letzte Vorlsg.: <u>Vektor(Kreuz)produkt:</u>

$$V \times V \to V \quad \leftarrow \text{3D eukl. VR}$$

$$\vec{a}, \vec{b} \mapsto \vec{a} \times \vec{b}$$

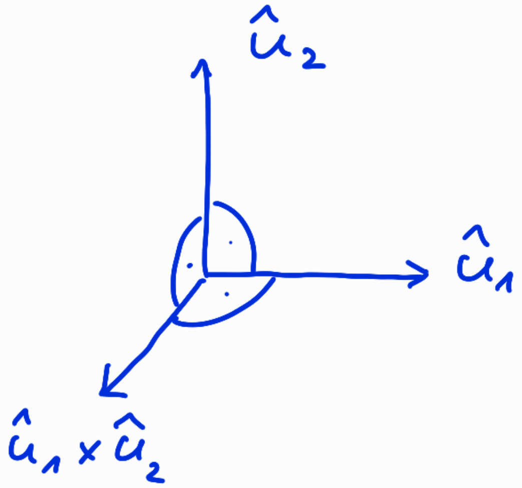
(V1) $\quad \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(V2) $\quad \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

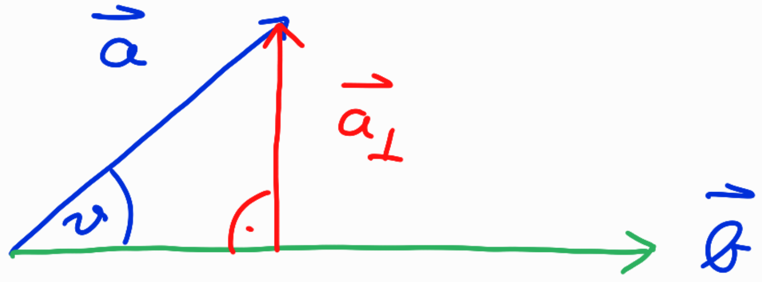
$\quad \vec{a} \times (\lambda \vec{b}) = \lambda (\vec{a} \times \vec{b})$

(V3) $\quad \hat{u}_1, \hat{u}_2$ orthonormal

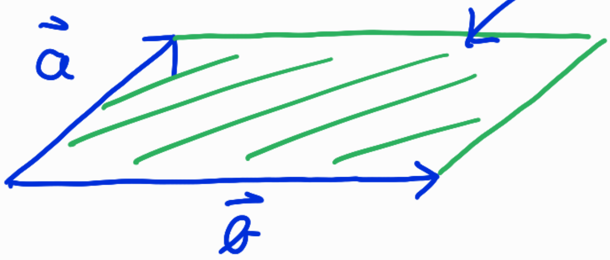
$\Rightarrow \quad \hat{u}_1, \hat{u}_2, \hat{u}_1 \times \hat{u}_2$ <u>rechtshdg. ONB</u>



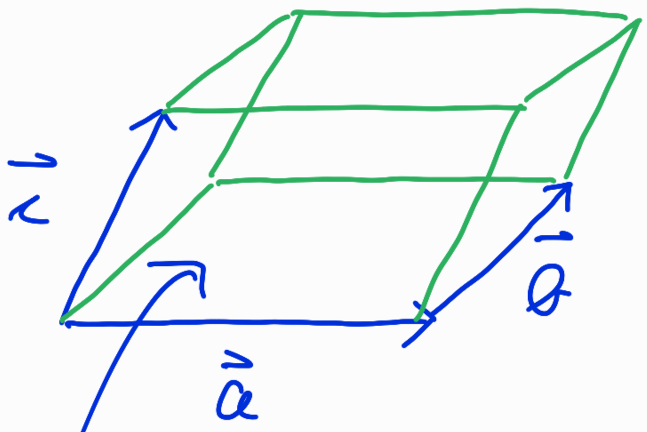
- $\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}$
- $\vec{a}_\perp = (\hat{b} \times \vec{a}) \times \hat{b}$



- $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\,|\sin\vartheta|$

- 

$A = |\vec{a} \times \vec{b}|$

- 

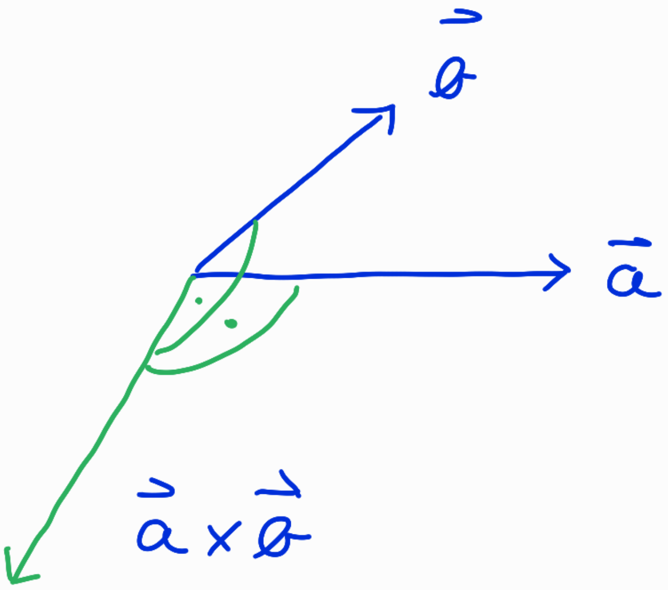
$V = |\langle \vec{a} \times \vec{b}, \vec{c} \rangle|$

$$\bullet \quad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_B \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_B = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}_B$$

$\uparrow$ ONB !

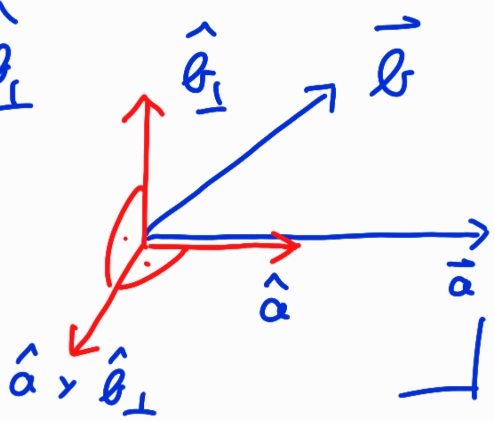
Nachtrag:

$\bullet$ $\boxed{\vec{a} \times \vec{b} \quad \text{orthogonal zu } \vec{a} \text{ und } \vec{b}}$



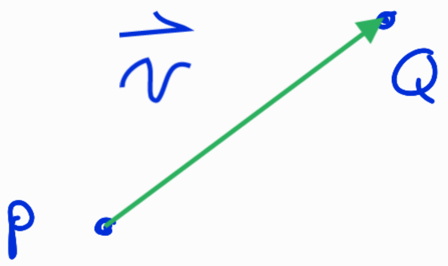
$$\text{denn } \vec{a} \times \vec{b} = \vec{a} \times (\vec{b}_\parallel + \hat{b}_\perp) = \overbrace{\vec{a} \times \vec{b}_\parallel}^{=\vec{0}} + \vec{a} \times \vec{b}_\perp$$

$$= |\vec{a}|\,|\vec{b}_\perp|\; \hat{a} \times \hat{b}_\perp$$



Koordinatensysteme für den euklidischen Raum E_3

$E_3 \;\hat{=}\;$ Gesamtheit aller Raumpunkte



$P, Q \in E_3$

$P + Q = ??$

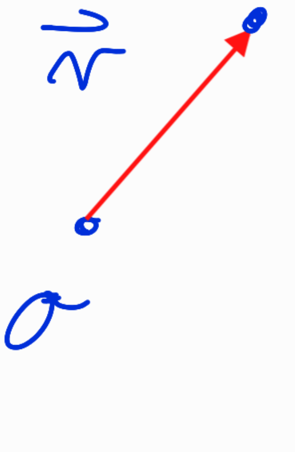
kein VR!

aber: je zwei Punkte $P, Q \in E_3$

def. einen Translationsvektor:

$$\vec{v} = \overrightarrow{PQ} \in V$$

umgekehrt: $O \in E_3$, $\vec{r} \in V$



$$P_{\vec{r}} = O + \vec{r} \in E_3$$

eind. best. durch

$$\vec{r} = \overrightarrow{O P_{\vec{r}}}$$

wähle <u>Bezugspunkt</u> (<u>Ursprung</u>) O

$\leadsto$ <u>bijektive</u> (eineindeutig) Abb.

$$\boxed{\begin{array}{ccccc} & V & \longrightarrow & E_3 & \\ & \vec{r} & \longmapsto & P_{\vec{r}} = O + \vec{r} & \\ \overrightarrow{OP} = & \vec{r}_P & \longleftarrow\!\shortmid & P & \end{array}}$$

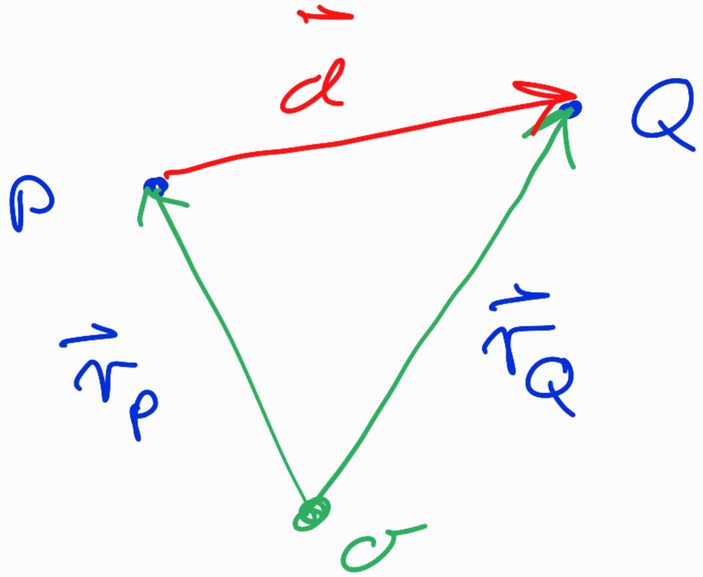
[Math.: E_3 ist <u>affiner</u> Raum mit 3D eukl. VR V = 3D eukl. Raum]

$$\boxed{\vec{r}_P = \text{Ortsvektor von Pkt } P \text{ bzgl. Bezugspkt } O}$$

$\leadsto$ rechne im E_3 mittels Ortsvektoren:

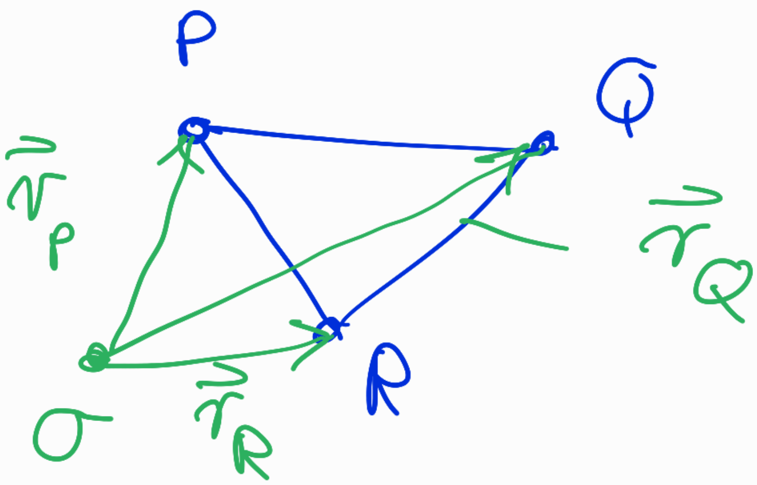
Beispiele:

1) Abstand zweier Punkte P, Q



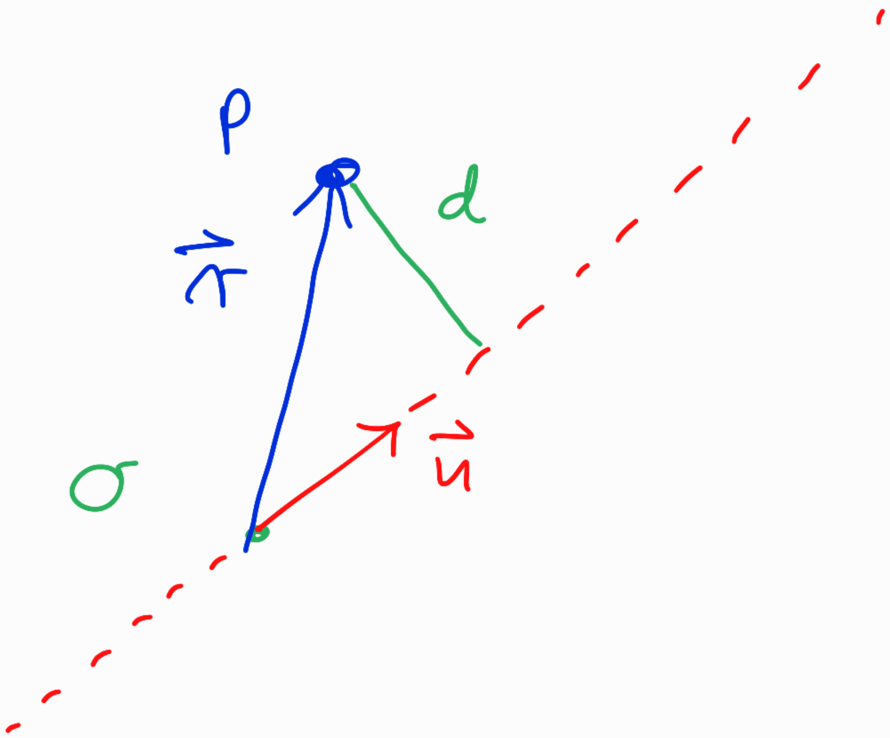
$$d = |\vec{d}| = |\vec{r}_Q - \vec{r}_P|$$

2) <u>Fläche</u> eines Dreiecks P, Q, R:



$$A = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = |(\vec{r}_Q - \vec{r}_P) \times (\vec{r}_R - \vec{r}_P)|\frac{1}{2}$$

3) Abstand Punkt P zu Geraden $\parallel \hat{u}$ durch $\mathcal{O}$:

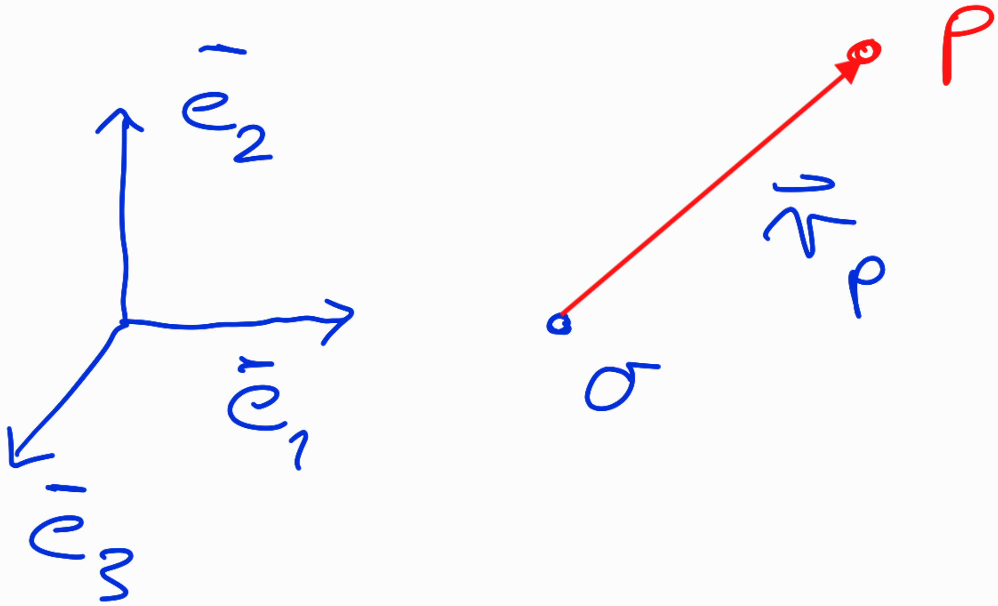


$$? = d = |\vec{r}_\perp| = |\hat{u} \times \vec{r}|$$

Kartesische Koordinaten

1) Bezugspunkt $\mathcal{O}$

2) ONB $B = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$

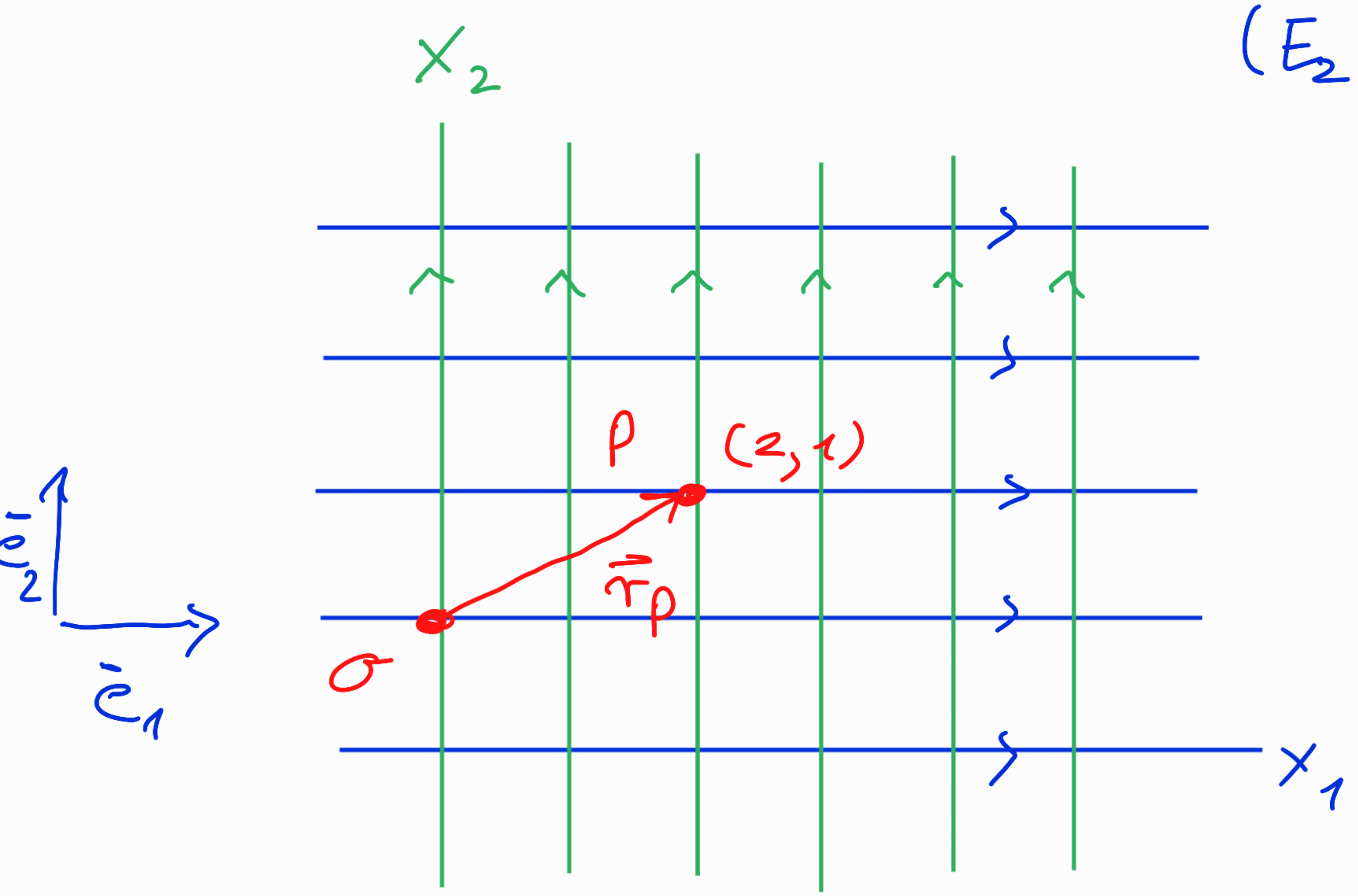


$$\rightsquigarrow \quad P \longleftrightarrow \vec{r}_P = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_B$$

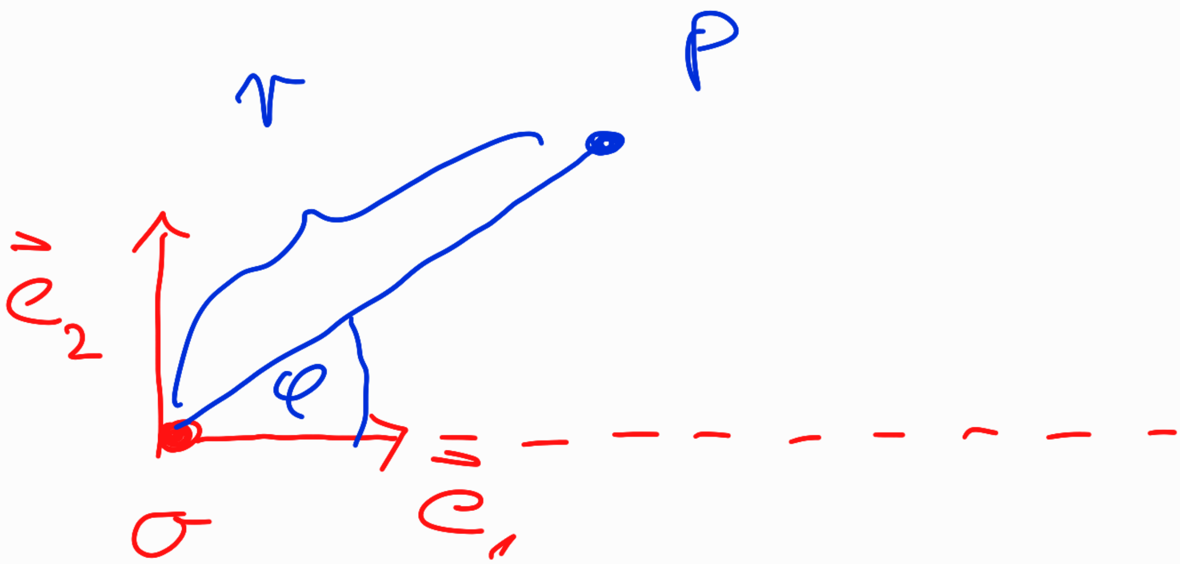
$\mathbb{R}^3 \ni \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ kartesischen Koordinaten von P (bzgl. O und ONB B)

graphisch: $\longrightarrow$

(E_2)



Polarkoordinaten für E_2



1) Bezugspunkt σ

2) Richtungsvektor $\bar{e}_1$

Polarkoordinaten von P:

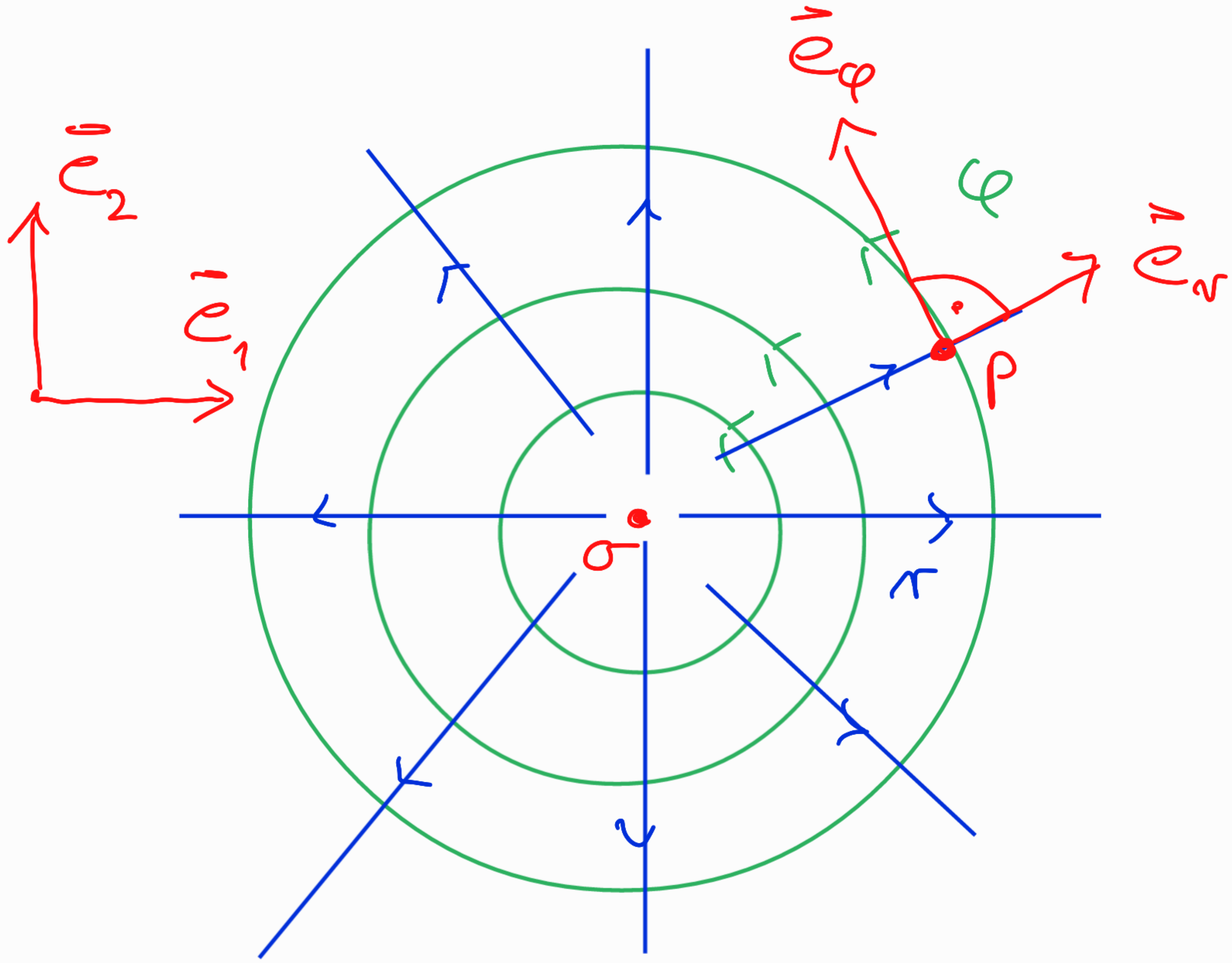
- <u>Radius</u> $r \in \mathbb{R}_+$
- <u>Winkel</u> $\varphi \in [0, 2\pi[$

$(r, \varphi) \longrightarrow$ kart. Koordinaten

(bzgl. σ, $B = (\bar{e}_1, \bar{e}_2)$)

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \end{pmatrix}$$

$\longrightarrow$



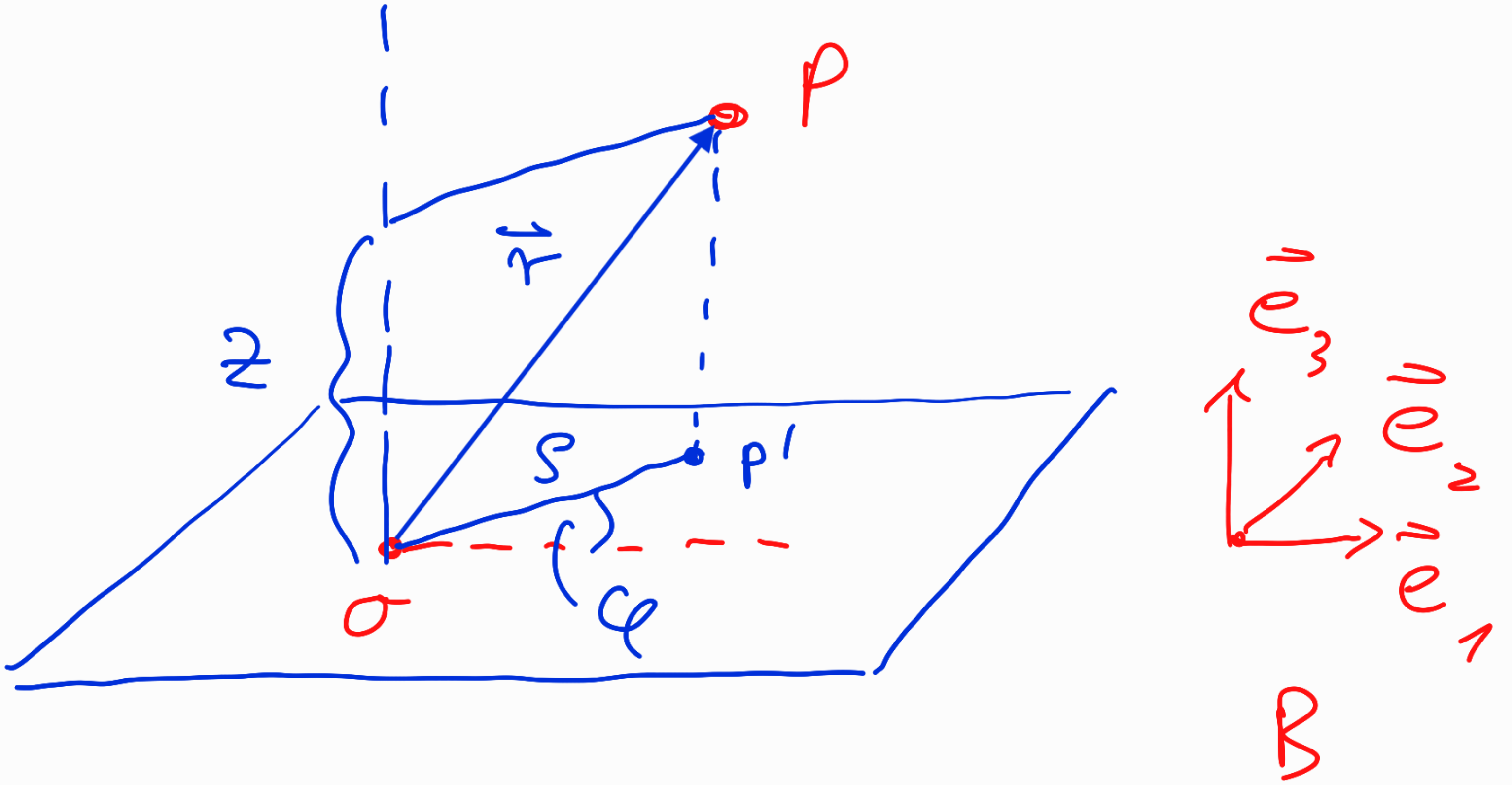
hilfreich: <u>Lokale</u> ONB

$$B_p = (\vec{e}_r, \vec{e}_\varphi)$$

$$\vec{e}_r = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}_B, \quad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}_B$$

$B = (\vec{e}_1, \vec{e}_2)$

Zylinderkoordinaten für E_3:



1) Bezugspunkt O

2) $B = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$

Zylinderkoordinaten von P: (ρ, φ, z)

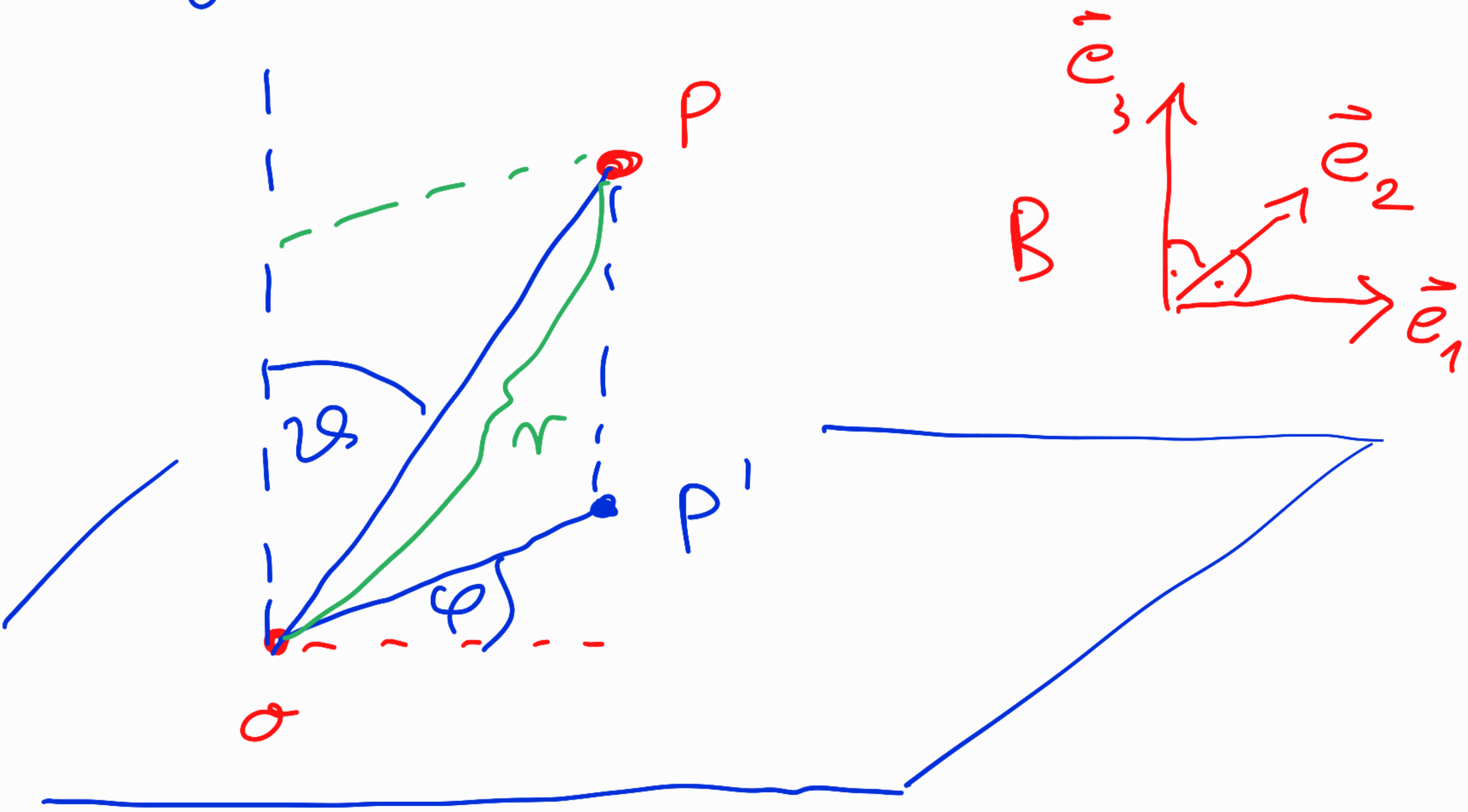
kart. Koordinaten:

$$(\varrho, \varphi, z) \longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \varrho\cos\varphi \\ \varrho\sin\varphi \\ z \end{pmatrix}$$

lokale ONB: $(\vec{e}_\varrho, \vec{e}_\varphi, \vec{e}_z)$:

$$\vec{e}_\varrho = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix}_B, \quad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}_B; \quad \vec{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_B$$

Kugelkoordinaten (sphärische Koord.)

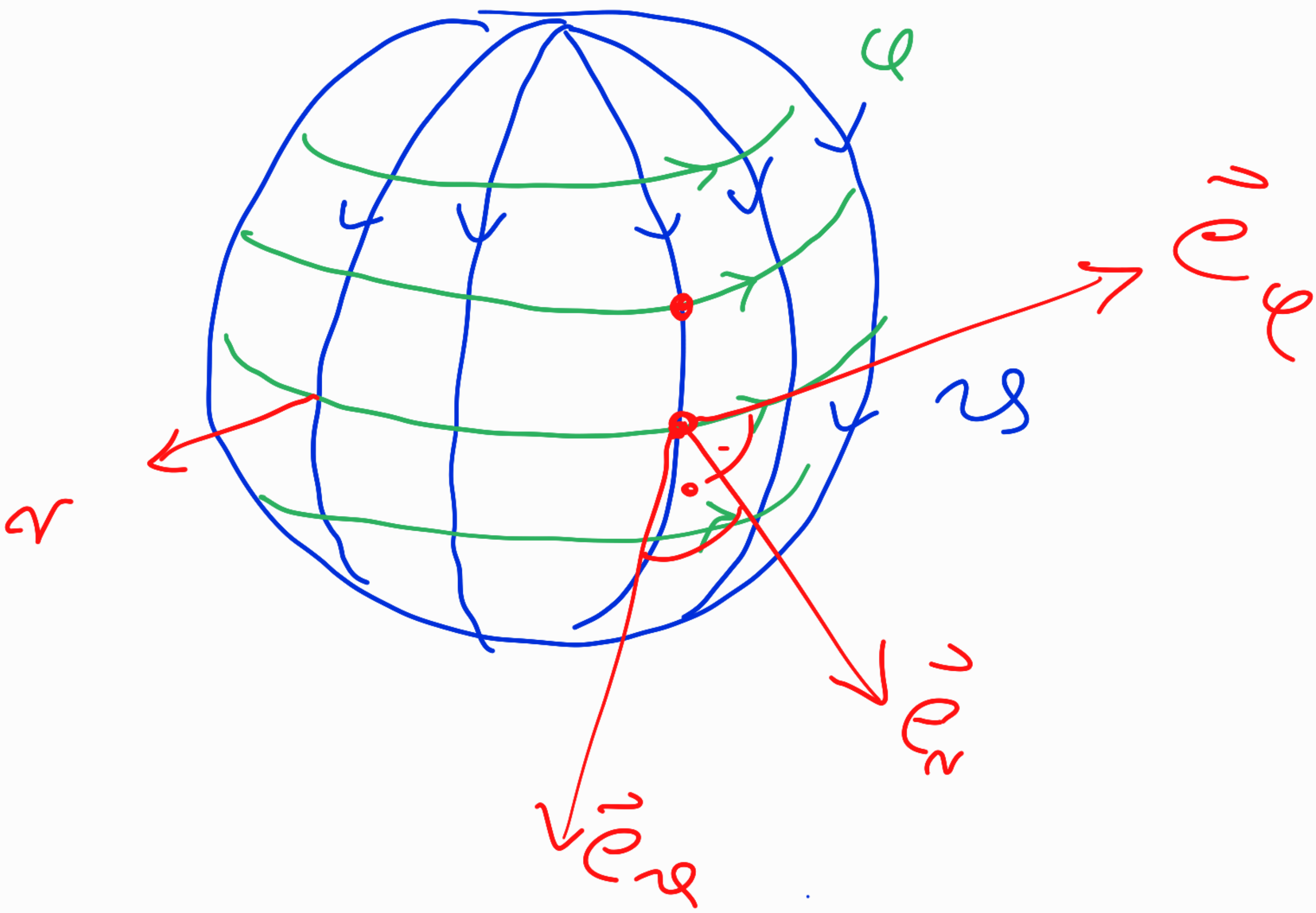


- Radius $r \in \mathbb{R}_+$
- Azimuthwinkel $\varphi \in [0, 2\pi[$
- Polarwinkel $\vartheta \in [0, \pi[$

↑

Kugelkoordinaten von P:

$$(r, \vartheta, \varphi) \longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} r\cos\varphi\sin\vartheta \\ r\sin\varphi\sin\vartheta \\ r\cos\vartheta \end{pmatrix}$$



Lokale ONB : $B_\rho = (\vec{e}_r, \vec{e}_\vartheta, \vec{e}_\varphi)$

$$\vec{e}_r = \begin{pmatrix} \cos\varphi \sin\vartheta \\ \sin\varphi \sin\vartheta \\ \cos\vartheta \end{pmatrix}_B \quad ; \quad \vec{e}_\vartheta = \begin{pmatrix} \cos\varphi \cos\vartheta \\ \sin\varphi \cos\vartheta \\ -\sin\vartheta \end{pmatrix}_B$$

$$\vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}_B$$