Robust quantum many-body teleportation

teleportation | weak measurements | wave function deformations

CRC183 ENTANGLED STATES OF MATTER

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University of Cologne

ML4Q MATTER AND LIGHT FOR QUANTUM COMPUTING

New Frontiers in Quantum Matter

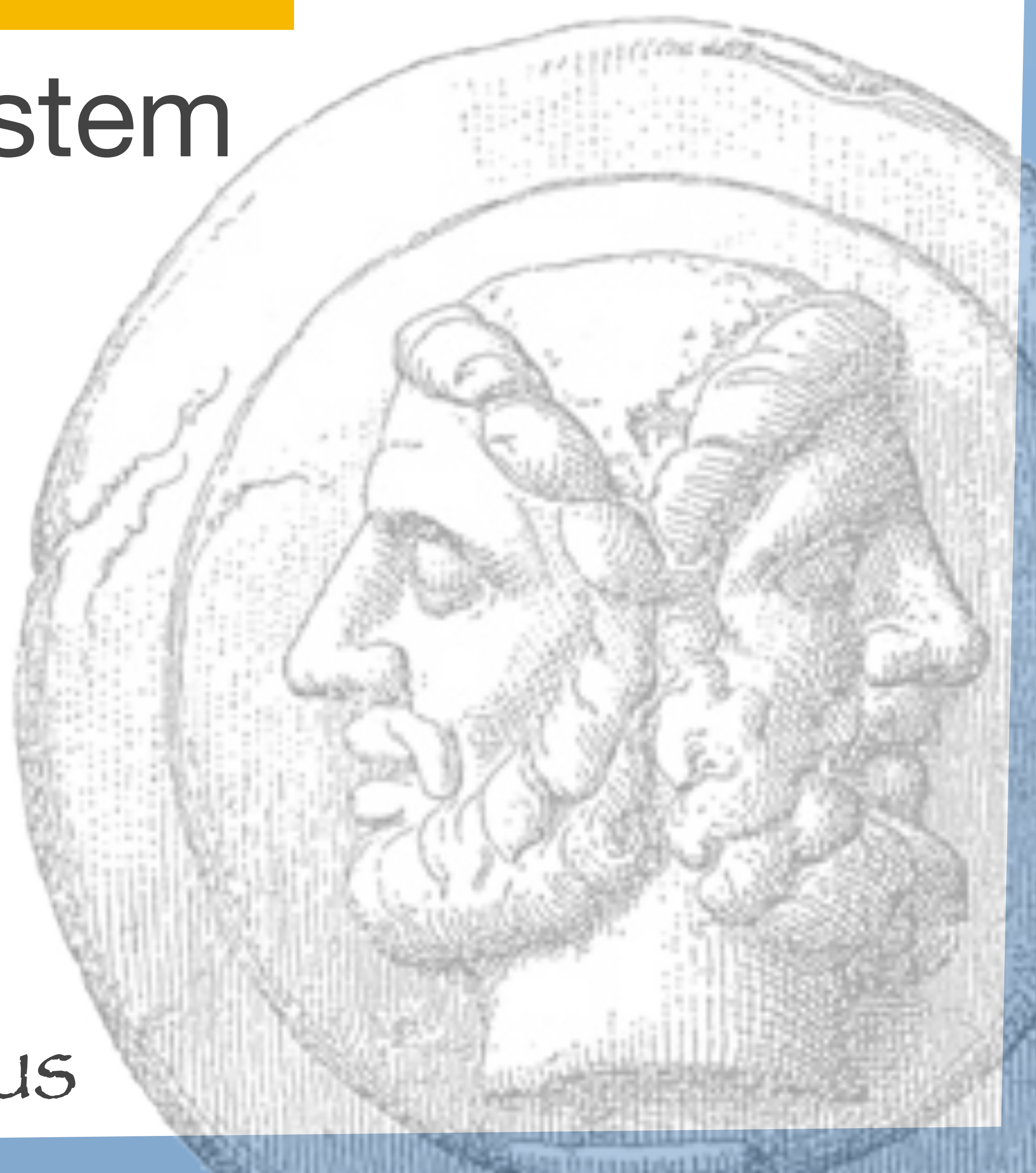
Weizmann Institute of Science, March 2024

quantum measurements

“About your cat, Mr. Schrödinger — I have good news and bad news.”

Quantum measurements can

- **extract information** from a system
- **shape entanglement** of a quantum system



double-faced Janus

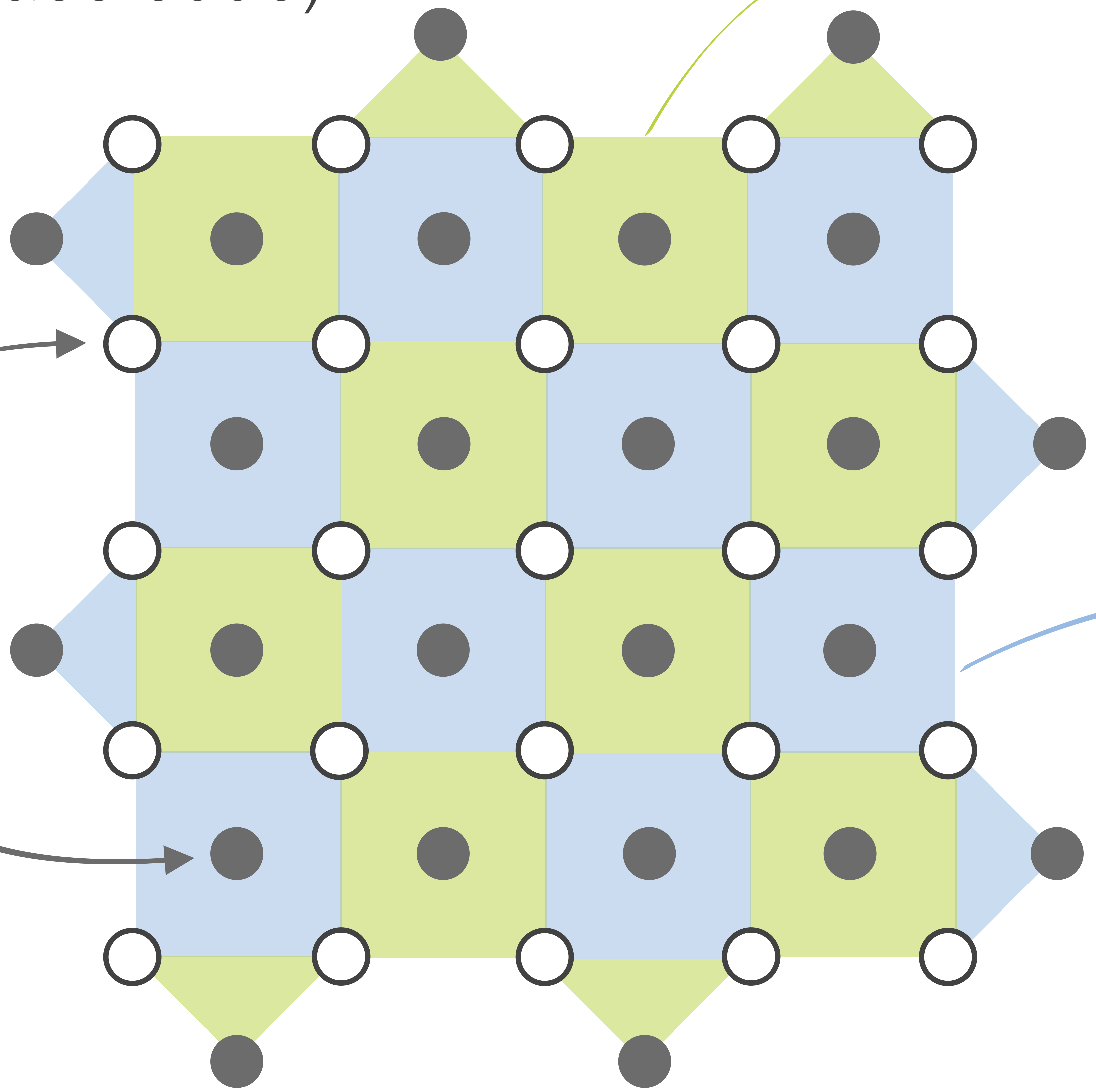
paradigmatic example: **surface code**

Kitaev (1997)

open boundaries
(rotated surface code)

computation
qubit

auxiliary
qubit

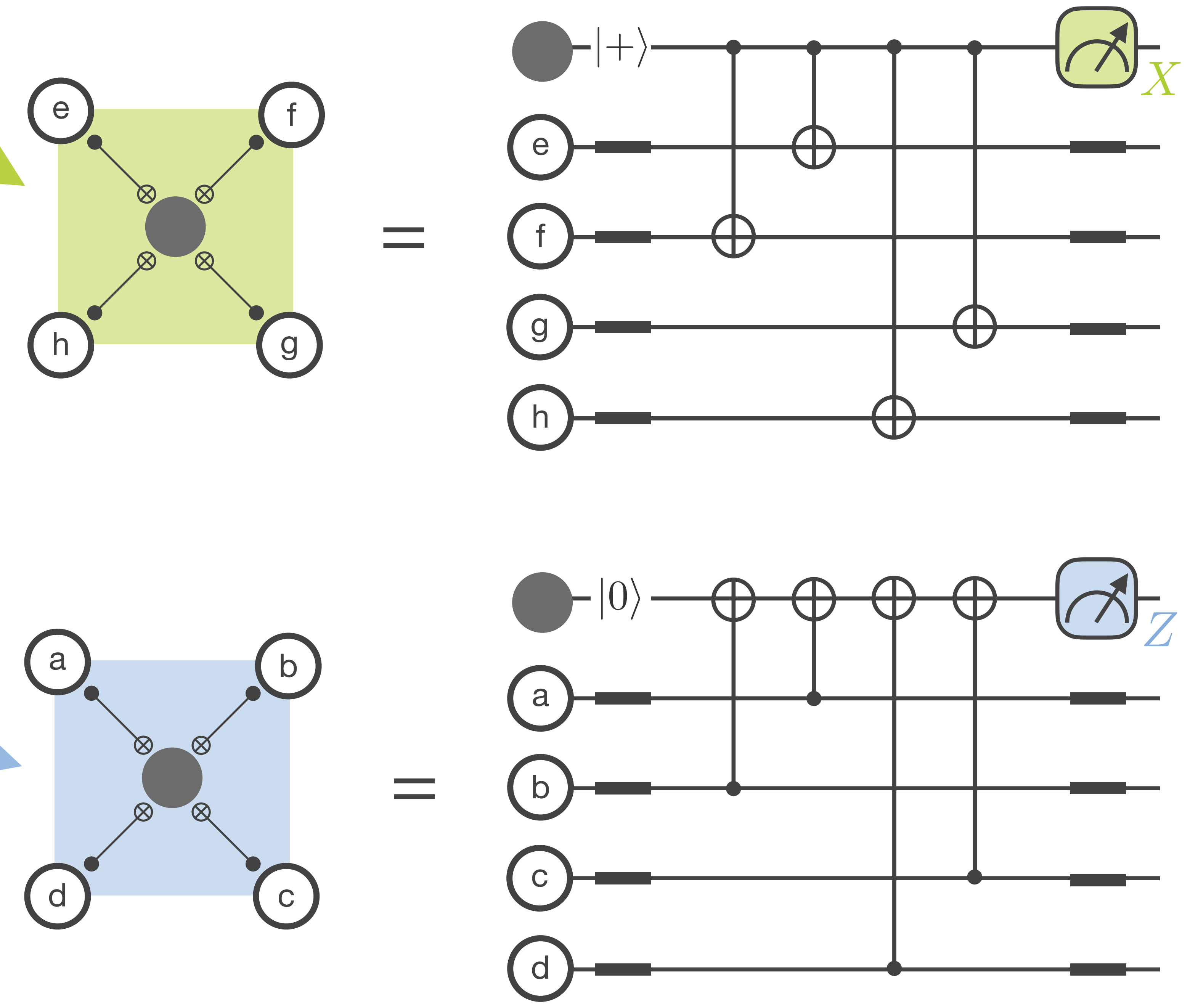


25 computational qubits
24 auxiliary qubits

d=5 code

X stabilizer

Z stabilizer

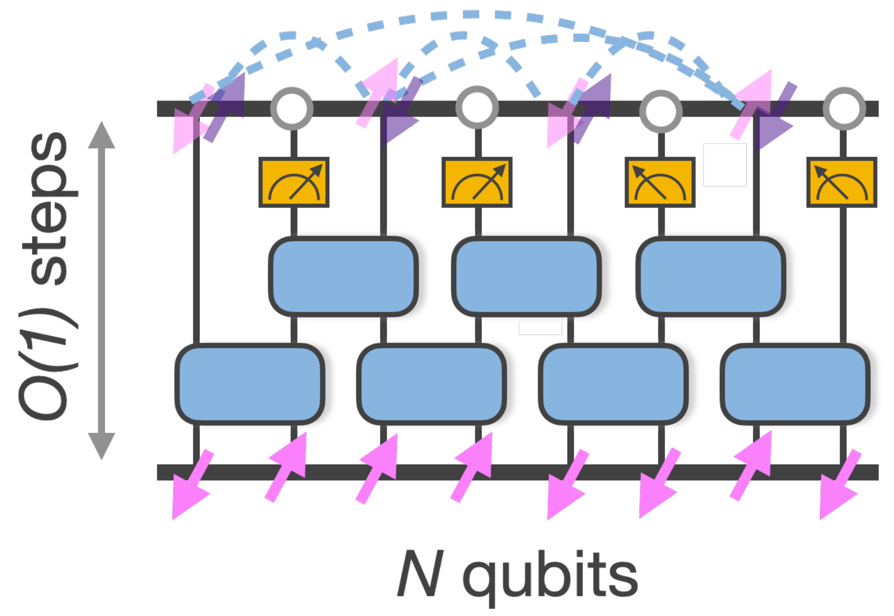


The toric/surface code was conceived as a **measurement protocol.**

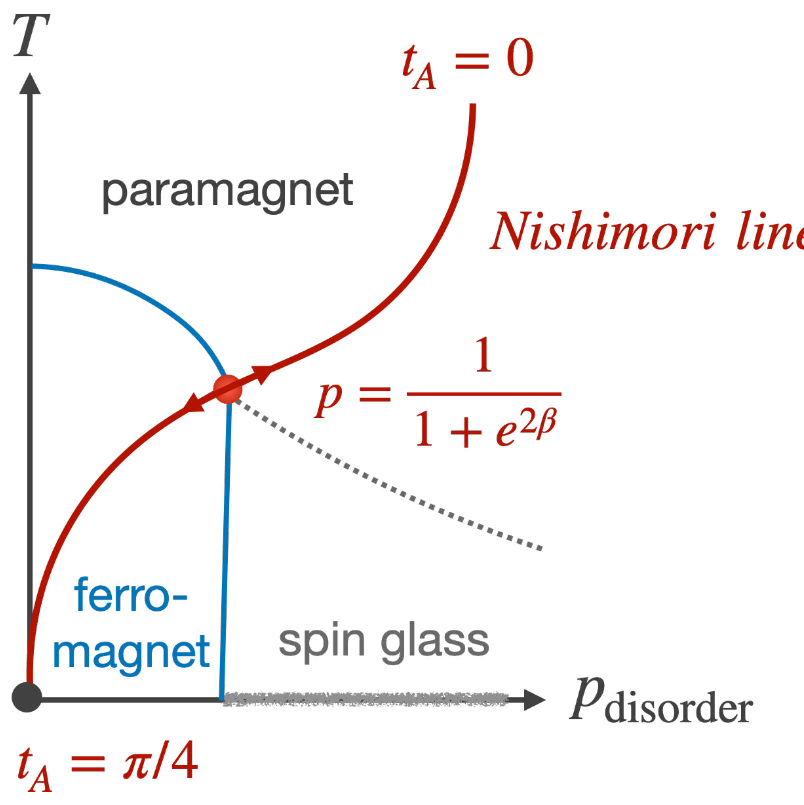
measurement & entanglement

Guo-Yi Zhu

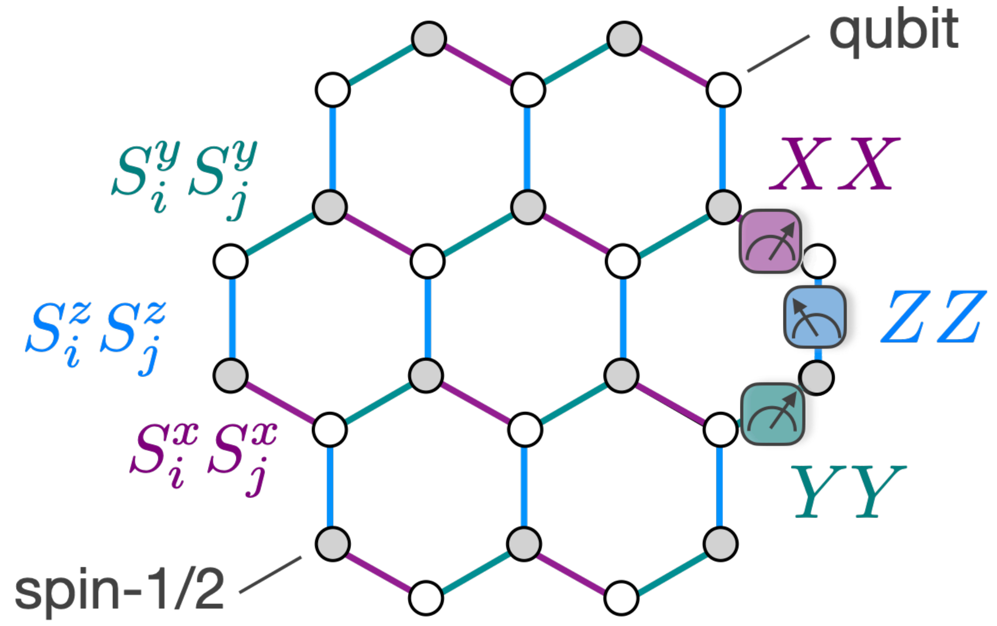
measurement-assisted **state preparation**



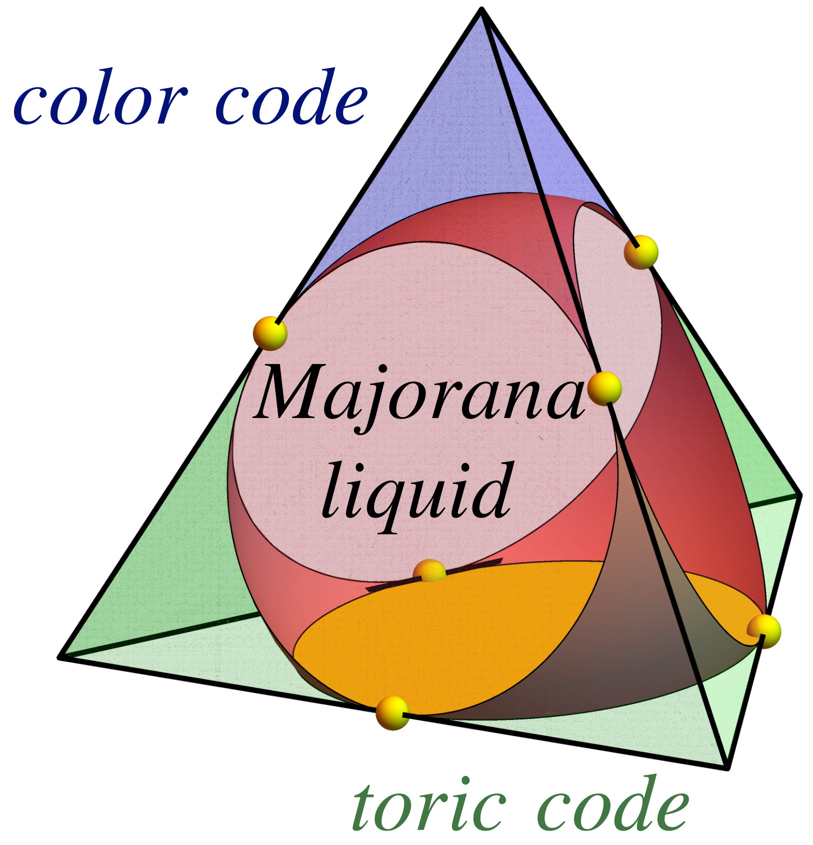
Nishimori's cat



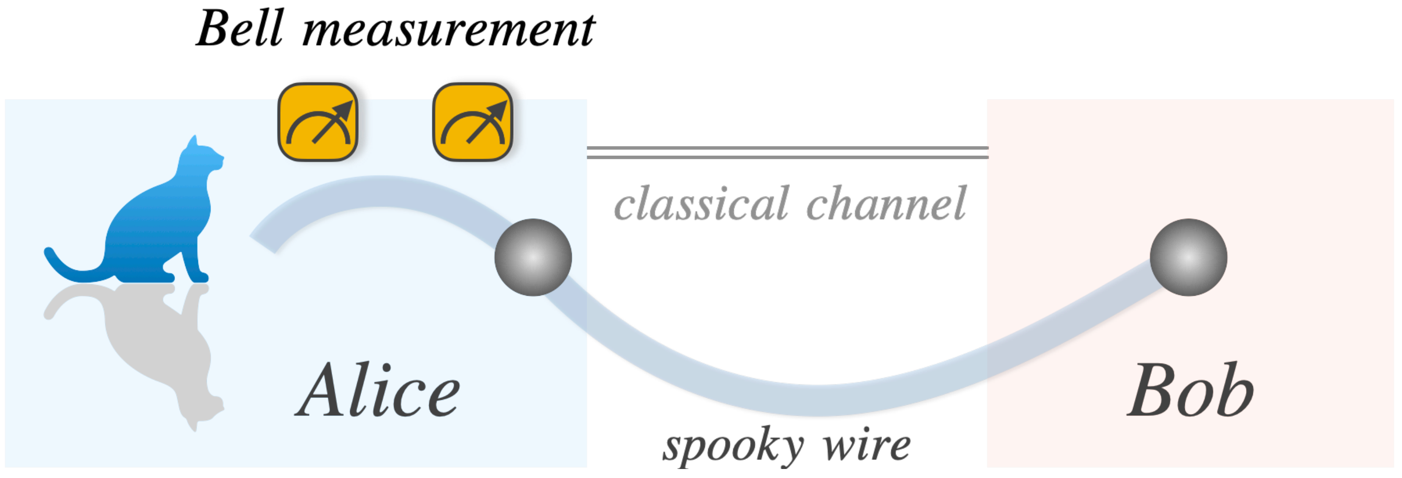
measurement-only quantum dynamics



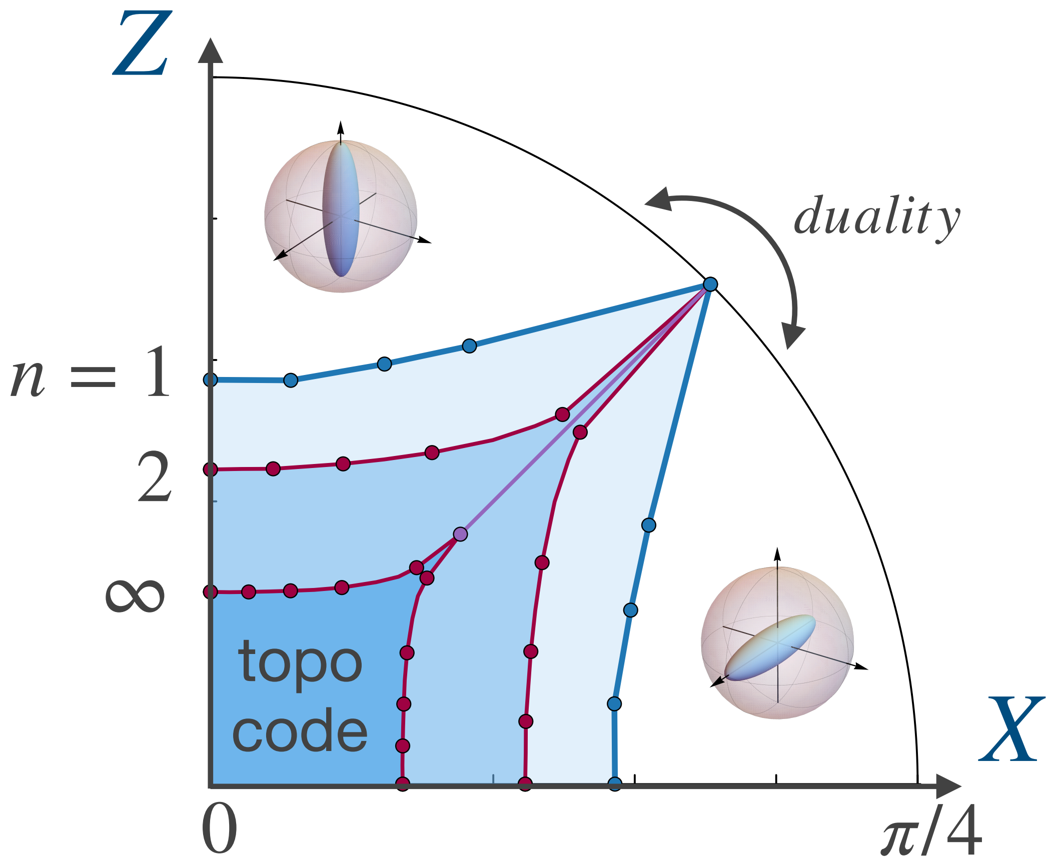
Kitaev circuits

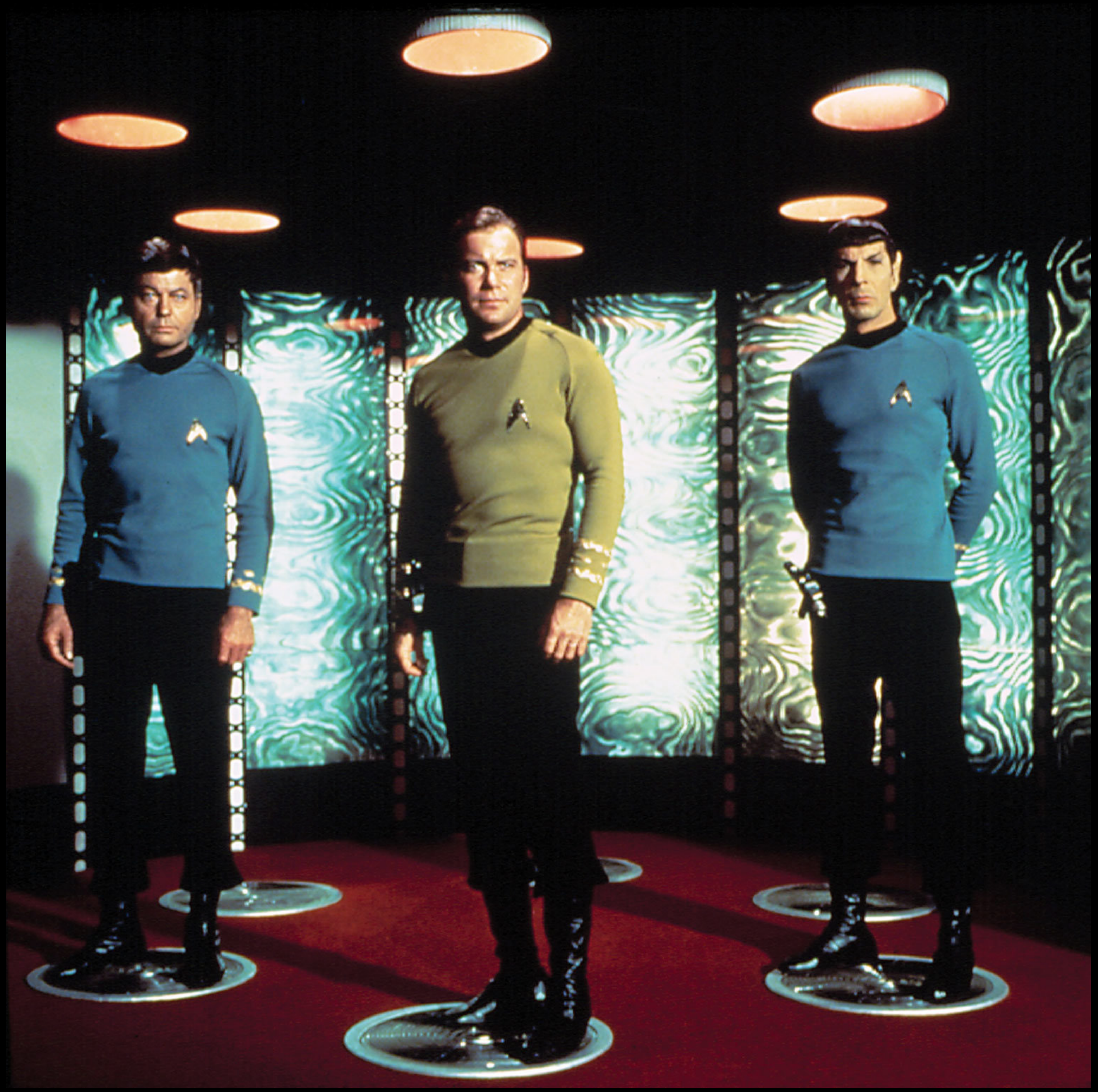


many-qubit **teleportation**



Bell decoders





science meets fiction

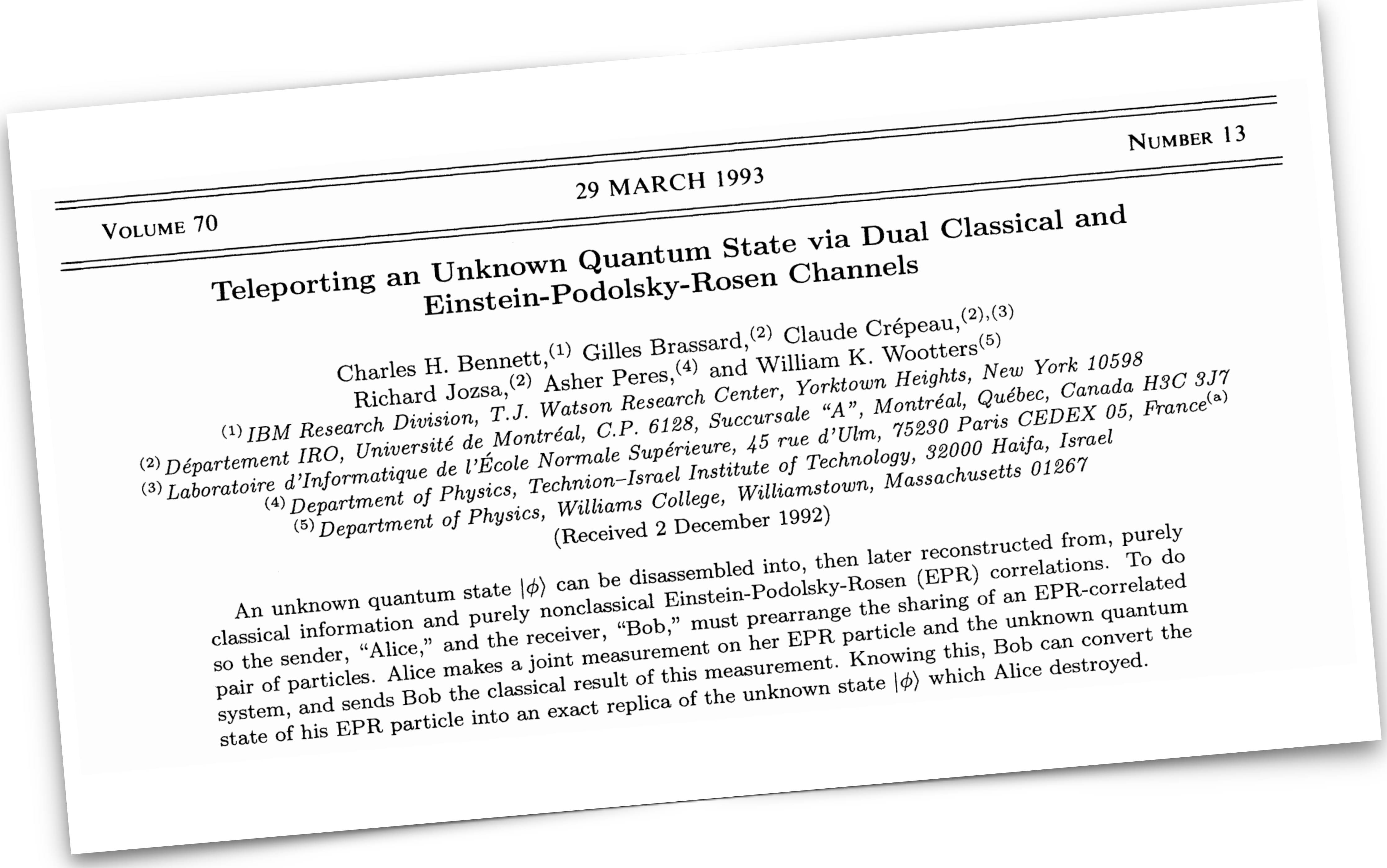
teleportation

quantum teleportation

teleportation?

quantum mechanics

to the rescue …

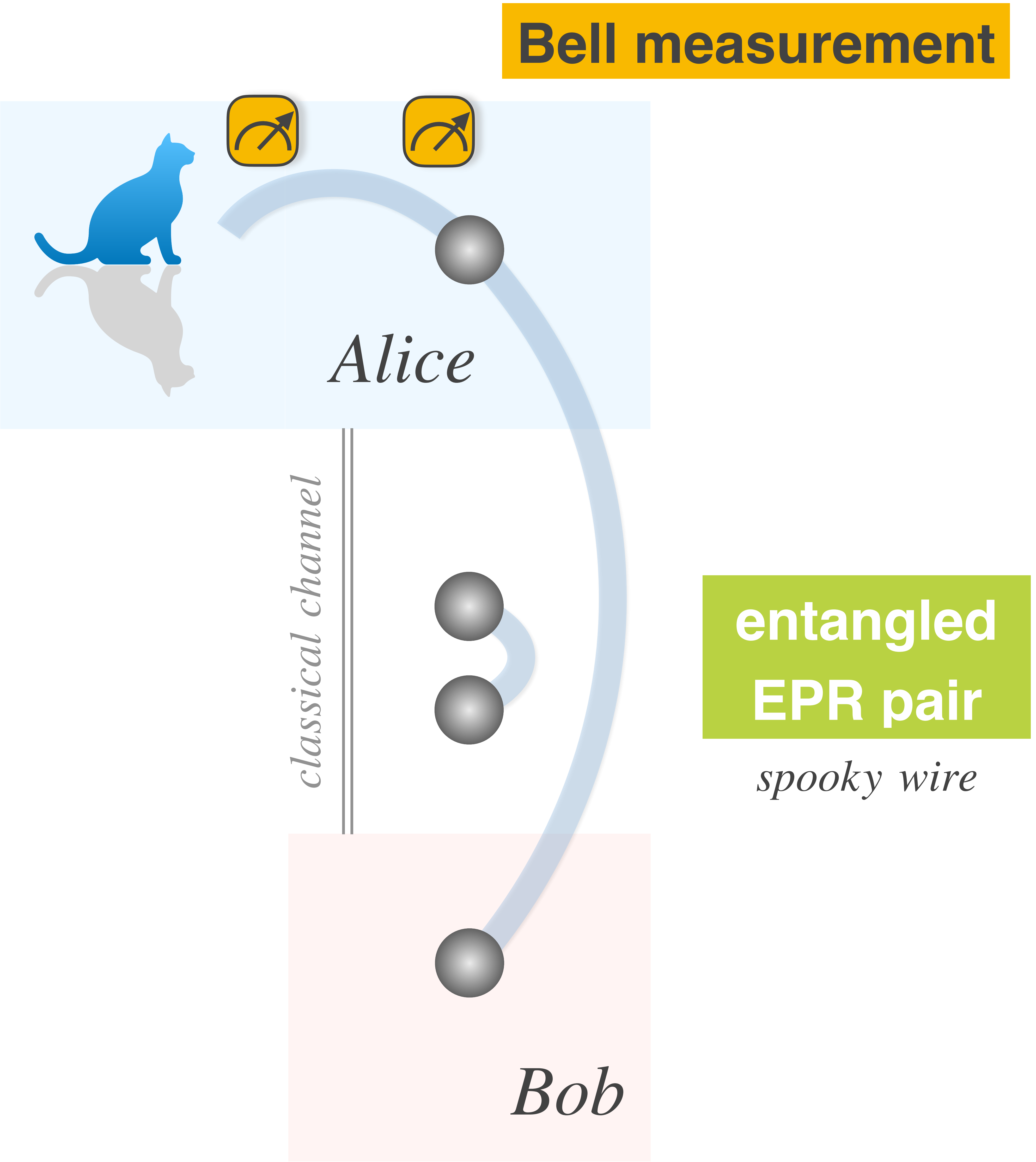


VOLUME 70 — 29 MARCH 1993 — NUMBER 13

Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

Charles H. Bennett,[1] Gilles Brassard,[2] Claude Crépeau,[2],[3] Richard Jozsa,[2] Asher Peres,[4] and William K. Wootters[5]
[1] *IBM Research Division, T.J. Watson Research Center, Yorktown Heights, New York 10598*
[2] *Département IRO, Université de Montréal, C.P. 6128, Succursale "A", Montréal, Québec, Canada H3C 3J7*
[3] *Laboratoire d'Informatique de l'École Normale Supérieure, 45 rue d'Ulm, 75230 Paris CEDEX 05, France*[(a)]
[4] *Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel*
[5] *Department of Physics, Williams College, Williamstown, Massachusetts 01267*
(Received 2 December 1992)

An unknown quantum state $|\phi\rangle$ can be disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations. To do so the sender, "Alice," and the receiver, "Bob," must prearrange the sharing of an EPR-correlated pair of particles. Alice makes a joint measurement on her EPR particle and the unknown quantum system, and sends Bob the classical result of this measurement. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state $|\phi\rangle$ which Alice destroyed.

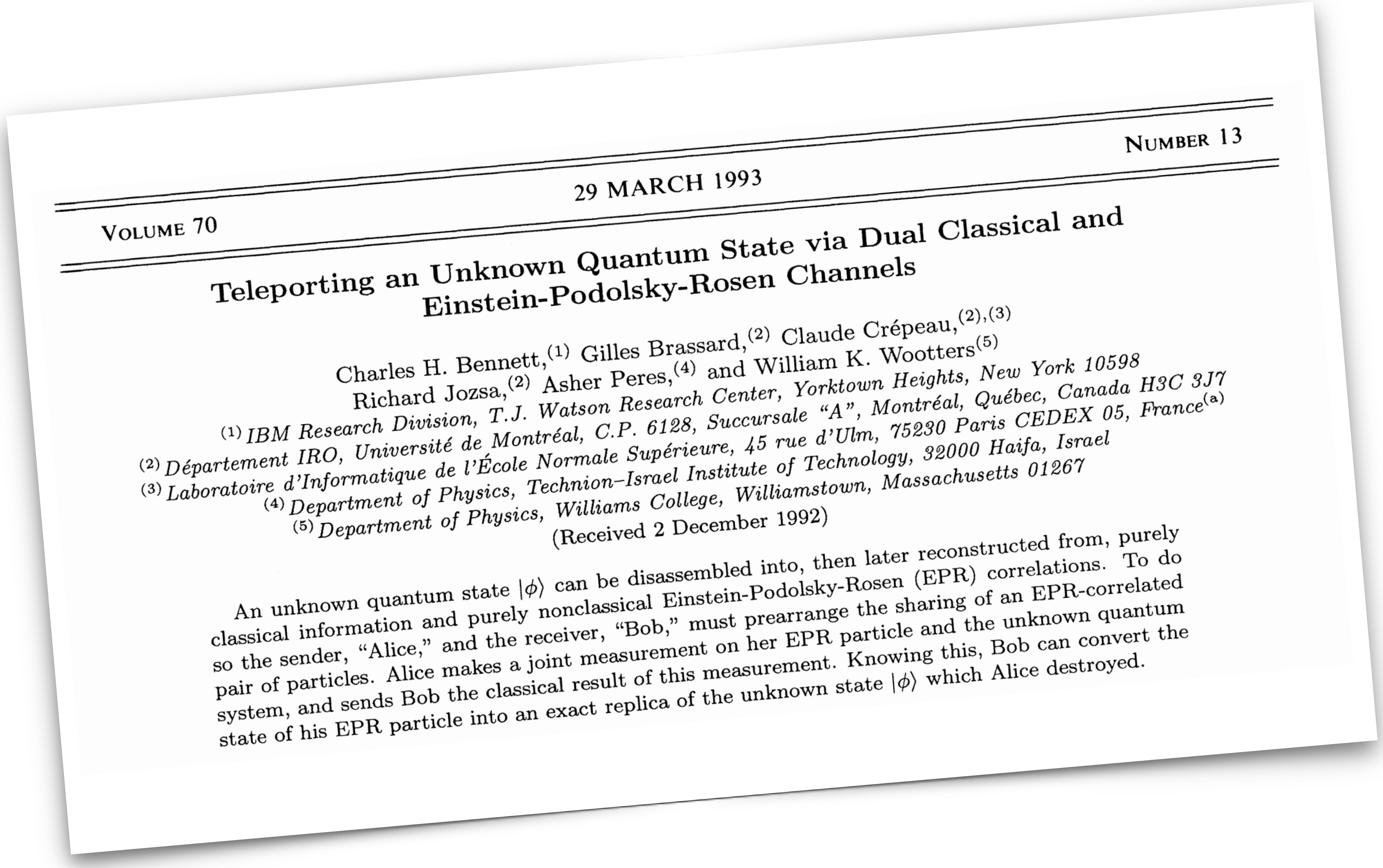


quantum teleportation

teleportation?

quantum mechanics

to the rescue …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experimental quantum teleportation (1997)

D. Bouwmeester, …., A. Zeilinger, Nature **390**, 575 (1997)

3 m teleportation on demand (2014)

The New York Times

W. Pfaff, …, R. Hanson, Science **345**, 532 (2014)

100 km optical fibre channel (2016)

Q.-C. Sun *et al.*, Nature Photonics **10**, 671 (2016)

1000 km earth-satellite channel (2017)

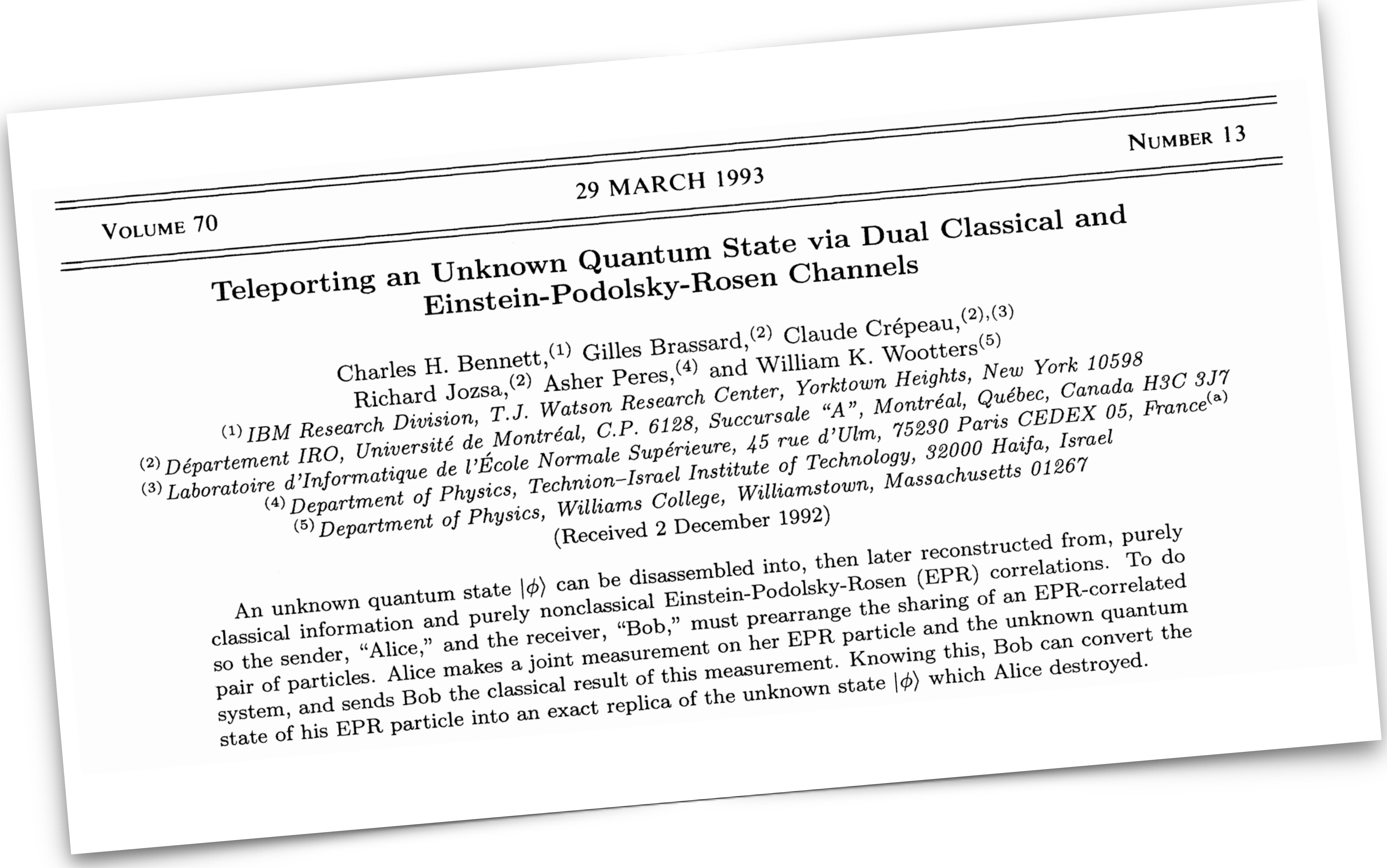
J.-G. Ren *et al.* Nature **549**, 70 (2017)

quantum teleportation

teleportation?

quantum mechanics

to the rescue …



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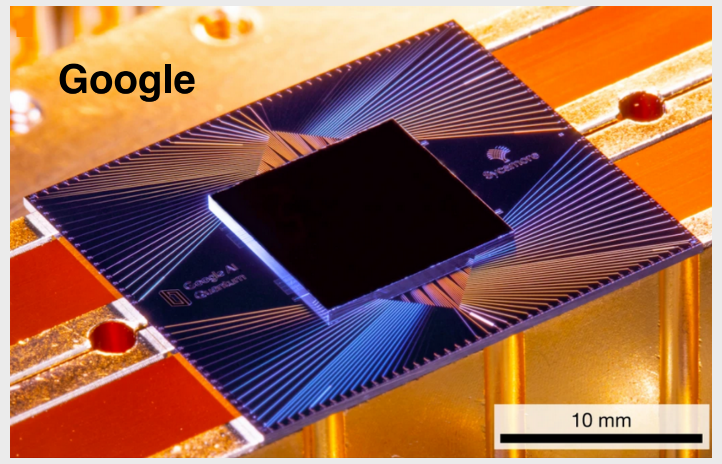
single qubit teleportation

few-qubit teleportation

noisy intermediate-scale

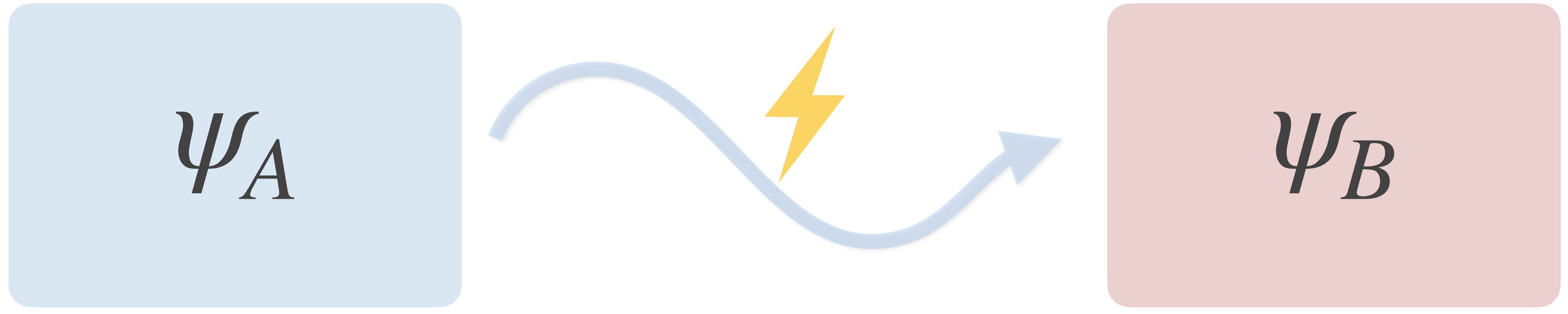
NISQ quantum devices



many-body teleportation

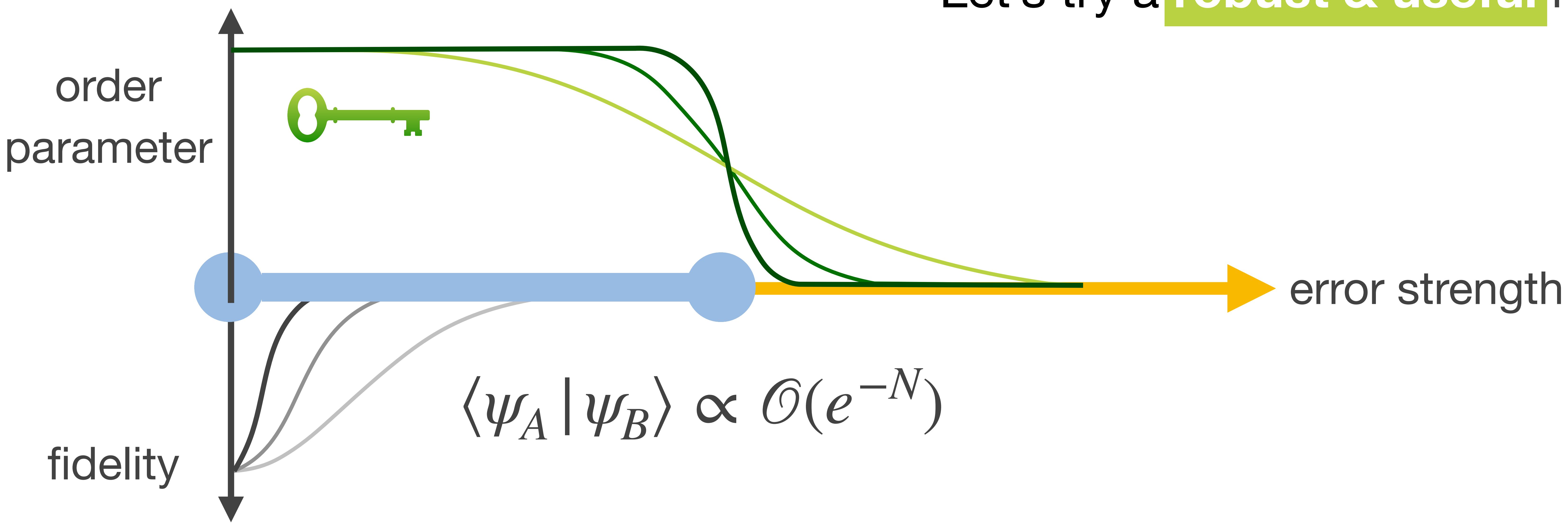
Star Trek

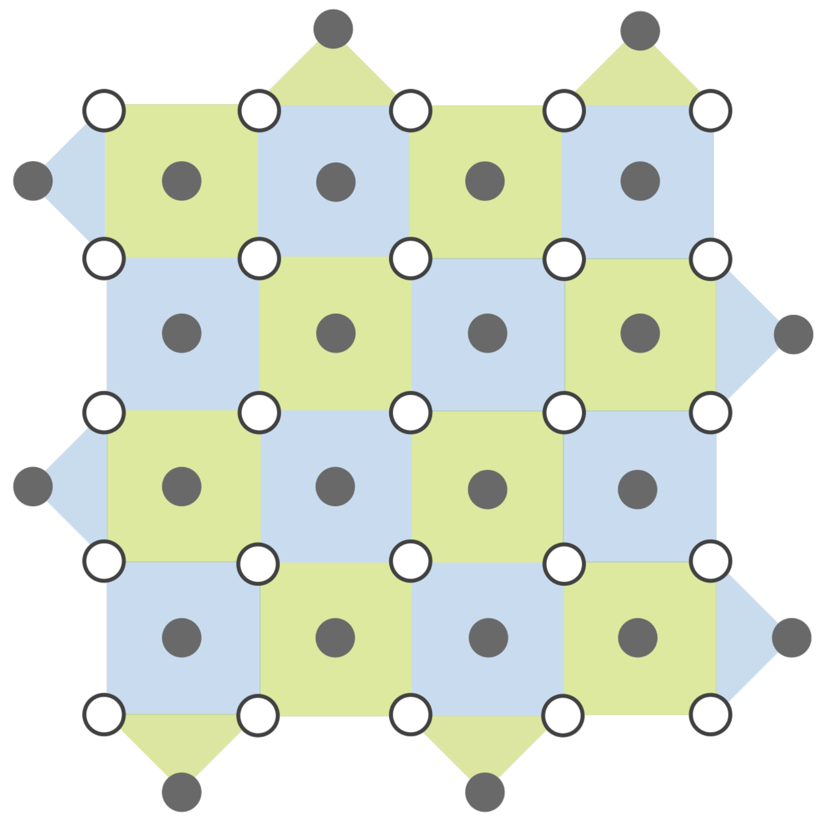
teleportation of **quantum matter**



Q: Can **quantum matter** be teleported under **coherent error** ?

Let's try a **robust & useful** many-body state





surface code / logical qubit

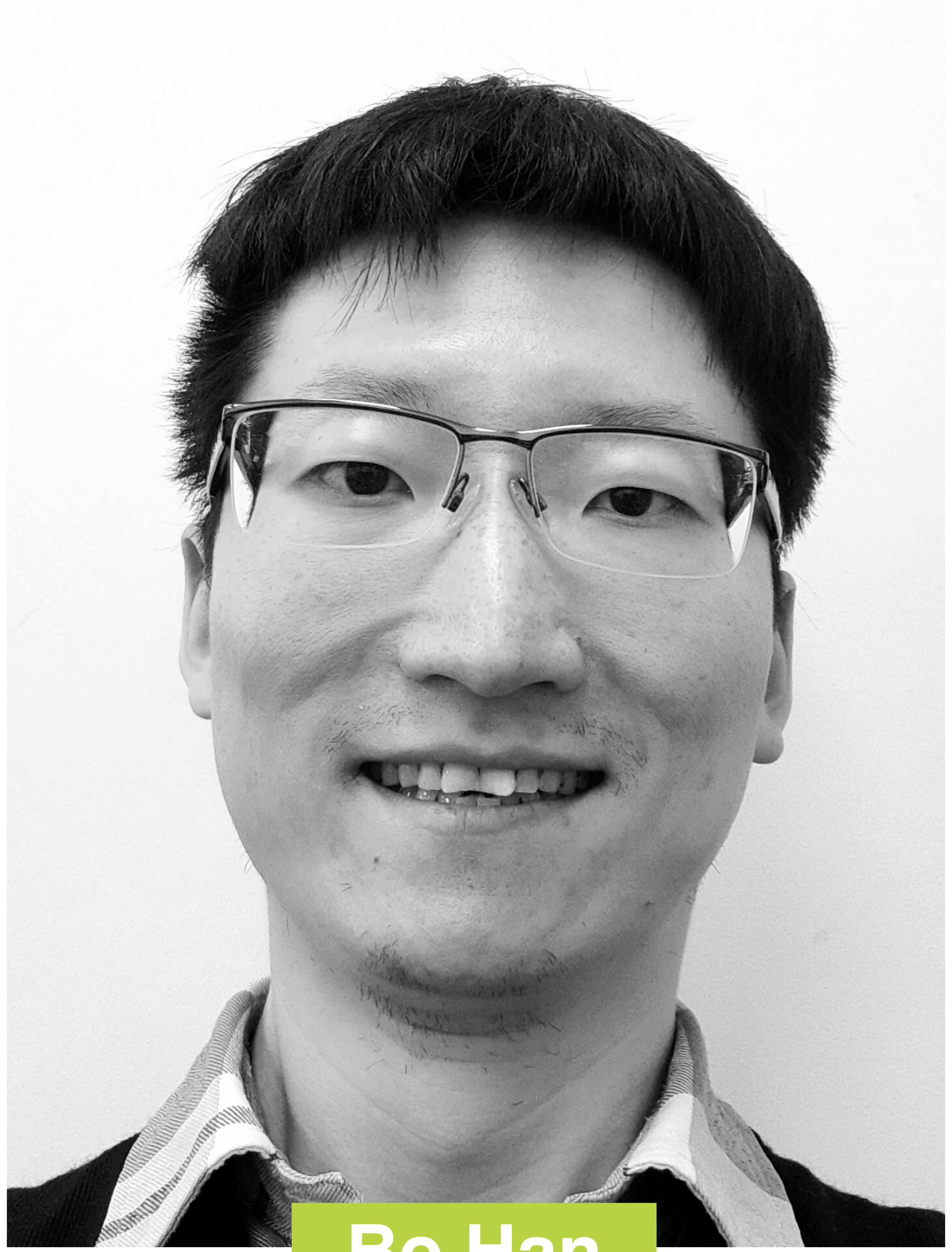
the **team**

Finn Eckstein

University of Cologne

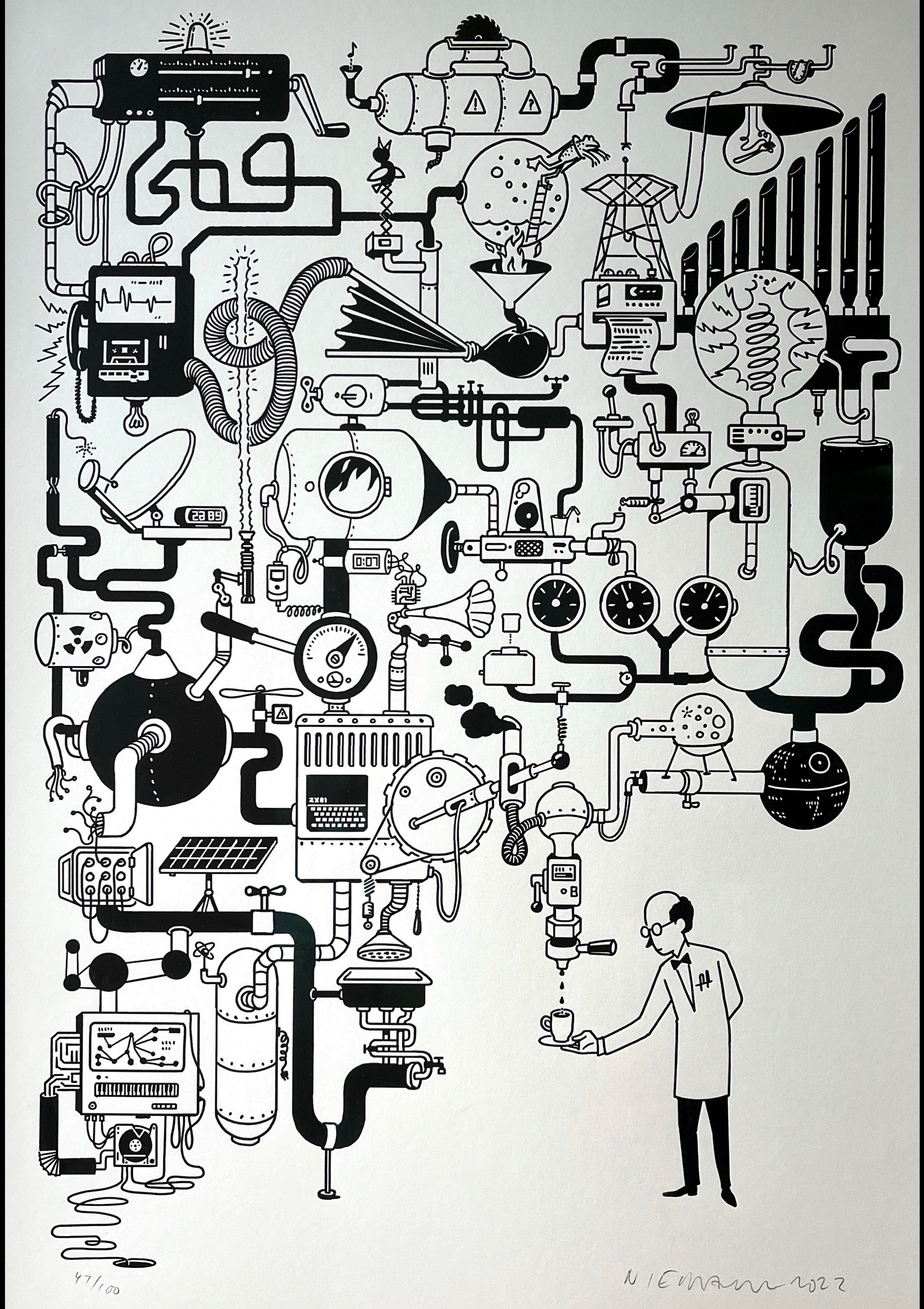
Guo-Yi Zhu

University of Cologne



Bo Han

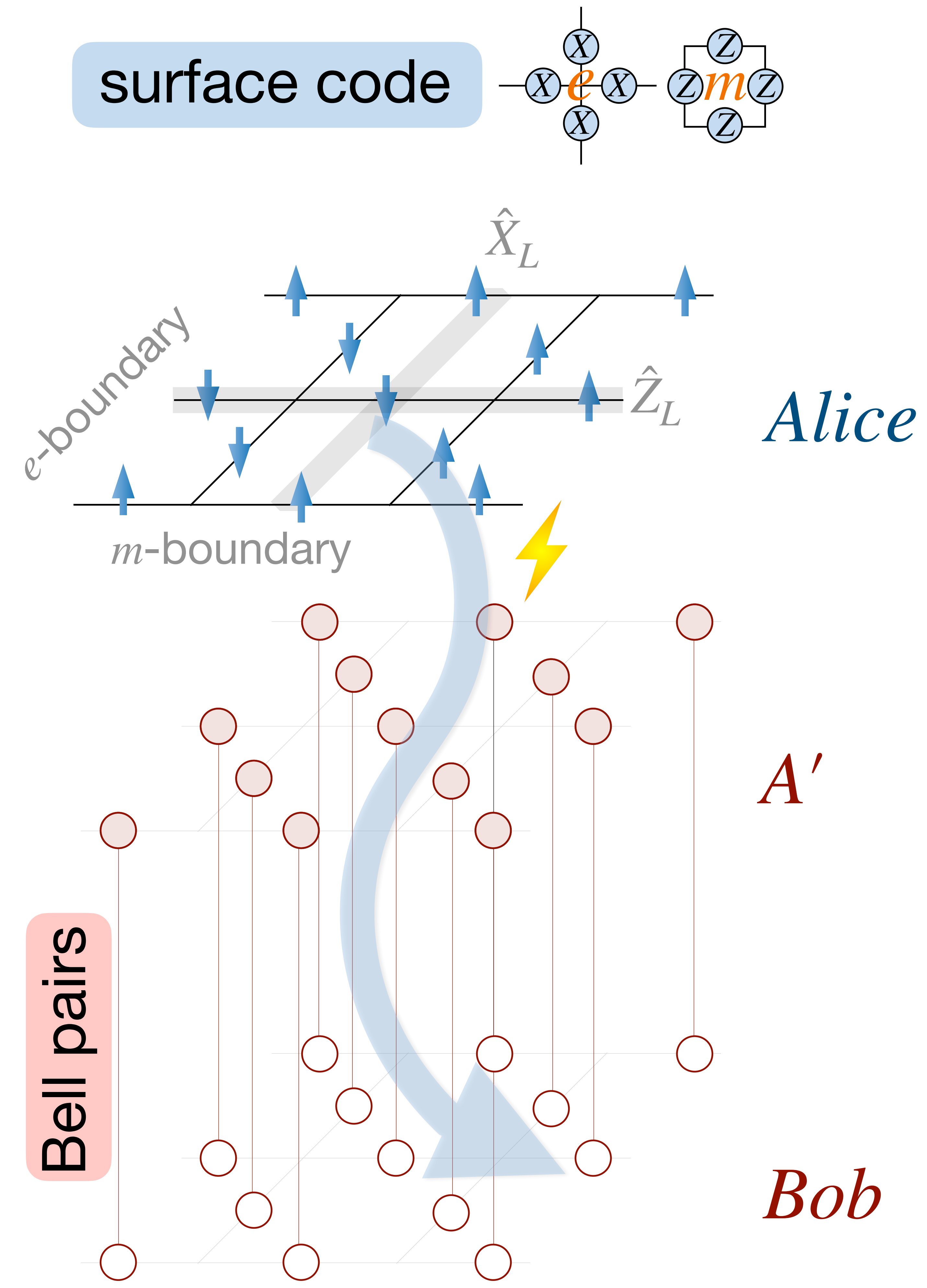
Weizmann Institute

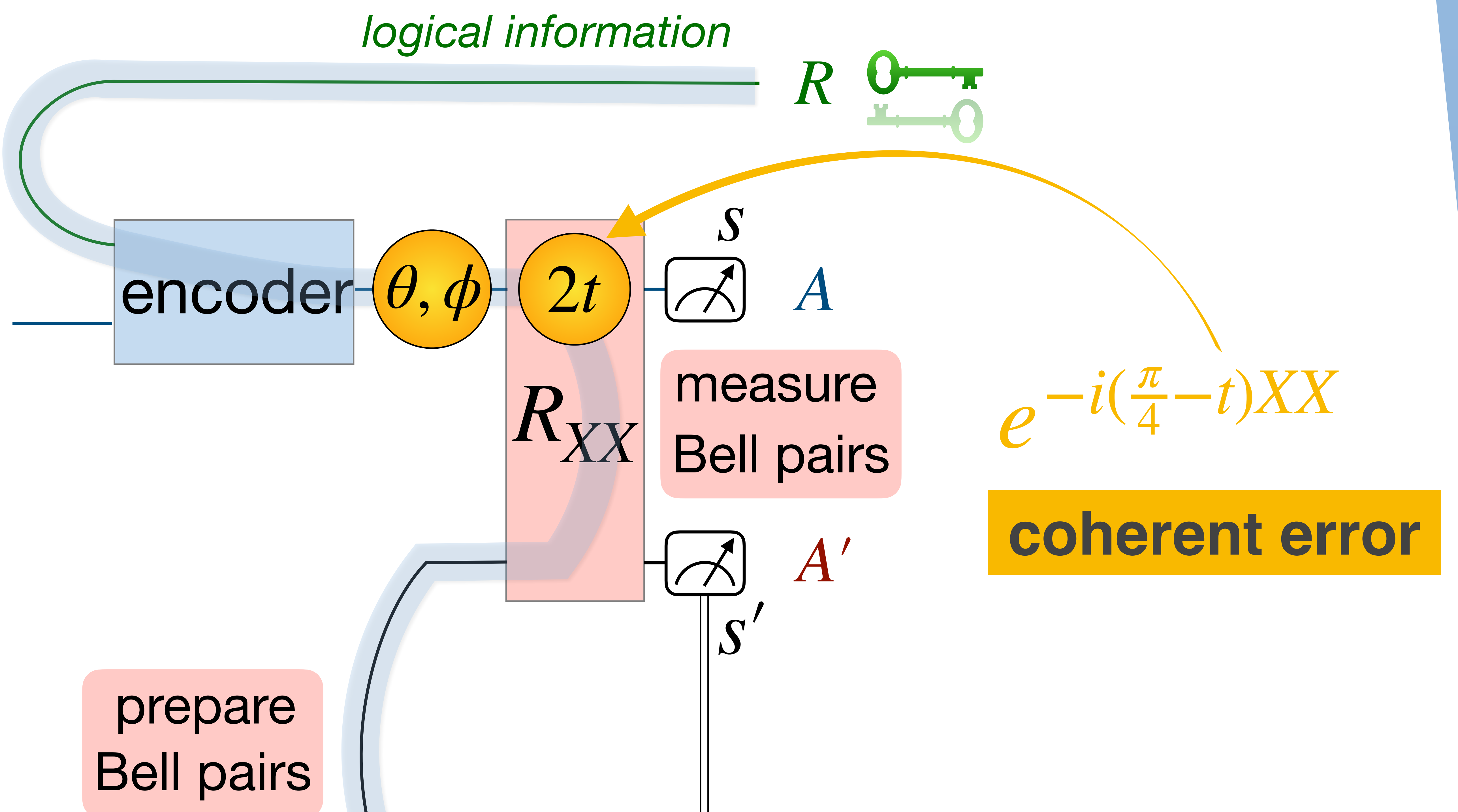


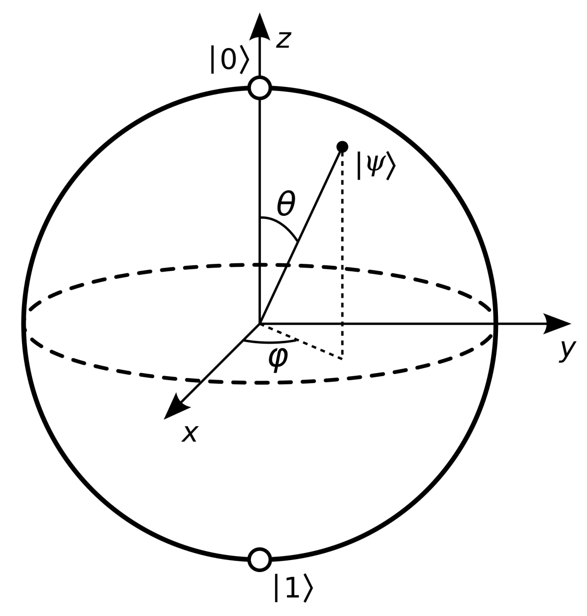
Christoph Niemann, New Yorker Cover

teleportation protocol

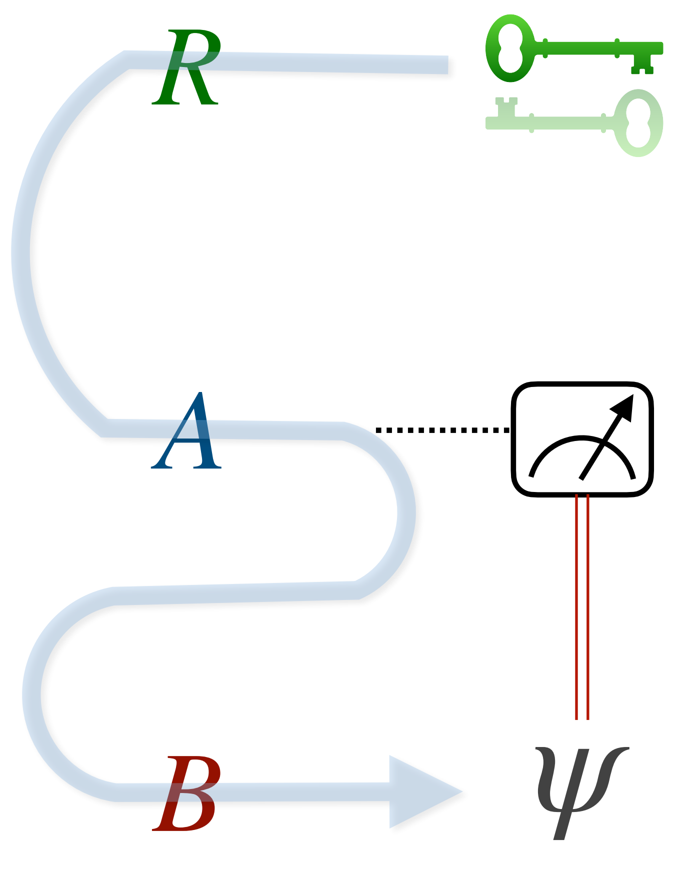
protocol & **quantum circuit**







diagnose robust teleportation



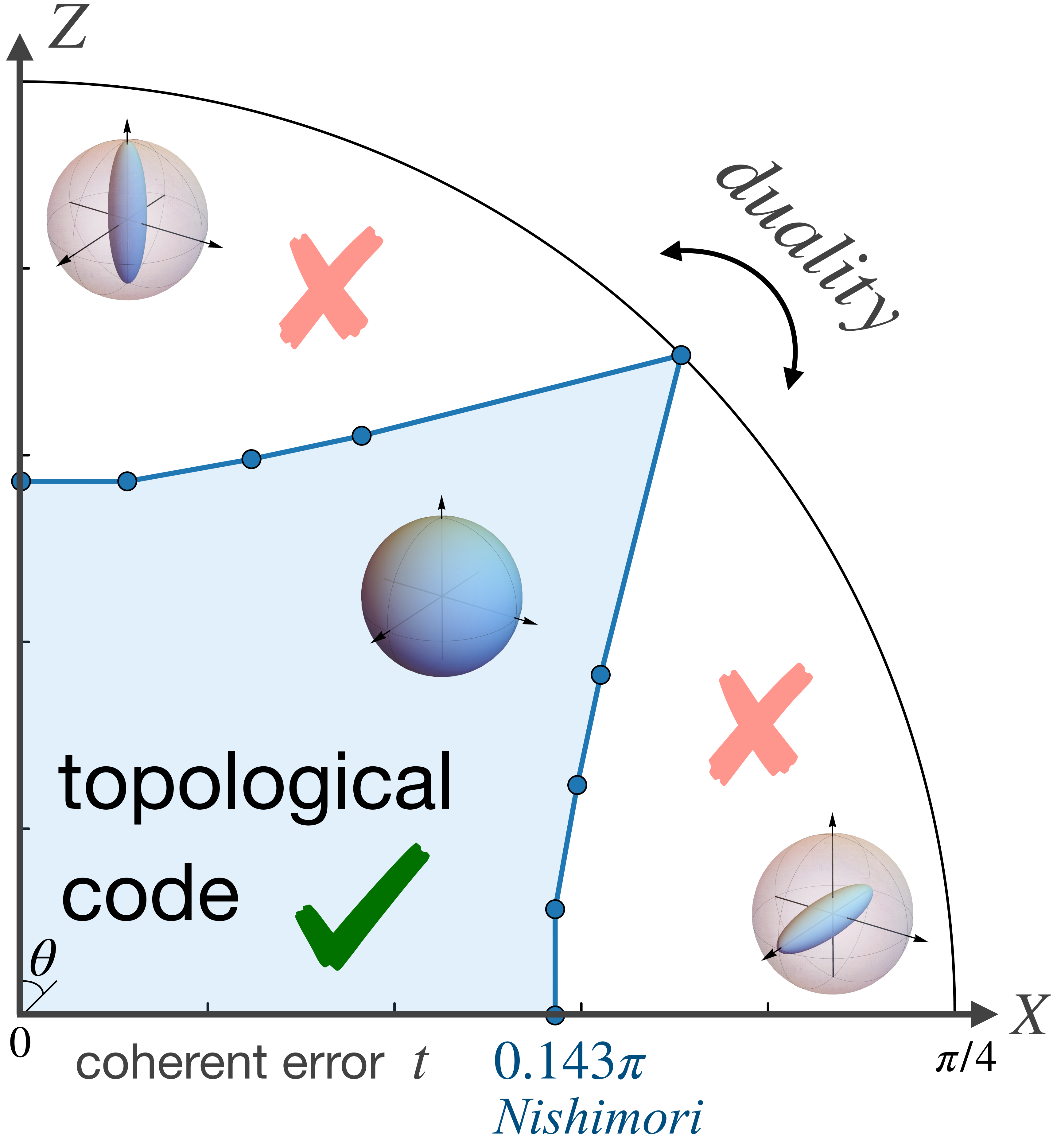
$$I_c = S_{RA} - S_A$$

$$= S_{AB} - S_{RAB}$$

$$= \sum_{\mathbf{s}} P(\mathbf{s}) S_B(\mathbf{s})$$

coherent information **channel capacity**

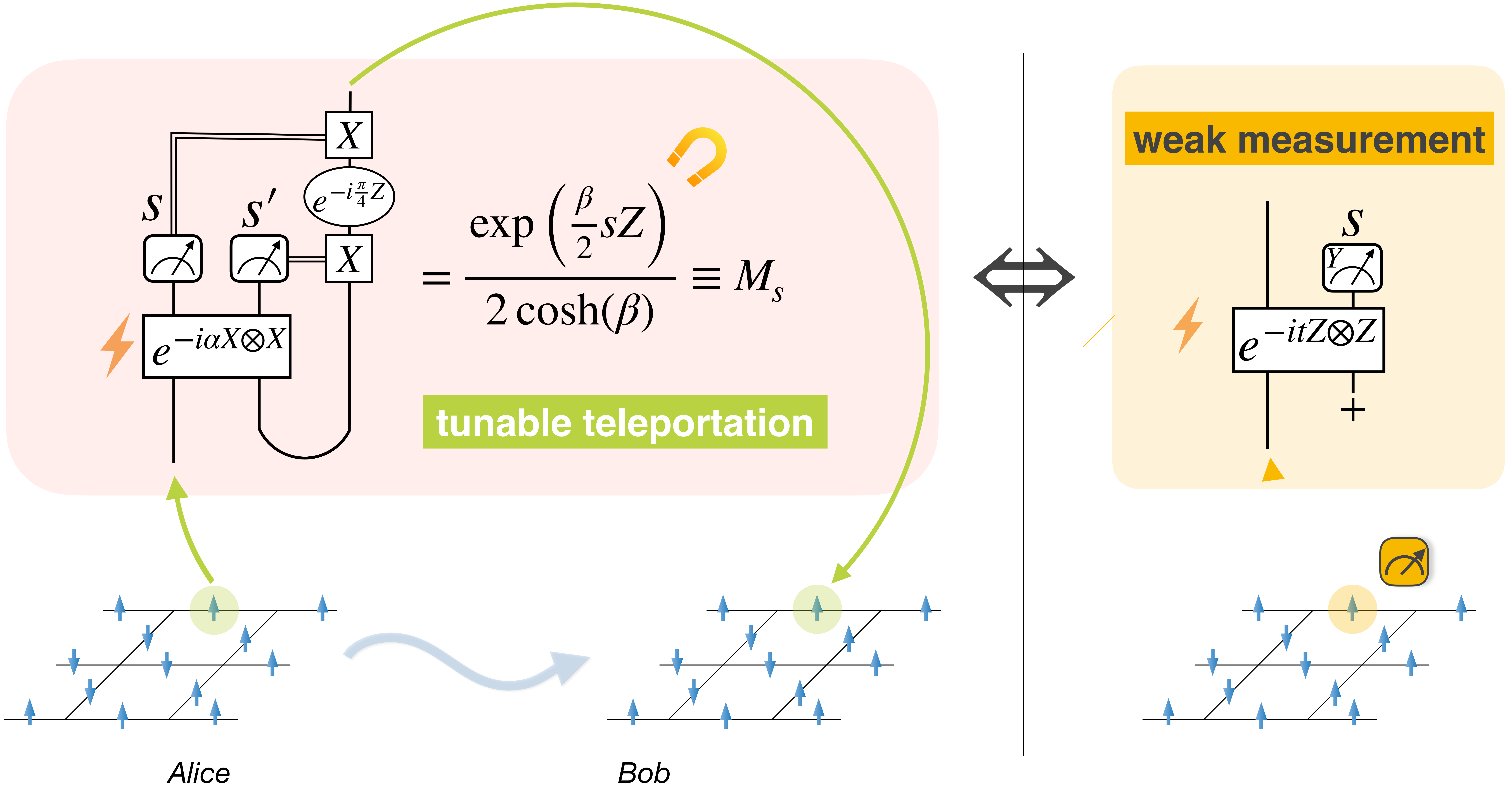
- Can **Alice steal** the key?
- Can **Bob decode** the key with classic info shared by Alice?
- **Ensemble average** of logical entropy (size of code space).



$$\rho_{RAB} = \sum_{\mathbf{s}} P(\mathbf{s}) \,|\, \Psi(\mathbf{s}) \rangle_{RB} \langle \Psi(\mathbf{s}) \,|\, \otimes \,|\, \mathbf{s} \rangle_A \langle \mathbf{s} \,|$$

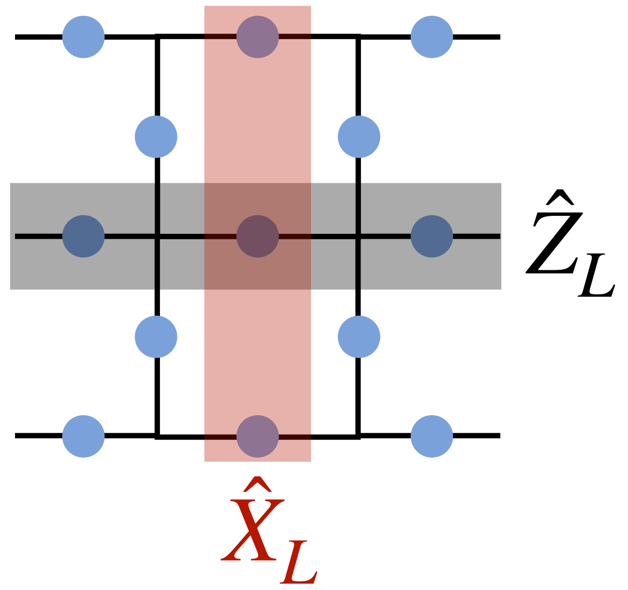
Schumacher, Nielsen 1996; Lloyd 1997; Gullans, Huse 2020;
Fan, Bao, Vishwanath, Altman 2023; Colmenarez, Huang, Diehl, Müller 2023

physical qubits / teleportation vs. measurement



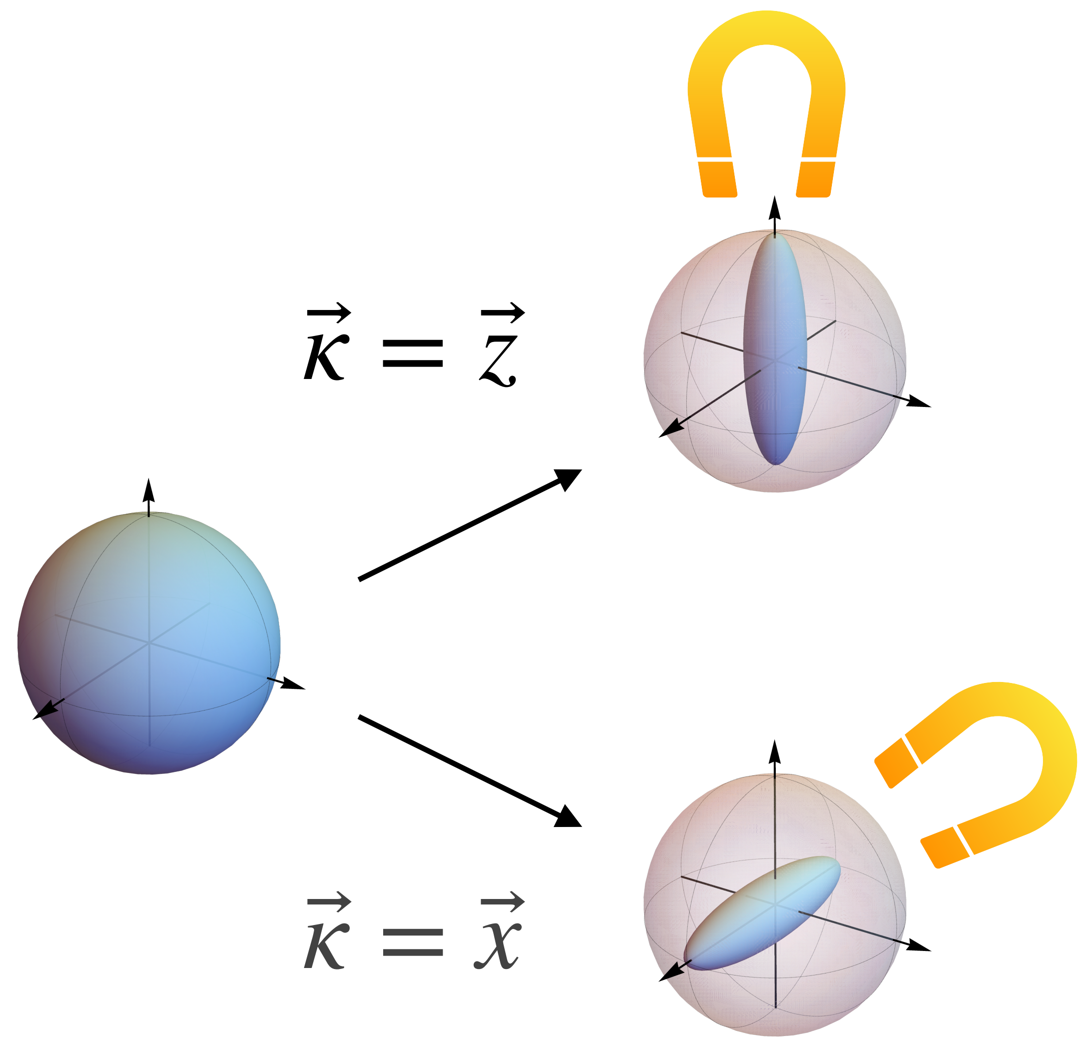
logical qubits / anyon condensation

- **deformation** in logical space $\quad P_{\mu\nu}(\mathbf{s}) := \langle \psi_\mu \,|\, M_{\mathbf{s}}^\dagger M_{\mathbf{s}} \,|\, \psi_\nu \rangle$



- logical **density matrix**

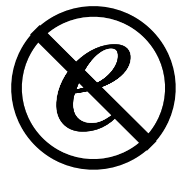
$$\rho_R(\mathbf{s}) = \frac{1}{2P(\mathbf{s})} \begin{pmatrix} P_{++}(\mathbf{s}) & P_{+-}(\mathbf{s}) \\ P^*_{+-}(\mathbf{s}) & P_{--}(\mathbf{s}) \end{pmatrix} = \frac{1 + \vec{\kappa}\cdot\vec{\sigma}}{2}$$



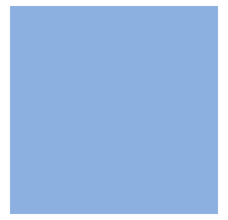
- **polarization / purification** of logical qubit

$$\vec{\kappa} = \left(\frac{P_{++} - P_{--}}{P_{++} + P_{--}}, \quad \frac{2\,|P_{+-}|}{P_{++} + P_{--}} \right)$$

- **anyon mechanism** $\quad$ *confinement* $\quad$ *condensation*

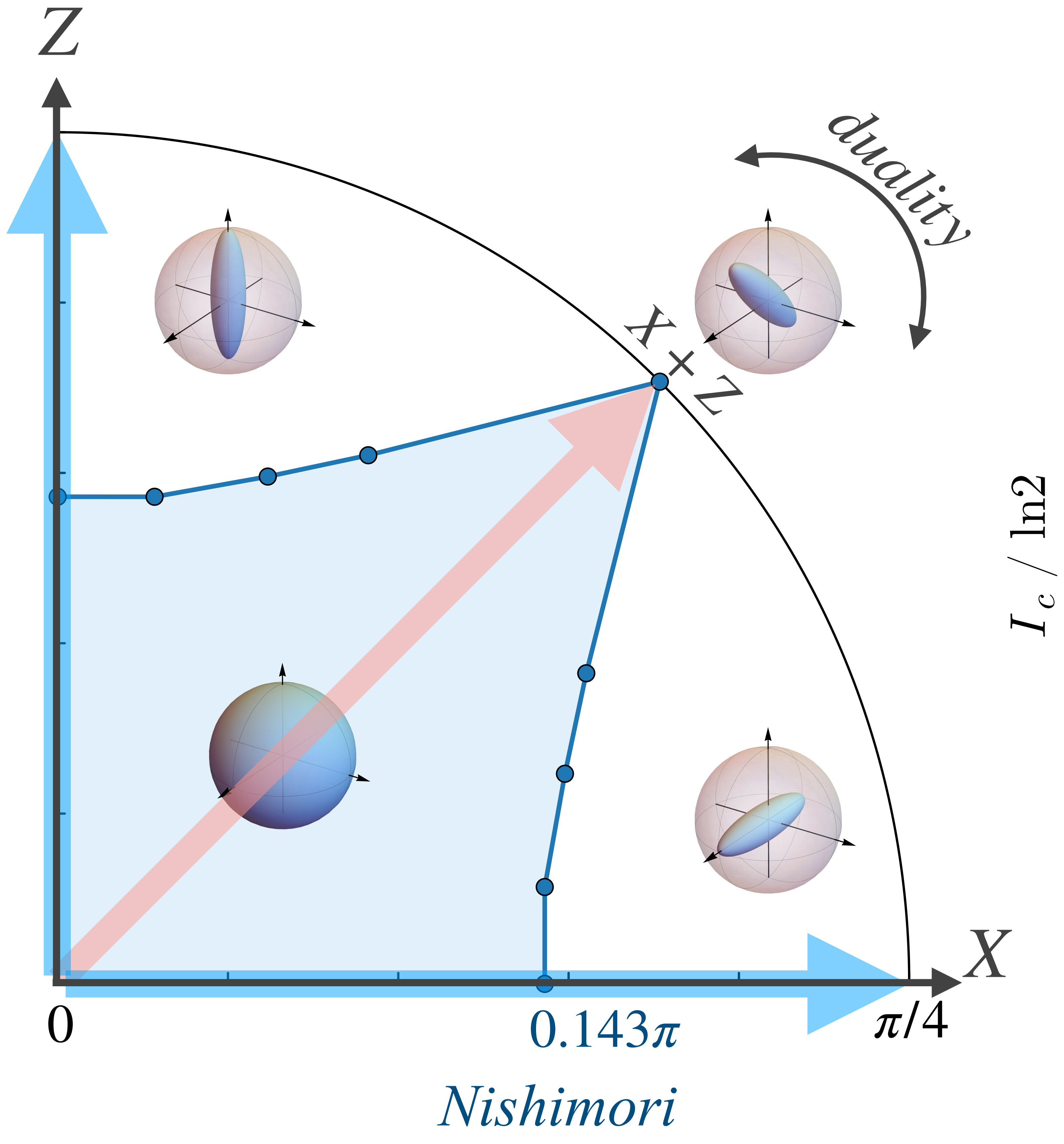


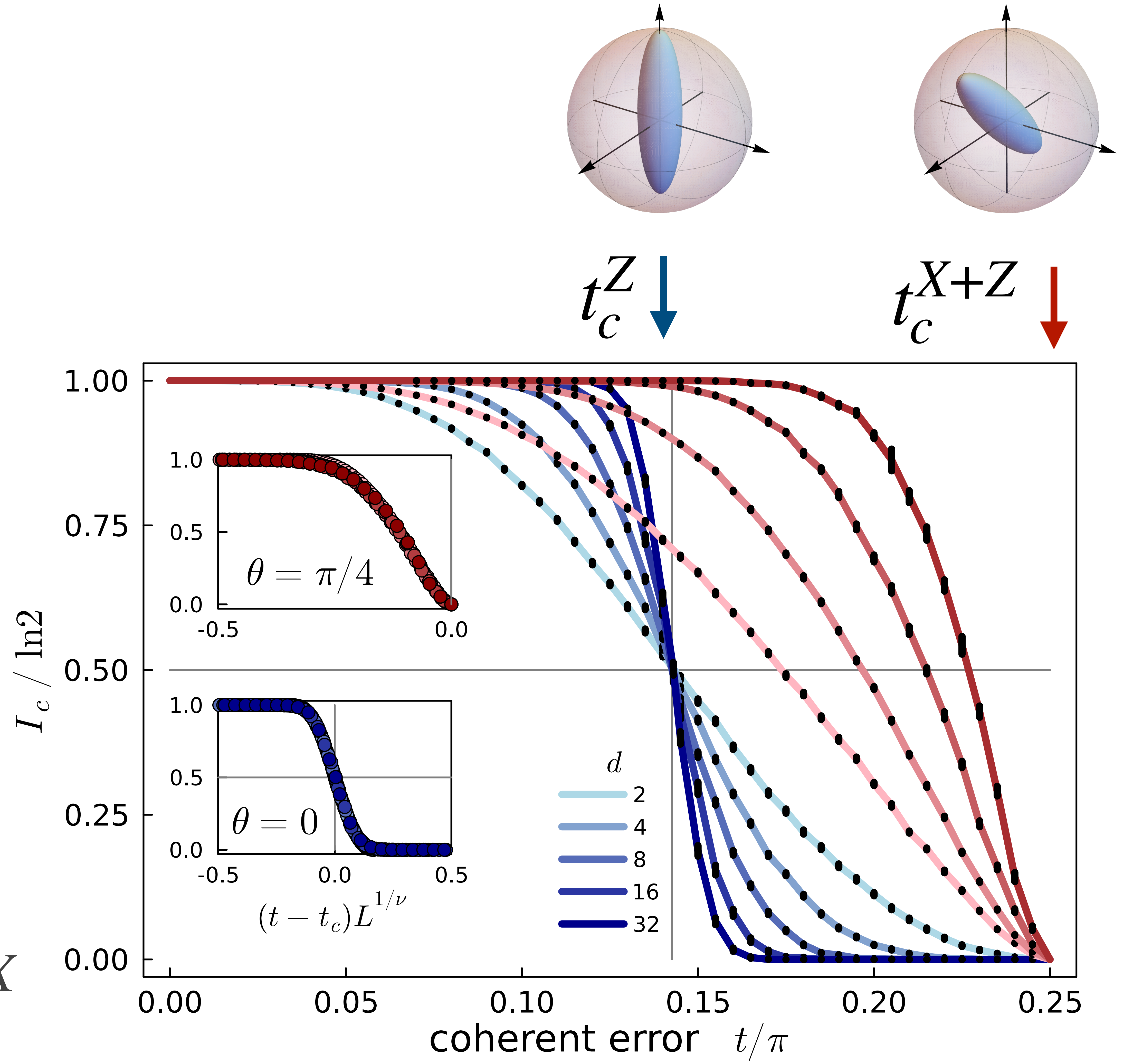
$$\psi \sim vac + e + ee + \ldots$$

$|\psi_+\rangle =$ 

$|\psi_-\rangle = e$  e

thresholds / phase transitions



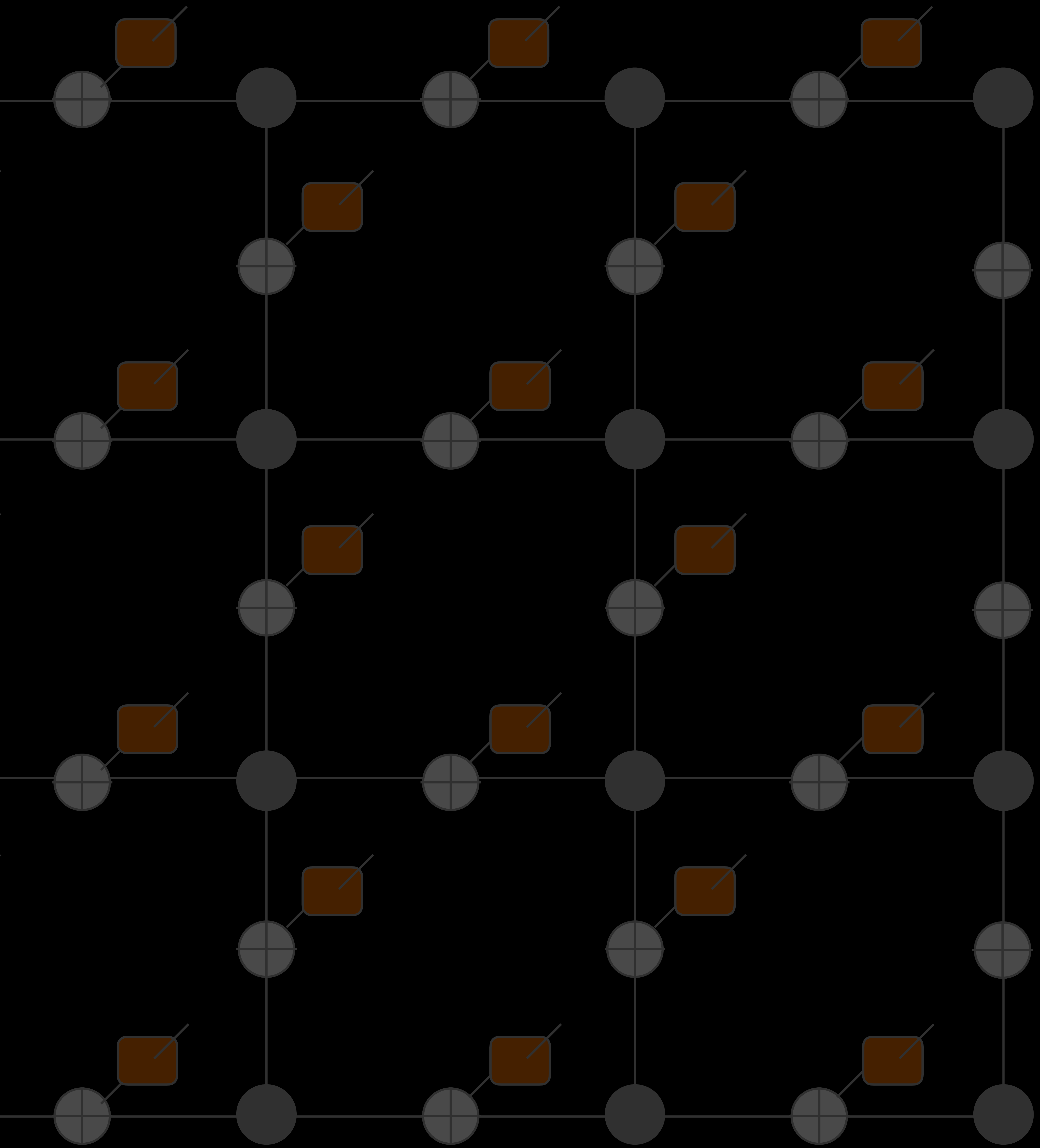


$\nu = 1.6(1)$ $\nu = 1.8(1)$

angle dependence due to **competition** of **anyon condensation**

self-dual direction
∞ threshold

teleportation succeeds even for **infinitesimal coupling**



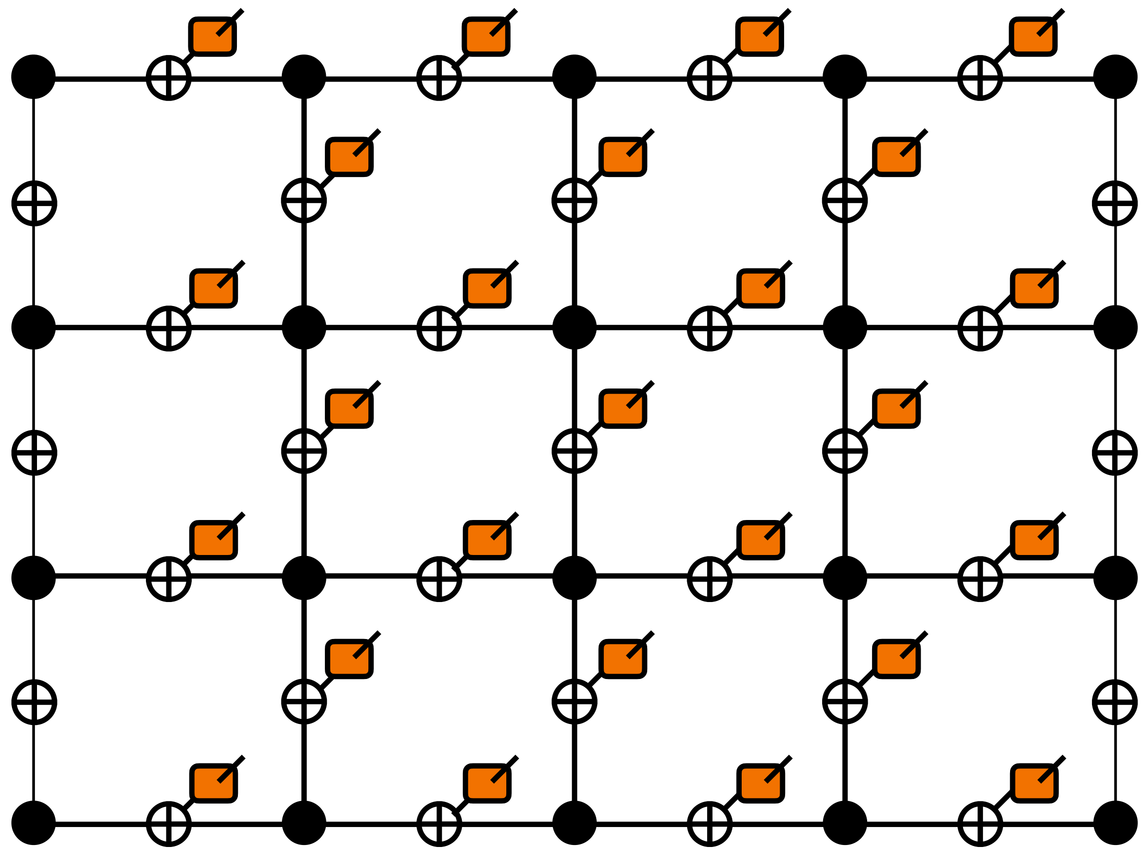
stat mech perspective

tensor network & statistical model

(2+0) dimensional

deformed wave function

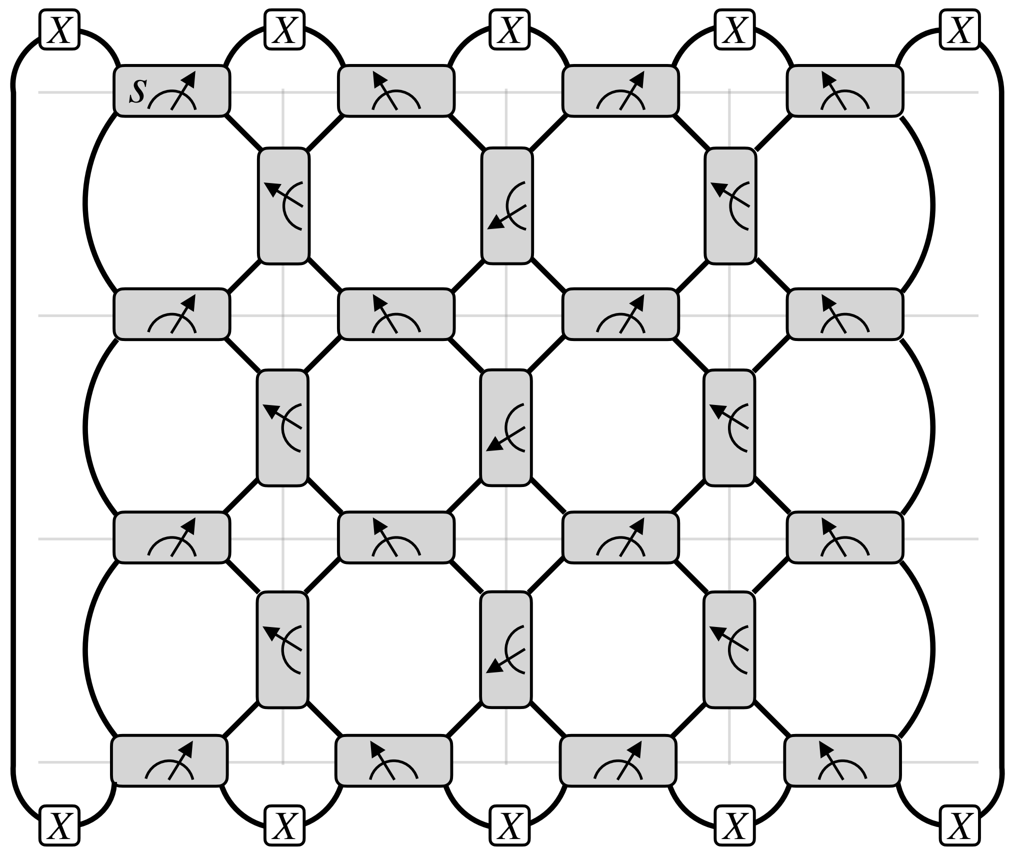
$|\psi(\mathbf{s})\rangle =$



(1+1) dimensional

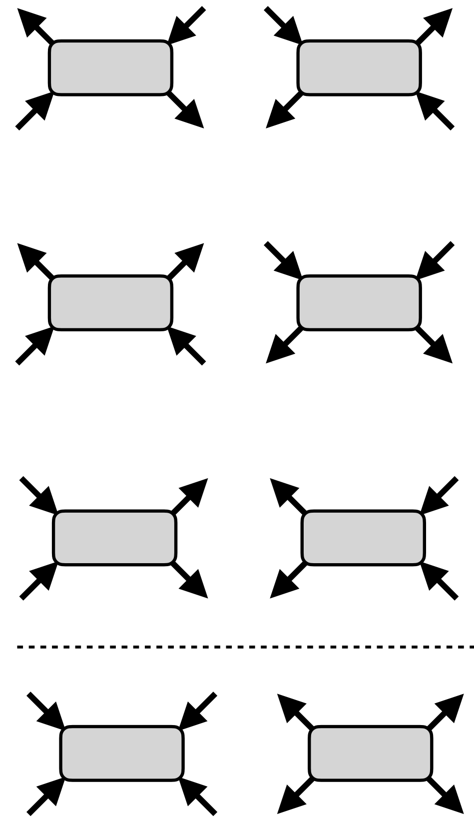
random circuit

$\langle\psi(\mathbf{s})|\psi(\mathbf{s})\rangle =$



2-dimensional

classical 8-vertex model

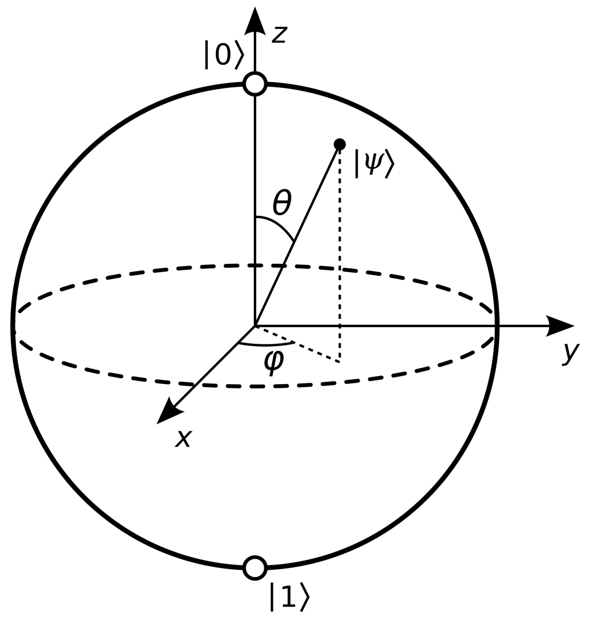


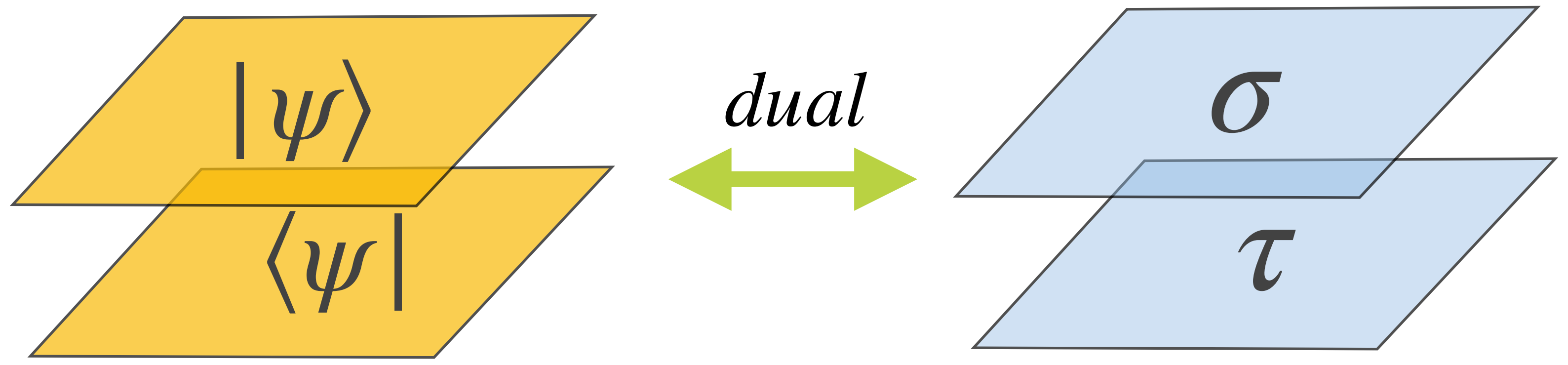
1

$s\sin(2t)\tan(\theta)\cos(\phi)$

$s\sin(2t)$

$is\sin(2t)\tan(\theta)\sin(\phi)$





$\tanh J = \sin(2t)\cos(\theta)$

$e^{-2K} = \sinh(J)\tanh(\theta)$

Ashkin-Teller model

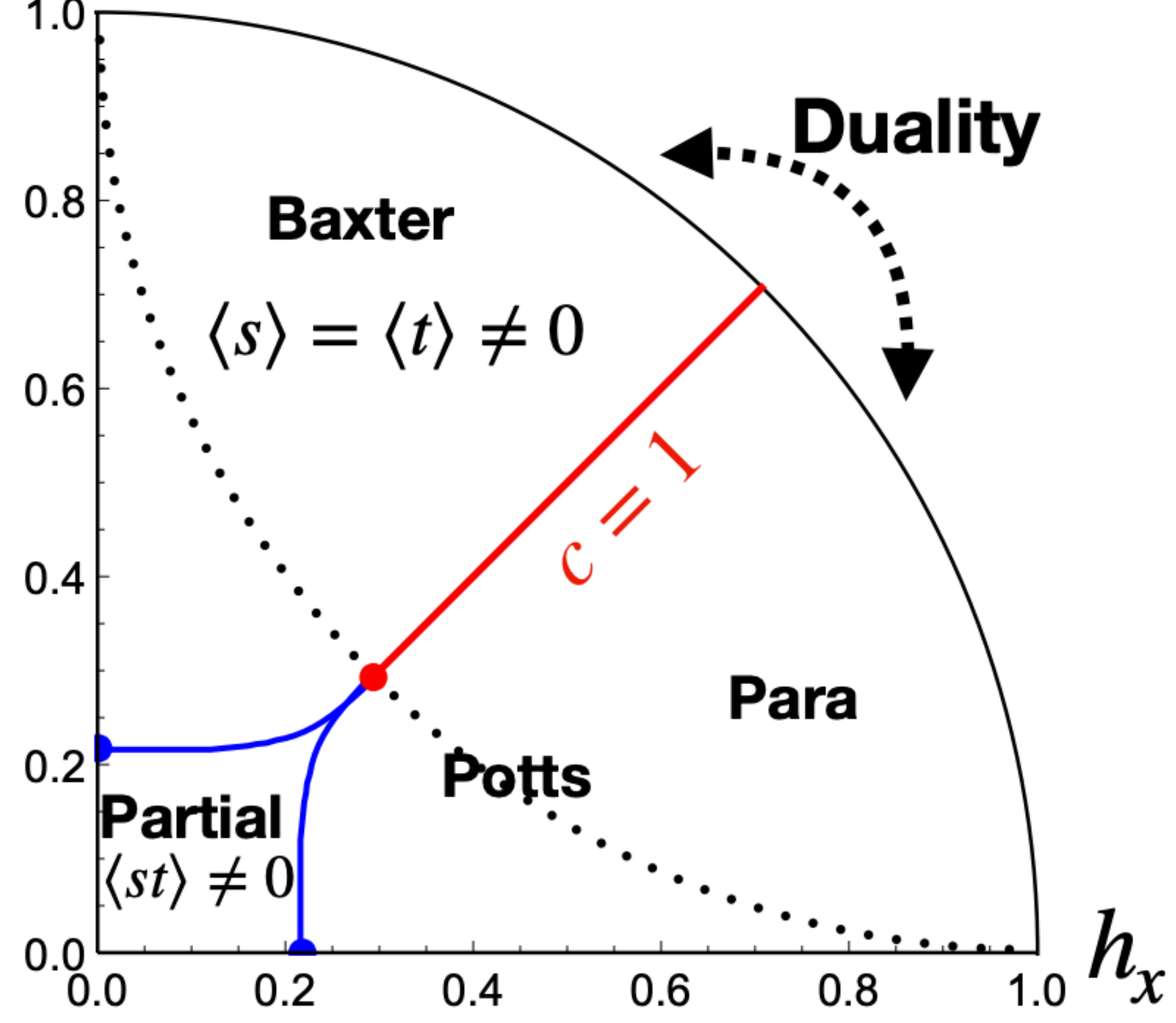
$$-E_{ij} = Js_{ij}\frac{\sigma_i\sigma_j + \tau_i\tau_j}{2} + i\phi\frac{\sigma_i\sigma_j - \tau_i\tau_j}{2} + \left(2K + i\pi\frac{1-s_{ij}}{2}\right)\frac{\sigma_i\sigma_j\tau_i\tau_j - 1}{2}$$

∞-**replica** model / post-selection

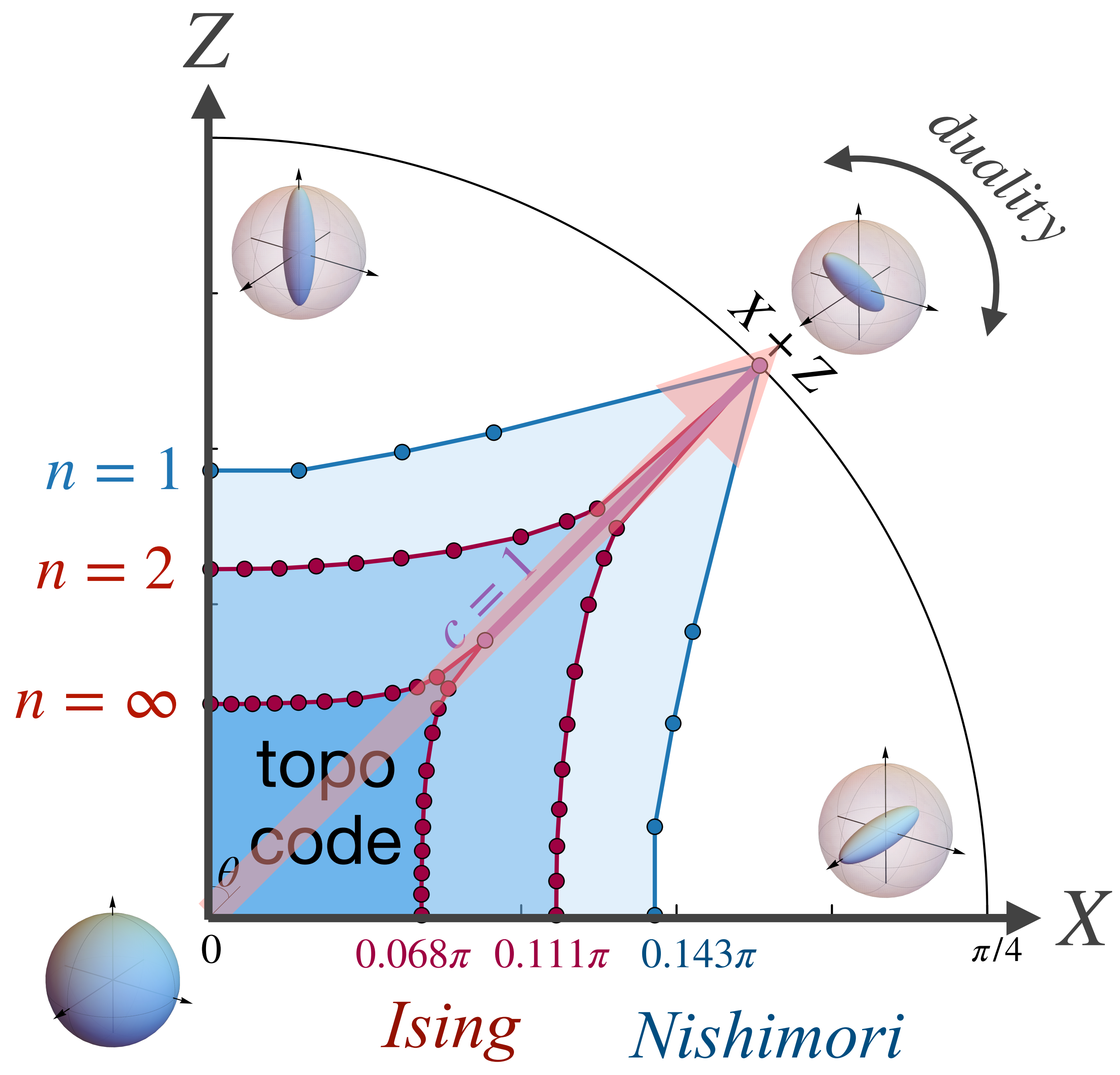
$\sum_{\mathbf{s}} P(\mathbf{s})^\infty$ distills out **most probable** configuration $s = +1$ → **no randomness**

exactly solvable

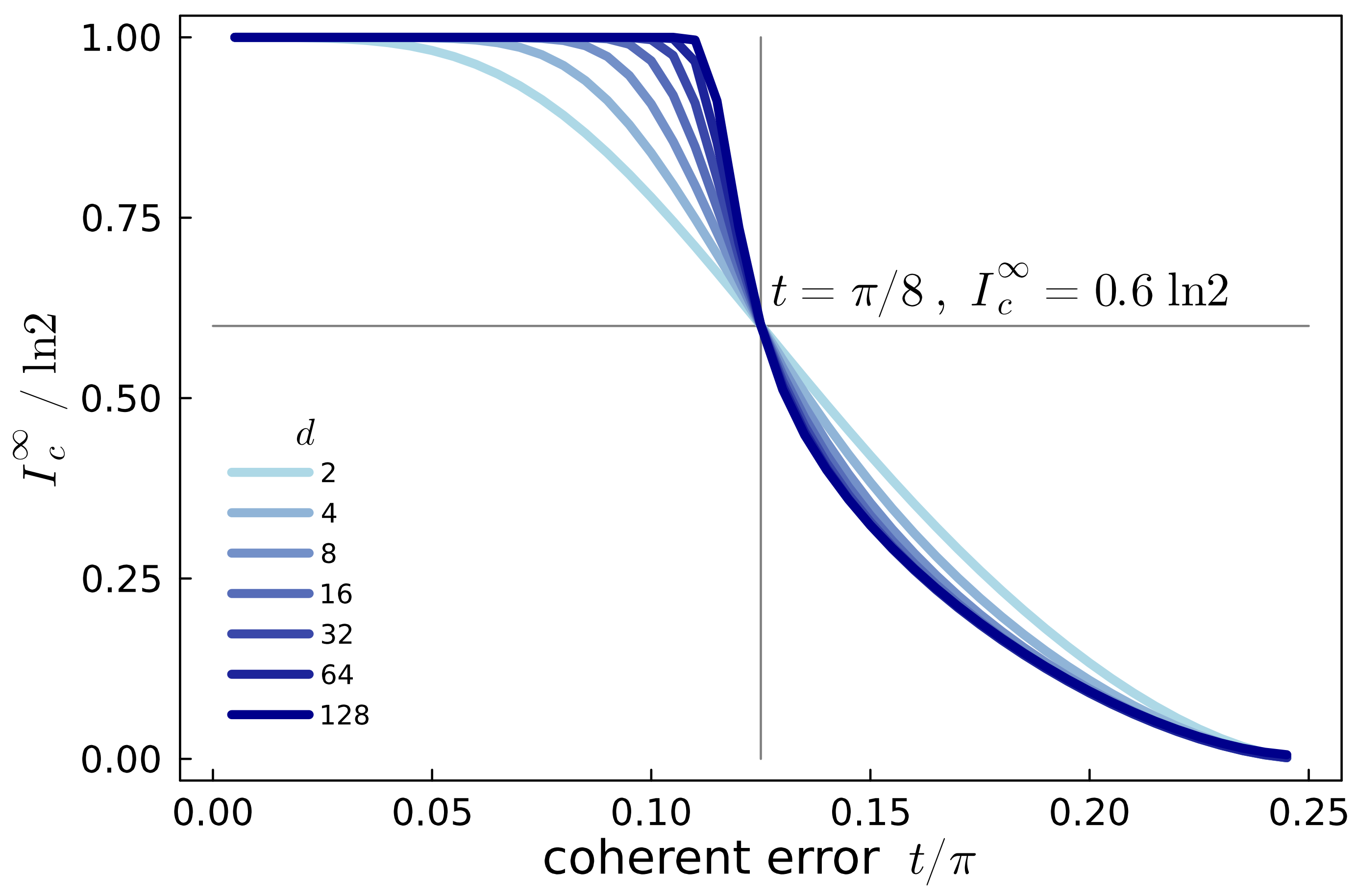
Ashkin-Teller model



Baxter 1982



Kosterlitz-Thouless transition



wave function deformation

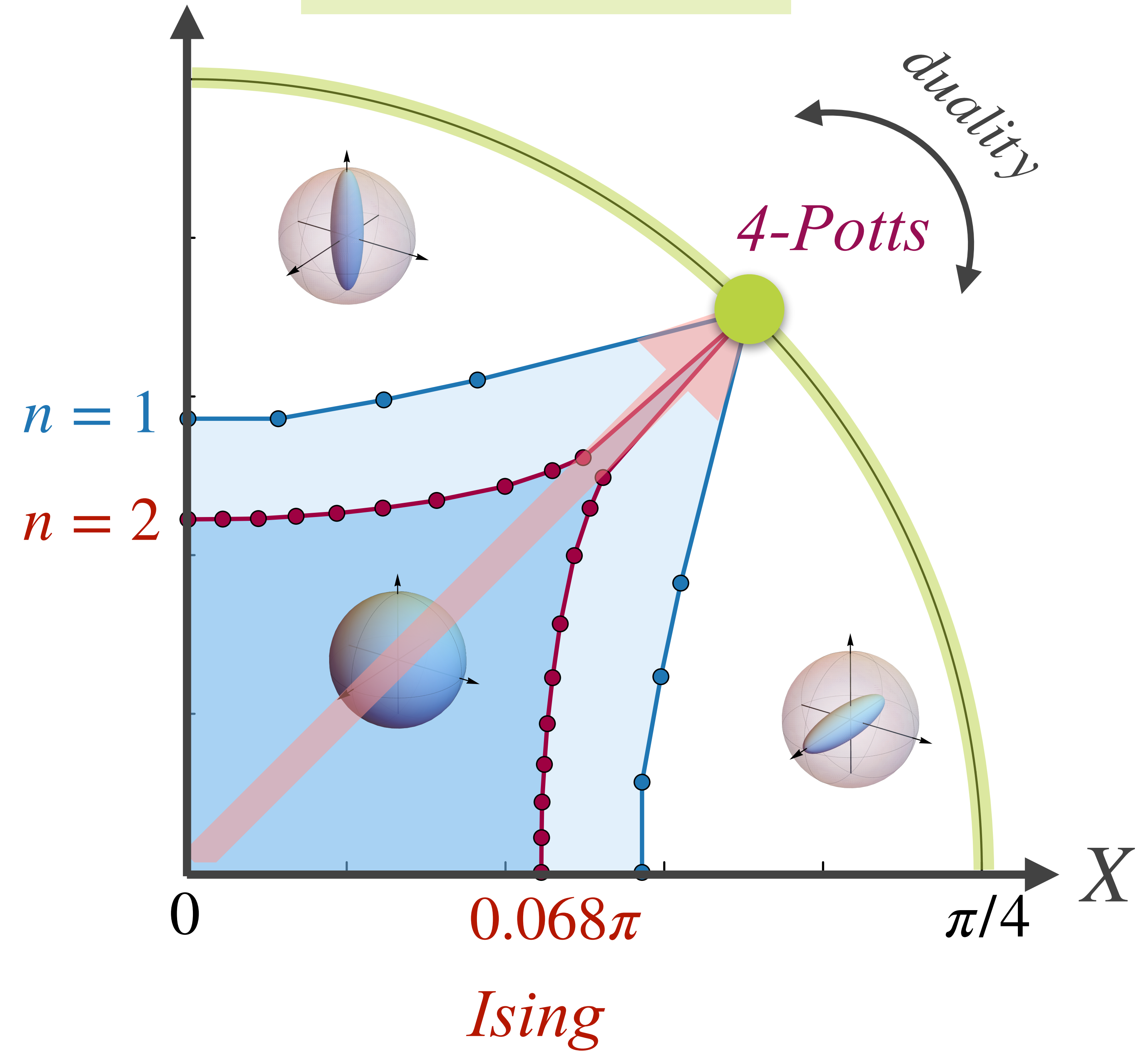
Ardonne, Fendley, Fradkin 2004; Zhu & Zhang, PRL 2019

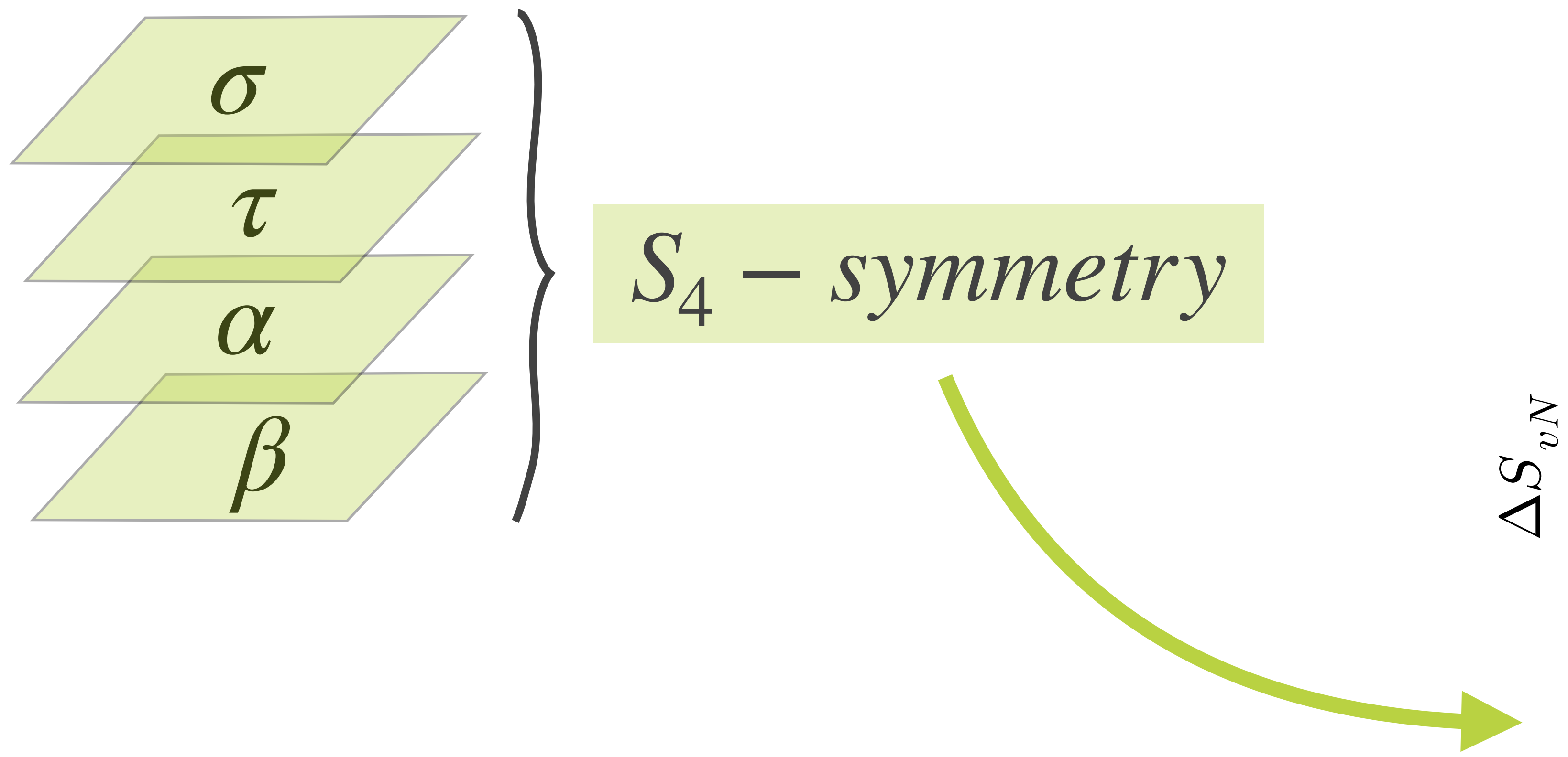
2-replica model

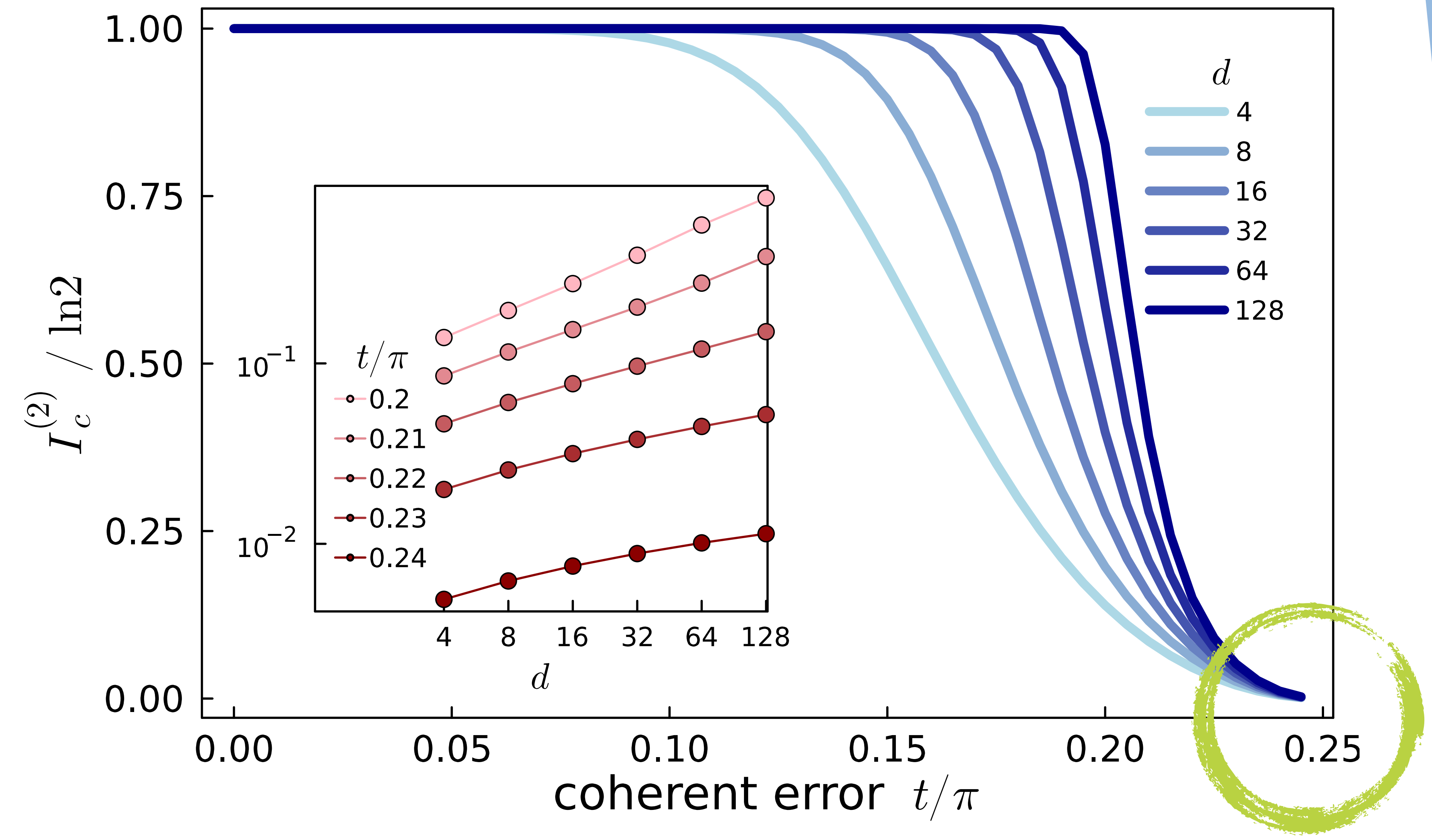
n-th order coherent information

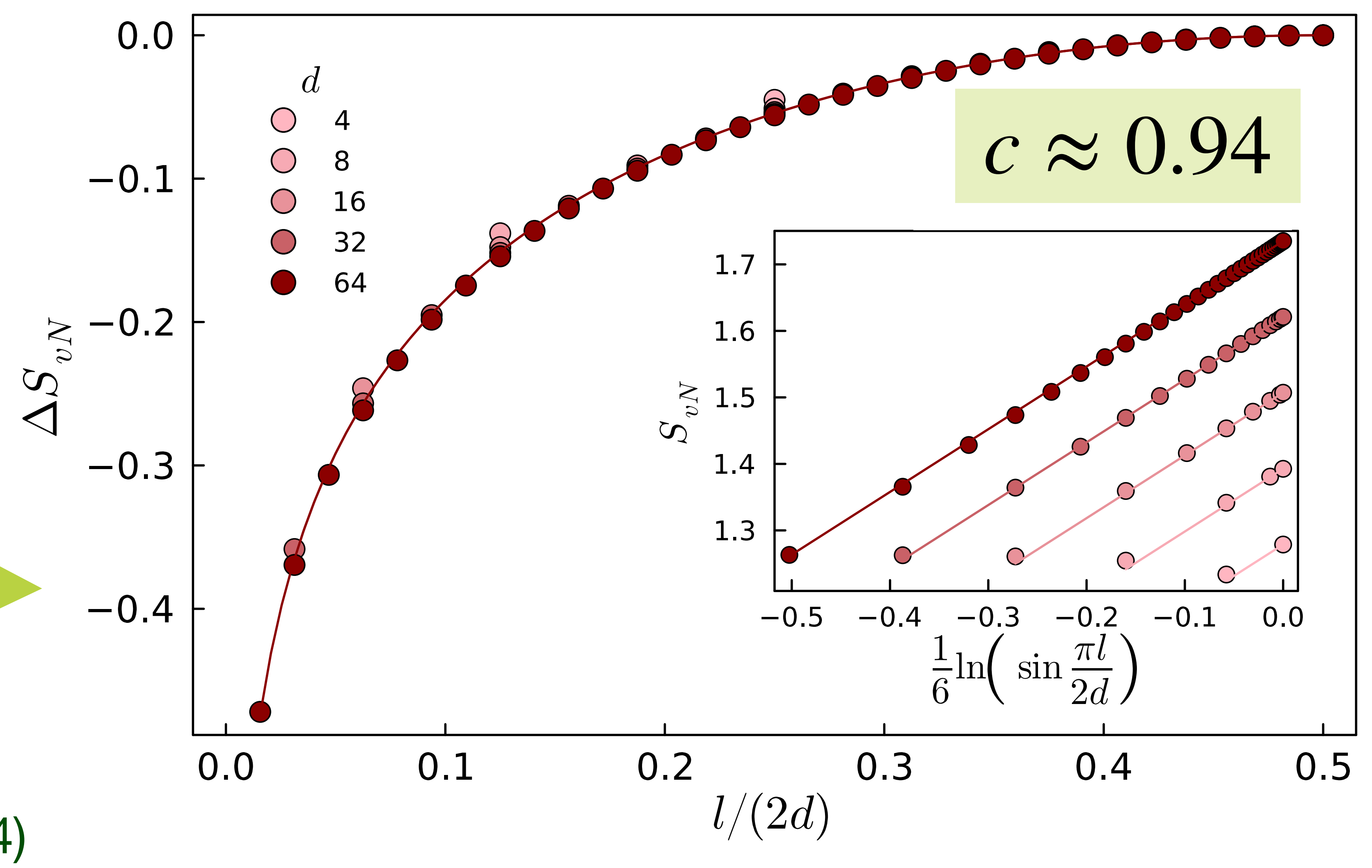
$$I_c^{(n)} = \frac{1}{1-n} \ln \frac{\mathrm{tr}(\rho_{RA}^n)}{\mathrm{tr}(\rho_A^n)} = \frac{1}{1-n} \ln[\mathrm{tr}\rho_R(\mathbf{s})^n]_n$$

n = 2 replica

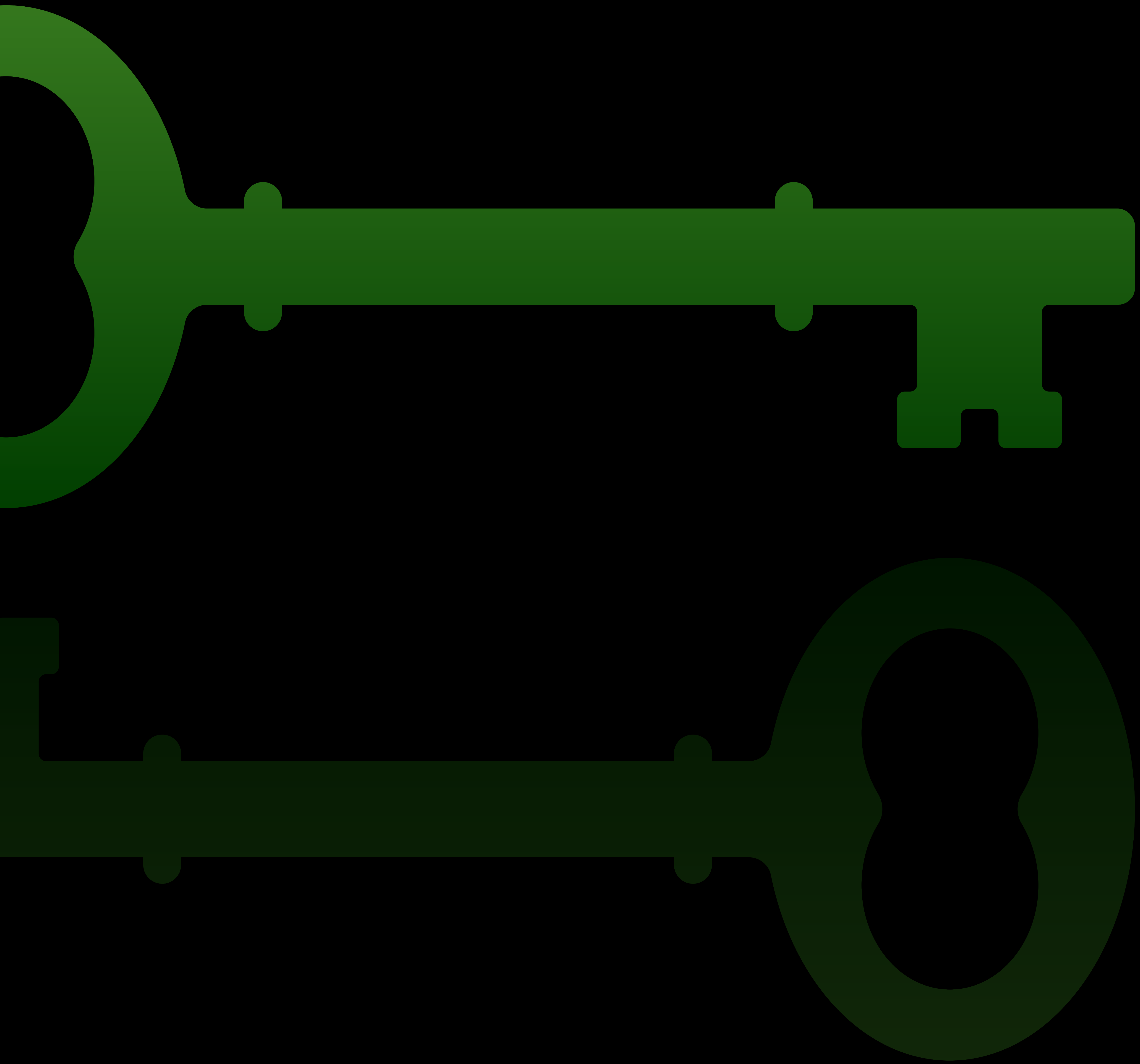








(2-replica study, see also Chen & Grover '24)

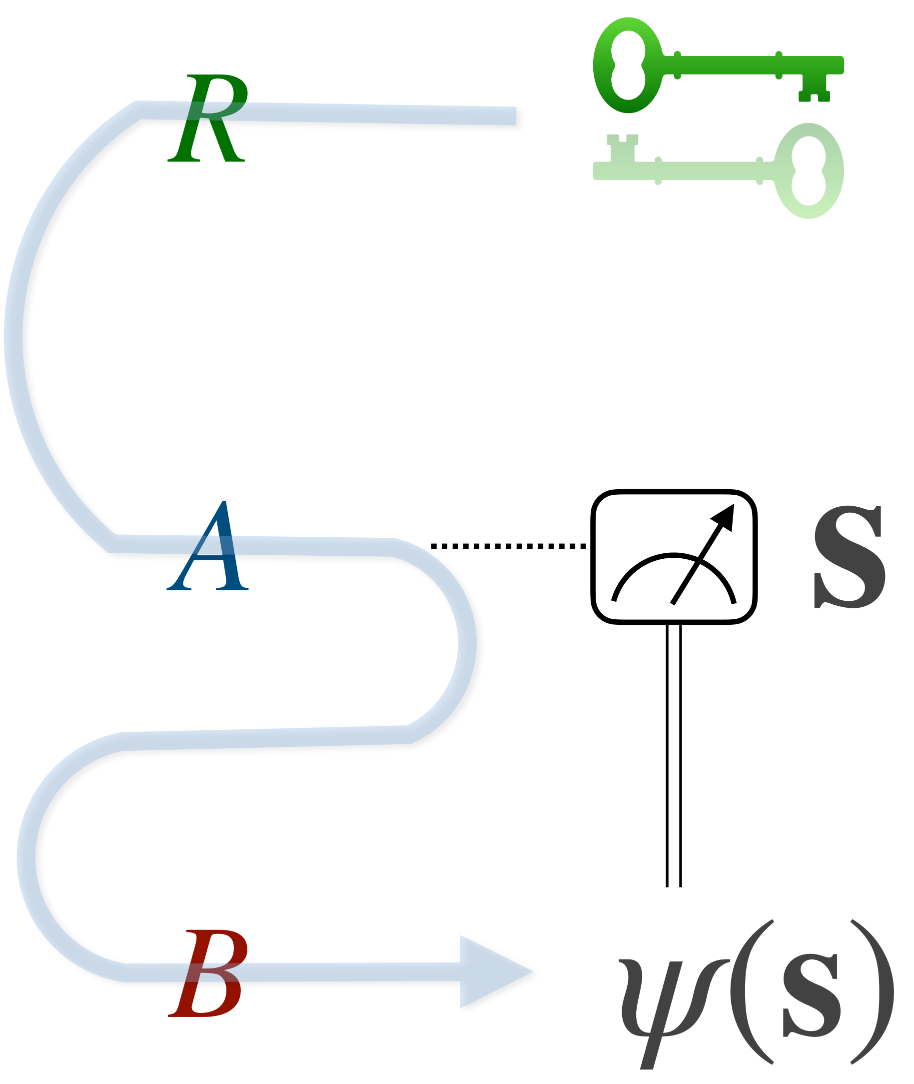


decoding
teleportation

decoding many-qubit teleportation

Who has the key / logical qubit?

- A quantum key **cannot be cloned.**
- Does the logical information **leak to Alice** or **flow to Bob** ?



Alice? $I_c = S_{RA} - S_A$

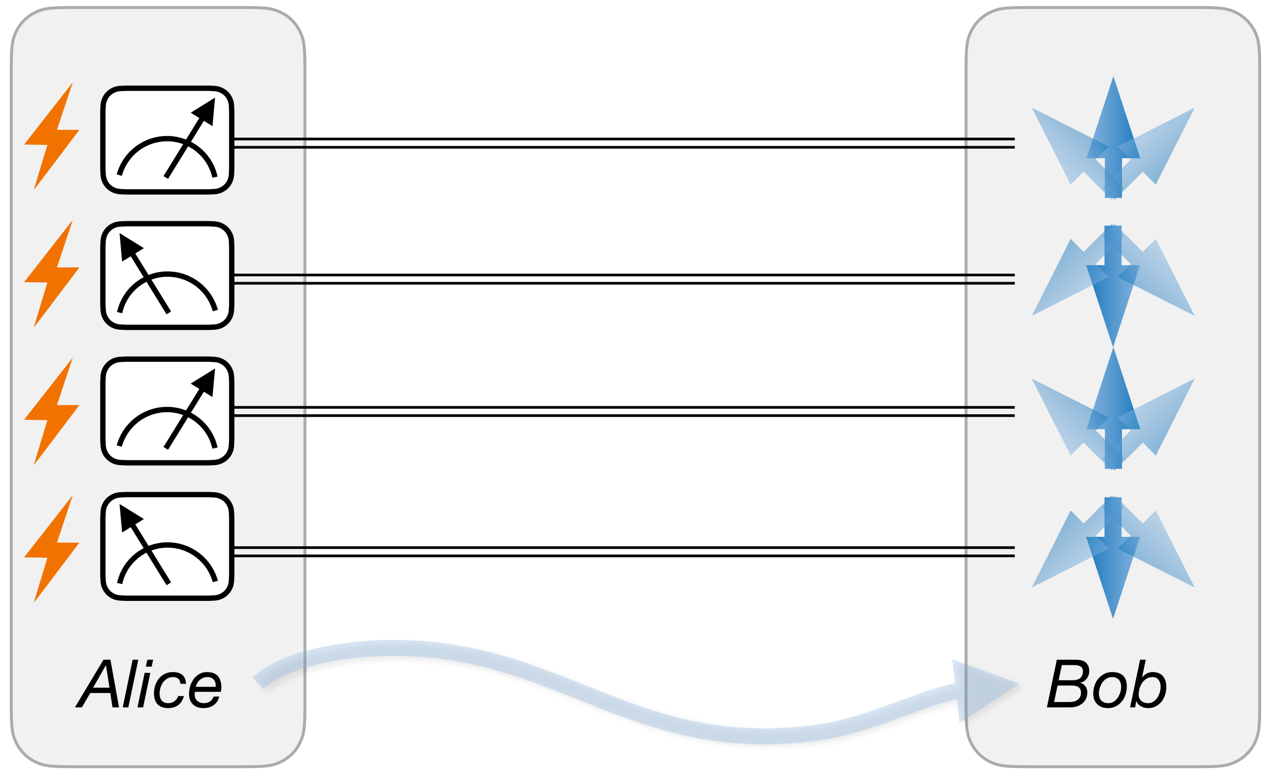
Scalable decoder
for entanglement transition
Gullans & Huse, PRL 2020

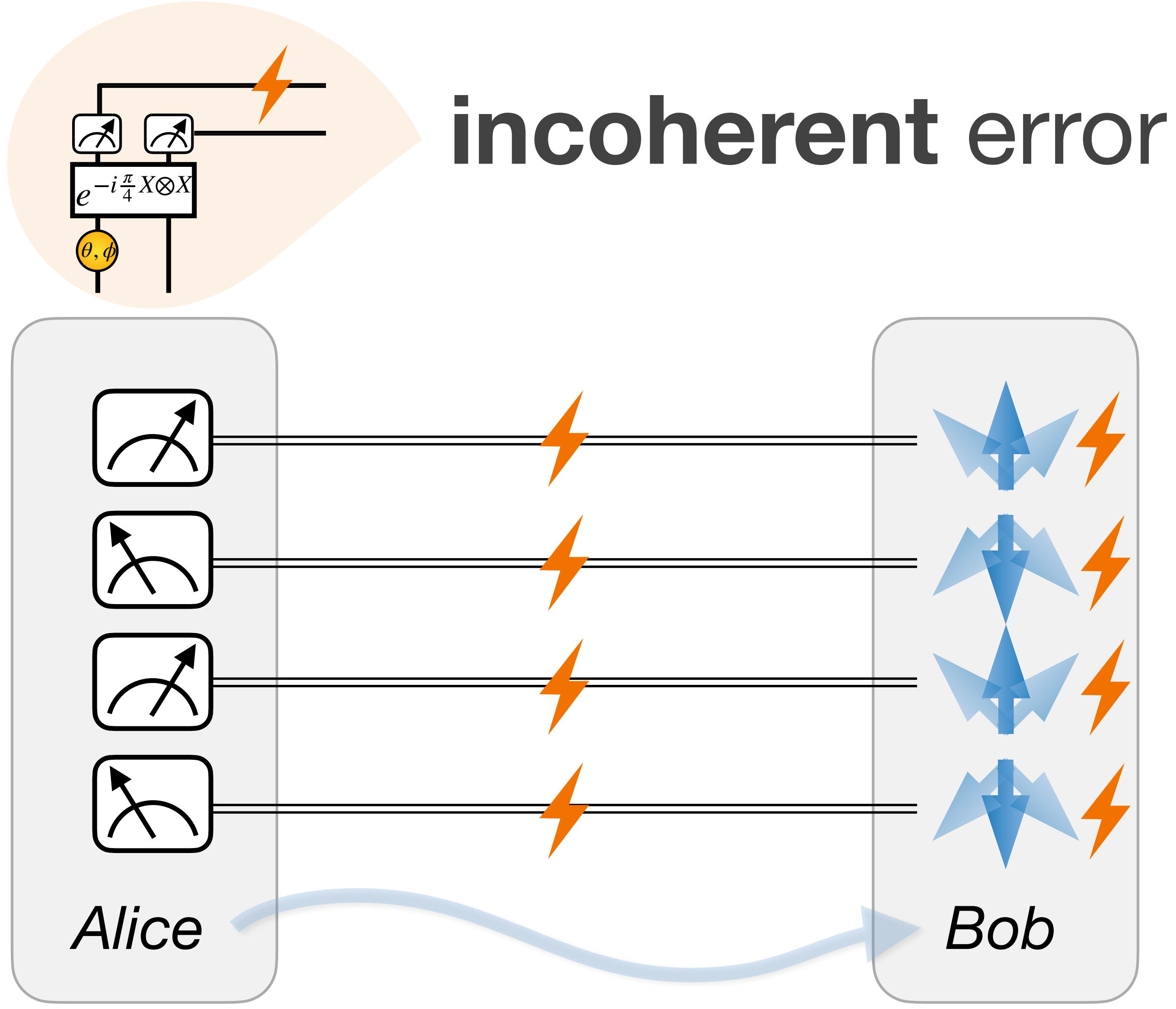
Bob? $I_c = S_{AB} - S_{RAB}$

error correction ?

decoding many-qubit teleportation

passive teleportation | **active** teleportation

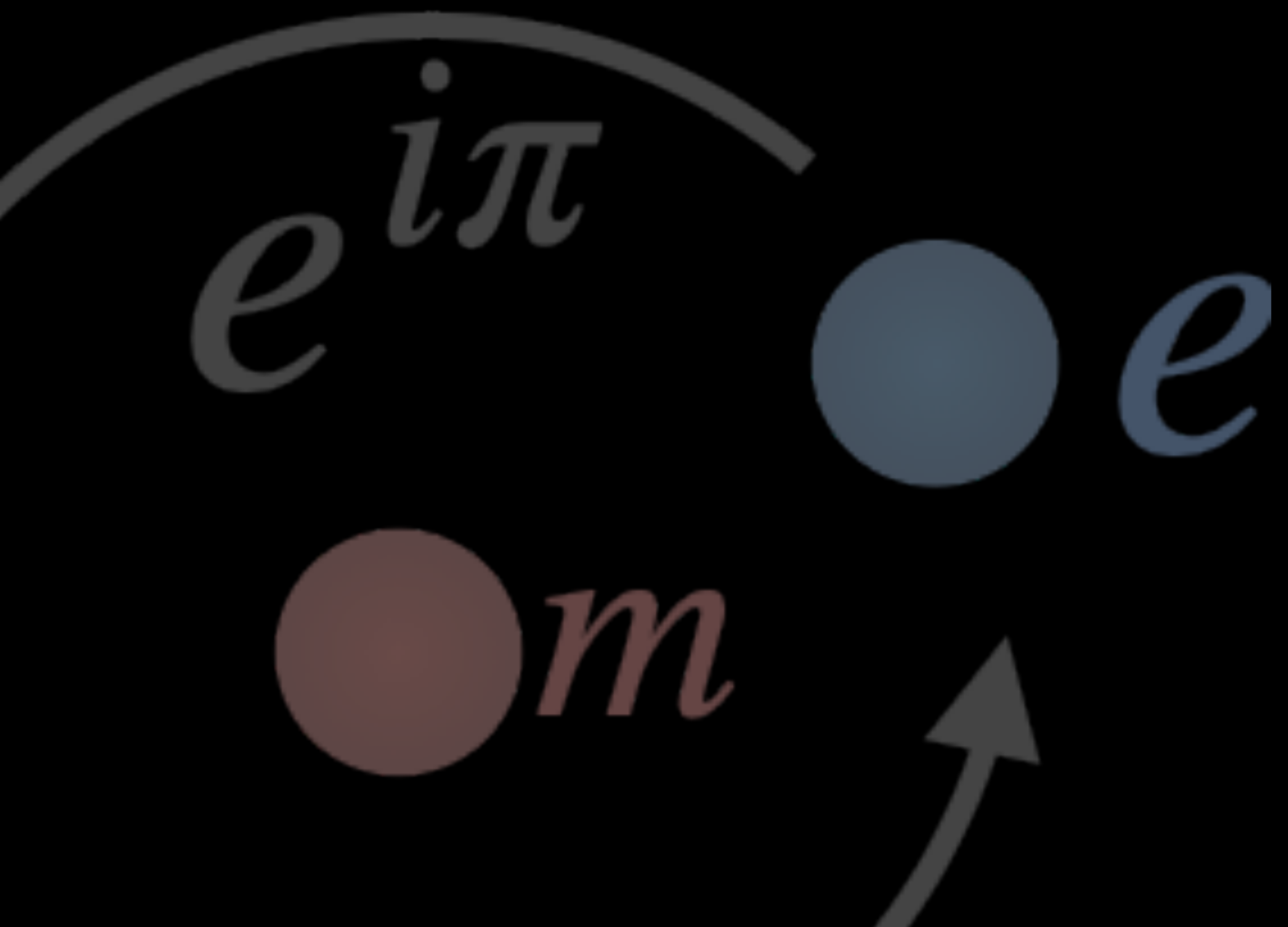




$p = \sin^2(t)$

	coherent error w/o decoder	**incoherent** error	**coherent** error w/ decoder
Z error $\theta = 0$	$t_c = 0.107\pi$	$p_c = 10.9\,\%$	$t_c = 0.143\pi$
$\theta = \pi/4$ $X + Z$ error	$t_c = \pi/4\,?$	$p_c = 50\%\,?$ (2-replica result, see also Chen & Grover '24)	$t_c = \pi/4$ (our finding)

summary

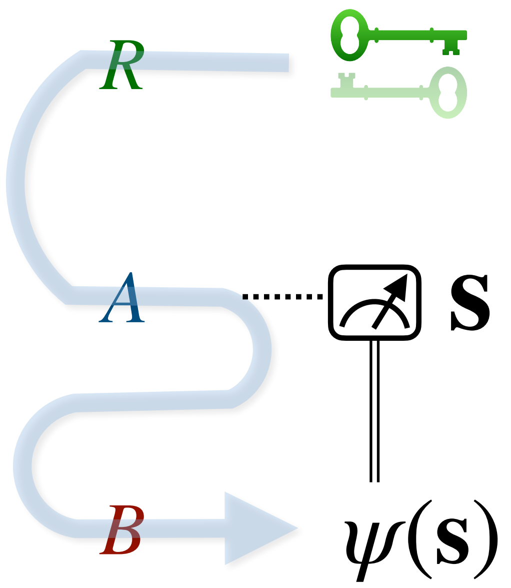


conclusions

Guo-Yi Zhu

- **teleportation of many-body state / logical qubit**

  self-duality ⇒ **optimal threshold**

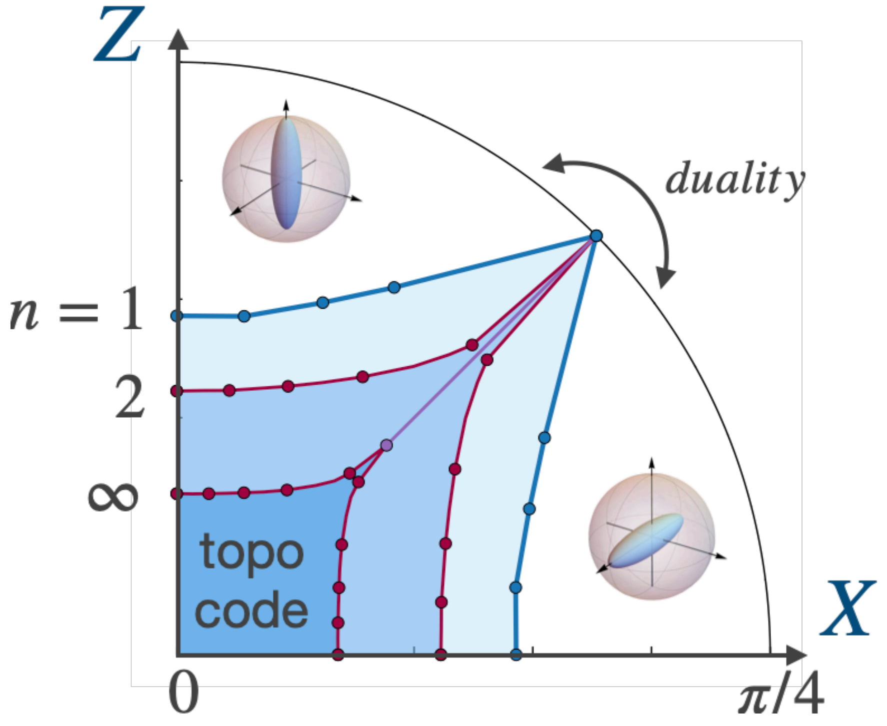


- **topological order**

  competing **anyon condensation** phase transitions

- **spin glass model**

  self-dual 4-state Potts | complex Ashkin-Teller model



- **experimentally feasible** in multiple NISQ platforms

tunable teleportation

↕

weak measurement

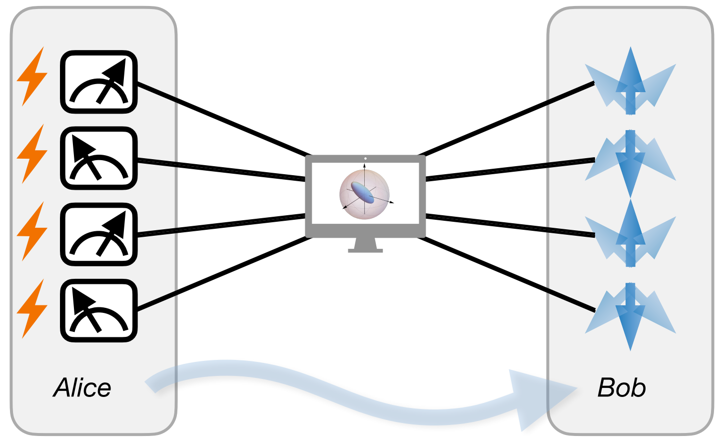
↕

wave function deformation

decoherence

Outlook

- non-unitary CFTs & non-Hermitian topology?
- coherent error + incoherent noise?



F. Eckstein, B. Han, ST, G.Y. Zhu, **arXiv: 2403.04767**

found by Guo-Yi