letzte Vorlesung:

- **Rotation (Wirbelstärke)**

$$\operatorname{rot} \vec{A}(\vec{r})$$

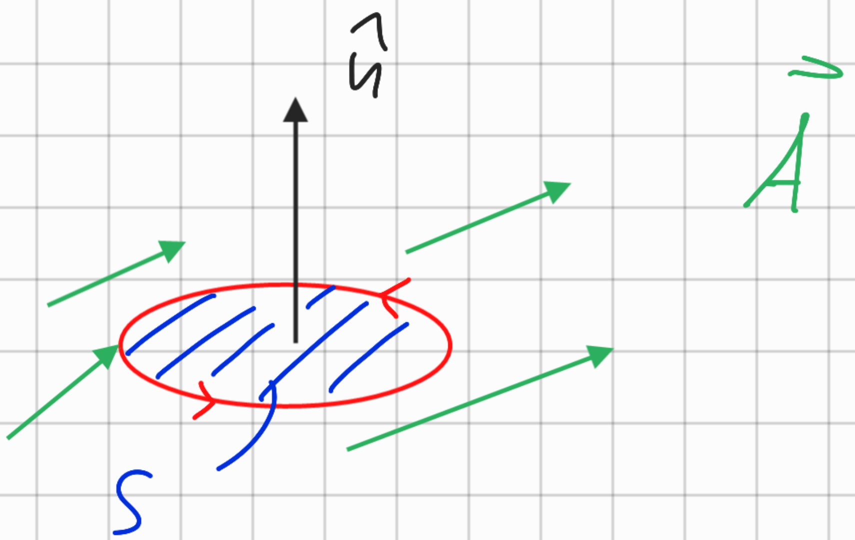
definiert durch

$$\boxed{\langle \hat{n}, \operatorname{rot} \vec{A}(\vec{r}) \rangle = \lim_{|S| \to 0} \frac{1}{|S|} \int_{\partial S} \vec{A} \, d\vec{\ell}}$$

S: Fläche bei $\vec{r}$, $\perp \hat{n}$

∂S: Rand von S

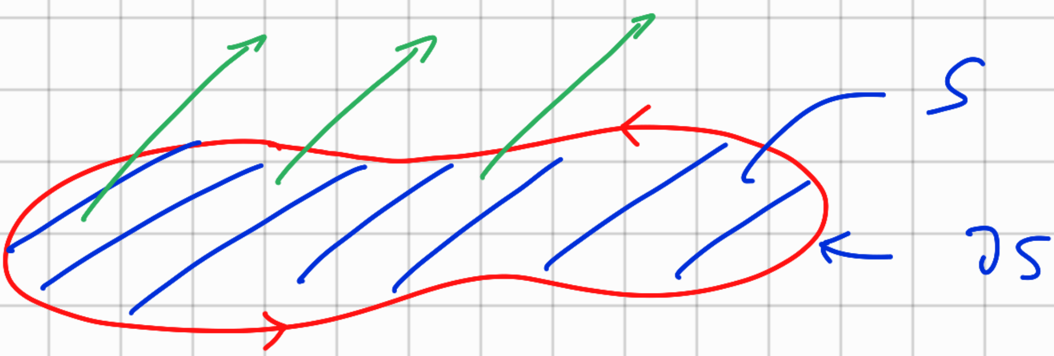
$|S|$: Flächeninhalt von S



$$\boxed{\operatorname{rot}\vec{A} = \vec{\nabla}\times\vec{A} = \begin{pmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{pmatrix} \times \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}}$$

- $\operatorname{rot}(\operatorname{grad} f) = \vec{0}$
- $\operatorname{rot}(f\vec{A})$
  $= (\operatorname{grad} f)\times\vec{A} + f\operatorname{rot}\vec{A}$
- <u>Satz von Stokes</u>

$$\boxed{\int_{\partial S} \vec{A}\, d\vec{l} \overset{!}{=} \int_S \operatorname{rot}\vec{A}\, d\vec{f}}$$



Magnetostatik:

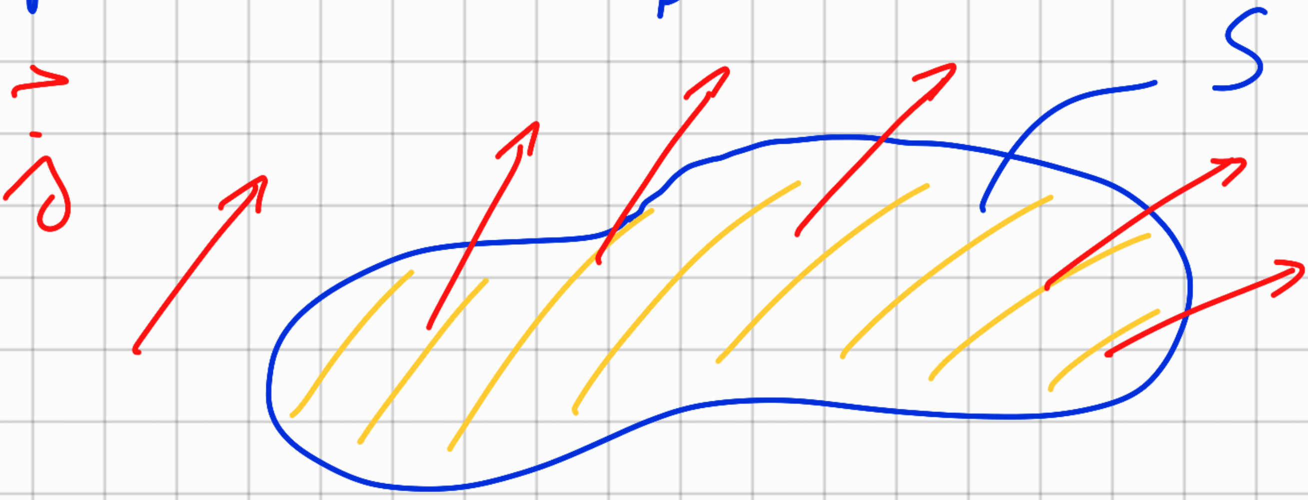
- Ampère: $\text{rot}\,\vec{B} \overset{!}{=} \mu_0 \vec{j}$

- Gauß: $\text{div}\,\vec{B} \overset{!}{=} 0$

Induktion:

- Faraday: $\text{rot}\,\vec{E} \overset{!}{=} -\frac{\partial \vec{B}}{\partial t}$

integrale Form der Gesetze von Ampère und Gauß:



$$\mu_0 I_S = \int_S \mu_0 \vec{j}\, d\vec{f} = \int_S \mathrm{rot}\,\vec{B}\, d\vec{f}$$

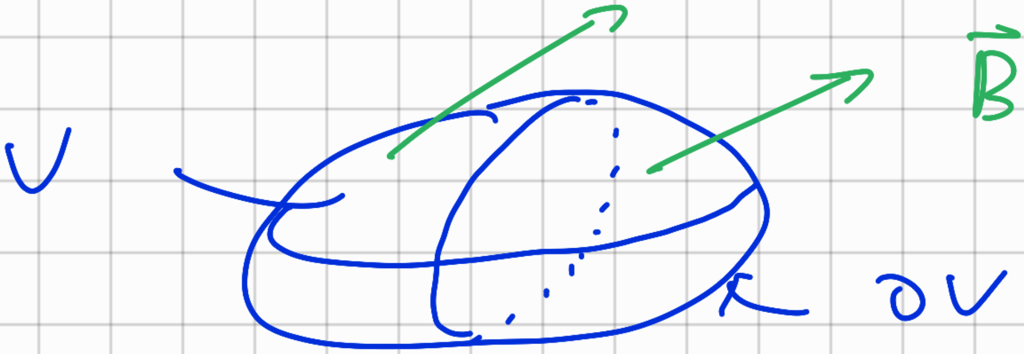
Ampère

$$= \int_{\partial S} \vec{B}\, d\vec{l}$$

Stokes!

Ampère

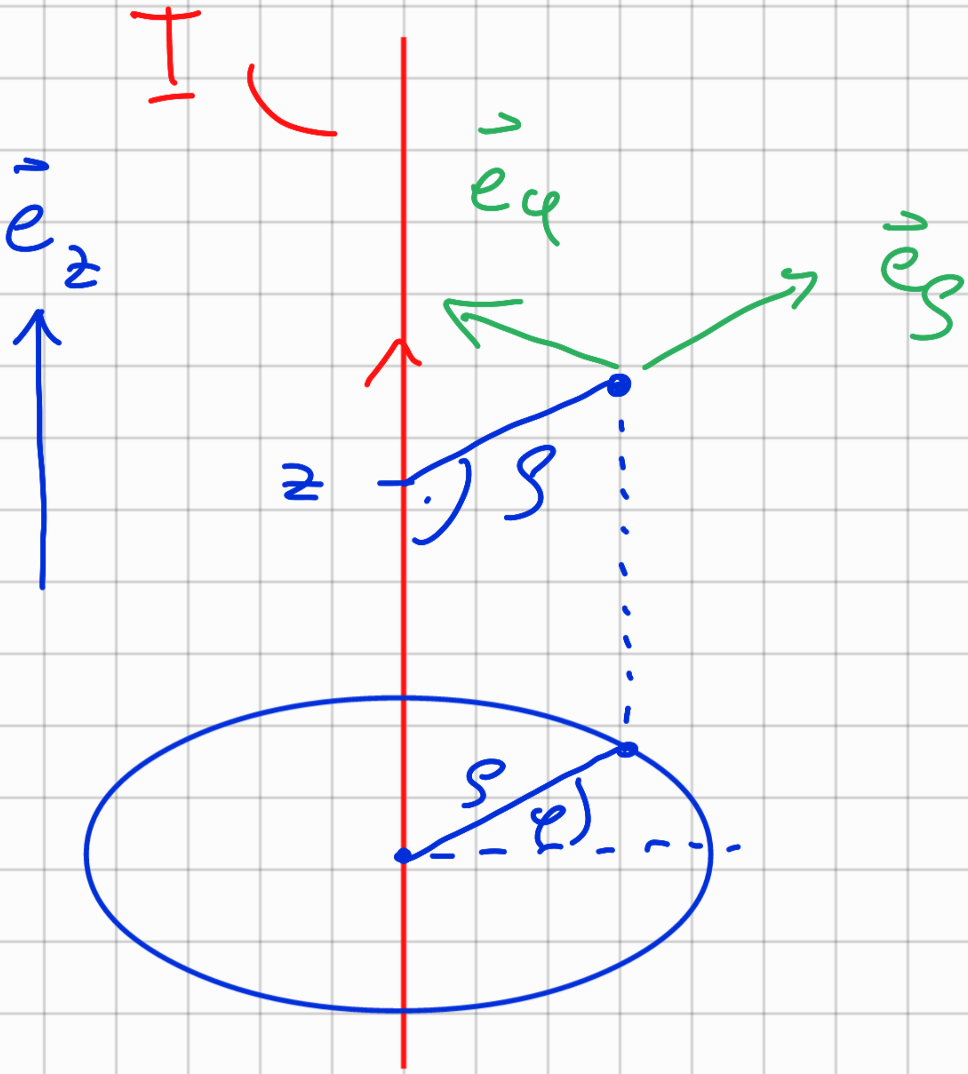
$$\boxed{\int_{\partial S} \vec{B}\, d\vec{l} = \mu_0 I_S}$$



Gauß

$$\boxed{\int_{\partial V} \vec{B}\, d\vec{f} = 0}$$

<u>Beispiel</u>: Magnetfeld eines Strom I führenden Drahts (gerade, unendlich lang)

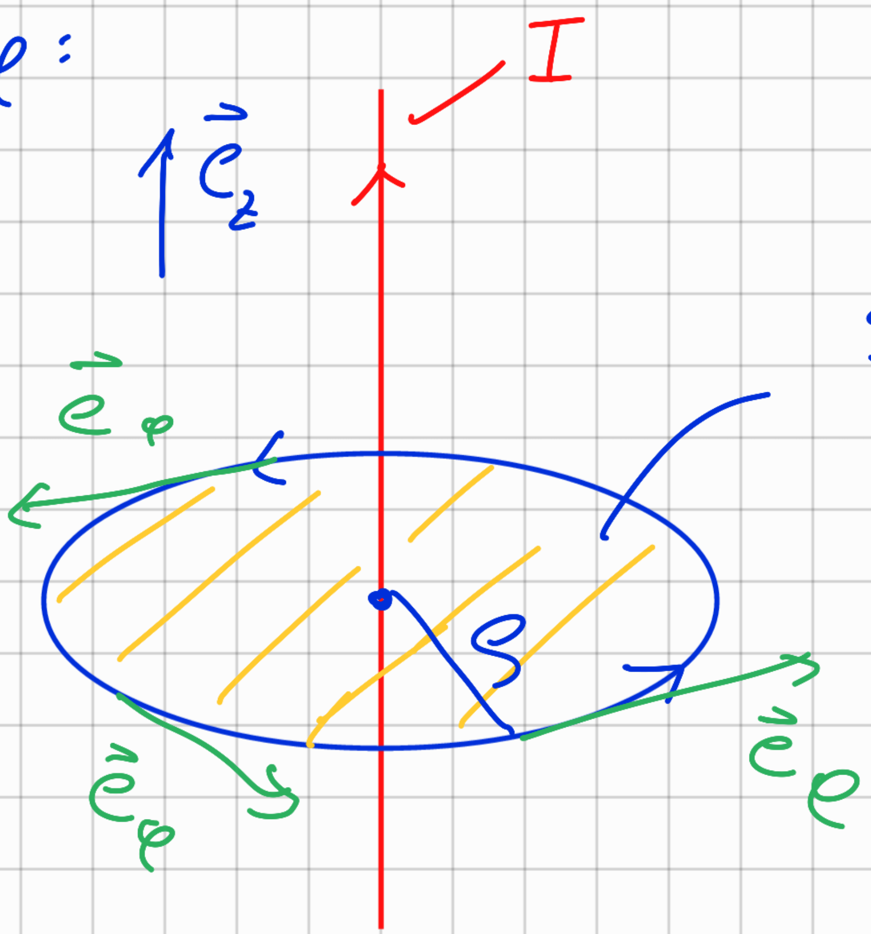


Symmetrie $\longrightarrow$ <u>Ansatz</u>

$$\vec{B}(\rho,\varphi,z) = B_\rho(\rho)\,\vec{e}_\rho + B_\varphi(\rho)\,\vec{e}_\varphi + B_z(\rho)\,\vec{e}_z$$

Bestimmung von B_ϱ, B_φ, B_z :

B_φ :



$B_\varphi(\varrho)$!

$S = K_\varrho$

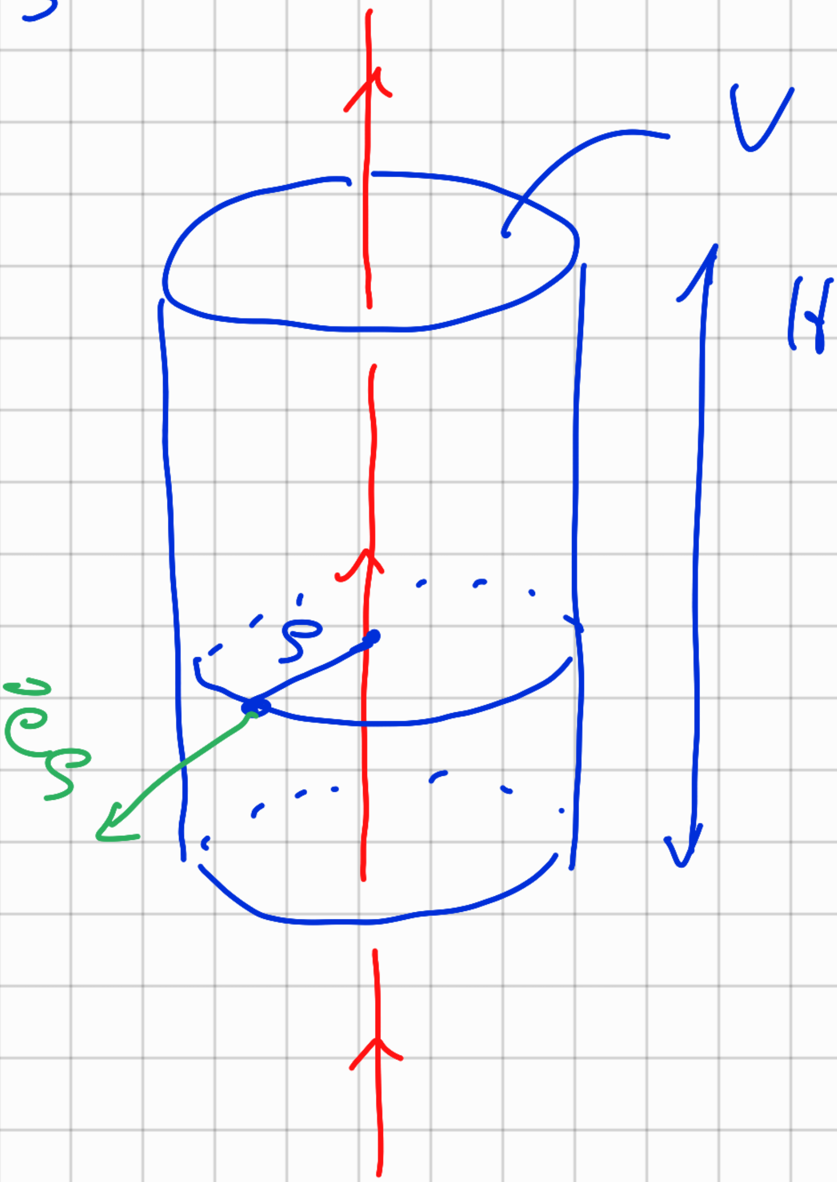
Ampère :

$$\mu_0 I = \int_{\partial S} \vec{B} \, d\vec{l} \overset{!}{=} 2\pi\varrho \; B_\varphi(\varrho)$$

$$\longrightarrow \quad \boxed{B_\varphi(\varrho) = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{\varrho}}$$

$B_\varrho(\varrho)$?

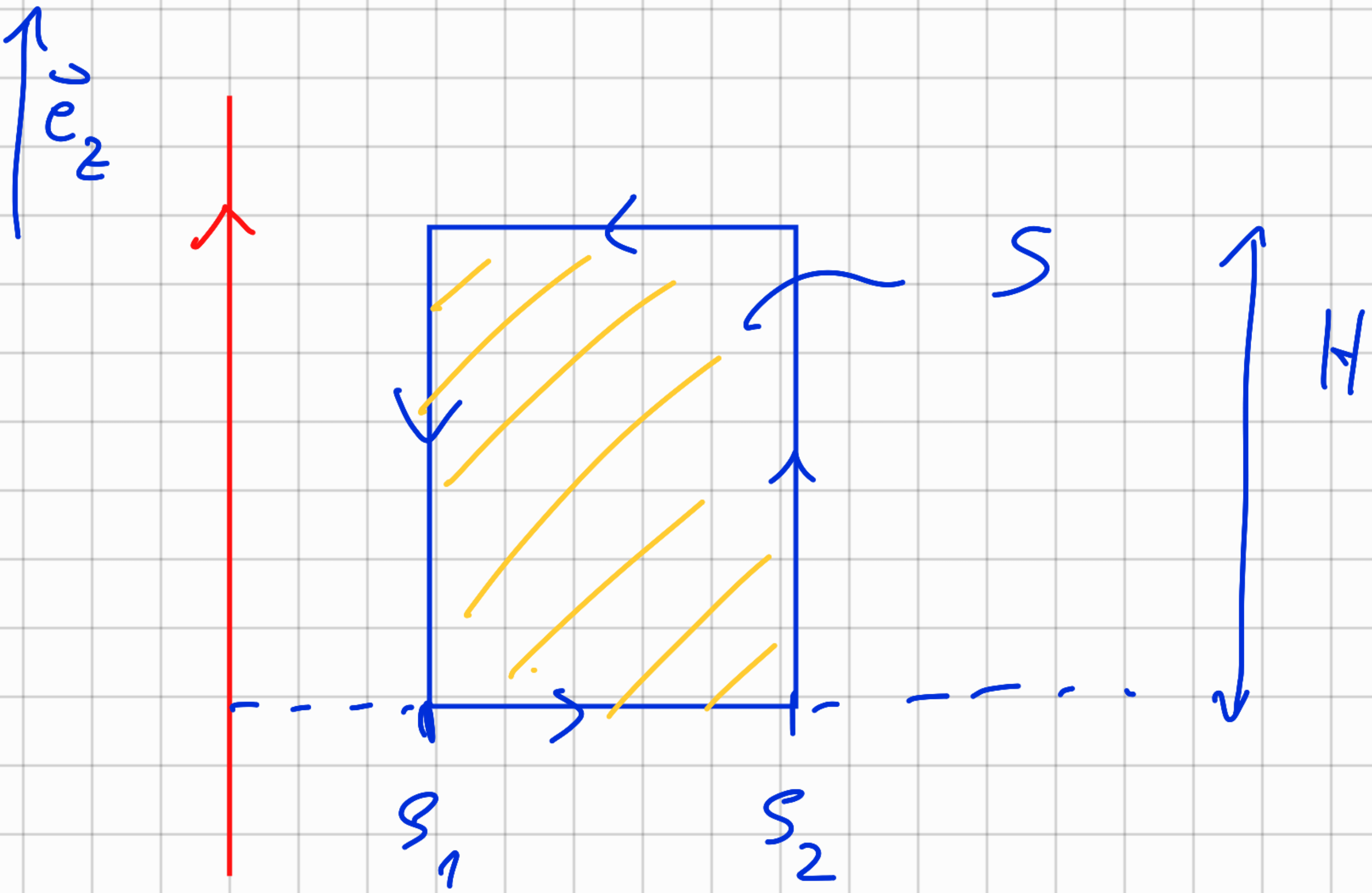
Gauß:



$$0 \underset{\text{Gauß}}{\overset{!}{=}} \oint_{\partial V} \vec{B}\, d\vec{f} = \underbrace{2\pi \varrho H}_{\neq 0}\, B_\varrho(\varrho)$$

$$\boxed{B_\varrho(\varrho) = 0}$$

$B_z(\rho)$: Ampère :



$$0 \overset{!}{=} \mu_0 I_S = \int_{\partial S} \vec{B}\, d\vec{l}$$

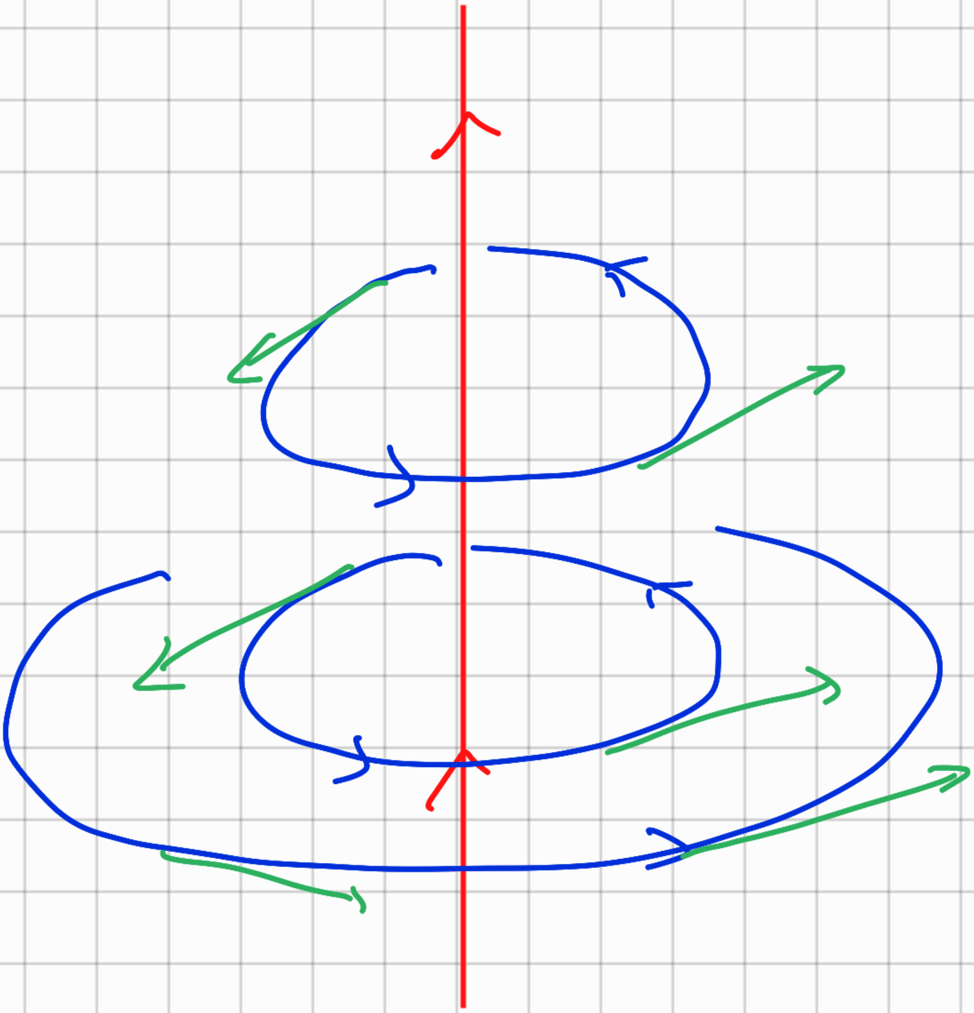
$$0 \overset{!}{=} H\left(B_z(\rho_2) - B_z(\rho_1) \right)$$

$$\rightsquigarrow \quad B_z(\rho) = \text{konst} \, !$$

$\rightsquigarrow$ falls $\vec{B} \to 0$ für $\varrho \to \infty$

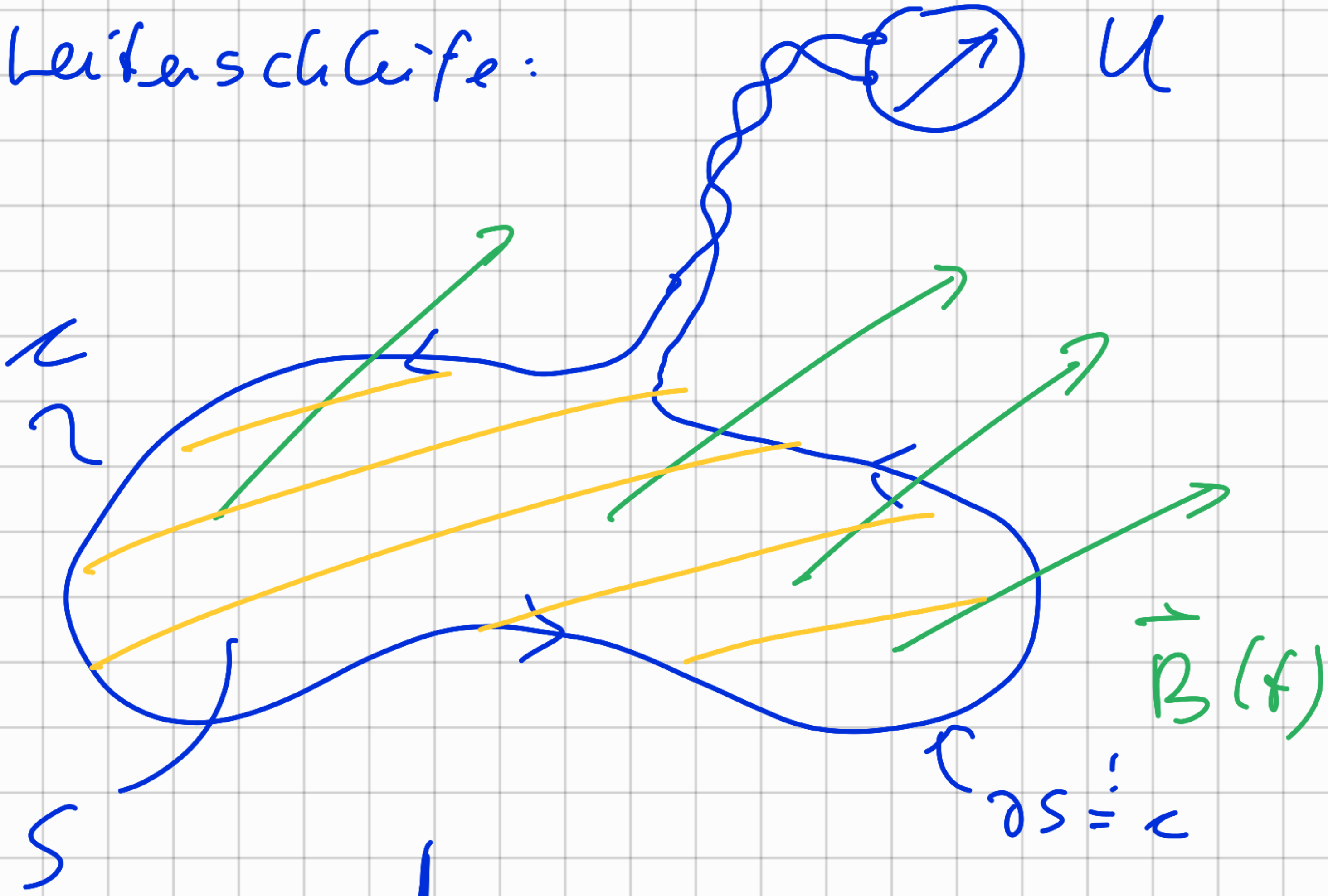
$$\boxed{B_z(\varrho) = 0}$$

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi} \frac{1}{\varrho} \vec{e}_\varphi}$$



Induktion:

Leiterschleife:



$$? = \underline{\underline{U}} \overset{!}{=} \int_c \vec{E}\, d\vec{l}$$

$$= \int_{\partial S} \vec{E}\, d\vec{l} \overset{\text{Stokes}}{=} \int_S \operatorname{rot} \vec{E}\, d\vec{f}$$

$$= - \int_S \frac{\partial \vec{B}}{\partial t} \, d\vec{f}$$

Faraday:

$$\mathrm{rot}\,\vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$= - \frac{d}{dt} \int_S \vec{B}(f)\, d\vec{f}$$

mag. Fluss durch Fl. S:

$$\Phi_S = \int_S \vec{B}\, d\vec{f}$$

$$\rightarrow \quad \boxed{U \overset{!}{=} - \frac{d}{dt} \Phi}$$