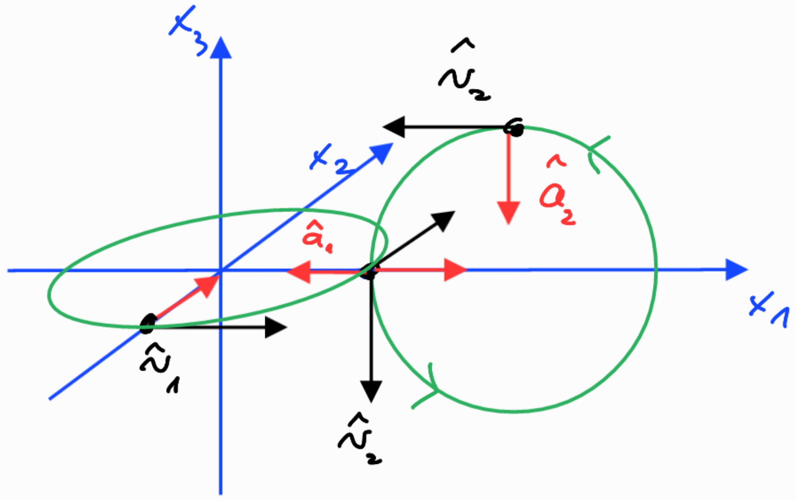
Lösungshinweise Blatt 6

28)



$$\vec{v}_1 = \omega_1 R \begin{pmatrix} -\sin \omega_1 t \\ \cos \omega_1 t \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \omega_2 R \begin{pmatrix} \sin \omega_2 t \\ 0 \\ -\cos \omega_2 t \end{pmatrix}$$

$$\vec{a}_1 = -\omega_1^2 R \begin{pmatrix} \cos \omega_1 t \\ \sin \omega_1 t \\ 0 \end{pmatrix}, \quad \vec{a}_2 = \omega_2^2 R \begin{pmatrix} \cos \omega_2 t \\ 0 \\ \sin \omega_2 t \end{pmatrix}$$

$\hookrightarrow$ $\vec{v}_1(0) = \omega R \vec{e}_2$, $\quad \vec{v}_2(0) = -\omega R \vec{e}_3$

$\vec{v}_1(.75s) = \omega R \vec{e}_1$, $\quad \vec{v}_2(.75s) = -\omega R \vec{e}_2$

$\vec{a}_1(0) = -\omega^2 R \vec{e}_1$, $\quad \vec{a}_2(0) = \omega^2 R \vec{e}_2$

$\vec{a}_1(.75s) = \omega^2 R \vec{e}_2$, $\quad \vec{a}_2(0.75s) = -\omega^2 R \vec{e}_3$

29)

$$0 \overset{!}{=} \frac{d}{dt} |\vec{v}|^2 = \frac{d}{dt} \langle \vec{v}, \vec{v} \rangle = \langle \dot{\vec{v}}, \vec{v} \rangle + \langle \vec{v}, \dot{\vec{v}} \rangle$$

$$= 2 \langle \vec{a}, \vec{v} \rangle \Longrightarrow \vec{a} \perp \vec{v}$$

30) Im Limes $h \to 0$:

$$\dot{\tau}(t) = \frac{1}{h}\left(\tau(t+h) - \tau(t)\right)$$

$$= \frac{1}{h}\left(T(\underbrace{\vec{r}(t+h)}_{\doteq\, \vec{r}(t) + \vec{v}h + o(\cancel{h^2})}) - T(\vec{r}(t))\right)$$

$$= \frac{1}{\cancel{h}}\left(\underbrace{T(\vec{r}(t) + \vec{v}h)}_{\doteq\, \cancel{T(\vec{r}(t))} + \langle \operatorname{grad} T(\vec{r}(t)), \vec{v}\cancel{h}\rangle + o(\cancel{h^2})} - \cancel{T(\vec{r}(t))}\right)$$

$$= \langle \operatorname{grad} T(\vec{r}(t)), \vec{v}(t)\rangle$$

31)

a) $$G'(x) = (\ln f(x))' = \frac{f'(x)}{f(x)}$$

b) $$\int_0^{\pi/4} \tan x \, dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\, dx = -\int_0^{\pi/4} \frac{(\cos x)'}{\cos x}\, dx$$

$$\overset{a)}{=} -\ln(\cos x)\Big|_0^{\pi/4}$$

$$= -\ln(\underbrace{\cos\tfrac{\pi}{4}}_{\doteq\, 1/\sqrt{2}}) = \frac{\ln 2}{2}$$

$$\int_a^b \frac{1}{x \ln x}\,dx = \int_a^b \frac{(\ln x)'}{\ln x}\,dx \overset{a)}{=} \ln|\ln x|\Big|_a^b$$

$$= \ln\left(\frac{\ln b}{\ln a}\right).$$

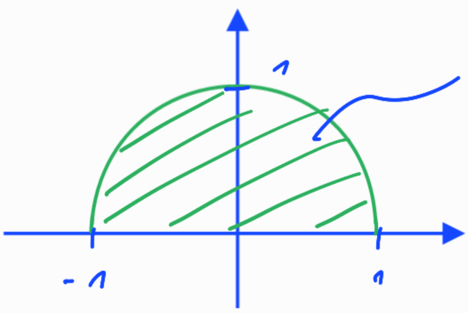
32)

a) $F'(x) = \frac{1}{2}\big(\cos^2 x \underbrace{- \sin^2 x + 1}_{\cos^2 x}\big) = \cos^2 x$ ✓

b) $$\int_{-1}^{1} \sqrt{1-x^2}\,dx \underset{x = \sin y}{=} \int_{-\pi/2}^{\pi/2} \underbrace{\sqrt{1-\sin^2 y}}_{\cos y}\,\cos y\,dy$$

$$\overset{a)}{=} \frac{1}{2}\big(\sin y \cos y + y\big)\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2}$$

1)



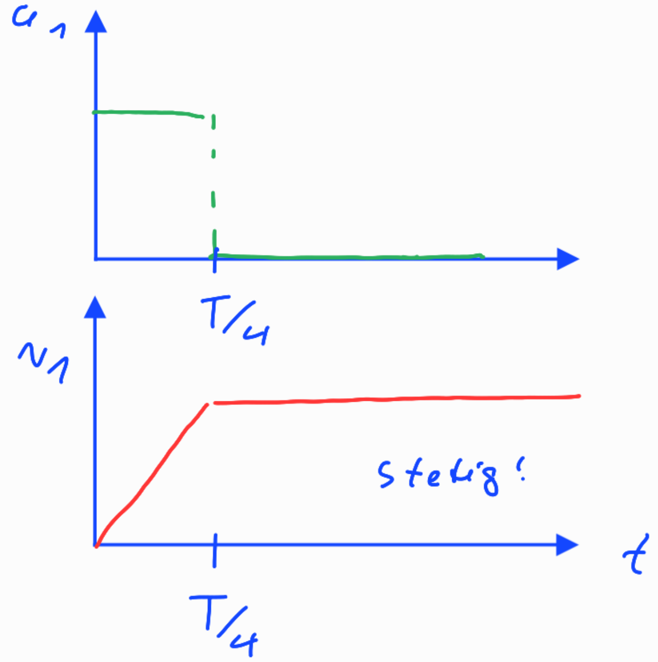
halbe Kreisscheibe vom Radius 1 $\rightsquigarrow$ Fläche $\frac{\pi}{2}$

33)

$$a_1(t) = \begin{cases} b : & t < T/4 \\ 0 : & t > T/4 \end{cases}$$

$\frac{d}{dt}$

$$v_1(t) = \begin{cases} bt : & t < T/4 \\ \frac{bT}{4} : & t > T/4 \end{cases}$$

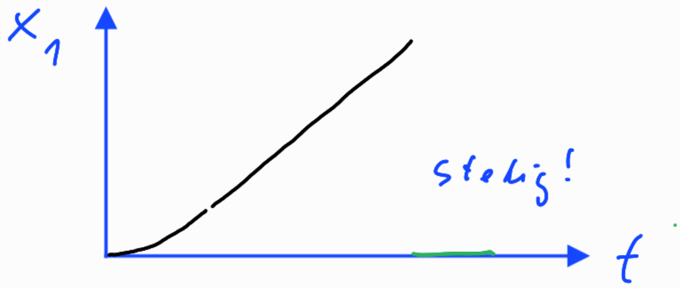


$\frac{d}{dt}$

$$x_1(t) = \begin{cases} \frac{b}{2} t^2 & : t < T/4 \\ \frac{b}{2}\left(\frac{T}{4}\right)^2 + \frac{bT}{4}\left(t - T/4\right) : & t > T/4 \end{cases}$$

$\hookrightarrow$ $v_1(T) = bT/4$

$x_1(T) = \frac{7}{2} b (T/4)^2$



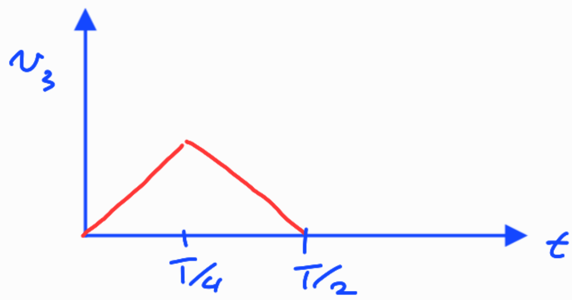
analog:

$$v_2(t) = \begin{cases} 0 : & t < 3T/4 \\ b\left(t - \frac{3T}{4}\right) : & t > 3T/4 \end{cases}$$

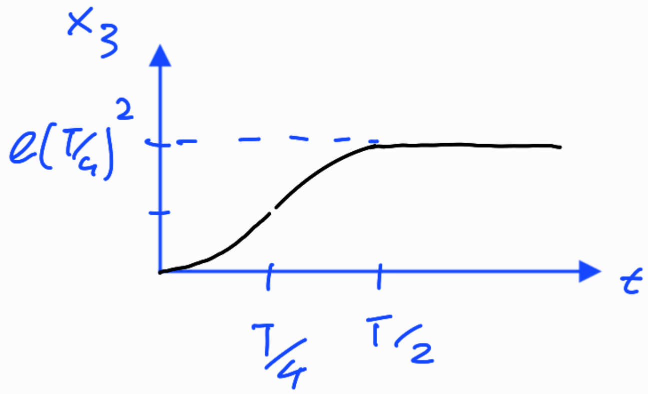
$$x_2(t) = \begin{cases} 0 : & t < 3T/4 \\ \frac{b}{2}\left(t - \frac{3T}{4}\right)^2 \end{cases}$$

$\Rightarrow v_2(T) = bT/4, \quad x_2(T) = \frac{b}{2}(T/4)^2$

$$v_3(t) = \begin{cases} b\,t & : \quad t < T/4 \\ b\,T/4 - b\,(t - T/4) & : \quad T/4 < t < T/2 \\ 0 & : \quad t > T/2 \end{cases}$$



$$x_3(t) = \begin{cases} \frac{b}{2}\,t^2 & : \quad t < T/4 \\ \frac{b}{2}\left(T/4\right)^2 + \frac{bT}{4}\left(t - T/4\right) - \frac{b}{2}\left(t - \frac{T}{4}\right)^2 & : \quad T/4 \le t < \frac{T}{2} \\ b\left(T/4\right)^2 & : \quad t \ge T/2 \end{cases}$$



$$\rightarrow \quad \vec{v}\,(T) = b\,T/4 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0.5\,\frac{m}{s} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\vec{r}\,(T) = b\left(T/4\right)^2 \begin{pmatrix} 7/2 \\ 1/2 \\ 1 \end{pmatrix} = 2{,}5\,m \begin{pmatrix} 7/2 \\ 1/2 \\ 1 \end{pmatrix}$$