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Nonequilibrium Physics: Problem Sheet 4

www.thp.uni-koeln.de/∼as/noneq13.html

7. Poker as a random walk problem

In the lecture we have argued that the number of chips of a Poker player in the absence of all-in bets is described by the discrete equation

$$x(t + \Delta t) = x(t) + \epsilon(t)b(t) \tag{1}$$

where $\epsilon(t)$ is a δ-correlated noise term with $\langle\epsilon(t)\rangle = 0$ and $b(t) = b_0 e^{t/t_0}$ is the blind at time t.

a) Perform the continuum limit in time ($\Delta t \to 0$) to derive a Langevin equation for $x(t)$.

b) Using the Langevin equation from a), derive the Fokker-Planck equation for $P(x,t)$, the number of remaining players with x chips at time t.
Hint: See the general procedure described earlier in the lecture.

c) Show that

$$P_\pm(x,t) = \frac{N_0}{\sqrt{2\pi\tau(t)}} \exp\left(-\frac{(x \pm x_0)^2}{2\tau(t)}\right)$$

are solutions of the Fokker-Planck equation if $\dot{\tau}(t) = \sigma^2 b^2(t)$ where σ is the rescaled noise strength (see problem a)).

d) Show that $P(x,t) = P_+(x,t) - P_-(x,t)$ is the solution which satisfies the (absorbing) boundary condition $P(x = 0, t) = 0$ and the initial condition $P(x, t = 0) = N_0\delta(x - x_0)$.
Hint: Determine τ by integrating the condition derived in c). Show $\tau(t) \to 0$ for $t \to 0$. Note that $x(t) \geq 0$ for the "physical" solution.