Quantum Information Theory

topics:

- quantum gates & circuits
  → quantum computer

- algorithms: Grover, Simon, Shor

  search (Grover) — integer factoriz. (Shor)

  → ~~RSA~~ !

- complexity: P, NP, BQP

- entanglement: Bell inequalities, quantum crypthography

- <u>noise</u> ("decoherence")

- quantum error correction (CSS, stabilizer, toric code)
- Fault tolerant quantum computing, threshold theorem
- quantum information: noiseless coding theorem
- quantum Shannon theory
- general aspects of entanglement

Quantum mechanics (recap)

postulates:

(P1) Statespace and states

statespace = complex vector space $\mathcal{H}$
with hermitian product

$$\langle \, , \, \rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$$

$$\varphi, \psi \mapsto \langle \varphi, \psi \rangle$$

$\dim \mathcal{H} < \infty$!

$$(\to \mathcal{H} = \mathbb{C}^d)$$

(pure) state = normalized vector
$\psi \in \mathcal{H}$

(P2) Measurements

Observable A = hermitian operator A $(= A^+)$

spectral decomposition:

$$A = \sum_{\ell=1}^{n} a_\ell P_\ell$$

- $a_1, a_2, \ldots, a_n$ real eigenvalues
- $P_1, P_2, \ldots, P_n$ orthogonal projections on eigensp. $V_1, \ldots, V_n$; $V_1 \oplus V_2 \oplus \ldots \oplus V_n = \mathcal{H}$

Born rule:

Measurement of A on system in state ψ yields result a_ℓ with probability $p_\ell = \langle \psi, P_\ell \psi \rangle$.

$\rightarrow$ expect. value $\langle A \rangle_\psi = \langle \psi, A\psi \rangle$

(P3) <u>Dynamics</u>

<u>Schrödinger eq.</u>

$$\dot{\psi}(t) = -i H \psi(t)$$

with <u>hermitian</u> op. H (Hamilton op.)

$$\longrightarrow \qquad \psi(t) = U_t \, \psi(0)$$

with <u>unitary</u> time evol. op.

$$U_t = e^{-iHt} \qquad (\hbar = 1)$$

Remarks & notations

1) <u>Why</u> statespace = <u>vector</u> space ?

<u>Superposition principle</u> !

QM: with A and B also superposition "$A + B$" state !

2) Hermitian inner product:

$$\langle \, , \, \rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$$

by def. (i) symmetric:

$$\langle \varphi, \psi \rangle = \langle \psi, \varphi \rangle^*$$

(ii) positive:

$$\langle \varphi, \varphi \rangle > 0 \qquad (\varphi \neq 0)$$

(iii) linear:

$$\langle \varphi, \psi_1 + \psi_2 \rangle = \langle \varphi, \psi_1 \rangle + \langle \varphi, \psi_2 \rangle$$

$$\langle \varphi, \lambda \psi \rangle = \lambda \langle \varphi, \psi \rangle$$

3) <u>Notation</u>:

$$\mathcal{H} \ni \psi = |\psi\rangle \quad \text{„ket"}$$

4) <u>Dual vector / -space</u>:

$$\mathcal{H} \ni \psi \longleftrightarrow \psi^* \in \mathcal{H}^*$$

for all $\varphi \in \mathcal{H}$ $\langle \psi, \varphi \rangle = \psi^* \varphi$

notation: $\psi^* = \langle \psi |$ „bra"

$\rightsquigarrow$

- $\langle \psi, \varphi \rangle = \psi^* \varphi = \langle \psi | \varphi \rangle$
- $A \varphi = A | \varphi \rangle$
- $\langle \psi, A\varphi \rangle = \langle \psi | A | \varphi \rangle$

etc.

5) <u>state measurement:</u>

"system in state $|\varphi\rangle$?"

$\Rightarrow$ $A = P_{|\varphi\rangle} = |\varphi\rangle\langle\varphi|$

$\rightsquigarrow$ measurement on system in state $|\psi\rangle$ positive ("1") with prob.

$$p = \langle\psi| P_{|\varphi\rangle} |\psi\rangle = |\langle\psi|\varphi\rangle|^2 .$$

6) <u>Ideal measurement</u>:

observable $A = \sum_{l=1}^{N} a_l P_l$;

after <u>ideal</u> measurement of A on system in state $|\psi\rangle$ with outcome a_k system in state

$$|\psi'\rangle = \frac{1}{\sqrt{p_k}} P_k |\psi\rangle$$

$(p_k = \langle\psi| P_k |\psi\rangle)$

"ideal measurement

$\hat{=}$ projection onto eigenspaces"

7) Linear dynamics:

given (P1) (linear statespace)

and (P2) (quantum measurement)

dynamics must be linear!

i.e. $\dot{\psi} \overset{!}{=} F \psi$,

linear operator

otherwise contradiction with

special relativity (→ "no super-luminal signals!")

Simon, Bužek, Gisin 2001

(PRL 87, 170405)

- conservation of norm $|\psi(t)|$

requiers

$$\mathcal{F} = -\mathcal{F}^{+} \quad ;$$

i.e. $\mathcal{F} = iH$ with

<u>hermitian</u> H

$\downarrow$

observable H <u>always conserved</u>

(since $[H, H] = 0$) and thus

identified with <u>energy</u> !

$\rightarrow$ Schrödinger Eq.

. .