Suppression of the Two-Neutrino Double-Beta Decay by Nuclear-Structure Effects

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(Received 14 July 1986)

The nuclear matrix element for 2ν double-beta decay is calculated within the quasiparticle random-phase approximation. It is shown that the decay matrix element passes through zero as a function of the strength g^{pp} of the particle-particle component of the spin-isospin polarization force, neglected previously. The analysis of electron capture/β^+ decay rates for semimagic neutron-deficient nuclei suggests values for g^{pp} in the very vicinity of this zero, which gives rise to long lifetimes. The qualitative features of nuclear $\beta\beta$ decay are illustrated with the example of an exactly soluble model.

PACS numbers: 23.40.Bw, 21.10.Pc, 23.40.Hc

There are two modes of $\beta\beta$ decay, one involving the emission of two neutrinos (2ν mode) and another associated with no neutrino emission (0ν mode) which requires the existence of massive Majorana neutrinos. The 2ν mode has two electrons and two antineutrinos in the final state and occurs in the standard theory in second order. In order to interpret the results of experiments on $\beta\beta$ decay, we need to understand the nuclear-structure part of the decay process. The 2ν mode, being independent of the neutrino properties, offers a sensitive test of our ability to deal with this problem.

Calculation of the nuclear matrix elements for the 2ν $\beta\beta$ decay is the topic of this Letter. The general theory has been reviewed recently by Haxton and Stephenson[1] In all previous investigations the calculated decay rates were faster than the experimental ones. The discrepancy was particularly disturbing in the case of ^{130}Te, where experiment and theory differ by a factor of 100. We offer here a possible solution to this problem.

The nuclear matrix element for the $2\nu\,\beta\beta$ decay connecting the 0^+ ground states of two even-even nuclei is, in the standard approximation, given by

$$M_{\rm GT}=\sum_m \frac{\langle f\,|\,\boldsymbol{\sigma}\tau^+\,|\,m\rangle\cdot\langle m\,|\,\boldsymbol{\sigma}\tau^+\,|\,i\rangle}{E_m-(M_i+M_f)/2}. \tag{1}$$

Like Vogel and Fisher,[2] we use the quasiparticle random-phase approximation (QRPA) to calculate the matrix elements connecting both the initial and the final 0^+ state with each of the intermediate 1^+ states. The energies of the intermediate states, E_m, are obtained at the same time.

The QRPA equations, and the QRPA matrices A and B, are of the form

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\omega\begin{pmatrix} x \\ y \end{pmatrix}, \tag{2}$$

$$A_{pn,p'n'}=(\varepsilon_p+\varepsilon_n)\delta_{pn,p'n'}+g^{ph}\bar{V}_{pn,p'n'}(u_pv_nu_{p'}v_{n'}+v_pu_nv_{p'}u_{n'})+g^{pp}V_{pn,p'n'}(u_pu_nu_{p'}u_{n'}+v_pv_nv_{p'}v_{n'}), \tag{3a}$$

$$B_{pn,p'n'}=g^{ph}\bar{V}_{pn,p'n'}(v_pu_nu_{p'}v_{n'}+u_pv_nv_{p'}u_{n'})-g^{pp}V_{pn,p'n'}(u_pu_nv_{p'}v_{n'}+v_pv_nu_{p'}u_{n'}). \tag{3b}$$

Here, ε_i are the quasiparticle energies and u,v the usual occupation amplitudes. There are two types of two-body interaction matrix elements in (3a) and (3b); the particle-hole matrix element $\bar{V}$ and the particle-particle matrix element V. We parametrize both of them by the respective matrix elements of a δ-function interaction $[V=\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\delta(\mathbf{r}_1-\mathbf{r}_2)]$. In a first-principles calculation these two matrix elements would be related by angular-momentum recoupling. However, our use of an *effective* shell-model space requires the use of an *effective* two-body interaction, and we are forced to take the two interaction constants g^{ph} and g^{pp} as independent. The particle-hole coupling constant g^{ph} is determined from the requirement that the energy of the giant Gamow-Teller (GT) resonance be correctly reproduced; we here obtain $g^{ph}=-333$ MeV fm^3, with an uncertainty of $\sim 10\%$.

The particle-particle interaction has been neglected in all previous treatments of $\beta\beta$ decay which use the QRPA and in most papers dealing with the β^- and β^+ decays. This interaction indeed has little effect on the giant GT resonance. However, as we shall see, it plays an essential role when one tries to calculate strongly suppressed quantities such as the matrix element $M_{\rm GT}$ in (1) or the β^+ decay rates in nuclei with a neutron excess.

In order to determine the value of the particle-particle coupling constant g^{pp}, we use the following procedure.

First, we note that the β^+ strength, which is suppressed by Pauli blocking, depends sensitively on g^{pp}. There is no experimental information on this strength from (n,p) reactions. However, in several semimagic neutron-deficient nuclei the electron capture/β^+ decay is expected to contain essentially the whole β^+ strength. We thus use the experimental ft values for the decays of ^{94}Ru, ^{96}Pd, ^{148}Dy, ^{150}Er, and ^{152}Yb (from Kleinheinz *et al.*,[3] Kurcewicz *et al.*,[4] and Nolte *et al.*[5]) to fix g^{pp}. In these nuclei the extreme shell model overestimates the β^+ strength by a factor of ~ 7. If we assume, following Towner,[6] that the interactions not accounted for (Δ isobar, etc.) can be included approximately by use of an axial-vector-current coupling constant $g_A = 1.0$ instead of 1.25, we obtain a good description of the experimental ft values with g^{pp} in a relatively narrow interval 128–144 MeV fm^3. The actual value of g^{pp} depends to some extent on the number of spherical subshells used in the calculation. (Addition of a pair of subshells changes the effective g^{pp} typically by $\sim 10\%$.) In our calculation we used all subshells within 10 MeV of the Fermi energy plus their spin-orbit partners. The single-particle energies were taken from a Wood-Saxon Coulomb-corrected potential, and the same number of subshells was used for neutrons and protons. QRPA calculations of $M_{\rm GT}$ are carried out for both the initial and final nuclei and the resulting matrix elements are averaged. The BCS equations were solved with a δ-function interaction of strength $g^{\rm pair} \sim -300$ MeV fm^3. It is interesting to note that the effective coupling constants $-3g^{pp}$ and $g^{\rm pair}$ are very close to each other.

We now turn to the evaluation of the 2ν $\beta\beta$ decay matrix element $M_{\rm GT}$. The matrix elements in Eq. (1) are obtained from

$$\langle 1_m^+ | \sigma\tau^+ | 0_i^+ \rangle = \sum_{pn} \langle p \| \sigma \| n \rangle (u_p v_n x_{pn}^m + v_p u_n y_{pn}^m), \quad (4a)$$

$$\langle 0_f^+ | \sigma\tau^+ | 1_m^+ \rangle = \sum_{pn} \langle p \| \sigma \| n \rangle (v_p u_n x_{pn}^m + u_p v_n y_{pn}^m). \quad (4b)$$

For nuclei with a neutron excess the complex conjugate of the β^+ matrix element, Eq. (4b), is small and very sensitive to the amount of pairing and ground-state correlations, characterized by the factors u, v, and the amplitudes y_{pn}^m, respectively. The terms proportional to x and y on the right-hand side of (4b) typically have opposite signs, a feature that can be understood from the following argument. We write the second of the QRPA equations (2) in the form $-Bx = (A+\omega)y$. Using the premise that the matrix $(A+\omega)$ is positive definite, we see that the relative sign of x and y is controlled by the sign of B. The latter matrix is largely positive, as a result of the repulsive nature of the particle-hole interaction, the attractive nature of the particle-particle interaction, and the way in which these interactions enter into the expression (3b). We note further that the two terms $v_p u_n x_{pn}^m$ and $u_p v_n y_{pn}^m$ are of the same order because the intrinsic smallness of y is compensated for by the difference in occupation amplitudes. As the value of g^{pp} increases, the second term in fact overpowers the first one, and each matrix element on the left-hand side of (4b) *passes through zero.* (For very large values of g^{pp} the QRPA becomes unstable and gives rise to complex frequencies.) As the β^- matrix elements (4a) are only weakly dependent on g^{pp}, $M_{\rm GT}$ passes through zero, too. Figure 1 illustrates this point. (The decay rate is proportional to the square of $M_{\rm GT}$; see Refs. 1 and 2 for the coefficient of proportionality.) We observe that the 2ν $\beta\beta$ decay matrix element $M_{\rm GT}$, and the decay rate, vanish for $g^{pp} \sim 110$ MeV fm^3 and vary very fast with g^{pp} in the vicinity of this value. This does not mean that the terms in Eq. (1) vanish all at once. Rather, the individual terms in the sum over intermediate states cancel each other. The closure approximation in which the E_m in Eq. (1) are replaced by a constant is of dubious value at this point, and the whole sum can be as small as the contribution of a single term, a feature noted[7] in the case of ^{128}Te. Quantitative prediction of the $\beta\beta$ decay lifetime now becomes very difficult. On the other hand, one can easily understand very long lifetimes in some nuclei, such as ^{130}Te.

The extreme sensitivity to g^{pp} may explain why the suppression of $M_{\rm GT}$ observed here went unnoticed in the shell-model work of Haxton and Stephenson as described in Ref. 1. Their G matrix definitely contains the particle-particle interaction that we find to be of crucial importance, but, because of the considerable uncertainty in present effective interactions, it is not clear whether the relevant interaction matrix elements are correctly

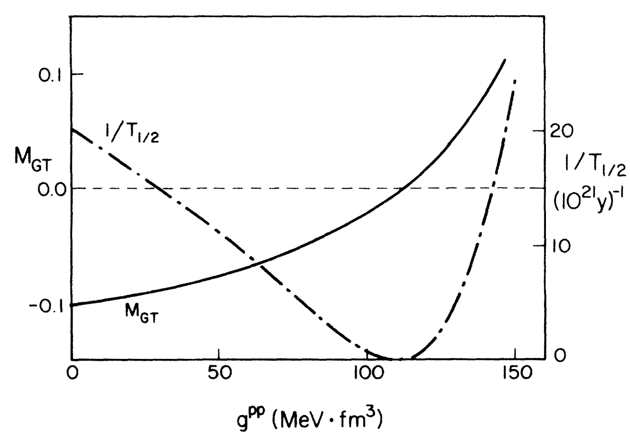

FIG. 1. The matrix element $M_{\rm GT}$ (in units of m_e^{-1}) and the decay rate for ^{130}Te as a function of the particle-particle coupling constant g^{pp}.

represented.

Is the described behavior of $M_{\rm GT}$ a genuine property of nuclei or just an artifact of the QRPA? To be able to give a convincing answer to this question, we have constructed a schematic model which can be solved exactly and yet contains most of the qualitative features of a more realistic description. We consider a set of single-particle states with the associated creation and time-reversed annihilation operators, $a^\dagger$ and $\tilde{a}$, which we label by their orbital angular momentum, spin, isospin, and the respective z components. Using LS coupling we then define sixteen particle-hole operators and twelve pair operators with definite spin and isospin; in all of them the orbital angular momenta are coupled to zero. These operators close under commutation; they form, in fact, the Lie algebra of SO(8).

From these operators we construct the following Hamiltonian:

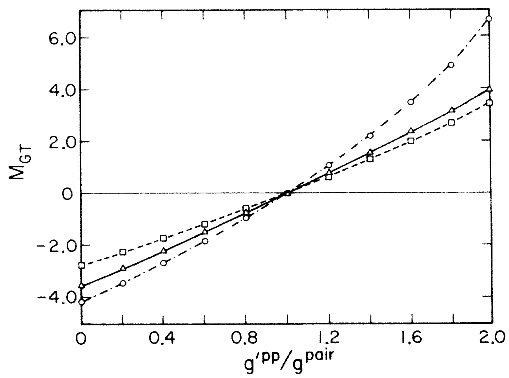

FIG. 2. Dependence of $M_{\rm GT}$ (in arbitrary units) on $g'^{pp}/g^{\rm pair}$ for the exactly soluble model described in the text. The full line connecting triangles is the exact solution, and the dashed line (dot-dashed line) connecting squares (circles) is the QRPA result for the initial (final) nucleus.

$$H = g^{\rm pair}(S^+_{nn}S^-_{nn} + S^+_{pn}S^-_{pn} + S^+_{pp}S^-_{pp}) + g'^{pp}P^+\cdot P^- + \tfrac{1}{2}g'^{ph}\sum_{\mu\nu}(-1)^{\mu+\nu}F^\nu_\mu F^{-\nu}_{-\mu}, \tag{5}$$

where S^+ is the pair creation operator with spin 0 and isospin 1 (S^- is the adjoint operator), P is the pair operator with spin 1 and isospin 0, and F^ν_μ is the particle-hole operator with spin 1, projection μ, and isospin 1, projection ν (Gamow-Teller operator). Note that this Hamiltonian is properly isoscalar and rotationally invariant. Moreover, it contains all of the features of the nuclear residual interaction directly relevant to $\beta\beta$ decay: pairing, the particle-hole interaction in the $J^\pi = 1^+$ channel, and the corresponding particle-particle interaction. What is missing from the Hamiltonian (5) is the nondegeneracy of the single-particle subshells in actual nuclei and especially the splitting of spin-orbit partners by the spin-orbit force.

The virtue of the Hamiltonian (5) is that its eigenvalues and eigenfunctions can be worked out *exactly*. In Fig. 2 we show the resulting $\beta\beta$ decay matrix element as a function of the ratio $g'^{pp}/g^{\rm pair}$. The parameters we use are $g^{\rm pair} = -0.30$, $g'^{ph} = +1.5$, and we take for the initial nucleus four proton particles and six neutron holes in a major shell with $l = 0,2,4$. Figure 2 is a clear and indisputable demonstration of the effect of increasing the value of g'^{pp}: It forces $M_{\rm GT}$ to pass through zero!

In the present model, the vanishing of $M_{\rm GT}$ has a clear-cut explanation. For $g^{\rm pair} = g'^{pp}$ and arbitrary g'^{ph}, H acquires a dynamical SU(4) symmetry (Wigner's supermultiplet symmetry), which leads to new conserved quantum numbers in addition to those of spin and isospin. Because of the pair superstructure of our model, it is useful to reinterpret SU(4) as SO(6), by use of the known isomorphism between these two algebras. It can then be shown that the ground state of H for $g^{\rm pair} = g'^{pp}$ is the highest-weight state of a totally symmetric irreducible representation of SO(6). Because the Gamow-Teller operator is one of the raising operators of

TABLE I. Lifetimes of the 2ν $\beta\beta$ decay. Lifetimes are given in years, coupling constants in megaelectronvolts per cubic femtometers.

Interaction parameters	$T_{1/2}^{\rm exper\ a}$	$T_{1/2}^{\rm calc}$ Schematic interaction $\chi = 23/A$	δ interaction $g^{pp} = 0$	δ interaction $g^{pp} = 144$	δ interaction $g^{pp} = 136$	δ interaction $g^{pp} = 128$
^{76}Ge	$> 8\times10^{19}$	1.8×10^{21}	1.6×10^{21}	5.0×10^{20}	1.8×10^{21}	6.3×10^{21}
^{82}Se	$> 1\times10^{20}$	3.3×10^{19}	7.4×10^{19}	5.5×10^{19}	2.5×10^{20}	6.3×10^{20}
^{96}Zr	...	...	2.8×10^{18}	5×10^{17} b	3.2×10^{17}	8.5×10^{18}
^{100}Mo	...	2.1×10^{18}	4.1×10^{18}	8×10^{17} b	2.8×10^{18}	3.7×10^{19}
^{116}Cd	...	...	7.5×10^{19}	9.3×10^{19}	3.4×10^{20}	1.9×10^{21}
^{128}Te	$> 7\times10^{24}$	1.1×10^{23}	1.5×10^{23}	1.0×10^{23}	2.7×10^{23}	8.2×10^{23}
^{130}Te	2.6×10^{21}	2.3×10^{19}	5.0×10^{19}	4.4×10^{19}	1.0×10^{20}	2.5×10^{20}
^{136}Xe	$> 2\times10^{19}$	2.3×10^{19}	1.5×10^{20}	2.0×10^{20}	5.1×10^{20}	1.5×10^{21}

[a]Experimental data from the following sources: Ge, Ref. 8; Se, Ref. 9, see also Ref. 10; Te, Ref. 10; Xe, Ref. 11.

[b]Unstable solution for the final nucleus; only the initial nucleus is used.

SO(6) ≃ SU(4), it follows that the β^+ strength of the final nucleus vanishes identically, and that $M_{\rm GT}$ must vanish, too. It is gratifying to see that the QRPA reproduces the location of this zero exactly. We have investigated several different parameter sets and find that the qualitative behavior is always similar to that seen in Fig. 2. In heavy nuclei the SU(4) symmetry is badly broken by the spin-orbit splitting. Nevertheless, we believe that the exact solution of the schematic model is instructive for two reasons. It shows that the QRPA yields a reasonable approximation to $M_{\rm GT}$, and it demonstrates the existence of a zero in $M_{\rm GT}$ in more general situations, as any perturbation to the Hamiltonian (5) can only change details of the curve in Fig. 2 but not its qualitative behavior.

In Table I we show the calculated $2\nu\,\beta\beta$ decay lifetimes for a number of spherical $\beta\beta$ decay candidates. For comparison we show in column 3 the lifetimes obtained with a schematic interaction,[2] which are very similar to those in column 4. The last three columns give lifetimes for three closely spaced values of g^{pp} in the predetermined interval. The extreme sensitivity is obvious. All lifetimes (except for column 3 where the effect of the Δ isobar was taken into account explicitly) were calculated with $g_A = 1.0$, as mentioned earlier. The experimental values are within, or close to, the boundaries of the calculated interval of lifetimes. It should be clear that the observed suppression makes $M_{\rm GT}$ susceptible to corrections arising from other components of the wave function such as quadrupole correlations, not considered here.

In conclusion, we have shown that the particle-particle interaction causes a large suppression of $M_{\rm GT}$. The existence of a zero in this matrix element is a consequence of general properties of the nuclear residual interaction. Is the same sensitivity to g^{pp} found here present for the 0ν mode as well? Detailed analysis will be necessary to answer this question.

It is a pleasure to thank S. E. Koonin, H. A. Weidenmüller, and A. Winther for valuable discussions. This work was supported by the U. S. Department of Energy under Contract No. DE-AT03-81ER40002 and by the National Science Foundation, under Grants No. PHY82-07332 and PHY85-05682.

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