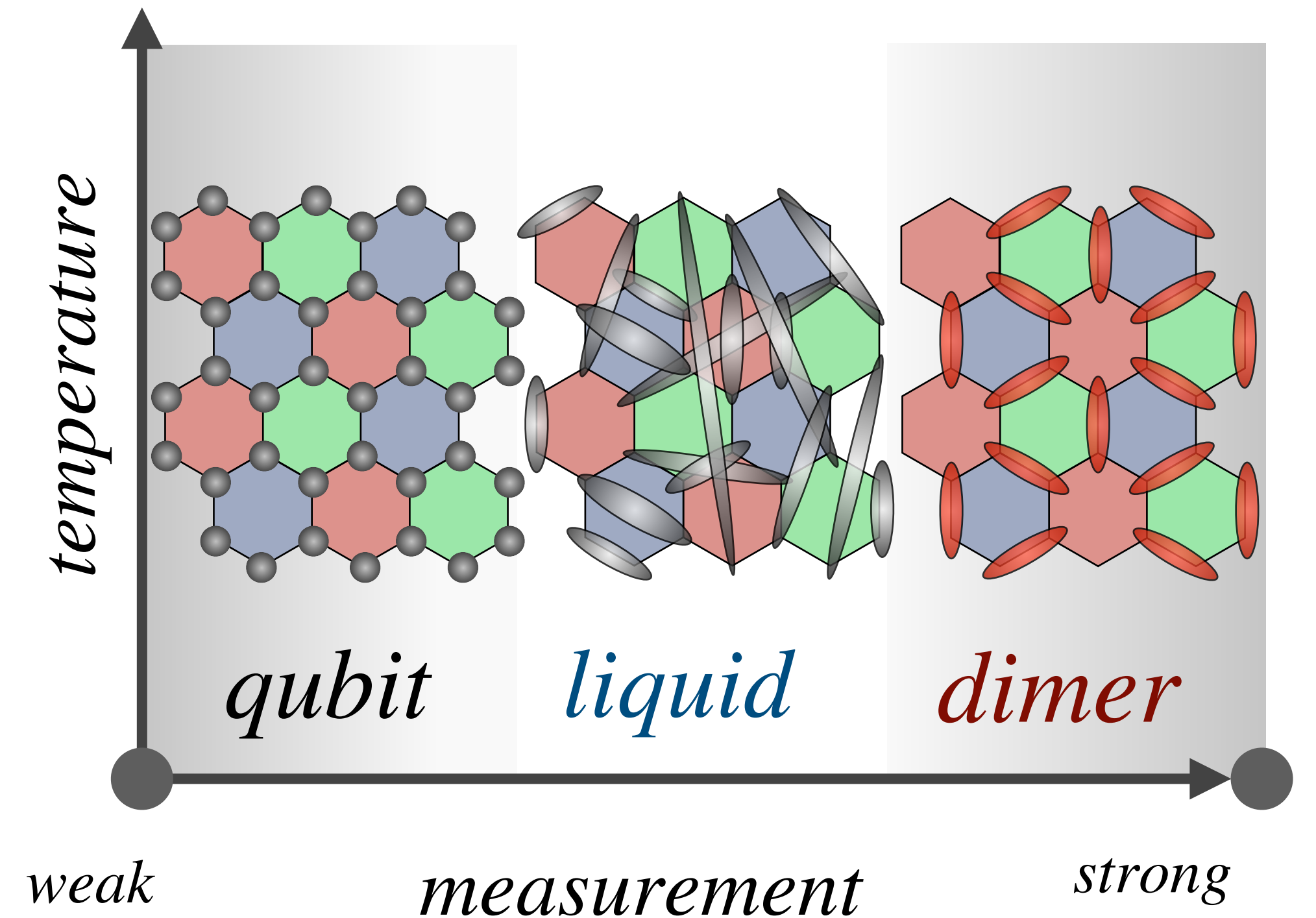
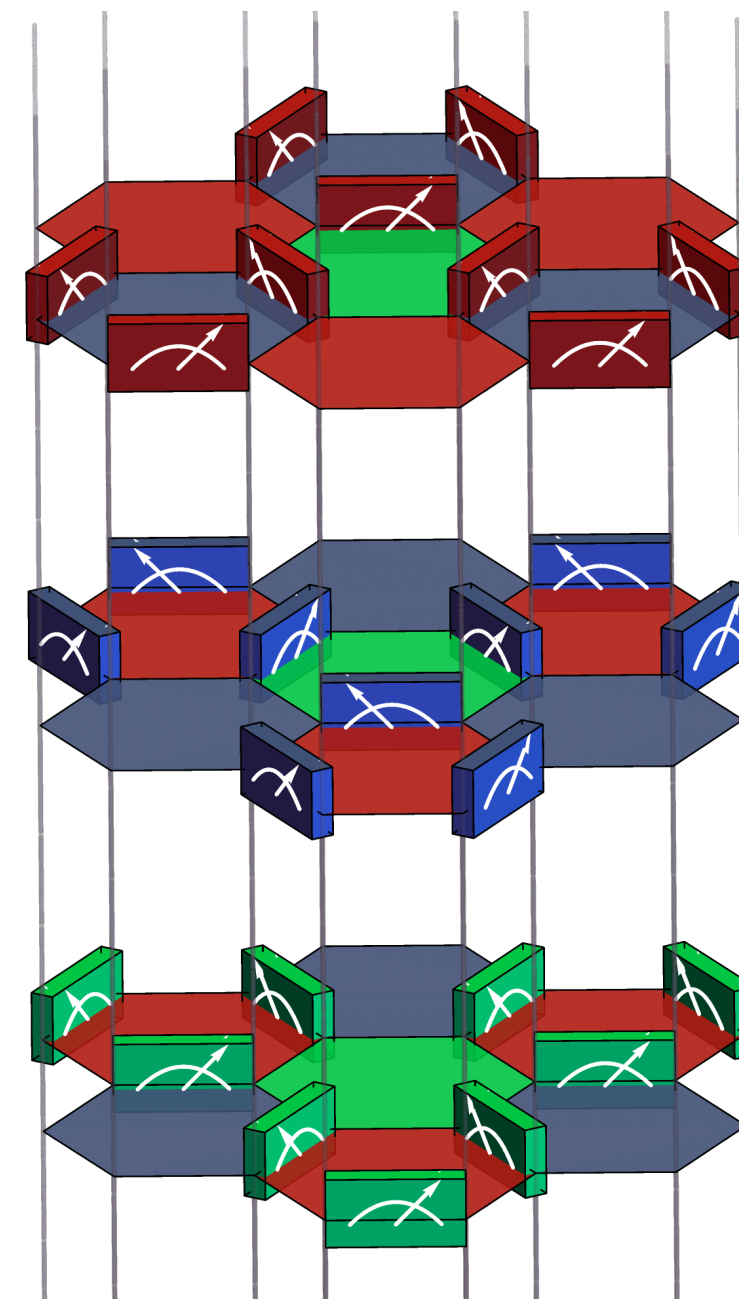
summary

Floquet code — conclusions



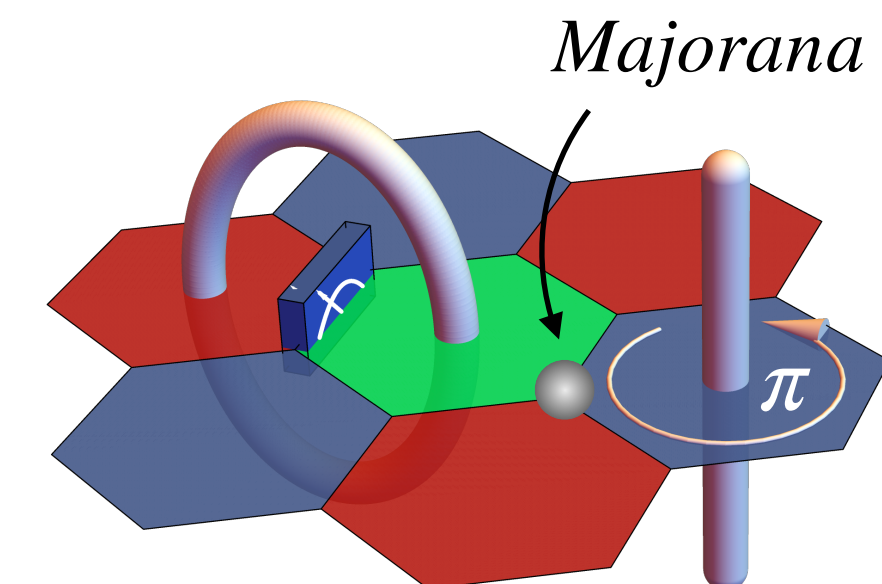
Guo-Yi Zhu

- **frustration** & **qubit fractionalization** by tunable weak measurement
- Floquet code **breakdown** to non-trivial state under coherent error
- **Majoranas** escape confinement and form **long-range entangled liquid**



Outlook

- **Feed-forward** deterministic preparation?
- topological phase transition from a parent **color code** (+ Majorana interaction)?



Guo-Yi Zhu & ST, arXiv: 2311.08450