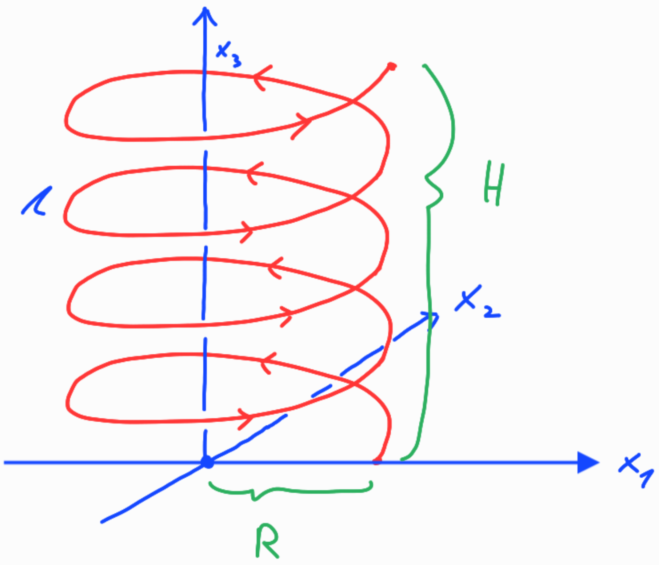
Lösungshinweise Blatt 11

53.



$$\vec{c}(\varphi) = \begin{pmatrix} R\cos\varphi \\ R\sin\varphi \\ H\varphi/8\pi \end{pmatrix}$$

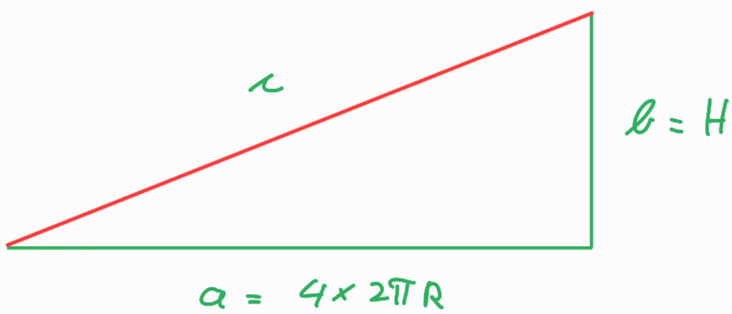
$$\varphi \in [0, 8\pi]$$

$$L = \int_c dl = \int_0^{8\pi} |\vec{c}\,'(\varphi)|\, d\varphi = \int_0^{8\pi} \left\| \begin{pmatrix} -R\sin\varphi \\ R\cos\varphi \\ H/8\pi \end{pmatrix} \right\| d\varphi$$

$$= \int_0^{8\pi} \left(R^2 + \left(H/8\pi\right)^2 \right)^{1/2} d\varphi$$

$$= \sqrt{(8\pi R)^2 + H^2}$$

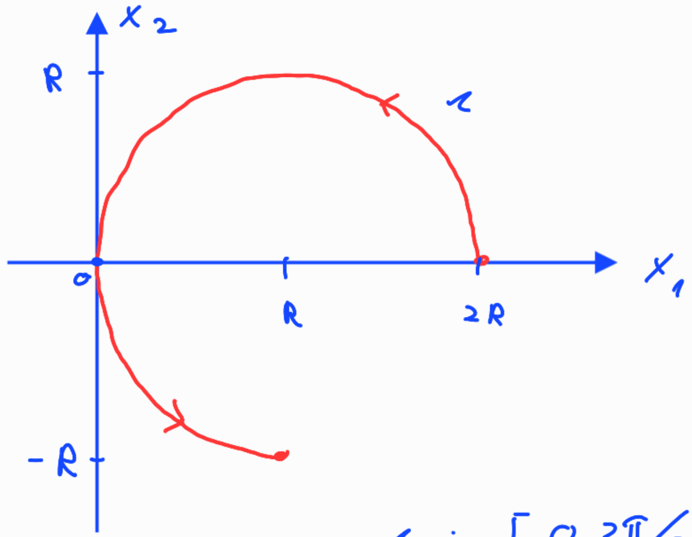
⌈ alternativ: Abwicklung auf Ebene ergibt:



$$L \overset{!}{=} c = \sqrt{a^2 + b^2}$$

$$= \sqrt{(8\pi R)^2 + H^2}$$ ⌋

54)



$$r : [0, 3\pi/2] \to \mathbb{R}^2$$

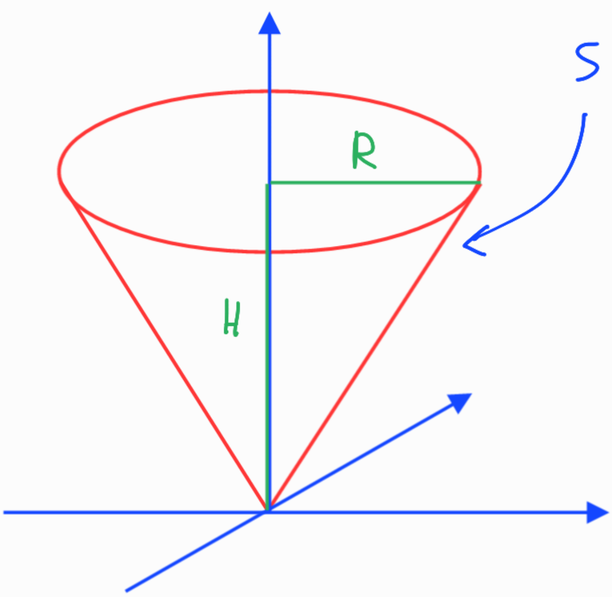
$$\varphi \mapsto \vec{r}(\varphi) = R\begin{pmatrix} 1+\cos\varphi \\ \sin\varphi \end{pmatrix}$$

$$\to \quad \vec{r}\,'(\varphi) = R\begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}$$

$$\to \int_r \vec{v}\cdot d\vec{\ell} = \int_0^{3\pi/2} \underbrace{\left\langle R\begin{pmatrix} 1+\cos\varphi \\ \sin\varphi \end{pmatrix}, R\begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}\right\rangle}_{= \; R^2(-\sin\varphi - \cancel{\cos\varphi\sin\varphi} + \cancel{\sin\varphi\cos\varphi})} d\varphi$$

$$= -R^2 \int_0^{3\pi/2} \sin\varphi \, d\varphi = R^2 \cos\varphi \Big|_0^{3\pi/2} = -R^2 .$$

55)



S = Kegelmantel

$$\frac{\partial \vec{S}}{\partial \varrho} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ H/R \end{pmatrix}, \quad \frac{\partial \vec{S}}{\partial \varphi} = \begin{pmatrix} -\varrho \sin\varphi \\ \varrho \cos\varphi \\ 0 \end{pmatrix}$$

$$\rightarrow \quad \frac{\partial \vec{S}}{\partial \varrho} \times \frac{\partial \vec{S}}{\partial \varphi} = \begin{pmatrix} -H\varrho/_R \cos\varphi \\ -H\varrho/_R \sin\varphi \\ \varrho \end{pmatrix}$$

$$\rightarrow \quad \left| \frac{\partial \vec{S}}{\partial \varrho} \times \frac{\partial \vec{S}}{\partial \varphi} \right| = \varrho \sqrt{H^2/_{R^2} + 1}$$

$$\rightarrow \quad F(S) = \int_S |d\vec{f}| = \int_0^R d\varrho \underbrace{\int_0^{2\pi} d\varphi \; \varrho \sqrt{H^2/_{R^2}+1}}_{2\pi \sqrt{H^2/_{R^2}+1} \, \varrho}$$

$$= \pi R^2 \sqrt{H^2/_{R^2} + 1} \; .$$

56) $S : [0,2] \times [0,1] \rightarrow \mathbb{R}^3$

$$(x_1, x_2) \longmapsto \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad \frac{\partial \vec{S}}{\partial x_1} = \vec{e}_1, \quad \frac{\partial \vec{S}}{\partial x_2} = \vec{e}_2, \quad \frac{\partial \vec{S}}{\partial x_1} \times \frac{\partial \vec{S}}{\partial x_2} = \vec{e}_3$$

$$\rightarrow \quad \int_S \vec{A} \, d\vec{f} = \int_0^2 dx_1 \int_0^1 dx_2 \, \underbrace{\langle \vec{A}(x_1, x_2, 0), \vec{e}_3 \rangle}_{= x_1 x_2} = \frac{x_1^2}{2}\Big|_0^2 \, \frac{x_2^2}{2}\Big|_0^1 = 1$$