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Quantum Information Theory – Sheet 6

Wintersemester 2021/22

Webpage: *http://www.thp.uni-koeln.de/∼rk/qit_22.html/*

Submission of solutions as pdf-file until Thursday, July 14, 12 pm, to
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20. cptp-map

5 Punkte

Show that a quantum operation

$$\rho \;\mapsto\; \mathcal{E}(\rho) \;=\; \sum_{k=1}^{K} E_k\,\rho\,E_k^\dagger$$

with Kraus operators E_k satisfying $\sum_{k=1}^{K} E_k^\dagger E_k = \mathbf{1}$ is *completely* positive and trace preserving.

21. Shor's 1-9 Code

8 Punkte

Use the error-correcting condition*

$$P E_k^\dagger E_{k'} P \propto P$$

to show that Shor's 1-9 code** also allows the correction of a single bit-flip X_j that appears *simultaneously* together with a single phase-flip Z_k. To this end you may check the above condition for the following set of error operators:

$$\{I_i,\; X_i,\; Z_i,\; X_j Z_k\,\}_{i,j,k=1,\ldots 9}\,.$$

[* P is the projection onto the code space, $\{E_k\}$ are Kraus-operators of the noise.
** which encodes logical qubit states $|0\rangle$ and $|1\rangle$ into 9-qubit states $\Big(\,|0\rangle^{\otimes 3} \pm |1\rangle^{\otimes 3}\,\Big)^{\otimes 3}/\sqrt{8}.$]

22. Random unit vectors in n dimensions

5 Punkte

Show that for large n an n-dimensional *random* unit vector is almost always almost orthogonal to any vector of a fixed orthogonal system.