Lösungshinweise Blatt 13

58)

$$\operatorname{rot}\vec{A} = \begin{pmatrix}\partial_1\\ \partial_2\\ \partial_3\end{pmatrix} \times \begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \vec{0}$$

$$\operatorname{rot}\vec{B} = \begin{pmatrix}\partial_1\\ \partial_2\\ \partial_3\end{pmatrix} \times \begin{pmatrix}a_2x_3 - a_3x_2\\ a_3x_1 - a_1x_3\\ a_1x_2 - a_2x_1\end{pmatrix} = \begin{pmatrix}2a_1\\ 2a_2\\ 2a_3\end{pmatrix} = 2\vec{a}$$

$$\operatorname{rot}\vec{C} = \begin{pmatrix}\partial_1\\ \partial_2\\ \partial_3\end{pmatrix} \times (a_1x_1 + a_2x_2 + a_3x_3)\begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix} = \begin{pmatrix}a_2b_3 - a_3b_2\\ a_3b_1 - a_1b_3\\ a_1b_2 - a_2b_1\end{pmatrix}$$

$$= \vec{a} \times \vec{b}$$

59a)

$$\operatorname{grad} f = \begin{pmatrix}x_2^2x_3^3\\ 2x_1x_2x_3^3\\ 3x_1x_2^2x_3^2\end{pmatrix}$$

$$\rightarrow \operatorname{rot}\operatorname{grad} f = \begin{pmatrix}6x_1x_2x_3^2 - 6x_1x_2x_3^2\\ 3x_2^2x_3^2 - 3x_2^2x_3^2\\ 2x_2x_3^3 - 2x_2x_3^3\end{pmatrix} = \vec{0}$$

b)

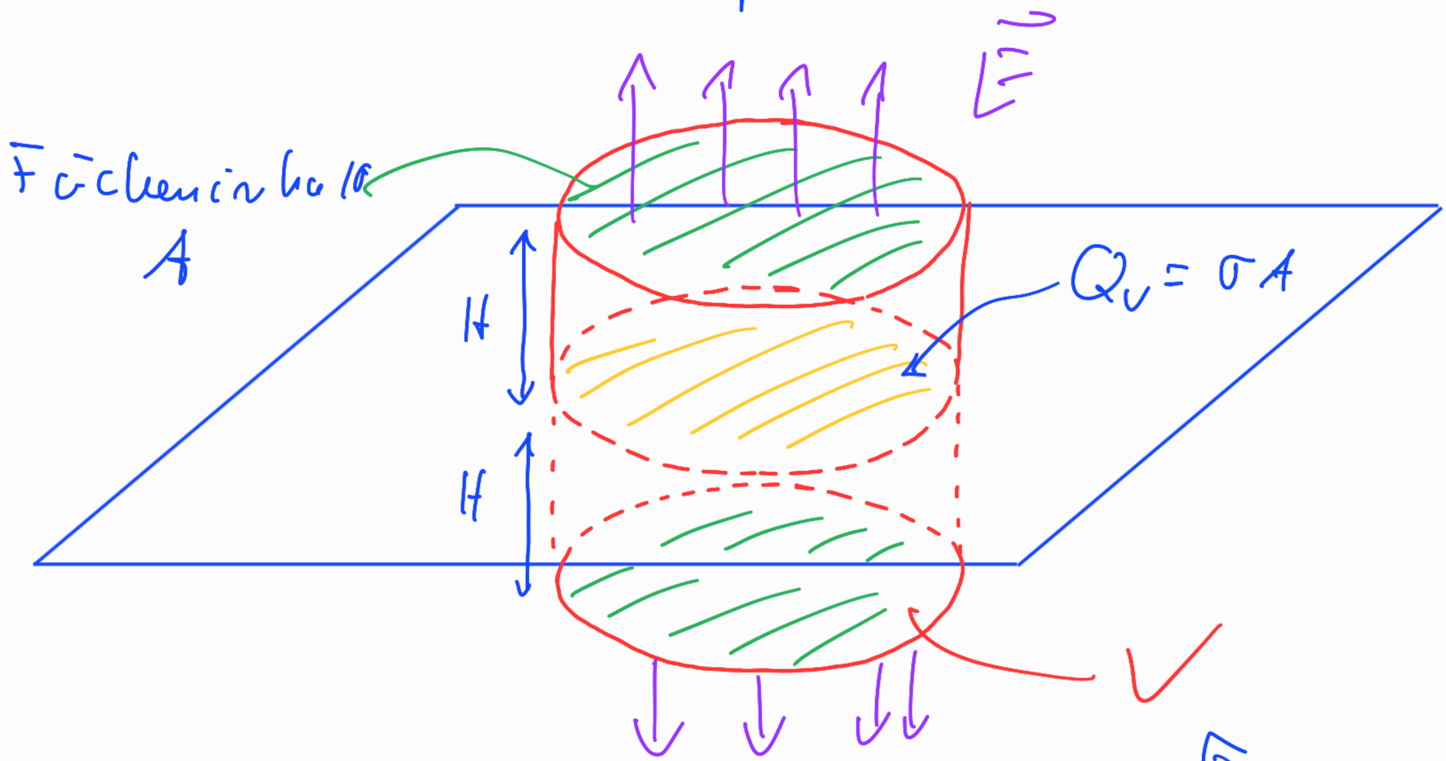
$$\operatorname{rot}\operatorname{grad} f = \begin{pmatrix}\partial_1\\ \partial_2\\ \partial_3\end{pmatrix} \times \begin{pmatrix}\partial_1 f\\ \partial_2 f\\ \partial_3 f\end{pmatrix} = \begin{pmatrix}\frac{\partial^2 f}{\partial x_2 x_3} - \frac{\partial^2 f}{\partial x_3 x_2}\\ \frac{\partial^2 f}{\partial x_3 x_1} - \frac{\partial^2 f}{\partial x_1 x_3}\\ \frac{\partial^2 f}{\partial x_1 x_2} - \frac{\partial^2 f}{\partial x_2 x_1}\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 0\end{pmatrix} = \vec{0}$$

Satz von Schwarz:

$$\frac{\partial^2 f}{\partial x_i x_j} \overset{!}{=} \frac{\partial^2 f}{\partial x_j x_i}$$

60) Für allg. V:

$$\int_{\partial V} \vec{E}\, d\vec{f} \underset{\text{S.v.G.}}{=} \int_V \operatorname{div} \vec{E}\, dV \underset{\text{Gauß. Gesetz}}{=} \int_V \rho/\varepsilon_0 \, dV = Q_V/\varepsilon_0$$



Für abbgebildetes Volumen V:

- $\int_{\partial V} \vec{E}\, d\vec{f} = 2A\, E(H)$
- $\frac{Q_V}{\varepsilon_0} = \sigma \cdot A / \varepsilon_0$

$= \;\rightarrow\; E(H) = \frac{\sigma}{2\varepsilon_0}$

unabhängig von H!

d.h. $\vec{E}(x_1, x_2, x_3) = \operatorname{sgn}(x_2) \cdot \frac{\sigma}{2\varepsilon_0} \cdot \vec{e}_3$.

G1) für allg. Flächenstück S:

$$\int_{\partial S} \vec{B}\, d\vec{l} \underset{\text{Stokes}}{=} \int_S \mathrm{rot}\,\vec{B}\, d\vec{f} \underset{\text{Ampère}}{=} \mu_0 \int_S \vec{j}\, d\vec{f} = \mu_0 I_S$$

wähle hier speziell $S = K_R$: Kreisscheibe vom Radius R, Mittelpkt. O, $\hat{n} \parallel \vec{e}_z$

$R < R_i$: $\quad \mu_0 I_{K_R} = \underline{\underline{0}} = \int_{\partial K_R} \vec{B}\, d\vec{l} = \underline{\underline{B^{(1)}(R)\, 2\pi R}}$

$\Rightarrow B^{(1)}(s) = 0$

$R_i < R < R_a$: $\quad \mu_0 I_{K_R} = \mu_0 \underline{\underline{I_0}} = \int_{\partial K_R} \vec{B}\, d\vec{l} = B^{(2)}(R)\, 2\pi R$

$$\Rightarrow \quad B^{(2)}(s) = \frac{\mu_0 I_0}{2\pi s}$$

$R_a < R$: $\quad \mu_0 I_{K_R} = \mu_0 (I_0 - I_0) = 0 = \int_{\partial K_R} \vec{B}\, d\vec{l} = B(R)\, 2\pi R$

$\Rightarrow B^{(3)}(s) = 0$

d.b.

$$\vec{B}(\varrho,\varphi,z) = \begin{cases} \vec{0} & : \varrho < R_i \\ \frac{\mu_0 I_0}{2\pi} \frac{1}{\varrho} \vec{e}_\varphi & : R_i < \varrho < R_a \\ \vec{0} & : \varrho > R_a \end{cases}$$