20.

obviously: $(\mathcal{E}\otimes \mathrm{I})(S) = \sum_k (E_k \otimes 1)\, S\, (E\otimes 1)^+ \geq 0$

$\uparrow$ $S \geq 0$

$\rightarrow$ $\mathcal{E}$ c.p. !

$$\mathrm{tr}\, \mathcal{E}(S) = \sum_k \mathrm{tr}\, E_k S E_k^+ = \mathrm{tr}\Big(\underbrace{\sum_k E_k^+ E_k}_{\geq 1}\Big) S = \mathrm{tr}\, S$$

$\rightarrow$ $\mathcal{E}$ t.p. !

21. –

22.

let $\vartheta_i \equiv \pi/2 + \Delta\varphi_i$ the angle of the random unit vector u and vector e_i of an ONB $(e_1, \dots, e_n)$ of $\mathbb{R}^n$:

$$\rightarrow \overline{|\sin \Delta\varphi_i|^2} = \overline{\cos^2(\pi/2 + \Delta\varphi_i)} = \overline{|\langle e_i, u\rangle|^2} = \overline{u_i^2} = \frac{1}{n}$$

$\uparrow$ $1 = \overline{|u|} = \sum_{i=1}^n \overline{u_i^2}$

$\rightarrow$ $|\vartheta_i - \pi/2| = O(1/\sqrt{n}) \to 0$ for $n \to \infty$