Mathematische Methoden WS 2011/12

Blatt 9

Aufgabe 1 · Determinanten

a)
$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \\ 0 & 1 & 0 \end{pmatrix} \rightarrow |A| = -10$$

b)
$$B = \begin{pmatrix} a & b & 0 & 0 \\ a & 0 & b & 0 \\ a & 0 & 0 & b \\ 0 & a & 0 & b \end{pmatrix} \rightarrow |B| = a^2b^2 + ab^3$$

c)
$$M = \begin{pmatrix} a_{11} & 0 & 0 & \dots & 0 \\ a_{21} & a_{22} & 0 & \dots & 0 \\ a_{31} & a_{32} & a_{33} & \dots & 0 \\ \vdots & & & \ddots & \\ a_{n1} & & \dots & & a_{nn} \end{pmatrix} \rightarrow |M| = a_{11} \cdot \det \begin{pmatrix} a_{22} & 0 & 0 \dots 0 \\ a_{32} & a_{33} & \\ \vdots & & \ddots \\ a_{n2} & & a_{nn} \end{pmatrix}$$

$$= a_{11} \cdot a_{22} \cdot \dots \cdot a_{nn} = \prod_{i=1}^{n} a_{ii}$$

Aufgabe 2 · lineares Gleichungssystem

$$\begin{aligned} 2x + 3y - z &= 2 \\ x - y + z &= 0 \\ -3x - 5y + 2z &= -1 \end{aligned} \quad \hat{=} \quad M\vec{x} = \vec{a} \quad \text{mit} \quad M = \begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \\ -3 & -5 & 2 \end{pmatrix}, \ \vec{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \vec{x} = M^{-1}\vec{a}$$

→ Matrixinversion · $(M^{-1})_{ij} = \frac{1}{|M|}(-1)^{i-j} M_{ji}$

↳ Unterdeterminanten der Matrix M

$$|M| = -1$$

$$M_{11} = \begin{vmatrix} -1 & 1 \\ -5 & 2 \end{vmatrix} = 3 \ , \ M_{12} = 5 \ , \ M_{13} = -8$$

$M_{21} = 1$, $M_{22} = 1$, $M_{23} = -1$, $M_{31} = 2$, $M_{32} = 3$, $M_{33} = -5$

$$\rightarrow \quad (M^{-1}) = \begin{pmatrix} -3 & 1 & -2 \\ 5 & -1 & 3 \\ 8 & -1 & 5 \end{pmatrix} \quad \Rightarrow \quad \vec{x} = M^{-1}\vec{a} = \begin{pmatrix} -4 \\ 7 \\ 11 \end{pmatrix}$$

Aufgabe 3: Eigenvektoren, Eigenwerte

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \det(A - \lambda \underline{\underline{1}}) = 2\lambda - \lambda^3$$

Eigenwerte: $\lambda_1 = -\sqrt{2}$, $\lambda_2 = 0$, $\lambda_3 = \sqrt{2}$

Eigenvektoren:

λ_1:
$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\sqrt{2}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{array}{ll} (\underline{\underline{I}}) & y = -\sqrt{2}x \\ (\underline{\underline{II}}) & x + z = -\sqrt{2}y \\ (\underline{\underline{III}}) & y = -\sqrt{2}z \end{array}$$

$(\underline{\underline{I}}) - (\underline{\underline{III}}) \Rightarrow x = z$ in $(\underline{\underline{II}})$: $y = -\sqrt{2}x$

normierte Eigenvektor: $\vec{x}_1 = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

analog: $\vec{x}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\vec{x}_3 = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$