Mathematische Methoden

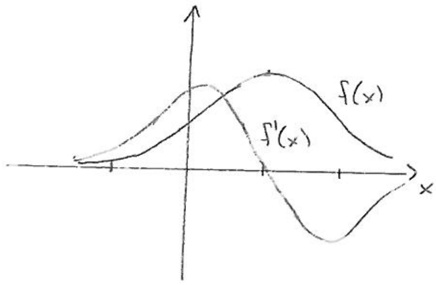
Blatt 2

Aufgabe 1: Funktionen

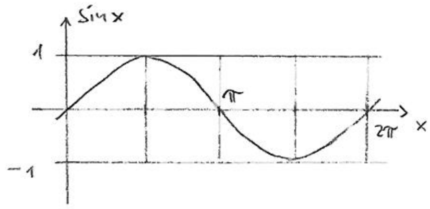
a) $f(x) = e^{-(x-1)^2}$

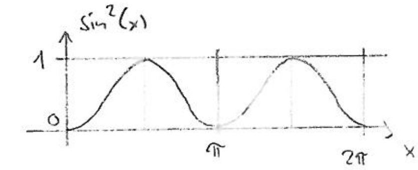
$f'(x) = 2(1-x)\,e^{-(x-1)^2}$

$f''(x) = 4(1-x)^2 e^{-(x-1)^2} - 2e^{-(x-1)^2}$



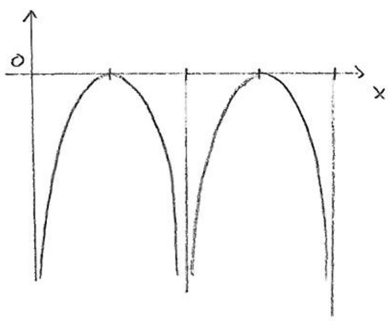
b) $f(x) = \ln(\sin^2(x))$





$\ln(x \to 0) = -\infty$

$\ln(1) = 0$



Aufgabe 2: Taylor-Reihe

a) $f(x) = a^x = e^{x\ln(a)} = \sum_{n=0}^{\infty} \frac{1}{n!}(x\ln(a))^n = \sum_{n=0}^{\infty} \frac{\ln(a)^n}{n!} x^n$

b) $f(x) = (1+x)^{-2}$

$f'(x) = -2(1+x)^{-3}$

$\vdots$

$f^{(n)}(x) = (-1)^n (n+1)!\,(1+x)^{-(n+2)} \quad \Rightarrow \quad f^{(n)}(0) = (-1)^n (n+1)!$

$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(n+1)!}{n!}(-1)^n x^n = \sum_{n=0}^{\infty}(n+1)(-1)^n x^n$

c) $f(x) = \sinh(x) = \frac{1}{2}(e^x - e^{-x})$ und d) $f(x) = \cosh(x) = \frac{1}{2}(e^x + e^{-x})$

es gilt: $e^{-x} = \sum_{n=0}^{\infty} \frac{1}{n!}(-x)^n = \sum_{n=0}^{\infty} \frac{1}{n!}(-1)^n x^n$

$$\Rightarrow \quad \frac{1}{2}\left(e^{x} + e^{-x}\right) = \sum_{n=0}^{\infty} \frac{1}{n!} x^{n} \underbrace{\frac{1}{2}\left(1 + (-1)^{n}\right)}_{= \begin{cases} 1 & : \text{ ungerade} \\ 0 & : n \text{ ungerade} \end{cases}} =$$

$$= \sum_{\substack{n=0 \\ n \text{ gerade}}}^{\infty} \frac{1}{n!} x^{n} = \sum_{m=0}^{\infty} \frac{1}{(2m)!} x^{2m} = \cosh(x)$$

analog : $\sinh(x) = \sum_{\substack{n=1 \\ n \text{ ungerade}}}^{\infty} \frac{1}{n!} x^{n} = \sum_{m=0}^{\infty} \frac{1}{(2m+1)!} x^{2m+1}$

e, $f(x) = e^{-x^2} = \sum_{n=0}^{\infty} \frac{1}{n!} (-x^2)^{n} = \sum_{n=0}^{\infty} \frac{1}{n!} (-1)^{n} x^{2n}$

Aufgabe 3 : partielle Ableitung

$$g(x,y,z) = x^2 \sin(xz) + z e^{y}$$

a, $\frac{\partial}{\partial x} g = 2x \sin(xz) + x^2 \cos(xz) z$

b, $\frac{\partial}{\partial y} g = z e^{y}$

c, $\frac{\partial}{\partial z} g = x^2 \cos(xz) x + e^{y} = x^3 \cos(xz) + e^{y}$

d, $\frac{\partial^2}{\partial x \partial y} g = \frac{\partial}{\partial x}(z e^{y}) = 0$

e, $\frac{\partial^2}{\partial z \partial y} g = \frac{\partial}{\partial z}(z e^{y}) = e^{y}$

f, $\frac{\partial^2}{\partial z^2} g = \frac{\partial}{\partial z}\left(x^3 \cos(xz) + e^{y}\right) = -x^4 \sin(xz)$