Albert-Ludwigs-Universität Freiburg — Johannes Berg
Physikalisches Institut

Information Theory: From Statistical Physics to Quantitative Biology

1. exercise class – 5. November 2008

1. Coin flips

A fair coin is flipped until the first head occurs. Let the random variable X denote the number of flips required.

a) Find the entropy $H(X)$ in bits. (25 pts)

b) A random variable is drawn according to this distribution. Find an efficient sequence of yes–no-questions to determine the value of the variable and compare the result with H. (25 pts) (Taken from Thomas & Cover.)

2. Maximal entropy subject to constraints

a) Find the probability function $p(i), i \in \{1, 2, \dots n\}$ that maximizes the entropy $H(i)$ subject to the constraint

$$E(i) \equiv \sum_{i=1}^{n} i p(i) = A > 0 \ . \tag{1}$$

(25pts)

b) N dice are cast [1]. Given the total number of eyes is αN, what proportion of the dice show i eyes, $i = 1, \dots, 6$? (25pts)

Hint: It helps to consider the number of ways N dice can achieve αN eyes with n_1 of them showing 1 eye, n_2 of them showing 2 eyes, etc. What are the values of $n_1, n_2, \dots$ which maximize this quantity? Use Stirling's formula and compare with a).

[1] $\equiv N$ Würfel werden geworfen.