Quantized Scaling of Growing Surfaces

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(Received 20 August 1997)

The Kardar-Parisi-Zhang universality class of stochastic surface growth is studied by exact field-theoretic methods. From previous numerical results, a few qualitative assumptions are inferred. In particular, height correlations should satisfy an operator product expansion and, unlike the correlations in a turbulent fluid, exhibit no multiscaling. These properties impose a quantization condition on the roughness exponent χ and the dynamic exponent z. Hence the exact values $\chi = 2/5, z = 8/5$ for two-dimensional and $\chi = 2/7, z = 12/7$ for three-dimensional surfaces are derived. [S0031-9007(98)05491-X]

PACS numbers: 64.60.Ht, 47.27.Eq, 68.35.Fx

Strongly driven dynamic systems offer some of the most intriguing realizations of statistical scale invariance. Hydrodynamic turbulence [1] or the growth of rough surfaces [2] are two classic examples, which turn out to be deeply connected from a theoretical point of view. In such systems, a stochastic force $\eta(\mathbf{r}, t)$ generates long-ranged correlations of a fluctuating dynamic field—the local velocity $\mathbf{v}(\mathbf{r}, t)$ of a fluid or the height $h(\mathbf{r}, t)$ of a surface. As typical differences $h(\mathbf{r}_1, t) - h(\mathbf{r}_2, t)$ or $\mathbf{v}(\mathbf{r}_1, t) - \mathbf{v}(\mathbf{r}_2, t)$ *increase* with the spatial separation $|\mathbf{r}_1 - \mathbf{r}_2|$, the scaling properties of these fields are generically more complex than those at a standard critical point. Indeed, the theoretical understanding of these universality classes far from equilibrium is still fragmentary.

The subject of this Letter is the simplest nonlinear model of stochastic surface growth, the famous Kardar-Parisi-Zhang equation

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta \tag{1}$$

for a d-dimensional surface [3]. The driving term $\eta(\mathbf{r}, t)$, which describes the random adsorption of molecules onto the surface, is taken to be Gauss distributed with correlations over only *microscopic* distances,

$$\overline{\eta(\mathbf{r}, t)\eta(\mathbf{r}', t')} = \sigma^2 \delta(t - t')\delta(\mathbf{r} - \mathbf{r}'). \tag{2}$$

The relation of this model to the theory of turbulence is manifest: Eq. (1) is formally equivalent to Burgers equation

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \nabla^2 \mathbf{v} + \nabla \eta \tag{3}$$

for the driven dynamics of a vortex-free velocity field $\mathbf{v}(\mathbf{r}, t) = \nabla h(\mathbf{r}, t)$ (with $\lambda = -1$) [4]. In a fluid, however, the driving force is correlated over *macroscopic* spatial distances. This leads to important differences in the scaling behavior [5], which are discussed below.

A surface growing from a flat initial state $h(\mathbf{r}, 0) = 0$ develops height correlations with an increasing correlation length $\xi_t \sim t^{1/z}$, which defines the dynamic exponent z [6]. A self-similar growth pattern, characterized, e.g., by the height difference moments

$$\langle [h(\mathbf{r}_1) - h(\mathbf{r}_2)]^k \rangle \sim |\mathbf{r}_{12}|^{-k\chi}, \tag{4}$$

(with $\mathbf{r}_{12} \equiv \mathbf{r}_1 - \mathbf{r}_2$), emerges on mesoscopic scales $\tilde{a} \ll |\mathbf{r}_{12}| \ll \xi_t$. For $|\mathbf{r}_{12}| \lesssim \tilde{a}$, the dissipation term $\nu \nabla^2 h$ in Eq. (1) breaks the asymptotic scale invariance. In the scaling regime (4), the height difference moments become *stationary*, i.e., independent of the correlation length ξ_t. They are characterized by a single critical index $\chi \geq 0$, the *roughness exponent* of the surface. The scaling relation $\chi + z = 2$ follows from the Galilei invariance of Eq. (1) [7]. For $d = 1$, one can show Eq. (4) to be valid with the roughness exponent $\chi = 1/2$, equal to that of the linear theory ($\lambda = 0$) [2]. In higher dimensions, however, little is known analytically. For $d > 2$, the rough state of the surface exists only if the rescaled driving amplitude $\lambda_0^2 \equiv \sigma^2 \lambda^2 / \nu^3$ exceeds a finite threshold value λ_c^2 [8]. Less rigorous theoretical arguments predict an upper critical dimension $d_> \leq 4$ beyond which Kardar-Parisi-Zhang surfaces are only logarithmically rough ($\chi = 0$) even in the strong-coupling regime $\lambda_0^2 > \lambda_c^2$ [9]. The numerical results presently available are consistent with Eq. (4). Extensive simulations yield $\chi \approx 0.39$ for $d = 2$, $\chi \approx 0.31$ for $d = 3$, and smaller positive values in higher dimensions, which are less reliable [10].

It has remained a challenge for theorists to calculate the rough asymptotic state of Kardar-Parisi-Zhang surfaces for $d > 1$ exactly or in a controlled approximation. In particular, standard perturbative renormalization about the linear theory fails to produce a fixed point belonging to this regime [11]—a notorious difficulty familiar from the theory of turbulence. In this Letter, a quite different approach is taken. Guided by numerical and experimental results, I make a few *qualitative* assumptions, namely, the existence of an operator product expansion (8) and of a stationary state (10) that is directed (i.e., it has no up-down symmetry). These assumptions turn out to constrain severely the possible solutions of Eq. (1). In particular, they naturally lead to a quantization condition for the

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roughness exponent

$$\chi = \frac{2}{k_0 + 2}, \qquad (5)$$

where k_0 is an odd integer for $d \geq 2$. Comparing with the above numerical estimates [12] and using the relation $\chi + z = 2$, then gives the main result of this Letter: the exact values $\chi = 2/5$, $z = 8/5$ for $d = 2$ and $\chi = 2/7, z = 12/7$ for $d = 3$.

The fundamental observables describing the equal-time surface configurations are the (connected) correlations

$$\langle h(\mathbf{r}_1) \cdots h(\mathbf{r}_n) \rangle_t = \int \mathcal{D}h\, h(\mathbf{r}_1) \cdots h(\mathbf{r}_n) P_t - \cdots, \qquad (6)$$

(the dots denoting the disconnected parts). The height probability distribution $P_t(\{h\})$ obeys the functional Fokker-Planck equation

$$\partial_t P_t = \left(\int d\mathbf{r} \left[\sigma^2 \frac{\delta^2}{\delta h(\mathbf{r})^2} - \frac{\delta}{\delta h(\mathbf{r})} J(\mathbf{r}) \right] P_t \right), \qquad (7)$$

where $J(\mathbf{r}) \equiv \nu \nabla^2 h(\mathbf{r}) + (\lambda/2)(\nabla h)^2(\mathbf{r})$ is the deterministic part of the current.

In the scaling regime ($\tilde{a} \ll |\mathbf{r}_{ij}| \ll \xi_t$ for $i, j = 1, \ldots, n$), the correlation functions (6) will generically become singular as some of the points approach each other. For $d < d_>$, these singularities are assumed to follow from an *operator product expansion*

$$h(\mathbf{r}_1) \cdots h(\mathbf{r}_k) = \sum_{\mathcal{O}} |\mathbf{r}_{12}|^{-kx_h + x_{\mathcal{O}}} \times C_k^{\mathcal{O}} \left(\frac{\mathbf{r}_{13}}{|\mathbf{r}_{12}|}, \ldots, \frac{\mathbf{r}_{1k}}{|\mathbf{r}_{12}|} \right) \mathcal{O}(\mathbf{r}_1). \qquad (8)$$

This identity is nothing but a consistency relation for the height correlations. Inserted in (6), it expresses any n-point function as a sum of $(n - k + 1)$-point functions in the limit $|\mathbf{r}_{ij}| \ll |\mathbf{r}_{il}| \ll \xi_t$ ($i, j = 1, \ldots, k$ and $l = k + 1, \ldots, n$). The notion of an operator product expansion is familiar in field theory [13] and has recently been applied successfully to nonequilibrium systems [14,15]. Of course, its status is still heuristic in that context. The sum on the right-hand side runs over all local scaling fields $\mathcal{O}(\mathbf{r})$. Each term contains a dimensionless scaling function $C_k^{\mathcal{O}}$ (a simple number for $k = 2$) and a power of $|\mathbf{r}_{12}|$ given by the scaling dimensions $x_{\mathcal{O}}$ and $x_h = -\chi$ (such that the overall dimension equals that of the left-hand side). The field $\mathcal{O}_k$ with the smallest dimension, x_k, determines, in particular, the asymptotic behavior of the k-point functions as $\xi_t \to \infty$,

$$\langle h(\mathbf{r}_1) \cdots h(\mathbf{r}_k) \rangle_t \sim \langle \mathcal{O}_k \rangle_t \sim \xi_t^{-x_k}. \qquad (9)$$

The amplitudes $\langle \mathcal{O}_k \rangle_t = \langle \mathcal{O}_k(\mathbf{r}) \rangle_t$ diverge with ξ_t, i.e., $x_k < 0$ [16]. They measure the *global* roughness, which increases as the surface develops higher mountains and deeper valleys. *Local* surface properties should, however, behave quite differently. For example, the gradient correlation functions are assumed to have a finite limit

$$\lim_{\xi_t \to \infty} \langle \nabla h(\mathbf{r}_1) \cdots \nabla h(\mathbf{r}_n) \rangle_t \equiv \langle \nabla h(\mathbf{r}_1) \cdots \nabla h(\mathbf{r}_n) \rangle. \qquad (10)$$

By writing $h(\mathbf{r}_i) - h(\mathbf{r}_i') = \int_{\mathbf{r}_i}^{\mathbf{r}_i'} \mathbf{ds} \cdot \nabla h(\mathbf{s})$, the same property follows for the height difference correlation functions $\langle \prod_{i=1}^n [h(\mathbf{r}_i) - h(\mathbf{r}_i')] \rangle_t$, in particular, for the moments (4). This implies a feature familiar from simulations: one cannot recognize the value of ξ_t from snapshots of the surface in a region much smaller than ξ_t.

By differentiating (8), one obtains an operator product expansion for the gradient field $\mathbf{v} \equiv \nabla h$ of the form

$$\mathbf{v}(\mathbf{r}_1) \cdots \mathbf{v}(\mathbf{r}_k) = \sum_{\mathcal{O}} |\mathbf{r}_{12}|^{-kx_{\mathbf{v}} + x_{\mathcal{O}}} \times \tilde{C}_k^{\mathcal{O}} \left(\frac{\mathbf{r}_{13}}{|\mathbf{r}_{12}|}, \ldots, \frac{\mathbf{r}_{1k}}{|\mathbf{r}_{12}|} \right) \mathcal{O}(\mathbf{r}_1), \qquad (11)$$

with new scaling functions $\tilde{C}_k^{\mathcal{O}}$ and the dimension $x_{\mathbf{v}} = -\chi + 1$. [Both sides of (11) are tensors of rank k whose indices are suppressed.] The fields $\mathcal{O}$ on the right-hand side govern the time-dependent amplitudes $\langle \mathbf{v}(\mathbf{r}_1) \cdots \mathbf{v}(\mathbf{r}_k) \rangle_t \sim \langle \mathcal{O} \rangle_t \sim \xi_t^{-x_{\mathcal{O}}}$ in analogy to (9). Hence, the stationarity condition (10) allows in (11) only fields $\mathcal{O}$ with a *non-negative* scaling dimension $x_{\mathcal{O}}$, such as **1** (the identity field), $(\nabla h)^2(\mathbf{r})$, etc. This in turn restricts the possible terms in (8): (a) *singular* terms involving fields $\mathcal{O}(\mathbf{r})$ with $x_{\mathcal{O}} \geq 0$; (b) *regular* terms, where the coefficient $|\mathbf{r}_{12}|^{-kx_h + x_{\mathcal{O}}} C_k^{\mathcal{O}}$ is a tensor of rank N in the differences $\mathbf{r}_{1i}$ ($i = 2, \ldots, k$). Such terms do not violate (10) since they have a vanishing coefficient $\tilde{C}_k^{\mathcal{O}}$ in (11) for $N < k$. They can readily be associated with composite fields of dimensions

$$x_{k,N} = -k_{\chi} + N. \qquad (12)$$

The leading ($N = 0$) term involves the (normal-ordered) field $\mathcal{O}_k(\mathbf{r}) = h^k(\mathbf{r})$ and governs the asymptotic singularity (9); the higher terms correspond to fields with k factors $h(\mathbf{r})$ and N powers of ∇.

It is useful to introduce the (normal-ordered) vertex fields $Z_q(\mathbf{r}) \equiv \exp[qh(\mathbf{r})]$, which are the generating functions of the fields $h^k(\mathbf{r})$. Equation (8) is then consistent with the operator product expansion

$$Z_{q_1}(\mathbf{r}_1) Z_{q_2}(\mathbf{r}_2) = \exp\left(\sum_{k,l} C_{k,l}^{1} w_1^k w_2^l \right) Z_{q_1 + q_2}(\mathbf{r}_1) + O(C_{k,l}^{\mathcal{O} \neq 1}), \qquad (13)$$

where $C_{k,l}^{\mathcal{O}} \equiv C_{k+l}^{\mathcal{O}}(0, \ldots, 0, \mathbf{r}_{12}/|\mathbf{r}_{12}|, \ldots, \mathbf{r}_{12}/|\mathbf{r}_{12}|)$ with the first k arguments equal to 0 and $w_i \equiv q_i |\mathbf{r}_{12}|^{\chi}$ [17,18]. Subleading singular terms (with positive-dimensional fields $\mathcal{O}$) and regular terms (with fields containing height gradients) are omitted. The vertex n-point functions $\langle Z_{q_1}(\mathbf{r}_1) \cdots Z_{q_n}(\mathbf{r}_n) \rangle_t$ behave asymptotically as $\exp(\xi_t^{\chi} \sum_{i=1}^n q_i)$. If $\sum_i q_i = 0$, they have a finite limit $\langle Z_{q_1}(\mathbf{r}_1) \cdots Z_{-q_1 - \cdots - q_{n-1}}(\mathbf{r}_n) \rangle$. Since these are precisely the vertex correlators that generate the height difference correlation functions and since (13) is analytic in the q_i, this leads back to the stationarity condition (10).

The operator product expansion (13) with the linear dimensions (12) is at the heart of the field theory for

Kardar-Parisi-Zhang systems. It is instructive to compare this theory with models of turbulence. Burgers equation (3) with force correlations

$$\overline{\eta(\mathbf{r},t)\eta(\mathbf{r}',t')} = \sigma^2 R^2 \delta(t - t')\Delta(|\mathbf{r} - \mathbf{r}'|/R) \quad (14)$$

over large distances R develops *multiscaling:* for example, the longitudinal velocity difference moments

$$\langle [v_\parallel(\mathbf{r}_1) - v_\parallel(\mathbf{r}_2)]^k \rangle \sim |\mathbf{r}_{12}|^{-kx_{\mathbf{v}} + \tilde{x}_k} R^{-\tilde{x}_k} \quad (15)$$

have a k-dependent singular dependence on $|\mathbf{r}_{12}|$ and R in the inertial scaling regime $\tilde{a} \ll |\mathbf{r}_{12}| \ll R$ [5,19]. Similar multiscaling is present in Navier-Stokes turbulence. Kolmogorov's famous argument predicts the exact scaling dimension of the velocity field, $x_{\mathbf{v}} = -1/3$, from dimensional analysis [20]. This determines the scaling of the third moment in (15) since $\tilde{x}_3 = 0$. The higher exponents $\tilde{x}_4, \tilde{x}_5, \ldots < 0$ cannot be obtained from dimensional analysis. Assuming the existence of an operator product expansion (11), the term (15) is generated by the lowest-dimensional field $\tilde{\mathcal{O}}_k$ with a singular coefficient [21]. Multiscaling thus implies the existence of a (presumably infinite) number of composite fields with anomalous negative dimensions. For the velocity vertex fields $\exp[qv(r)]$ of Burgers turbulence in one dimension, Polyakov has conjectured an operator product expansion similar to (13) and consistent with multiscaling [14]. The distinguishing feature of Kardar-Parisi-Zhang surfaces is the absence of multiscaling [22]. Notice that the resulting properties (12) and (13) have been derived solely from the assumptions (8) and (10) without using Eq. (1) explicitly.

To establish the consistency of the operator product expansion with the underlying dynamic equation, one has to construct correlation functions that remain finite in the continuum limit $\tilde{a} \to 0$. With the probability distribution (7), the height correlations (6) develop singularities dictated by their normalization in the linear regime ($|\mathbf{r}_{ij}| \ll \tilde{a}$). The existence of a well-defined asymptotic scaling regime for $|\mathbf{r}_{ij}| \gg a$ implies that these singularities can be absorbed by a change of variables

$$h(\mathbf{r}) \to Z_h(\tilde{a}/r_0)h(\mathbf{r}), \qquad t \to Z_t(\tilde{a}/r_0)t\,, \quad (16)$$

such that the "renormalized" correlations (6) satisfy normalization conditions independently of $\tilde{a}$ at some mesoscopic scale r_0 [18,23]. The Z factors have the asymptotic behavior $Z_h \sim (\tilde{a}/r_0)^{\chi - \chi_0}$ and $Z_t \sim (\tilde{a}/r_0)^{z - z_0}$ as $\tilde{a}/r_0 \to 0$, where $\chi_0 = (2 - d)/2$ and $z_0 = 2$ are the exponents in the linear regime. Of course, I do not assume perturbative renormalizability (i.e., that the Z factors are analytic functions of λ_0^2). Since the scaling dimensions (12) are linear in k, the renormalization (16) also removes the singularities from correlations of the fields $h^k(\mathbf{r})$ and $Z_q(\mathbf{r})$, ensuring a finite limit of the coefficients C in (8), (11), and (13) and of the amplitudes $\langle \mathcal{O}_k \rangle_t$ in (9). The substitution (16) also leads to new coefficients in (1) and (7):

$$\begin{aligned} \nu(\tilde{a}/r_0) &\sim Z_t^{-1} \simeq \nu^* \times (\tilde{a}/r_0)^{\chi}, \\ \sigma^2(\tilde{a}/r_0) &\sim Z_h^2 Z_t^{-1} \simeq \sigma^{*2} \times (\tilde{a}/r_0)^{d-2+3\chi}, \\ r_0^{\chi_0}\lambda(\tilde{a}, r_0) &\sim Z_t^{-1} Z_h^{-1} \simeq g^*. \end{aligned} \quad (17)$$

Galilei invariance is expressed by the asymptotic scale invariance of the dimensionless coupling $r_0^{\chi_0}\lambda$, while the other coefficients become irrelevant as $\tilde{a}/r_0 \to 0$. However, as explained in Ref. [14] for Burgers turbulence, the equation of motion for the renormalized correlation functions is quite subtle due to anomalies dictated by the operator product expansion. To exhibit the anomalies for the height correlations, I introduce the smeared vertex fields $Z_q^a(\mathbf{r}) \equiv \exp[q \int d\mathbf{r}' \delta_a(\mathbf{r} - \mathbf{r}')h(\mathbf{r}')]$ [where $\delta_a(\mathbf{r})$ is a normalized function with support in the sphere $|\mathbf{r}| < a$] and the abbreviations $Z_i^a \equiv Z_{q_i}^a(\mathbf{r}_i), Z_i \equiv Z_{q_i}(\mathbf{r}_i)Z$. Using (6), (7), and (17), it is straightforward to derive

$$\partial_t \langle Z_1^a \cdots Z_n^a \rangle_t = \sum_{i=1}^n q_i \langle Z_1^a \cdots J Z_i^a \cdots Z_n^a \rangle_t\,, \quad (18)$$

where $JZ_i^a \equiv [q_i \sigma^2 \delta_a(0) + J(\mathbf{r}_i)]Z_i^a$. The singularity structure of the current is determined by (8) and (17):

$$JZ_i^a = g^* \hat{Z}_i + a^{2\chi - 2}\left(\sum_{k=1}^{\infty} c_k a^{k\chi} q_i^k\right) Z_i + O(a^{\chi})\,, \quad (19)$$

for $a, \tilde{a} \to 0$ with $a/\tilde{a}$ kept constant. The field $\hat{Z}_q(\mathbf{r}) \equiv (\nabla h)^2 Z_q(\mathbf{r})$ denotes the finite part of the operator product $(\nabla h)^2(\mathbf{r})Z_q^a(\mathbf{r})$ for $a \to 0$, and $\hat{Z}_i \equiv \hat{Z}_{q_i}(\mathbf{r}_i)$. The finite dissipation term $(\nabla^2 h)Z_{q_i}(\mathbf{r}_i)$ becomes irrelevant in this limit since $\nu \sim a^{\chi}$. The singular part of (19) is a power series in q_i with asymptotically constant coefficients $c_1 = \sigma^{*2} a^d \delta_a(0) + \nu^* c_{1,1} + g^* c_{2,1}$ and $c_k = \nu^* c_{1,k} + g^* c_{2,k}$ for $k = 2, 3, \ldots$. The terms of order $a^{(2+k)\chi - 2}$ originate from operator products $\nabla^2 h(\mathbf{r}_i)h(\mathbf{r}_1') \cdots h(\mathbf{r}_k') \sim \mathbf{1}$ and $(\nabla h)^2(\mathbf{r}_i)h(\mathbf{r}_1') \cdots h(\mathbf{r}_k') \sim \mathbf{1}$; their respective coefficients $c_{1,k}$ and $c_{2,k}$ are integrals over the scaling functions in (8) and the regularizing functions $\delta_q(\mathbf{r}_i - \mathbf{r}_j')$. Of course, divergent terms have to cancel so that Eq. (18) has a finite continuum limit

$$\partial_t \langle Z_1 \cdots Z_n \rangle_t = \sum_{i=1}^n q_i \langle Z_1 \cdots J Z_i \cdots Z_n \rangle_t\,, \quad (20)$$

with $JZ_i = \lim_{a \to 0} JZ_i^a$. For generic values of χ, this implies $JZ_i = g^* \hat{Z}_i$. However, if χ satisfies the condition (5) for some integer k_0, the dissipation current contributes an anomaly:

$$JZ_i = g^* \hat{Z}_i + \nu^* c_{1,k_0} q_i^{k_0} Z_i\,. \quad (21)$$

Equations (20) and (21) govern, in particular, the stationary state of the surface. For $d = 1$, the stationary height distribution is known, $P \sim \exp[-(\sigma^2/\nu)\int d\mathbf{r}(\nabla h)^2]$. It equals that of the linear theory, thus restoring the up-down symmetry $h(\mathbf{r}) - \langle h \rangle_t \to -h(\mathbf{r}) + \langle h \rangle_t$ broken by the nonlinear term in (1). The exponent $\chi = 1/2$ satisfies (5) with $k_0 = 2$ but the up-down symmetry forces the anomaly to vanish ($c_{1,2} = 0$). In higher dimensions, this symmetry is expected to remain broken in the stationary regime. The surface has rounded hilltops and steep valleys, just like the upper side of a cumulus cloud [24]. Hence, the local slope is correlated with the relative height, resulting in nonzero odd moments $\langle (\nabla h)^2(\mathbf{r}_1)[h(\mathbf{r}_1) - h(\mathbf{r}_2)]^k \rangle$. However, this is consistent with Eqs. (20) and (21) only for odd values of k_0, where

$$\langle \hat{Z}_q(\mathbf{r}_1) Z_{-q}(\mathbf{r}_2) \rangle - \langle \hat{Z}_{-q}(\mathbf{r}_1) Z_q(\mathbf{r}_2) \rangle = -(\nu^*/g^*) c_{1,k_0} q^{k_0} \langle Z_q(\mathbf{r}_1) Z_{-q}(\mathbf{r}_2) \rangle , \quad (22)$$

and, hence, for odd values of $k \geq k_0$,

$$\langle (\nabla h)^2(\mathbf{r}_1)[h(\mathbf{r}_1) - h(\mathbf{r}_2)]^k \rangle = -(\nu^*/g^*) c_{1,k_0} \langle [h(\mathbf{r}_1) - h(\mathbf{r}_2)]^{k-k_0} \rangle . \quad (23)$$

The directedness of the stationary growth pattern thus requires a nonzero anomaly c_{1,k_0} with an odd integer k_0. The roughness exponent is then determined by Eq. (5). The values $k_0 = 3$ for $d = 2$ and $k_0 = 5$ for $d = 3$ give the exponents quoted above, in reasonable agreement with the numerical results [10,12].

In summary, the scaling of growing surfaces has been determined by requiring consistency of the effective large-distance field theory subject to a few phenomenological constraints. The Galilei symmetry of the dynamic equation conspires with these constraints to allow only discrete values of the roughness exponent in two and three dimensions. The underlying solutions of Eq. (1) are distinguished by a dynamical anomaly in the strong-coupling regime: The dissipation term contributes a finite part to the effective equation of motion (20) despite being formally irrelevant. The anomaly manifests itself in identities such as (23) between stationary correlation functions. The quantization rule (5) is analogous to the exact Kolmogorov scaling of the third velocity difference moment in Navier-Stokes turbulence. The deeper reason for this rigidity is yet to be explained.

I am grateful to D. Wolf for useful discussions.

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