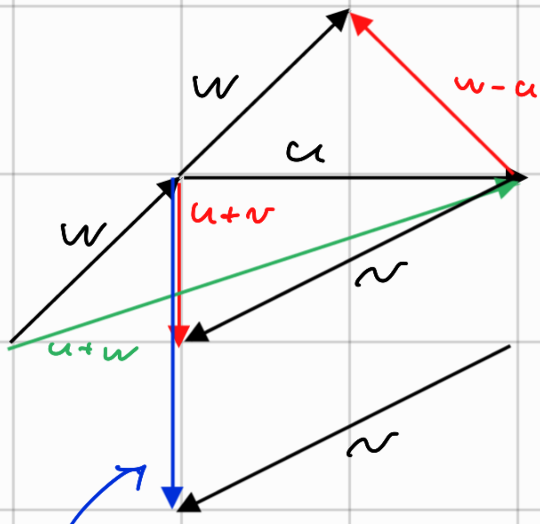
Lösungshinweise Blatt 1

1.



$$r = w + 2\left(u+v-\tfrac{1}{2}w\right) = 2(u+v)$$

2.

a)

$$\vec{w} = 1\vec{e}_1 + 1\vec{e}_2 = \begin{pmatrix}1\\1\end{pmatrix}_B$$

$$\vec{v} = -2\vec{e}_1 - 1\vec{e}_2 = \begin{pmatrix}-2\\-1\end{pmatrix}_B$$

$$\vec{u} = 2\vec{e}_1 + 0\vec{e}_2 = \begin{pmatrix}2\\0\end{pmatrix}_B$$

b)

$$\vec{y} = \begin{pmatrix}1\\2\end{pmatrix}_B = \vec{e}_1 + 2\vec{e}_2 = \vec{z}$$

4.

a) $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}_C = \vec{f}_1 + \vec{f}_2 + \vec{f}_3 = 3\vec{e}_1 + \vec{e}_2 - \vec{e}_3 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}_B$

$\rightarrow \quad \vec{u} + \vec{v} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}_B \; ; \quad \vec{u} - \vec{v} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}_B$

b) $$\left.\begin{aligned} \vec{e}_1 &= \tfrac{1}{2}\left(\vec{f}_1 + \vec{f}_3\right) \\ \vec{e}_2 &= \tfrac{1}{2}\left(\vec{f}_2 - \vec{f}_3\right) \\ \vec{e}_3 &= \tfrac{1}{2}\left(\vec{f}_1 - \vec{f}_2\right) \end{aligned}\right\}$$

$$\vec{w} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}_B = \vec{e}_1 - 2\vec{e}_2 + \vec{e}_3 = \vec{f}_1 - \tfrac{3}{2}\vec{f}_2 + \tfrac{3}{2}\vec{f}_3 = \begin{pmatrix} 1 \\ -3/2 \\ 3/2 \end{pmatrix}_C$$

5. a) $\dim P_3 = 4$

b) Nein, denn $x(x-1) = x^2 - x = f_2(x) - f_1(x)$,

d.h. $\tilde{f}_0 = f_2 - f_1$, $\rightarrow \tilde{f}_0, f_1, f_2, f_3$

linear abhängig und damit

<u>keine</u> Basis.

r)

$$g(x) = 1 + x^2,$$

$$g + h - \ell = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}_B = -f_1$$

d. h. $(g+h-\ell)(x) = -f_1(x) = -x$