Lösungshinweise Blatt 7

35) Bzgl. ONB $\vec{e}_1, \vec{e}_2, \vec{e}_3$: $\quad \vec{f}(x) = \sum_i f_i(x)\vec{e}_i$,

$$\vec{g}(x) = \sum_i g_i(x)\vec{e}_i$$

$$\rightarrow \langle \vec{f}, \vec{g} \rangle' = \sum_i (f_i g_i)' = \sum_i (f_i' g_i + f_i g_i')$$

$$= \sum_i f_i' g_i + \sum_i f_i g_i' = \langle \vec{f}', \vec{g} \rangle + \langle \vec{f}, \vec{g}' \rangle$$

$$(\vec{f} \times \vec{g})_1' = (f_2 g_3 - f_3 g_2)' =$$

$$= f_2' g_3 + f_2 g_3' - f_3' g_2 - f_3 g_2'$$

$$= \underbrace{f_2' g_3 - f_3' g_2}_{} + \underbrace{f_2 g_3' - f_3 g_2'}_{}$$

$$= (\vec{f}' \times \vec{g})_1 + (\vec{f} \times \vec{g}')_1$$

Komponenten $(\vec{f} \times \vec{g})_2'$, $(\vec{f} \times \vec{g})_3'$ analog.

36)

- $|1 + 2i| = \sqrt{5}$, $\quad \dfrac{1}{1+2i} = \dfrac{1-2i}{(1+2i)(1-2i)} = \dfrac{1-2i}{5}$

  $\operatorname{Re}(1+2i) = 1$, $\quad \operatorname{Im}(1+2i) = 2$

- $\left| \dfrac{1}{1+i} \right| = \dfrac{1}{|1+i|} = \dfrac{1}{\sqrt{2}}$, $\quad \left(\dfrac{1}{1+i}\right)^{-1} = 1+i$

  $\dfrac{1}{1+i} = \dfrac{1-i}{(1+i)(1-i)} = \dfrac{1-i}{2} \rightarrow \operatorname{Re}\dfrac{1}{1+i} = \dfrac{1}{2}$, $\quad \operatorname{Im}\dfrac{1}{1+i} = -\dfrac{i}{2}$

- $\left|\frac{1}{i}\right| = \frac{1}{|i|} = 1$, $\left(\frac{1}{i}\right)^{-1} = i$,

  $\frac{1}{i} = \frac{-i}{i\cdot(-i)} = \frac{-i}{-i^2} = -i$

  $\rightarrow$ $\mathrm{Re}\,\frac{1}{i} = 0$, $\mathrm{Im}\,\frac{1}{i} = -1$

- $z = (1+2i)(1-3i) = 1 - i - 2\cdot 3\, i^2 = 7 - i$

  $\rightarrow$ $|z| = \sqrt{8} = 2\sqrt{2}$, $\frac{1}{z} = \frac{7+i}{8}$

  $\mathrm{Re}\, z = 7$, $\mathrm{Im}\, z = -1$

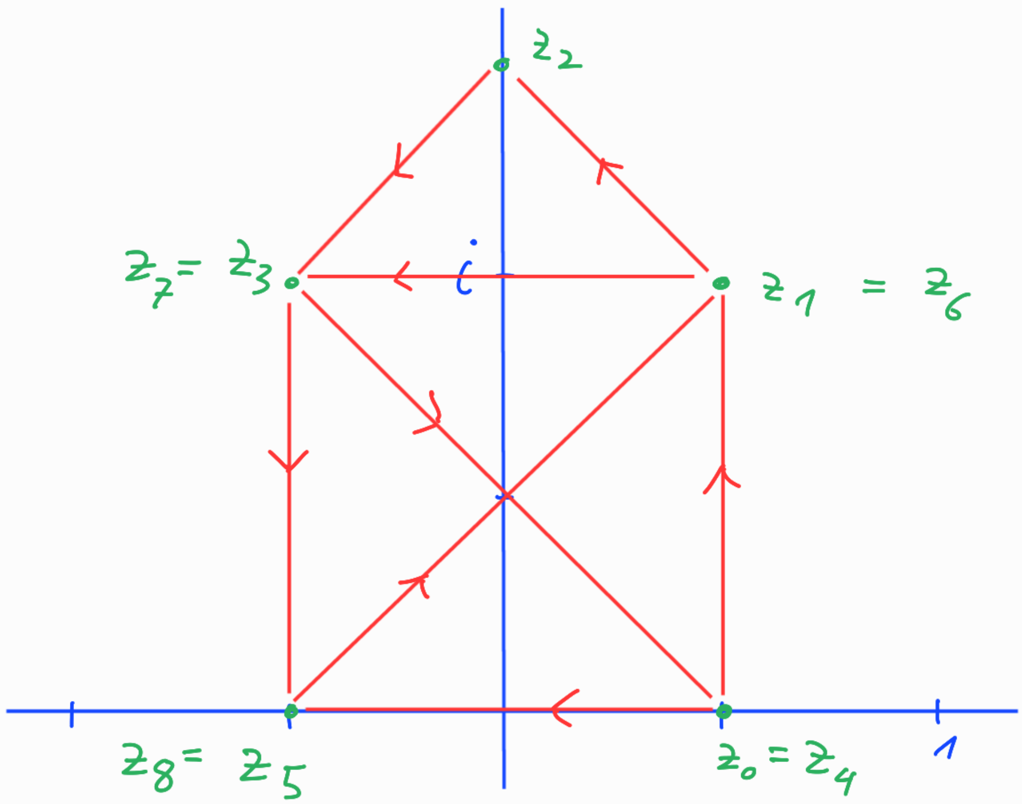
- $e^{i\pi/4} = \cos\pi/4 + i\sin\pi/4 = \frac{1}{\sqrt{2}}(1+i)$

  $|e^{i\pi/4}| = 1$, $\left(e^{i\pi/4}\right)^{-1} = e^{-i\pi/4}$

  $\mathrm{Re}\, e^{i\pi/4} = 1/\sqrt{2} = \mathrm{Im}\, e^{i\pi/4}$

- $z = \sqrt{-9} = 3i$, $|z| = 3$, $\frac{1}{z} = -\frac{i}{3}$

  $\mathrm{Re}\, z = 0$, $\mathrm{Im}\, z = 3$



38)

a) $e^{ix} + e^{-ix} \overset{E.F.}{=} \cos x + i\sin x + \cos(-x) + i(\sin(-x))$

$= 2\cos x \quad ;$

$e^{ix} - e^{-ix} \overset{E.I}{=} \cos x + i\sin x - \cos(-x) - i(\sin(-x))$

$= 2i \sin x \quad .$

b) $\cos^2 x = \frac{1}{4}(e^{ix} + e^{-ix})^2$

$= \frac{1}{4}(e^{2ix} + e^{-2ix} + 2)$

$\sin^2 x = \frac{-1}{4}(e^{ix} - e^{-ix})^2$

$= -\frac{1}{4}(e^{2ix} + e^{-2ix} - 2)$

$\rightarrow \cos^2 x + \sin^2 x = 1 \quad ,$

$$(\cos x)' = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)' = \frac{1}{2}\left(i e^{ix} - i e^{-ix}\right)$$

$$= -\frac{1}{2i}\left(e^{ix} - e^{-ix}\right) = -\sin x \quad ;$$

$$\uparrow$$

$$i = -\frac{1}{i}$$

$$(\sin x)' = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)' = \frac{1}{2i}\left(i e^{ix} + i e^{-ix}\right)$$

$$= \frac{1}{2}\left(e^{ix} + e^{-ix}\right) = \cos x \quad .$$

39)

a) komplexer Parameter $\lambda = a - ib$ im

Integranden $e^{-\lambda x}$ wird auf gleiche

Weise wie ein reeller Parameter behandelt:

$$\int_0^b e^{-\lambda x}\,dx = -\frac{e^{-\lambda x}}{\lambda}\Big|_0^b = \frac{1}{\lambda} - \frac{e^{-\lambda b}}{\lambda}$$

wegen $e^{-\lambda b} \underset{\lambda = a - ib}{=} e^{-ab}\, e^{ib} \xrightarrow[b \to \infty]{} 0$ also

$$\int_0^\infty e^{(-a+ib)x}\,dx = \frac{1}{\lambda} = \frac{1}{a - ib} = \frac{a + ib}{a^2 + b^2}$$

b) wegen $\operatorname{Re} e^{(-a+ib)x} = e^{-ax} \operatorname{Re} e^{ibx} \overset{\text{E.I.}}{=} e^{-ax} \cos bx$

bzw. $\operatorname{Im} e^{(-a+ib)x} = e^{-ax} \operatorname{Im} e^{ibx} \overset{\text{E.I.}}{=} e^{-ax} \sin bx$

ist $\displaystyle\int_0^\infty \cos(bx)\, e^{-ax}\,dx = \operatorname{Re} \int_0^\infty e^{(-a+ib)x}\,dx \overset{a)}{=} \frac{a}{a^2 + b^2}$

und $\displaystyle\int_0^\infty \sin(bx)\, e^{-ax}\,dx = \operatorname{Im} \int_0^\infty e^{(-a+ib)x}\,dx \overset{a)}{=} \frac{b}{a^2 + b^2}$