Solution to Problem Set 9

December 17, 2019

1 Mean-field free energy of the ferromagnet

$$F_{MF} = -\frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)\sum_{\sigma}\ln\left[1+e^{-\beta(\epsilon-\mu-\sigma\phi_0/2)}\right]+\frac{\phi_0^2}{2J} \tag{1}$$

(a) We take the limit $\beta\to\infty$,

$$\begin{aligned}F_0 &= -\lim_{\beta\to\infty}\frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)\sum_{\sigma}\ln\left[1+e^{-\beta(\epsilon-\mu)}\right]\\ &= -2\lim_{\beta\to\infty}\frac{1}{\beta}\int_0^{\mu} d\epsilon\ \nu(\epsilon)\ln\left[1+e^{-\beta(\epsilon-\mu)}\right],\end{aligned}$$

where $\nu(\epsilon)=\sum_{\mathbf{k}}\delta(\epsilon-\epsilon_{\mathbf{k}})$ is the density of states *per site per spin* of the ferromagnet. This leads to,

$$F_0 = -2\lim_{\beta\to\infty}\frac{1}{\beta}\left[\int_0^{\mu} d\epsilon\ \nu(\epsilon)\ln\left[1+e^{\beta(\epsilon-\mu)}\right]+\int_0^{\mu} d\epsilon\ \nu(\epsilon)\ln\left[e^{-\beta(\epsilon-\mu)}\right]\right].$$

The first term goes to zero, while the second one yields,

$$F_0 = 2\int_0^{\mu} d\epsilon\ \nu(\epsilon)(\epsilon-\mu). \tag{2}$$

(b) We will perform the expansion at finite temperature and then take the limit $\beta\to\infty$. To this end, first note that

$$F_{MF} = \frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)G[\phi_0]+\frac{\phi_0^2}{2J}, \tag{3}$$

where $G[\phi_0] = -\sum_{\sigma}\ln\left[1+e^{-\beta(\epsilon-\mu-\sigma\phi_0/2)}\right]$.

Therefore, we will first compute the derivatives of $G[\phi_0]$,

$$G'[\phi_0] = \beta\sum_{\sigma}\Big(-\frac{\sigma}{2}\Big)f(\epsilon-\mu-\sigma\phi_0/2),$$

where $f(\epsilon)=\frac{1}{e^{\beta(\epsilon-\mu)}+1}$ is the Fermi function.

Further derivatives are straightforward to compute in terms of the derivatives of the Fermi function. We then have,

$$G''[\phi_0] = \beta\sum_{\sigma}\Big(-\frac{\sigma}{2}\Big)^2 f'(\epsilon-\mu-\sigma\phi_0/2),$$
$$G'''[\phi_0] = \beta\sum_{\sigma}\Big(-\frac{\sigma}{2}\Big)^3 f''(\epsilon-\mu-\sigma\phi_0/2),$$
$$G''''[\phi_0] = \beta\sum_{\sigma}\Big(-\frac{\sigma}{2}\Big)^4 f'''(\epsilon-\mu-\sigma\phi_0/2).$$

Notice that the odd derivatives of G vanish at $\phi_0 = 0$. The integrand can now be Taylor expanded about $\phi_0 = 0$ up to fourth order as follows,

$$F_{MF} \approx \frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)\left[G[0] + G''[0]\frac{\phi_0^2}{2!} + G''''[0]\frac{\phi_0^4}{4!}\right] + \frac{\phi_0^2}{2J}, \tag{4}$$

This leads to

$$F_{MF} = \frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)\left[-\sum_{\sigma}\ln\left[1+e^{-\beta(\epsilon-\mu)}\right] + 2\beta\Big(\frac{1}{2}\Big)^2 f'(\epsilon-\mu)\frac{\phi_0^2}{2!} + 2\beta\Big(\frac{1}{2}\Big)^4 f'''(\epsilon-\mu)\frac{\phi_0^4}{4!}\right] + \frac{\phi_0^2}{2J}.$$

Now we take the limit $\beta\to\infty$, while remembering that in this limit the Fermi function becomes a theta function $f(\epsilon) = \Theta(\mu-\epsilon)$ (with derivatives $f'(\epsilon) = -\delta(\epsilon-\mu)$ etc.),

$$F_{MF} = F_0 + \frac{1}{\beta}\int d\epsilon\ \nu(\epsilon)\left[-2\beta\Big(\frac{1}{2}\Big)^2\delta(\epsilon-\mu)\frac{\phi_0^2}{2!} - 2\beta\Big(\frac{1}{2}\Big)^4\delta''(\epsilon-\mu)\frac{\phi_0^4}{4!}\right] + \frac{\phi_0^2}{2J},$$

which, via integration by parts, immediately yields,

$$F_{MF} = F_0 + \frac{\phi_0^2}{2}\left[\frac{1}{J} - \frac{\nu(\mu)}{2}\right] - 2\frac{\nu''(\mu)}{4!}\left(\frac{\phi_0}{2}\right)^4. \tag{5}$$

(c) The condition for a phase transition in the mean field limit is that the coefficient of ϕ_0^2 become negative and that of ϕ_0^4 remain positive (in order to have a Mexican hat potential). This leads to the Stoner criterion,

$$\boxed{\frac{J\nu(\mu)}{2} > 1} \tag{6}$$

If one assumes free dispersion in 3D to be $\omega \propto k^2$, then the density of states $\nu(\epsilon)\propto\sqrt{\epsilon}$, thereby leading to $\nu''(\mu) < 0$.

If $\nu''(\mu) > 0$, then one will not obtain a minima with a fourth order expansion. This leads to two options: 1. Either one can go to a higher order mean field expansion that can lead to more mean field solutions, 2. There is no second order transition, and instead the transition becomes first order (cf. Piers Coleman, Chapter 13, page 473-474).

2 Variational BCS Wave Function

The correct form of the Hamiltonian here is

$$\hat{K} = \sum_{\mathbf{k}\sigma}\xi_{\mathbf{k}}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} + \frac{1}{\mathcal{V}}\sum_{\mathbf{k},\mathbf{l}}{}' V_{\mathbf{kl}}c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}c_{-\mathbf{l}\downarrow}c_{\mathbf{l}\uparrow} \tag{7}$$

where $\mathcal{V}$ is the total volume, and the prime over the summation means that the sum extends over $\mathbf{k} \neq \mathbf{l}$.

We are again working at $T = 0$.

(a) It is easy to see that

$$\langle \Psi | c^\dagger_{\mathbf{k}} c_{\mathbf{k}} | \Psi \rangle = |v_{\mathbf{k}}|^2 \tag{8}$$

Also, for $\mathbf{k} \neq \mathbf{l}$, we need $\langle \Psi | c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow} | \Psi \rangle$, which is equal to

$$\langle 0 | (u^*_{\mathbf{l}} + v^*_{\mathbf{l}} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow})(u^*_{\mathbf{k}} + v^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}) \; c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow} \; (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})(u_{\mathbf{l}} + v_{\mathbf{l}} c^\dagger_{\mathbf{l}\uparrow} c^\dagger_{-\mathbf{l}\downarrow}) | 0 \rangle$$

In the above product, the $c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow}$ in the center will contract with the corresponding creation operators on the right and the $c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}$ will contract with the corresponding destruction operators on the left to simplify the matrix element to

$$\langle 0 | (u^*_{\mathbf{l}} + v^*_{\mathbf{l}} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow}) v^*_{\mathbf{k}} \; (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}) v_{\mathbf{l}} | 0 \rangle = u^*_{\mathbf{l}} v^*_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{l}}$$

This yields,

$$E_{\rm sc} = \langle \Psi | \hat{K} | \Psi \rangle = 2 \sum_{\mathbf{k}} \xi_{\mathbf{k}} v^2_{\mathbf{k}} + \frac{1}{\mathcal{V}} \sum_{\mathbf{k},\mathbf{l}}{}' V_{\mathbf{kl}} v^*_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{l}} u^*_{\mathbf{l}} \tag{9}$$

Writing $u_{\mathbf{k}} = \sin\theta_{\mathbf{k}}$ and $v_{\mathbf{k}} = \cos\theta_{\mathbf{k}}$, we have

$$E_{\rm sc} = \langle \Psi | \hat{K} | \Psi \rangle = 2 \sum_{\mathbf{k}} \xi_{\mathbf{k}} \cos^2\theta_{\mathbf{k}} + \frac{1}{4\mathcal{V}} \sum_{\mathbf{k},\mathbf{l}}{}' V_{\mathbf{kl}} \sin 2\theta_{\mathbf{k}} \sin 2\theta_{\mathbf{l}} \tag{10}$$

Minimizing the energy, i.e. setting $\frac{\partial E_{\rm sc}}{\partial \theta_{\mathbf{k}}} = 0$, we get the following condition,

$$\tan 2\theta_{\mathbf{k}} = \frac{1}{\mathcal{V}} \sum_{\mathbf{l}}{}' V_{\mathbf{kl}} \frac{\sin 2\theta_{\mathbf{l}}}{2\xi_{\mathbf{k}}} \tag{11}$$

(b) Writing

$$\Delta_{\mathbf{k}} = -\frac{1}{2\mathcal{V}} \sum_{\mathbf{l}}{}' V_{\mathbf{kl}} \sin 2\theta_{\mathbf{l}},$$

we have,

$$\tan 2\theta_{\mathbf{k}} = -\frac{\Delta_{\mathbf{k}}}{\xi_{\mathbf{k}}}.$$

And hence,

$$\cos 2\theta_{\mathbf{k}} = -\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}},$$

$$\sin 2\theta_{\mathbf{k}} = \frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}}},$$

where $E_{\mathbf{k}} = \sqrt{\Delta^2_{\mathbf{k}} + \xi^2_{\mathbf{k}}}$. This means,

$$\Delta_{\mathbf{k}} = -\frac{1}{2\mathcal{V}} \sum_{\mathbf{l}}{}' V_{\mathbf{kl}} \sin 2\theta_{\mathbf{l}} = -\frac{1}{2\mathcal{V}} \sum_{\mathbf{l}}{}' V_{\mathbf{kl}} \frac{\Delta_{\mathbf{l}}}{E_{\mathbf{l}}}. \tag{12}$$

If one chooses to explore the $\Delta_{\mathbf{k}} = \Delta$ solution, the equation for Δ becomes,

$$\Delta = V\frac{1}{2}\int_{-\omega_D}^{\omega_D} d\xi\ \nu(\xi)\frac{\Delta}{\sqrt{\Delta^2+\xi^2}}. \tag{13}$$

For a non-zero solution for Δ, we must have

$$1 = V\frac{1}{2}\int_{-\omega_D}^{\omega_D} d\xi\ \nu(\xi)\frac{1}{\sqrt{\Delta^2+\xi^2}}.$$

Assuming $\omega_D \ll \mu$, we approximate the density of states *per spin per volume* with ν (i.e. the density of states at Fermi energy for the free Hamiltonian), and we have,

$$1 \approx V\int_{0}^{\omega_D} d\xi\ \nu\frac{1}{\sqrt{\Delta^2+\xi^2}}$$

$$1 \approx \nu V \ln\left[\frac{\omega_D+\sqrt{\Delta^2+\omega_D^2}}{\Delta}\right] \approx \nu V \ln\left[\frac{2\omega_D}{\Delta}\right] \tag{14}$$

Thus we find,

$$\boxed{\Delta \approx 2\omega_D e^{-\frac{1}{\nu V}}} \tag{15}$$

(Note that tan determines an angle only modulo π, and hence there is a sign ambiguity in the values of cos and sin. The signs have been so chosen that the variational solution produces a non-trivial value of $\Delta_{\mathbf{k}} = \Delta$ at zero temperature.)

Also,

$$u_{\mathbf{k}}^2 = \frac{1}{2}(1-\cos 2\theta_{\mathbf{k}}) = \frac{1}{2}\left[1+\frac{\xi_{\mathbf{k}}}{\sqrt{\Delta^2+\xi_{\mathbf{k}}^2}}\right] \tag{16}$$

$$v_{\mathbf{k}}^2 = \frac{1}{2}(1+\cos 2\theta_{\mathbf{k}}) = \frac{1}{2}\left[1-\frac{\xi_{\mathbf{k}}}{\sqrt{\Delta^2+\xi_{\mathbf{k}}^2}}\right] \tag{17}$$

(c) $E_{\text{sc}} = \langle\Psi|\hat{K}|\Psi\rangle = 2\sum_{\mathbf{k}}\xi_{\mathbf{k}}|v_{\mathbf{k}}|^2 - \frac{V}{\mathcal{V}}\sum_{\mathbf{k},\mathbf{l}}{}' v_{\mathbf{k}}^* u_{\mathbf{k}} v_{\mathbf{l}} u_{\mathbf{l}}^*$. The latter sum is only over an energy range ω_D around the Fermi surface.

Also, $E_{FS} = 2\sum_{\xi_{\mathbf{k}}<0}\xi_{\mathbf{k}}$.

We therefore obtain,

$$\delta E_{\text{cond}} = E_{sc} - E_{FS} \approx \sum_{|\xi_{\mathbf{k}}|<\omega_D}\xi_{\mathbf{k}}\left[1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right] - \frac{V}{\mathcal{V}}\sum_{\mathbf{k}}\frac{\Delta_{\mathbf{k}}}{2E_{\mathbf{k}}}.\sum_{\mathbf{l}}\frac{\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}} - 2\sum_{-\omega_D<\xi_{\mathbf{k}}<0}\xi_{\mathbf{k}}, \tag{18}$$

wherein we've ignored the non-extensive contribution to the energy coming from the states with $\mathbf{k} = \mathbf{l}$ in the second summation (which is clearly ignorable in the thermodynamic limit). Using the equation for Δ, and as before approximating density of states *per spin per volume* around the Fermi energy (of the free electron Hamiltonian) with ν, we can now simply write this as,

$$
\begin{aligned}
\delta E_{\text{cond}} &\approx \sum_{|\xi_{\mathbf{k}}|<\omega_D} \xi_{\mathbf{k}}\left[1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right] - \frac{\mathcal{V}}{V}\Delta^2 - 2\sum_{-\omega_D<\xi_{\mathbf{k}}<0} \xi_{\mathbf{k}}. \\
&\approx \mathcal{V}\int_{-\omega_D}^{\omega_D} d\xi\ \nu\xi\left[1-\frac{\xi}{\sqrt{\Delta^2+\xi^2}}\right] - \frac{\mathcal{V}}{V}\Delta^2 - 2\mathcal{V}\int_{-\omega_D}^{0} d\xi\ \nu\xi \\
&\approx -\mathcal{V}\int_{-\omega_D}^{\omega_D} d\xi\ \nu\frac{\xi^2}{\sqrt{\Delta^2+\xi^2}} - \frac{\mathcal{V}}{V}\Delta^2 + \mathcal{V}\nu\omega_D^2 \\
&\approx -2\mathcal{V}\nu\left[\frac{\omega_D\sqrt{\Delta^2+\omega_D^2}}{2} - \frac{\Delta^2}{2V\nu}\right] - \frac{\mathcal{V}}{V}\Delta^2 + \mathcal{V}\nu\omega_D^2 \\
&\approx -\mathcal{V}\nu\omega_D\sqrt{\Delta^2+\omega_D^2} + \mathcal{V}\nu\omega_D^2, \qquad (19)
\end{aligned}
$$

which is clearly negative, implying that the superconducting state has lower energy than the metallic state!

(d) This is easier to do with the Bogoliubov operators for the BCS state,

$$
\begin{pmatrix} \alpha_{\mathbf{k}\uparrow} \\ \alpha^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{pmatrix} u_{\mathbf{k}} & v_{\mathbf{k}} \\ -v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix}.
$$

We also have

$$
\begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} = \begin{pmatrix} u_{\mathbf{k}} & -v_{\mathbf{k}} \\ v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} \alpha_{\mathbf{k}\uparrow} \\ \alpha^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix}.
$$

The Bogoliubov operators annihilate the BCS state − using this fact and representing the number operator in terms of the Bogoliubov operators we get,

$$
\begin{aligned}
N|\Psi\rangle &= \sum_{\mathbf{k}} (c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k}\uparrow} + c^\dagger_{\mathbf{k}\downarrow}c_{\mathbf{k}\downarrow})|\Psi\rangle \\
&= \sum_{\mathbf{k}} [(u^*_{\mathbf{k}}\alpha^\dagger_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}}\alpha_{-\mathbf{k}\downarrow})(u_{\mathbf{k}}\alpha_{\mathbf{k}\uparrow} - v_{\mathbf{k}}\alpha^\dagger_{-\mathbf{k}\downarrow}) + (u_{-\mathbf{k}}\alpha^\dagger_{\mathbf{k}\downarrow} + v_{-\mathbf{k}}\alpha_{-\mathbf{k}\uparrow})(u^*_{-\mathbf{k}}\alpha_{\mathbf{k}\downarrow} + v^*_{-\mathbf{k}}\alpha^\dagger_{-\mathbf{k}\uparrow})]|\Psi\rangle \\
&= \sum_{\mathbf{k}} [-(u^*_{\mathbf{k}}\alpha^\dagger_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}}\alpha_{-\mathbf{k}\downarrow})v_{\mathbf{k}}\alpha^\dagger_{-\mathbf{k}\downarrow} + (u_{-\mathbf{k}}\alpha^\dagger_{\mathbf{k}\downarrow} + v_{-\mathbf{k}}\alpha_{-\mathbf{k}\uparrow})v^*_{-\mathbf{k}}\alpha^\dagger_{-\mathbf{k}\uparrow}]|\Psi\rangle \\
&= \sum_{\mathbf{k}} [-2u_{\mathbf{k}}v_{\mathbf{k}}\alpha^\dagger_{\mathbf{k}\uparrow}\alpha^\dagger_{-\mathbf{k}\downarrow} + 2v^2_{\mathbf{k}}]|\Psi\rangle.
\end{aligned}
$$

The norm of the state $N|\Psi\rangle$ is,

$$
\langle\Psi|N^2|\Psi\rangle = \sum_{\mathbf{k}}(2u_{\mathbf{k}}v_{\mathbf{k}})^2 + (\sum_{\mathbf{k}} 2v^2_{\mathbf{k}})^2. \qquad (20)
$$

Using $\langle\Psi|N|\Psi\rangle = \sum_{\mathbf{k}} 2|v_{\mathbf{k}}|^2$, we get that

$$
\delta N = \sqrt{\langle N^2\rangle - \langle N\rangle^2} = \sqrt{\sum_{\mathbf{k}}(2u_{\mathbf{k}}v_{\mathbf{k}})^2} \leq \sqrt{\sum_{\mathbf{k}}(2v_{\mathbf{k}})^2} = \sqrt{N}.
$$

Hence in the thermodynamic limit

$$
\lim_{N\to\infty}\frac{\delta N}{N} \sim \frac{1}{\sqrt{N}} \to 0. \qquad (21)
$$

3 Tunneling density of states of a superconductor

Consider the retarded Gorkov Green function

$$[G^R(\mathbf{k},\epsilon)]^{-1} = \begin{bmatrix} \epsilon + i\eta - \xi_{\mathbf{k}} & -\Delta \\ -\Delta & \epsilon + i\eta + \xi_{\mathbf{k}} \end{bmatrix}. \tag{22}$$

Inverting it we get,

$$[G^R(\mathbf{k},\epsilon)]_{11} = \frac{\epsilon + i\eta + \xi_{\mathbf{k}}}{(\epsilon + i\eta)^2 - \xi_{\mathbf{k}}^2 - \Delta^2} = \frac{u_{\mathbf{k}}^2}{\epsilon + i\eta - E_{\mathbf{k}}} + \frac{v_{\mathbf{k}}^2}{\epsilon + i\eta + E_{\mathbf{k}}}. \tag{23}$$

From this we get the tunneling density of states as,

$$\nu(\epsilon) = -\frac{1}{\pi}\frac{1}{\mathcal{V}}\sum_{\mathbf{k}} \mathrm{Im}[G^R(\mathbf{k},\epsilon)]_{11} = \sum_{\mathbf{k}} u_{\mathbf{k}}^2 \delta(\epsilon - E_{\mathbf{k}}) + \sum_{\mathbf{k}} v_{\mathbf{k}}^2 \delta(\epsilon + E_{\mathbf{k}}) \tag{24}$$

Next, we calculate $\nu(\epsilon)$ for $\epsilon > 0$. Approximating density of states *per spin per volume* around the fermi energy of the free electron Hamiltonian as ν_0, we get

$$\nu(\epsilon) = \nu_0 \int d\xi \; \frac{1}{2}\left[1 + \frac{\xi}{\sqrt{\Delta^2 + \xi^2}}\right] \delta(\epsilon - \sqrt{\Delta^2 + \xi^2}) \tag{25}$$

The zeros of the delta function are at $\xi = \pm\sqrt{\epsilon^2 - \Delta^2}$ if $\epsilon > \Delta$. This yields,

$$\nu(\epsilon) = \frac{\nu_0}{2}\left[\sum_{\xi = \pm\sqrt{\epsilon^2 - \Delta^2}} \left|\frac{\sqrt{\Delta^2 + \xi^2}}{\xi}\right|\right] \Theta(\epsilon - \Delta) = \nu_0 \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2}} \Theta(\epsilon - \Delta). \tag{26}$$

Likewise, one gets exactly the same answer for ν at $-\epsilon$ for $\epsilon > 0$. Thus we have

$$\boxed{\nu(\epsilon) = \nu_0 \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2}} \Theta(|\epsilon| - \Delta)} \tag{27}$$

Note that 11 component yields the tunneling density of states for the spin up electrons. Likewise, one gets the same result for spin down electrons too.

The density of states is obviously zero at Fermi energy, because all the excitations have been pushed into the range $\geq \Delta$. Since a lot of excitations that were close to the Fermi surface have been pushed close to Δ, one sees a divergence of density of states near $\epsilon = \pm\Delta$. In the limit $\Delta \to 0$, one gets the tunneling density of states for a normal metal.