Lösungshinweise Blatt 5

23 a)

$$A(e_1) = \begin{pmatrix} 0 \\ a_3 \\ -a_2 \end{pmatrix}, \quad A(e_2) = \begin{pmatrix} -a_3 \\ 0 \\ a_1 \end{pmatrix}, \quad A(e_3) = \begin{pmatrix} a_2 \\ -a_1 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad A = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

23 b)

$$B = \big(B(e_1), \; B(e_2), \; B(e_3) \big) = (b_1, \; b_2, \; b_3) = b^T$$

23 c)

$$C = b\, b^T = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} (b_1 \; b_2 \; b_3) = \begin{pmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_2 b_1 & b_2^2 & b_2 b_3 \\ b_3 b_1 & b_3 b_2 & b_3^2 \end{pmatrix}$$

23 d)

$$D = B A = (b_1 \; b_2 \; b_3) \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

$$= (b_2 a_3 - b_3 a_2, \; b_3 a_1 - b_1 a_3, \; b_1 a_2 - b_2 a_1)$$

24 a) $\quad Rg(A) = \dim \operatorname{Im} A = \dim \operatorname{Span}(1, x, x^2) = 3$

b)

$$_B A_B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

c)

mit $B' = (x, \; x^2, \; x^3, \; 1), \quad \mathcal{C} = (1, \; 2x, \; 3x^2, \; x^3)$

folgt

$$A x = 1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{C}}, \quad A x^2 = 2x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{C}},$$

$$A x^3 = 3x^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}_{\mathcal{C}}, \quad A 1 = 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{\mathcal{C}}$$

$$\rightarrow \quad {}_{\mathcal{C}} A_{B'} = \left(\begin{array}{c|c} \mathbb{1}_3 & 0 \\ \hline 0 & 0 \end{array} \right)$$

d) nach Vrlsg. $\quad e^{\frac{d}{dx}}(f)(x) = f(x+1)$

$$\left.\begin{array}{l} \rightarrow \quad T\,1 = 1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}_C , \quad T\,x = x+1 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}_C \\ \quad T\,x^2 = (x+1)^2 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}_C , \quad T\,x^3 = (x+1)^3 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}_C \end{array}\right\} (*)$$

d.h.

$${}_C T_B = \mathbb{1}_4$$

⌜ (*) "zu Fuß": $\quad T|_{P_3} = 1 + \frac{d}{dx} + \frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{6}\frac{d^3}{dx^3}$

$\rightarrow \quad T\,1 = 1, \quad T\,x = x+1, \quad T\,x^2 = x^2 + 2x + 1 = (x+1)^2$

$T\,x^3 = x^3 + 3x^2 + 3x + 1 = (x+1)^3$ ⌟

25) $\quad A\,b_1 = \begin{pmatrix}9\\9\\9\end{pmatrix} = 9\,b_1 , \quad A\,b_2 = \begin{pmatrix}12\\-24\\12\end{pmatrix} = 12\,b_2$

$A\,b_3 = \begin{pmatrix}2\\0\\-2\end{pmatrix} = 2\,b_3 \quad \rightarrow \quad {}_B A_B = \begin{pmatrix}9&0&0\\0&12&0\\0&0&2\end{pmatrix}$

26 a) $\quad (B^{-1}A^{-1})(A\,B) = B^{-1}(A^{-1}A)\,B = B^{-1}B = \mathbb{1}$

$(A\,B)(B^{-1}A^{-1}) = A\,(B\,B^{-1})\,A^{-1} = A\,A^{-1} = \mathbb{1}$

$\rightarrow \quad B^{-1}A^{-1} = (A\,B)^{-1}$

b)

$$M\,M^{-1} = \frac{1}{ad-bc}\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix}ad-bc & 0\\ 0 & -bc+ad\end{pmatrix} = \mathbb{1}_2 \quad ✓$$

c)

$$A^{-1} = \begin{pmatrix}1 & & & \\ & \frac{1}{2} & & 0 \\ & & \frac{1}{3} & \\ 0 & & & \frac{1}{4}\end{pmatrix}, \quad B^{-1} = \begin{pmatrix}0 & & & 1 \\ & & \frac{1}{2} & \\ & \frac{1}{3} & & \\ \frac{1}{4} & & & 0\end{pmatrix}, \quad C^{-1} = \begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{pmatrix}, \quad D^{-1} = \begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}$$