Mathematische Methoden – WS 2011/12

Blatt 4

Aufgabe 1: Integralfunktionen

a) Gamma-Funktion: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$

$\longrightarrow \Gamma(n+1) = \int_0^\infty t^n e^{-t}\, dt = \dots$ $\qquad f_1(t) = t^n \qquad f_2(t) = e^{-t}$

$f_1'(t) = n t^{n-1} \qquad F_2(t) = -e^{-t}$

$$\dots = \underbrace{-\left[t^n e^{-t}\right]_0^\infty}_{0} + n \underbrace{\int_0^\infty t^{n-1} e^{-t}\, dt}_{=\Gamma(n)} = n\,\Gamma(n) \quad \checkmark$$

b) Fehlerfunktion: $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

$\int_0^x y^2 e^{-y^2}\, dy = \dots$ $\qquad f_1(y) = y \qquad f_2(y) = y e^{-y^2}$

$f_1'(y) = 1 \qquad F_2(y) = -\frac{1}{2} e^{-y^2}$

$$\dots = \underbrace{-\frac{1}{2}\left[y e^{-y^2}\right]_0^x}_{= x e^{-x^2}} + \underbrace{\frac{1}{2} \int_0^x e^{-y^2}\, dy}_{= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x)} = -\frac{1}{2} x e^{-x^2} + \frac{1}{4}\sqrt{\pi}\, \operatorname{erf}(x)$$

Aufgabe 2: Komplexe Zahlen: Multiplikation

$z_1 = 1 + i$, $z_2 = -2 + 2i$, $z_3 = -i$

a) $p_j = i z_j$

$p_1 = -1 + i$

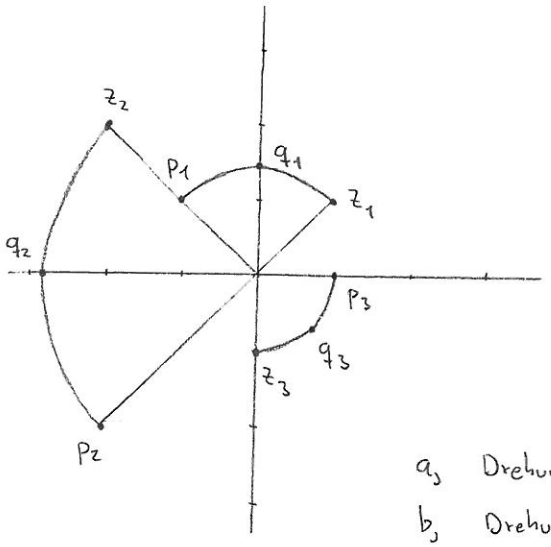
$p_2 = -2 - 2i$

$p_3 = 1$

b) $q_1 = \sqrt{2}\, i$ $\qquad q_j = \frac{1}{\sqrt{2}}(1+i) z_j$

$q_2 = -2\sqrt{2}$

$q_3 = \frac{1}{\sqrt{2}}(1-i)$



a) Drehung um $\pi/2$

b) Drehung um $\pi/4$

Aufgabe 3 : Komplexe Zahlen : $e^{i\varphi}$

a) $2e^{-i\pi/4} = 2\frac{1}{\sqrt{2}}(1-i) = \sqrt{2} - \sqrt{2}i$

b) $ie^{\pi i} = -i$

c) $e^{n\pi i} = (-1)^n$

d) $2 - 2i = 2\sqrt{2}\frac{1}{\sqrt{2}}(1-i) = 2\sqrt{2}e^{-i\pi/4}$

e) siehe c)

f) $(1+i)^n = \left[\sqrt{2}\underbrace{\frac{1}{\sqrt{2}}(1+i)}_{=e^{i\pi/4}}\right]^n = 2^{n/2}e^{in\pi/4}$

Aufgabe 4 : Komplexe Zahlen : Division

a) $\frac{1}{1-i} = \frac{1+i}{(1-i)(1+i)} = \frac{1}{2} + \frac{1}{2}i$

b) $\frac{1+i}{(2-i)^2} = \frac{1+i}{3-4i} = \frac{(1+i)(3+4i)}{25} = \frac{1}{25}(3+7i-4) = -\frac{1}{25} + \frac{7}{25}i$

c) $\frac{i-2}{2+e^{i\pi/4}} = \frac{i-2}{2+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i} = \frac{(i-2)(2+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i)}{(2+\frac{1}{\sqrt{2}})^2+\frac{1}{2}} =$

$= \frac{1}{5+4/\sqrt{2}}\left(-4-\frac{1}{\sqrt{2}} + i\left(2+\frac{3}{\sqrt{2}}\right)\right)$