Comment on "Critical behavior of a traffic flow model"

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(April 6, 2000)

We show that the dynamical structure factor investigated by L. Roters et al. (Phys. Rev. **E59**, 2672 (1999)) does not allow the determination of the precise nature of the transition in the Nagel-Schreckenberg cellular automata model for traffic flow. We provide evidence for the existence of a crossover instead of a critical point.

PACS numbers: 05.40.-a, 02.50.Ey, 89.40.+k, 05.60.-k

In a recent paper L. Roters et al. [1] have investigated the dynamical structure factor

$$S_v(k,\omega) \;=\; \frac{1}{NT}\left\langle\left|\sum_{n,t} v_{n,t}\, e^{i(kn-\omega t)}\right|^2\right\rangle, \qquad (1)$$

of the Nagel-Schreckenberg (NaSch) model [2] of traffic flow. This turned out to be an interesting quantity to look at. Along a ridge with negative slope – corresponding to backwards moving jams – an algebraic behavior $S_v(k,\omega)\big|_{\omega/k=v_{\rm j}} \sim k^{-\gamma}$ of the structure factor is found above a transition density.

This was interpreted as an indication of critical behavior. Moreover, the transition was suggested to be at a well-defined critical point. In Ref. [1] it is also claimed that the results of [3] support their interpretation that "traffic jams occur on all time scales".

However, in [3] only the so-called *cruise-control limit* was investigated which is known to organize itself into a critical state. For the unmodified NaSch model of [2] it has been shown earlier [4] that a cutoff in the lifetime distribution of jams near $\tau_c = 10000$ exists for $v_{max} = 5$ (see Fig. 1). Only for times smaller than τ_c the distribution appears to decay algebraically. Note that density $\rho = 0.1$ is already well above the transition density[1]. We want to emphasize that the existence of the cutoff is no finite-size or finite-time effect [4] but an intrinsic feature of the NaSch model.

For the calculation of the structure factor in [1] a discretization $k = 2\pi m_k/N$ and $\omega = 2\pi m_\omega/T$ (with integers m_k, m_ω) is necessary. From Fig. 4 of [1] one sees that the largest value of T considered was $T = 2048$. Therefore the longest lifetime contributing to the structure factor is $\tau = 2048$. This is much smaller than the cutoff found in [4] and lies well in the region where an algebraic decay is found. Therefore the results of [1] and [4] are consistent, but the algebraic decay is not to be interpreted as an evidence for the existence of a critical point in the NaSch model. In order to see the cutoff times $T > 10^4$ have to considered. Therefore one should look at the long-time behavior of very large systems. Here one finds a cutoff which depends only on the randomization p, but not on L, N or T.

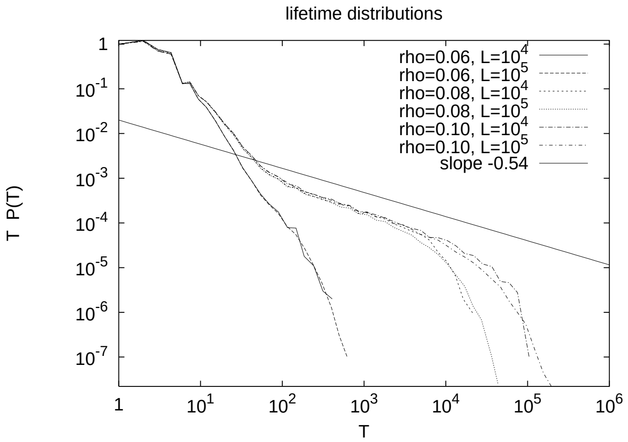

FIG. 1. Lifetime distribution in the NaSch model for $v_{max} = 5$ and $p = 0.5$. For details, see [4].

There are several results which provide evidence for the existence of a crossover, i.e. a change from the free-flow to the jammed phase without any nonanalytic behavior:

- Cutoff in the lifetime distribution [4]. This has been discussed above. In [4] each car having a velocity less than v_{max} before the randomization step is jammed. Therefore the cutoff in the lifetimes should also be observable for all definitions of jams which count only a subset of those in [4], e.g. the one used [1]. We expect the long-time behavior of the lifetime distribution to be independent of the definition, whereas the behavior for small times might depend strongly on this definition.
- Spatial correlations do not show algebraic decay [7]. Density-density correlations decay exponen-

[1] Ref. [1] does not give a numerical value for the critical density. From [5–7] (see also [8]) we know that the transition occurs below the density of maximum flow which is $\rho_{max} = 0.085 \pm 0.005$ for $v_{max} = 5$. This is also consistent with the data presented in Fig. 3 of [9].

tially for all densities. The maximal correlation length diverges only in the limit $p \to 0$. This is consistent with the existence of a critical point at density $\rho_c = 1/(v_{max}+1)$ in the deterministic case $p = 0$ [10]. For $p > 0$ also the distribution of jam sizes shows an exponential behavior. Therefore one does not find any indication of critical behavior in the spatial direction in this case.

- Measurements of the relaxation time [5–7]. The relaxation time suggested in [5] shows interesting behavior which is difficult to interpret in terms of critical point phenomena. While the maximal value of the relaxation time increases as a function of the system size, the width of the transition region does not seem to shrink. (Note that the problems with apparently negative relaxation times occur only far above the transition.) This would mean that – if the relaxation time indeed goes to infinity for $L \to \infty$ – the divergence occurs in an interval rather than at a specific (critical) point. However, in view of the lifetime measurements, one is tempted to think that probably the relaxation times also converge to a large but finite asymptotic value for which the system sizes used in the simulations were not sufficiently large. Complicated interactions between finite lifetime jams could in principle lead to divergent relaxation times (as originally suggested in [5]), however, it would still remain to be explained why the algebraic behavior in the structure factor is observed in the whole high-density regime and not only where the relaxation time diverges. The most plausible interpretation of the results in [1] is that the structure factor shows an apparent scaling behavior since the measurement times have been significantly shorter than the characteristic lifetimes.

Another result quoted in [1] in support of their conclusions, namely the interpretation of the observation of two peaks in the distance-headway distribution in terms of two-phase coexistence [11], was after the publication of [1] withdrawn in [12] by pointing out limitations of the analogy with the gas-liquid coexistence.

Furthermore we like to point out that for higher velocities v_{max} the transition region becomes much narrower. This can e.g. be seen in the behavior of the fundamental diagrams (see Fig. 3(a) of [13]). Therefore it is more difficult to detect the difference between a crossover and a sharp transition for large v_{max}.

In conclusion, the careful analysis of the method of [1] shows that their results, though interpreted as indication of a sharp phase transition, are compatible with a crossover type jamming transition conjectured using several other techniques.

Part of this work has been performed within the research program of the SFB 341 (Köln–Aachen–Jülich). Thanks are due to partial support by OTKA and Humboldt Stiftung extended to JK.

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