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Depinning in a random medium

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Abstract. We develop a renormalized continuum field theory for a directed polymer interacting with a random medium and a single extended defect. The renormalization group is based on the operator algebra of the pinning potential; it has novel features due to the breakdown of hyperscaling in a random system. There is a second-order transition between a localized and a delocalized phase of the polymer; we obtain analytic results on its critical pinning strength and scaling exponents. Our results are directly related to spatially inhomogeneous Kardar–Parisi–Zhang surface growth.

Low-dimensional manifolds in media with quenched disorder are objects encountered in a large variety of different physical systems. Obvious examples are interfaces in disordered bulk media and random field systems [1, 2] or magnetic flux lines in dirty superconductors [3], but there is also a deep connection to the problem of non-equilibrium surface growth [4, 5] and randomly driven hydrodynamics [6]. Furthermore, the theory serves as a simple paradigm for more complicated, fully frustrated random systems such as spin glasses [7].

A one-dimensional manifold in a random medium is a phenomenological continuum model for a (single) magnetic flux line in type-II superconductors with impurities [3, 14], where the flux line interacts with an ensemble of quenched *point* defects (represented by a random potential). In addition to these point impurities there may also be *extended* (e.g *columnar* or *planar*) defects in the system. Experiments that systematically probe the effect of this kind of impurities have recently become possible in high-temperature superconductors [8]. The statistics of the line configurations is governed by an energetic competition: point defects tend to *roughen* the flux line; it performs large transversal excursions in order to take advantage of locally favourable regions. An attractive extended defect, on the other hand, suppresses these excursions and, if it is sufficiently strong, *localizes* the line to within a finite transversal distance $\xi_\perp$. The two regimes are separated by a second-order phase transition where the localization length $\xi_\perp$ diverges. In contrast to temperature-driven transitions, it involves the competition of two different configuration energies rather than energy and entropy and is hence governed by a *zero-temperature* renormalization group fixed point.

The system is described by an effective Hamiltonian

$$\mathcal{H} = \int \mathrm{d}t \left\{ \frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 - V(r,t) + \rho\,\Phi(t) \right\}. \tag{1}$$

Here $r(t)$ denotes the displacement vector of the flux line (also called *directed polymer*) in d' transversal dimensions, as a function of the longitudinal 'timelike' coordinate t.

The random potential $V(r,t)$, Gauss-distributed with $\overline{V(r,t)} = 0$ and $\overline{V(r,t)V(r',t')} = 2\sigma^2 \delta^{d'}(r-r')\delta(t-t')$, models the quenched point disorder. Averages over disorder are denoted by an overbar, thermal averages by angular brackets $\langle \cdots \rangle$. The last term in the Hamiltonian describes the interaction with the extended defect. In this paper, we concentrate on columnar defects, $\Phi(t) = \delta^{d'}(r(t))$, but most of the results can straightforwardly be generalized to planar defects.

This model gains additional interest, since it is related to non-equilibrium critical phenomena [9]. If $Z(r,t)$ denotes the restricted partition sum over all paths ending at a fixed given point (r,t), the 'height field' $h(r,t) = \beta^{-1} \log Z(r,t)$ obeys the evolution equation

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + V - \rho\, \delta^{d'}(r) \tag{2}$$

with $\nu = (2\beta)^{-1}$ and $\lambda = 1$. This is the Kardar–Parisi–Zhang (KPZ) equation of directed surface growth† with an additional term describing a local inhomogeneity in the rate of mass deposition onto the surface.

Quite a few authors have studied these models. Early numerical work for $d' = 1$ indicates a delocalization transition at a finite defect strength ρ_c [10, 11]. In other large scale simulations [12], however, it is found that arbitrarily weak defects localize the polymer for $d' = 1$, but a finite defect strength is necessary for $d' > 1$. This result is supported by an approximate renormalization treatment for the problem on a hierarchical lattice [12], by scaling arguments [13], and by an approximate functional renormalization [14]. In [15] a Wilson-type renormalization is discussed, but its consistency is unclear. All of these approaches rely on non-systematic approximations; and the status of the transition has remained controversial. The problem has so far defied attempts at an exact solution even for $d' = 1$, in contrast to the related problem of disorder-induced depinning from a rigid wall [10, 16]. However, in a recent paper [17], the mapping onto the KPZ equation (2) is exploited to construct a mode coupling approximation in $d' = 1$, which leads to results in agreement with ours.

Our paper is devoted to a field-theoretic study of the delocalization transition. The large-scale behaviour of directed lines in a random potential (with $\rho = 0$) is governed by a zero-temperature renormalization group fixed point‡ with two basic exponents [2], the *roughness exponent* ζ and the anomalous dimension of the *disorder-averaged* free energy $-\omega$, whose definitions are recalled in equations (3) and (4) below. In a first step, we construct the renormalized continuum field theory for the zero-temperature fixed point. We then take this theory as the starting point for a systematic perturbation theory in the pinning potential, involving an ε-expansion with borderline dimension $d' = 1$. In contrast to standard cases like ϕ^4-theory, here even the unperturbed system is a field theory with complicated multipoint correlation functions, due to the non-thermal averaging over the disorder. Nevertheless, two fundamental properties of the local pinning field $\Phi(t)$ can be obtained in terms of the exponents ζ and ω: (i) its scaling dimension and (ii) the form of its operator product expansion, see equations (8) and (10) below. These properties determine the renormalization group equations for the pinning strength to leading order, and hence the phase diagram of the system. We find a transition at $\rho = 0$ for $d' \leqslant 1$, and at a finite

† The two basic exponents of the KPZ universality class for $\rho = 0$, the roughness exponent χ and the dynamical exponent z, are defined by $\langle\langle (h(0,0) - h(r,t))^2 \rangle\rangle \sim |r|^{2\chi} f(|tr^{-z}|)$, and are related to the polymer exponents by $\chi = \omega/\zeta$ and $z = 1/\zeta$.

‡ This statement is true for any finite strength σ^2 of the disorder if $d' \leqslant 2$, which is assumed in what follows for notational simplicity.

(non-universal) pinning strength ρ_c for $d' = 2$. The renormalization may equivalently be carried out in the framework of the KPZ dynamics, equation (2). This is discussed at the end of this paper, together with some implications for inhomogeneous growth processes.

In the unperturbed random system (for $\rho = 0$), the large-scale asymptotics of disorder-averaged correlation functions is due to the sample-to-sample fluctuations of the polymer *ground states*, i.e. the paths of minimal energy [18]†. Typical transversal excursions of the paths, given, e.g., by the two-point function

$$\Delta^2(t_1 - t_2) \equiv \overline{\langle (r(t_1) - r(t_2))^2 \rangle} \sim |t_1 - t_2|^{2\zeta} \tag{3}$$

are characterized by the *roughness exponent* ζ [2]. It is larger than for thermal fluctuations, namely $\zeta = 2/3$ for $d' = 1$ and $\zeta \approx 5/8$ for $d' = 2$ (see, e.g., [5] and references therein). The exponent $-\omega$ is the anomalous dimension of the *disorder-averaged* free energy $\overline{F} = -\beta^{-1}\overline{\log \operatorname{Tr} \exp(-\beta \mathcal{H})}$, whose universal part has the scaling form

$$\overline{F}(T, R) \sim T^{\omega} \mathcal{F}(R/T^{\zeta}) \tag{4}$$

in a finite system of transversal size R and longitudinal size T [2]. A 'Galilei' invariance enforces the relation $\omega = 2\zeta - 1$ between these exponents (see, e.g., [18, 21, 5]). In an ordinary universality class governed by thermal fluctuations, the universal free energy is scale-invariant (i.e. $\omega = 0$), which implies a set of hyperscaling relations [22]. Such relations are no longer valid for quenched averages.

In the continuum theory (1), the large-distance scaling (3) and (4) is reached in a crossover from free thermal behaviour described by the Gaussian fixed point ($\sigma^2 = \rho = 0$) on small scales. This crossover is parametrized by the effective strength $\sigma^2\beta^3$ of the randomness and has the characteristic longitudinal length $\xi_\parallel = \beta(\sigma^2\beta^3)^{-2/(2-d')}$. The transversal displacement and the finite-size free energy have the form $\Delta^2(t, \beta^{-1}, \sigma^2) = \beta^{-1} t \, \mathcal{D}(t/\xi_\parallel)$ and $\overline{F}(T, R, \beta^{-1}, \sigma^2) = \beta^{-1}\mathcal{F}(T/\beta R^2, T/\xi_\parallel)$, respectively, with scaling functions that are finite in the limit $t, T \ll \xi_\parallel$. In the opposite limit $t, T \gg \xi_\parallel$, comparison with (3) and (4) exhibits the singular dependence of $\overline{F}$ and Δ^2 on the 'bare' parameters β^{-1} and σ^2. We absorb these singularities into the definition of the renormalized quantities $r_R = (\beta/\beta_R)^{1/2} r$ and $\overline{F}_R = (\beta/\beta_R)\overline{F}$ with $\beta_R = \xi_\parallel^{-\omega}$ (recall that $\omega = 2\zeta - 1$). This also entails a renormalization of the defect, $\Phi_R = \delta^{d'}(r_R) = (\beta/\beta_R)^{-d'/2}\Phi$ and $\rho_R = (\beta/\beta_R)^{1+d'/2}\rho$. The renormalized displacement function and free energy remain finite in the continuum limit $\xi_\parallel \to 0$ (i.e. $\beta^{-1} \to 0$ or $\sigma^2 \to \infty$), and the renormalized temperature β_R^{-1} is an irrelevant coupling constant of dimension $-\omega$. This is why the renormalized theory may be called a zero-temperature fixed point. The existence of a zero-temperature continuum limit is crucial if the ensemble of ground states generated by the quenched disorder is to have universal features.

The crossover respects the Galilei invariance. In particular, the *connected* displacement function can be shown to equal that of the Gaussian theory [18, 21]

$$\overline{\langle (r(t_1) - r(t_2))^2 \rangle^c} \sim \beta^{-1}|t_1 - t_2| \,. \tag{5}$$

This equation takes the identical form in renormalized variables. Hence at zero temperature, the full renormalized displacement function Δ_R^2 equals its *thermally disconnected* part $\overline{\langle r_R(t_2) - r_R(t_1) \rangle^2}$; the connected part (5) is a correction to scaling obtained by expanding the renormalized crossover form $\Delta_R^2(t, \beta_R^{-1}) = |t|^{2\zeta}\tilde{\mathcal{D}}(\beta_R^{-1} t^{-\omega})$ in the temperature β_R^{-1},

$$\Delta_R^2(t, \beta_R^{-1}) = |t|^{2\zeta} + \tilde{c}\,\beta_R^{-1}\,|t|^{2\zeta - \omega} + \cdots \,. \tag{6}$$

† In the framework of the continuum theory, one can show that two independent paths of minimal energy in a given sample occur with probability 0. Related questions are discussed in [19] and on the lattice in [20].

The zero-temperature continuum field theory serves as the point of expansion for a perturbative renormalization of the defect problem along the lines of [23]. The variables $r_R, \overline{F}_R, \beta_R, \Phi_R, \rho_R$ now take the role of bare fields and couplings, and we drop the subscript. The universal part of the disorder-averaged free energy density $\overline{f} \equiv \lim_{T\to\infty} \partial_T \overline{F}$ in a system of transversal size $R \equiv L^\zeta$ can be expanded in powers of the defect strength

$$\overline{f}(\rho, L) - \overline{f}(0, L) = -\beta^{-1} \lim_{T\to\infty} \partial_T \overline{\log\left\langle \exp\left[-\beta\rho \int \mathrm{d}t\ \Phi(t)\right]\right\rangle}$$

$$= -\beta^{-1} \sum_{m=1}^{\infty} \frac{(-\beta\rho)^m}{m!} \int \mathrm{d}t_2 \cdots \mathrm{d}t_m \overline{\langle \Phi(0)\Phi(t_2)\cdots\Phi(t_m)\rangle^c} . \tag{7}$$

The scale L acts as infrared cutoff and will generate the renormalization group flow.

A weak defect potential distorts the minimal energy paths of the unperturbed system; the dominant paths reorganize exploiting the low-lying excitations. The statistics of these excitations is encoded in the *connected* correlation functions of the local pinning field $\Phi(t)$ at the disorder fixed point that appear in (7). To derive the renormalization group equation for the defect strength, we now have to study the short-distance structure of these objects.

The one-point function $\overline{\langle\Phi(t)\rangle}$ gives the probability density that at time t, the polymer is at the origin $r = 0$, averaged over thermal and disorder fluctuations. In the limit $T \to \infty$ and with periodic boundary conditions in the transverse direction, one has by translational invariance

$$\overline{\langle\Phi(t)\rangle} = L^{-x} \qquad \text{with} \quad x = d'\zeta \tag{8}$$

where the exponent x is the *scaling dimension* of the field Φ at the disorder fixed point.

The *full* multipoint correlation functions $\overline{\langle\Phi(t_1)\cdots\Phi(t_m)\rangle}$ give the probability density that a (single) path crosses the line $r = 0$ at given times $t_1, \ldots, t_m$. To discuss their short distance properties, specifically consider the two-point function $\overline{\langle\Phi(t_1)\Phi(t_2)\rangle}$ for $|t_1 - t_2| \ll L$. In this limit, it depends on the infrared cutoff as L^{-x}, i.e. in the same way as the one-point function (8). Hence asymptotically, it factorizes into $\overline{\langle\Phi(t_1)\rangle}$ and the L-independent 'return probability' to the origin (which is simply the inverse spread of the paths $\sim \Delta(t)^{-d'}$ given by (6))

$$\overline{\langle\Phi(t_1)\Phi(t_2)\rangle} \sim |t_1 - t_2|^{-x}(1 - \tilde{c}\beta^{-1}|t_1 - t_2|^{-\omega} + \cdots)\overline{\langle\Phi(t_1)\rangle} . \tag{9}$$

Again the leading singularity is due to sample-to-sample fluctuations of the minimal energy paths, while the correction term is due to thermal fluctuations around these paths. At zero temperature, the field $\Phi(t)$ can be replaced by its thermal expectation value $\langle\Phi(t)\rangle$; hence $\overline{\langle\Phi(t_1)\Phi(t_2)\rangle}$ equals its thermally disconnected part $\overline{\langle\Phi(t_1)\rangle\langle\Phi(t_2)\rangle}$ † and the connected part $\overline{\langle\Phi(t_1)\,\Phi(t_2)\rangle^c}$ vanishes, just as the connected displacement function (5) does. Only the subleading singularity survives in $\overline{\langle\Phi(t_1)\,\Phi(t_2)\rangle^c}$. An analogous argument applies to the singularities in any correlation function $\overline{\langle\cdots\Phi(t)\Phi(t')\cdots\rangle}$ as $|t - t'| \to 0$. Therefore the relation

$$\Phi(t)\Phi(t') \sim c\,\beta^{-1}|t - t'|^{-x-\omega}\,\Phi(t) \tag{10}$$

(with a constant $c > 0$) is valid as an operator identity, i.e. inserted in an arbitrary *connected* correlation function $\overline{\langle\cdots\Phi(t)\Phi(t')\cdots\rangle^c}$. The notion of an *operator algebra* that encodes

† This equality uses the uniqueness of the minimal energy path in a given sample[19,20]. In systems with such paths that are degenerate, the disconnected part $\overline{\langle\Phi(t_1)\rangle\langle\Phi(t_2)\rangle}$ receives an additional contribution at zero temperature from configurations with one path crossing $r = 0$ at time t_1 and *another one* at time t_2. However, this contribution is expected to scale as $\overline{\langle\Phi\rangle}^2$ and hence to be non-singular in the limit $t_2 \to t_1$. Hence the leading singularity of $\overline{\langle\Phi(t_1)\Phi(t_2)\rangle^c}$ is still given by (10).

the universal short-distance properties of correlation functions is familiar in field theory†. The new feature of (10) is that the *leading* singularity is governed by a correction-to-scaling exponent. This is a consequence of the breakdown of hyperscaling at the disorder fixed point.

The operator algebra (10) dictates the leading ultraviolet singularities of the integrals in (7). Analytically continued to arbitrary d', they show up as poles in

$$\varepsilon(d') \equiv 1 - x - \omega = 2 - (d'+2)\zeta(d'), \tag{11}$$

which serves as expansion parameter. Inserting equations (8) and (10) in (7), we find to second order that

$$\overline{f}(\rho, L) - \overline{f}(0, L) = L^{-\varepsilon}\overline{\langle\Phi\rangle}\left(w - \frac{c}{\varepsilon}w^2\right) + \mathrm{O}\left(w^3, \varepsilon^0 w^2\right). \tag{12}$$

where $w \equiv \rho L^{\varepsilon}$ is the dimensionless defect coupling. The pole in ε can be absorbed into a renormalized coupling $W = Z(W)w$ with $Z(W) = 1 - (c/\varepsilon)W + \mathrm{O}\left(W^2\right)$. The renormalization group flow‡

$$L\partial_L W = \varepsilon W - cW^2 + \mathrm{O}\left(W^3\right) \tag{13}$$

determines the large scale behaviour of the perturbed system. For $\varepsilon > 0$, i.e. for $d' < 1$, the perturbation is relevant: for any attractive bare defect potential, the renormalized coupling is driven towards large attractive values. Hence the flux line is localized by an arbitrary weak attractive columnar defect. The localization length diverges as $\xi_\perp \sim |\rho|^{-\nu_\perp}$ with $\nu_\perp = \zeta/\varepsilon$ when the defect strength approaches zero from below. In the borderline dimension $d' = 1$ an attractive defect potential is marginally relevant: the line is still localized by an arbitrary weak columnar defect, but with an essential singularity in the localization length $\xi_\perp \sim \exp(2/3c|W|)$.

For $\varepsilon < 0$, i.e. for $d' > 1$, a *weak* defect is an irrelevant perturbation. The transition to a localized state now takes place at a *finite* critical strength ρ_c (which however depends on the microscopic scales of the system and is hence non-universal). It is governed by the non-trivial fixed point $W^* = \varepsilon/c < 0$ of (13). Close to the transition, the localization length diverges as $\xi_\perp \sim |\rho - \rho_c|^{-\nu_\perp}$, where $\nu_\perp = \zeta/y^*$ and y^* is given by the ε-expansion $y^* = -\varepsilon + \mathrm{O}(\varepsilon^2)$.

Additional insight into this problem may be gained by the mapping onto the KPZ equation (2). From this stochastic equation, one constructs in a standard way the generating functional $\mathrm{Tr}\exp(-S[h, \tilde{h}])$ of the dynamic correlation functions (denoted by $\langle\langle\cdots\rangle\rangle$) in terms of the height field h and the 'conjugate' field $\tilde{h}$ [25]. Insertions of this field generate response functions, e.g. $\langle\langle h(r,t)\,\Pi_j \tilde{h}(r_j, t_j)\rangle\rangle = \langle\langle \delta\, h(r,t)/\Pi_j \delta\, V(r_j, t_j)\rangle\rangle$. The local defect in the rate of mass deposition leads to a term $S_I = \rho \int \mathrm{d}t\, \tilde{h}(0, t)$ in the dynamic action, the analogue of the defect term in (1). The disorder-averaged free energy density $\overline{f}(\rho, L)$ equals (minus) the *stationary growth velocity* $v(\rho, L) \equiv \langle\langle\partial_t h\rangle\rangle(\rho, L)$. The excess growth rate caused by the defect can be calculated perturbatively about the KPZ fixed point

$$v(\rho, L) = \lim_{T\to\infty} \partial_T \left\langle\!\!\left\langle h(r, T)\exp\left[-\rho \int \mathrm{d}t\ \tilde{h}(0, t)\right]\right\rangle\!\!\right\rangle. \tag{14}$$

Equating this series term by term with (7), we obtain

$$\beta^{m-1}\ \overline{\langle\Phi(t_1)\cdots\Phi(t_m)\rangle^{c}} = \lim_{T\to\infty} \langle\langle h(r, T)\tilde{h}(0, t_1)\cdots\tilde{h}(0, t_m)\rangle\rangle \tag{15}$$

† For example, the algebra of the pinning field at the Gaussian fixed point, which is relevant to temperature-driven unbinding transitions, reads $\Phi(t)\Phi(t') \sim |t - t'|^{-x_0}\Phi(t) + \cdots$ with $x_0 = d'/2$. See [23].

‡ Further primitive singularities are expected at higher orders in the perturbation expansion; hence in contrast to thermal depinning [24], the flow equation does not terminate at this order.

and a resulting flow equation that is identical with (13). From (15), we identify $\beta\Phi(t)$ with $\widetilde{h}(0, t)$, which hence is a field of scaling dimension $\widetilde{x} = x + \omega$. Equation (10) then yields without further calculation the short-distance algebra

$$\widetilde{h}(0,t)\widetilde{h}(0,t') \sim c\,|t-t'|^{-\widetilde{x}}\,\widetilde{h}(0,t') + \cdots . \qquad (16)$$

Its leading singularity is no longer a correction-to-scaling exponent; the peculiarity of the correlation functions written in terms of the $\widetilde{h}$ fields is rather that they have to be computed in the non-trivial 'vacuum' state $h(r, t)$. This makes the *non-unitarity* of this theory manifest, which is generated by the averaging over disorder. We note that the form of the operator algebra (16) also follows from the mode coupling approach of [17] for $d' = 1$.

In the thermodynamic limit $L \to \infty$, the stationary growth velocity becomes independent of ρ for $\rho > \rho_c$ but increases with decreasing ρ for $\rho < \rho_c$ (i.e. for sufficiently strong excess mass deposition at the origin), as follows from the mapping onto the polymer free energy density. In one dimension, the surface has an approximately triangular stationary profile $H(r) = \langle\langle h(r,t)\rangle\rangle - vt$ in the phase of enhanced growth, $\rho < 0$ [9]. From a simple scaling argument, we obtain that for $\xi_\perp(W) \ll L^\zeta$, the excess velocity scales as $(v(W, L) - v(0, L)) \sim \xi_\perp^{(-1+\omega)/\zeta} \sim \exp(2/3cW)$. The same essential singularity shows up in the slope $|\partial H/\partial r| \sim (v(W, L) - v(0, L))^{1/2}$. The response function has the form $\langle\langle h(r,t)\widetilde{h}(r_1,t_1)\rangle\rangle = \xi_\perp^{-1}\mathcal{G}(r_1/\xi_\perp)$ for $t \to \infty$ and $r, \xi_\perp \ll R$. All of these quantities are accessible in numerical simulations which could provide a useful test of the results discussed in this paper.

In summary, we have shown that a class of field theories with quenched randomness shares with conventional field theories the notion of a short-distance algebra of its scaling operators. That is the basis of a renormalization group which we believe to be applicable quite generically to perturbed random systems.

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