Lösungshinweise Blatt 13

54) $$x_H(t) = e^{iHt/\hbar}\, x\, e^{-iHt/\hbar} = e^{\frac{it}{\hbar}[H,\dots]}\, x$$

$$= x + \frac{it}{2m\hbar}\underbrace{[p^2, x]}_{-2pi/\hbar} + 0 = x + \frac{p}{m}t$$

$H = p^2/2m$

$$p_H(t) = p \, ,$$

$$\rightarrow \langle x \rangle_{\psi(t)} = \langle x_H(t) \rangle_{\psi_0} = \langle x \rangle_{\psi_0} + \frac{\langle p \rangle_{\psi_0}}{m} \cdot t$$

$$[x_H(t), x_H(t')] = [x + pt/m, x + pt'/m]$$

$$= [x, p] \cdot \frac{t'-t}{m} = \frac{i\hbar}{m}(t'-t) \, .$$

55) a)

$$a_H^+(t) = e^{\frac{it}{\hbar}[H,\dots]}\, a^+ = e^{i\omega t[aa^+,\dots]}\, a^+$$

$$= a^+ + i\omega t\, a^+ + \frac{(i\omega t)^2}{2!} a^+ + \frac{(i\omega t)^3}{3!} a^+ + \dots$$

$$= e^{i\omega t}\, a^+ \, ,$$

offenbar $(a_H^+(t))^+ = a_H(t) \rightarrow a_H(t) = e^{-i\omega t}\, a$

b)

$$\frac{x}{\ell} = \frac{a^+ + a}{\sqrt{2}} \rightarrow \frac{1}{\ell} x_H(t) = \frac{1}{\sqrt{2}}\left(e^{i\omega t} a^+ + e^{-i\omega t} a\right)$$

$$= \frac{1}{2} e^{i\omega t}\left(\frac{x}{\ell} - i\frac{\ell}{\hbar} p\right) + \frac{1}{2} e^{-i\omega t}\left(\frac{x}{\ell} + \frac{i\ell}{\hbar} p\right)$$

$$= \frac{x}{\ell} \cos\omega t + \frac{\ell}{\hbar} p \sin\omega t$$

d.h. $$x_H(t) = x \cos\omega t + \frac{p}{m\omega} \sin\omega t$$

mit $\frac{\hbar p}{\ell} = i \frac{a^+ - a}{\sqrt{2}}$ folgt analog

$$p_H(t) = -m\omega x \sin\omega t + p \cos\omega t$$

$$\rightarrow \langle x\rangle_{\psi(t)} = \langle x_H(t)\rangle_{\psi_0} = \langle x\rangle_{\psi_0} \cos\omega t + \frac{\langle p\rangle_{\psi_0}}{m\omega} \sin\omega t$$

$$\langle p\rangle_{\psi(t)} = \langle p_H(t)\rangle_{\psi_0} = -m\omega\langle x\rangle_{\psi_0} \sin\omega t + \langle p\rangle_{\psi_0} \cos\omega t.$$

5̄6) $$P^{(1)}_{mn} = \frac{u^2}{\hbar^2}\left|\int_{-\infty}^{+\infty} dt\; e^{i(m-n)\omega t} \frac{e^{-\frac{t^2}{2\tau^2}}}{\sqrt{2\pi\tau^2}}\right|^2 \left|\langle m|\tfrac{x}{\ell}|n\rangle\right|^2$$

- $$\left|\langle m|\tfrac{x}{\ell}|n\rangle\right|^2 = \frac{1}{2}\left|\langle m|a^+ + a|n\rangle\right|^2 = \begin{cases} \frac{n+1}{2} & : m = n+1 \\ n/2 & : m = n-1 \\ 0 & : \text{sonst} \end{cases}$$

$\rightarrow$ nur Übergänge nach $m = n \pm 1$,

in diesem Fall $\left| \int_{-\infty}^{\infty} dt \; e^{i\omega t} \frac{e^{-t^2/2\tau^2}}{\sqrt{2\pi}\,\tau} \right|^2 = e^{-\omega^2\tau^2}$

$$\rightarrow \quad P^{(1)}_{n+1,n} = \frac{u^2}{2\hbar^2} (n+1)\, e^{-(\omega\tau)^2}$$

$$P^{(1)}_{n-1,n} = \frac{u^2}{2\hbar^2} \, n \, e^{-(\omega\tau)^2}$$