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Exercise sheet online at http://www.thp.uni-koeln.de/∼jinhong/qft2

Quantum Field Theory II
Problem Set 5

Please hand in your solution of the exercise marked with (*), for which we offer correction, in the mail box until Monday 12am.

1. Weinberg angle (*)

In the standard model, the gauge group is $SU(2) \times U(1)$. We have three gauge bosons W^a_λ associated with $SU(2)$ and one gauge boson B_λ associated with $U(1)$, and two different gauge couplings, g for $SU(2)$ and g' for $U(1)$. The covariant derivative for gauge group $SU(2) \times U(1)$ is written as

$$D_\lambda = \left(\partial_\lambda + igW^a_\lambda T^a + ig'B_\lambda Y\right), \tag{1}$$

where Y is the hypercharge operator and when acting on a left-handed doublet, $T^a \equiv \tau^a/2$ becomes the $SU(2)$ generators. The charge operator is given by $Q = T^3 + Y$. Introduce a new configuration of fields

$$\begin{aligned} A_\lambda &= \sin(\theta_W)W^3_\lambda + \cos(\theta_W)B_\lambda, \\ Z_\lambda &= \cos(\theta_W)W^3_\lambda - \sin(\theta_W)B_\lambda, \end{aligned} \tag{2}$$

such that A_λ only couples to Q.

a) What is the value of θ_W?

b) What are the coupling constant e of A_λ to the electron (i.e. the electron charge e) as a function of g and g'?

c) What values of the hypercharge of left and right-handed electron, Y_L and Y_R, have to be chosen such that both fields carry the same electric charge e? Use that the covariant derivative for the right handed electron is given as

$$D_\lambda \psi_R = \left(\partial_\lambda + ig'B_\lambda Y_R\right)\psi_R. \tag{3}$$

d) How does Z_λ couple to a left-handed and to a right-handed electron?

2. Higgs mechanism

a) What Feynman diagrams describe the coupling of the Higgs particle to the leptons e, μ, and τ? Which of these couplings is the strongest? Use the Yukawa coupling.

b) The discovery of the Higgs particle in 2012 at an energy of 126GeV was using the decay of the Higgs particle i.e. $h \to ZZ$ and WW. Let us calculate the mass of W and Z boson. The covariant derivative of a two-component scalar field ϕ is defined as

$$D_\mu \phi = \left(\partial_\lambda + igW_\lambda^a T^a + ig' B_\lambda Y\right) \phi. \tag{4}$$

Note that the hypercharge of the field ϕ is $Y = 1/2$. Expand the Lagrangian $\left(D^\lambda \phi\right)^\dagger D_\lambda \phi$ around the Higgs expectation value using

$$\phi = \begin{pmatrix} 0 \\ \rho_0/\sqrt{2} \end{pmatrix} + \begin{pmatrix} 0 \\ \delta\rho/\sqrt{2} \end{pmatrix}. \tag{5}$$

You may use Eq. (2) and the answer of the problem 1a). Why does a Higgs particle decay into a pair of Z bosons or a pair of W bosons? What are the coupling constants for these processes?

c) The expectation value of the coefficient of the quadratic term in the gauge bosons W or Z corresponds to the mass square of the W or Z, respectively. Note that the gauge boson A_λ is completely decoupled with ϕ, so that it remains massless. What is the ratio of the mass of the W and Z boson? The Weinberg angle θ_W is determined from the ratio and fixed with $\sin^2 \theta_W \simeq 0.23$ by experiments.

3. Standard model

Count the number of free parameters in the standard model.