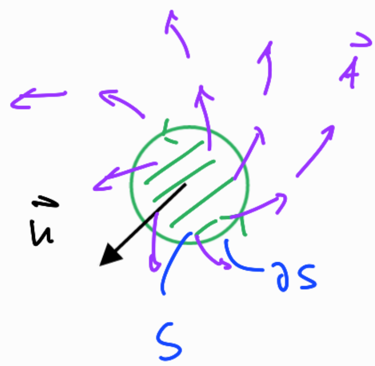
Wiederholung:

Rotation (Wirbelstärke) von Vf $\vec{A}$ in $\vec{r}_0$:

Vektor $\text{rot}\,\vec{A}(\vec{r}_0)$ bestimmt durch

$$\boxed{\langle \hat{n}, \text{rot}\,\vec{A}(\vec{r}_0)\rangle = \lim_{|S|\to 0} \frac{1}{|S|} \int_{\partial S} \vec{A}\, d\vec{\ell}}$$



S: Fläche, $\vec{r}_0 \in S$

∂S: Rand der Fläche

$|S|$: Flächeninhalt

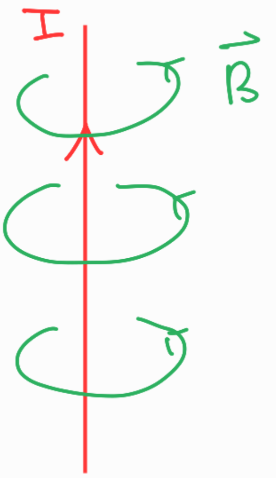
$\hat{n}$: Flächennormale

in kart. Koordinaten:

$$\text{rot}\,\vec{A} = \vec{\nabla} \times \vec{A} = \begin{pmatrix} \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3} \\ \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1} \\ \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2} \end{pmatrix}$$

Anwendungen:

Magnetostatik: „elektrische Ströme verursachen magnetische Wirbelfelder"

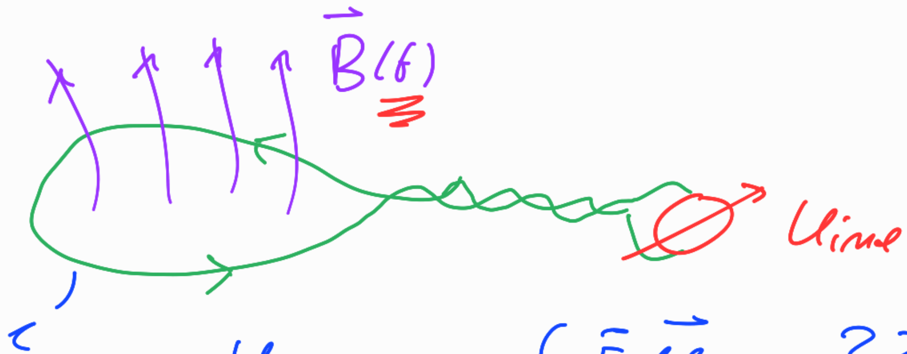


Stromdichte: $\vec{j}(\vec{r}) \xrightarrow{?} \vec{B}(\vec{r})$ Magnetfeld

Ampère: $$\boxed{\text{rot}\,\vec{B} \overset{!}{=} \mu_0 \vec{j}}$$

$\vec{j} \longrightarrow \vec{B}$?!

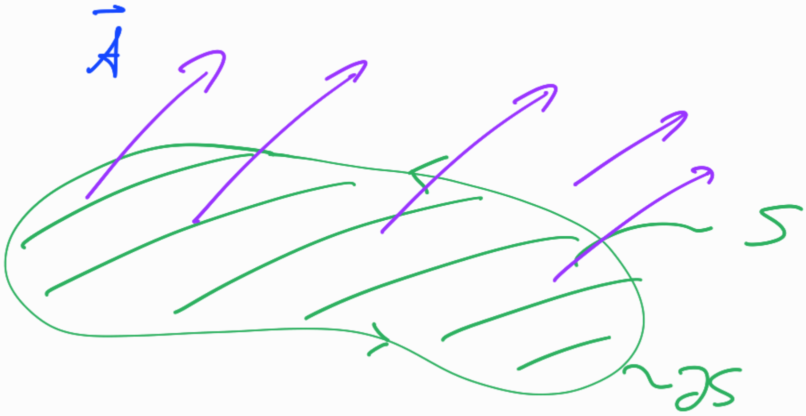
• Induktionsgesetz: „zeitlich veränderliche Magnetfelder erzeugen elektrische Wirbelfelder"



$$U_{ind} = -\int_C \vec{E}\, d\vec{l} = ??$$

Faraday: $$\boxed{\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

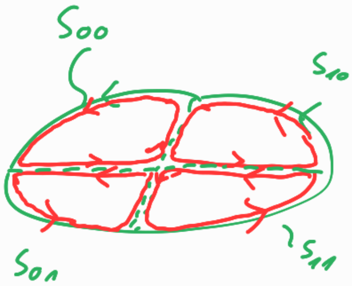
Satz von Stokes



$$\boxed{\int_{\partial S} \vec{A}\, d\vec{\ell} \overset{!}{=} \int_{S} \mathrm{rot}\,\vec{A}\, d\vec{f}}$$

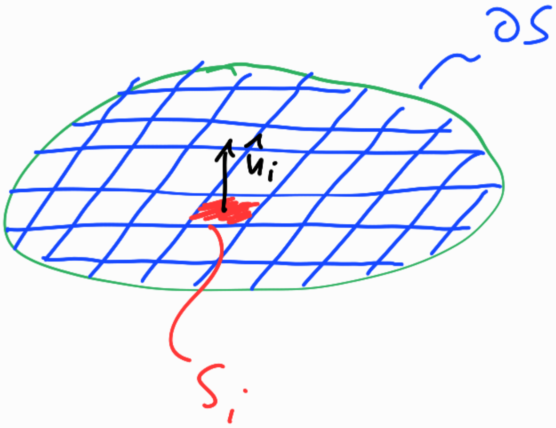
⌈ „Physikerbeweis":

$$\int_{\partial S} \vec{A}\, d\vec{\ell} \overset{!}{=} \int_{\partial S_0} \vec{A}\, d\vec{\ell} + \int_{\partial S_1} \vec{A}\, d\vec{\ell}$$



$$\int_{\partial S} \vec{A}\, d\vec{\ell} = \int_{\partial S_{00}} \vec{A}\, d\vec{\ell} + \int_{\partial S_{01}} \vec{A}\, d\vec{\ell} + \int_{\partial S_{10}} \vec{A}\, d\vec{\ell} + \int_{\partial S_{11}} \vec{A}\, d\vec{\ell}$$

⋮

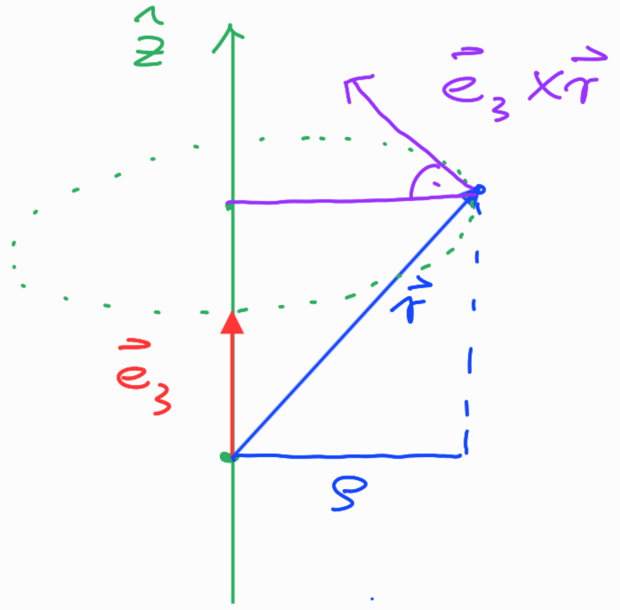


$$\oint_{\partial S} \vec{A}\, d\vec{l} = \sum_i \underbrace{\frac{1}{|S_i|} \oint_{\partial S_i} \vec{A}\, d\vec{l}}_{\langle \hat{n}_i ,\, \operatorname{rot} \vec{A}(\vec{r}_i) \rangle} \cdot |S_i|$$

$$= \sum_i \langle \operatorname{rot} \vec{A}(\vec{r}_i) ,\, \underbrace{\hat{n}_i\, |S_i|}_{\vec{\delta S_i}} \rangle$$

$$\overset{!}{=} \int_S \operatorname{rot} \vec{A} \cdot d\vec{f} \qquad !$$

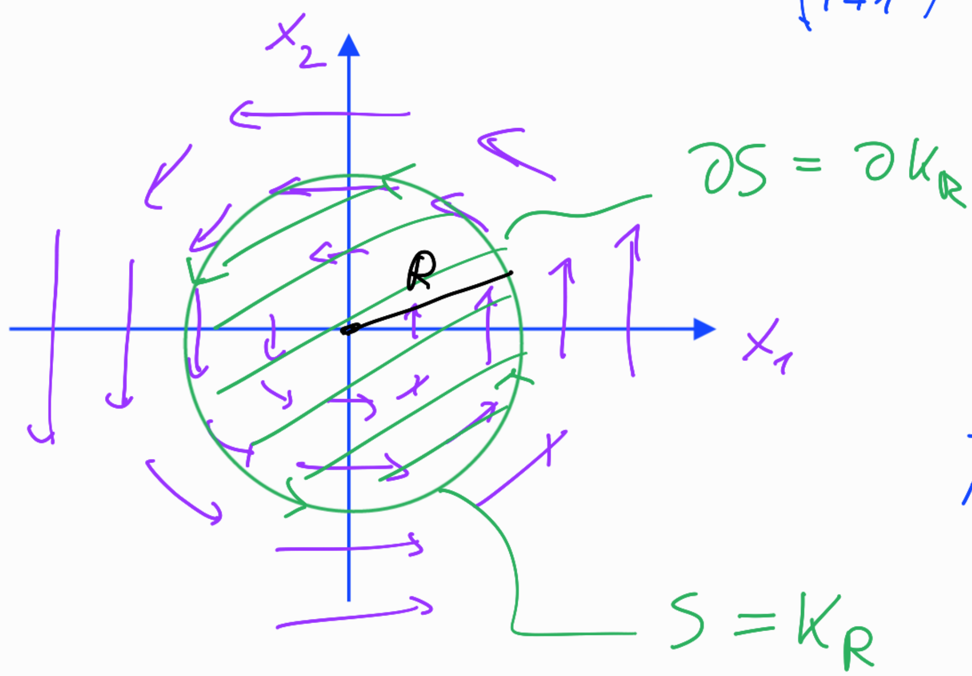
Beispiel:
$$\vec{A}(\vec{r}) = \vec{e}_3 \times \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \\ 0 \end{pmatrix}$$



$$= \varrho \, \vec{e}_\varphi$$

$$\mathrm{rot}\, \vec{A} = \vec{\nabla} \times \begin{pmatrix} -x_2 \\ x_1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = 2\vec{e}_3$$



$$d\vec{l} \parallel \vec{e}_\varphi$$

$$\vec{A} = \varrho \vec{e}_\varphi \parallel \vec{e}_\varphi$$

Fläche $S = K_R \perp \vec{e}_3$:

$$\underbrace{= \int_{\partial K_R} \vec{A}\, d\vec{l}}_{} = \int_{\partial K_R} \underset{=}{R}\, \vec{e}_\varphi \cdot \underbrace{d\vec{l}}_{|d\vec{l}|} = R \underbrace{\int_{\partial K_R} dl}_{2\pi R} = \underline{\underline{2\pi R^2}}$$

Stokes

$$= \int_{K_R} \operatorname{rot} \vec{A}\, d\vec{f} = \int_{K_R} 2\, \underbrace{\vec{e}_3 \cdot d\vec{f}}_{|d\vec{f}|} = \underset{=}{2} \underbrace{\int_{K_R} |d\vec{f}|}_{\pi R^2}$$

$$= \underline{\underline{2\pi R^2}}$$

Magnetostatik

Ampère :

$$\boxed{\operatorname{rot} \vec{B} = \mu_0 \vec{j}}$$

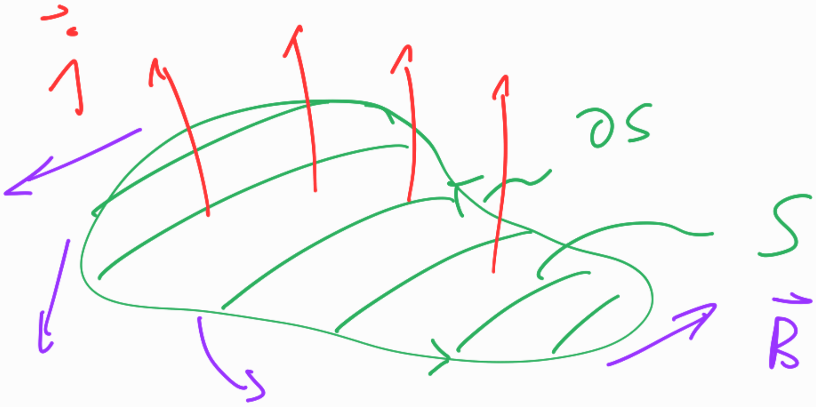
$\mu_0 = \frac{1}{c^2} \cdot \frac{1}{\varepsilon_0}$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{As}{Vm}$

$c = 3{,}00 \cdot 10^8 \, m$

Lichtgeschw.

Integrale Form



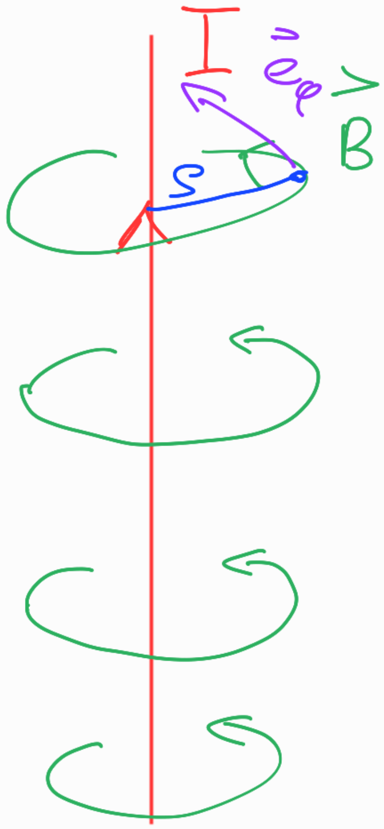
$$\int_{\partial S} \vec B\, d\vec \ell \underset{\text{S. v. Stokes}}{=} \int_S \underbrace{\text{rot}\,\vec B}_{=\,\mu_0 \vec j}\, d\vec f \underset{\text{Ampère}}{=} \mu_0 \underbrace{\int_S \vec j\, d\vec f}_{I_S}$$

I_S: elektr. Strom durch Fläche S

$$\boxed{\int_{\partial S} \vec B\, d\vec \ell = \mu_0 I_S} \qquad (*)$$

Ampère Gesetz, integrale Form

$\Rightarrow$ Magnetfeld eines langen geraden, stromführenden Drahts:



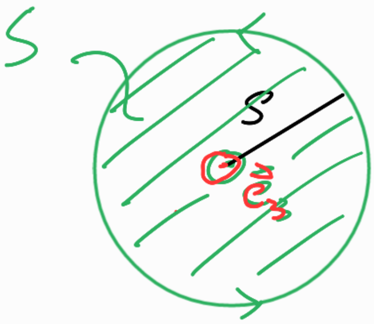
Ansatz:

$$\boxed{\vec{B}(s,\varphi,z) \overset{!}{=} B(s)\,\vec{e}_\varphi}$$

$$\left[\cancel{B_z(s)\,\vec{e}_z} + \cancel{B_s(s)\,\vec{e}_s}\right]$$

$$B(s) = ?$$

Ampère (*):



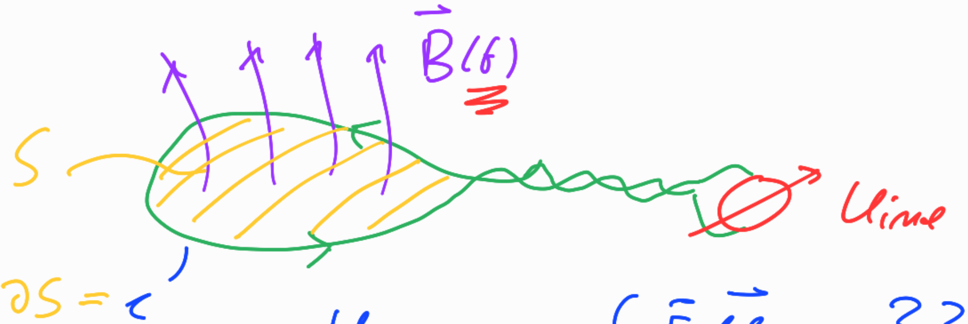
$$\int_{\partial S} \vec{B}\,d\vec{l} = \mu_0 I_S = \mu_0 I$$

$$\|$$

$$\int_{\partial S} B(s)\,\underbrace{\vec{e}_\varphi \cdot d\vec{l}}_{|d\vec{l}|} = B(s)\cdot \underbrace{\int_{\partial S} |d\vec{l}|}_{2\pi s} = B(s)\,2\pi s\,.$$

$$\rightarrow \quad \boxed{\vec{B}(s) = \frac{\mu_0 I}{2\pi}\,\frac{1}{s}\,\vec{e}_\varphi}$$

• Induktionsgesetz: „zeitlich veränderliche Magnetfelder erzeugen elektrische Wirbelfelder"



$$U_{ind} = -\int_{\mathcal{C}} \vec{E}\, d\vec{\ell} = ??$$

Faraday: $\boxed{\operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ (*)

$$U_{ind} = -\int_{\mathcal{C}} \vec{E}\, d\vec{\ell} \underset{\mathcal{C} \stackrel{!}{=} \partial S}{=} -\int_{\partial S} \vec{E}\, d\vec{\ell} \underset{\text{Stokes !}}{=} -\int_S \operatorname{rot}\vec{E}\, d\vec{f}$$

$$\underset{\text{Faraday}}{=} + \int_S \frac{\partial \vec{B}}{\partial t}\, d\vec{f} = + \frac{d}{dt} \Phi_S(t)$$

magn. Fluss durch Fläche S

$$\Phi_S = \int_S \vec{B}\, d\vec{f}\ .$$

Induktionsgesetz: $\boxed{U_{ind} = \frac{d}{dt} \Phi_S(t)}$