Solid State Theory

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WS 2012/13

Sheet 1: return on: Monday, Oct 15, 2012, 12:00 (SR THP)

Exercise 1: Bravais lattice

Show that the lattice L defined as

$$L = L_1 \cup L_2 \ , \ \text{with} \ \ L_\mu = \{\vec{R}_{\vec{n}} + \vec{R}_\mu | n_i \in \mathbb{Z}\} \ , \ \mu = 1, 2 \ ,$$

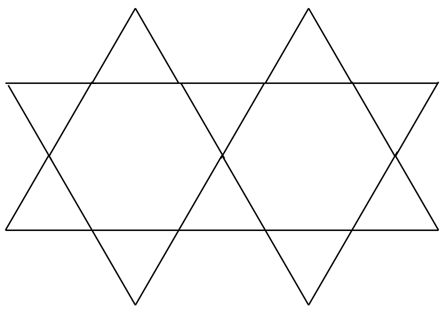
$$\vec{R}_{\vec{n}} = n_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + n_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + n_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ , \quad \text{and} \ \ \vec{R}_1 = \vec{0} \ , \ \vec{R}_2 = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \ ,$$

is a Bravais lattice. (5 points)

Exercise 2: 2d honeycomb lattice

Show that the 2d honeycomb lattice (see the lecture for a definition) is *not* a Bravais lattice . (No rigorous proof is required here.) (3 points)

Exercise 3: Kagome lattice



The Kagome lattice consists of a regular arrangement of corner-sharing triangles, as shown in the figure. The lattice points are the corners of the triangles.

a) What is the expression for the lattice points $\vec{R}_{\vec{n},\mu}$? (The Kagome lattice is not a Bravais lattice.) (2 points)

b) How does the unit cell look like? (1 point)

c) Determine the area of the unit cell. (1 point)

Exercise 4: 2d triangular lattice

The 2d triangular lattice is defined as

$$L = \{\vec{R}_{\vec{n}} | n_i \in \mathbb{Z}\} \ , \ \text{with } \vec{R}_{\vec{n}} = n_1\vec{a}_1 + n_2\vec{a}_2 \ ,$$

$$\text{and } \vec{a}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \ \vec{a}_2 = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \ .$$

Consider the following linear combination of the primitive vectors $\vec{a}_1$ and $\vec{a}_2$:

$$\begin{aligned} \vec{b}_1 &= k_1\vec{a}_1 + k_2\vec{a}_2 \ , \\ \vec{b}_2 &= l_1\vec{a}_1 + l_2\vec{a}_2 \ . \end{aligned}$$

Under which conditions are the two vectors $\vec{b}_1$ and $\vec{b}_2$ primitive vectors of the same 2d triangular lattice? (3 points)