Lösungshinweise Blatt 7

32 a)

$$\langle \hat{p} \rangle_{\chi_0} = \int dx \; e^{-ip_0 x/\hbar} \psi_0^*(x) \underbrace{\left(-i\hbar \frac{\partial}{\partial x}\right) e^{ip_0 x/\hbar} \psi_0(x)}_{-i\hbar e^{ip_0 x/\hbar}\psi_0'(x) \;+\; p_0 e^{ip_0 x/\hbar}\psi_0(x)}$$

$$= \langle \hat{p} \rangle_{\psi_0} + p_0 \, .$$

b) mit $k_0 = p_0/\hbar$:

$$\tilde{\chi}(k) = \int dx \; \psi_0(x) \, e^{ik_0 x} e^{-ikx} = \int dx \; \psi_0(x) \, e^{-i(k-k_0)x} = \tilde{\psi}(k-k_0)$$

$$\rightsquigarrow \quad \chi(x,t) = \int \frac{dk}{2\pi} \, e^{-i\frac{\hbar k^2}{2m}t} \, e^{ikx} \, \tilde{\psi}(k-k_0)$$

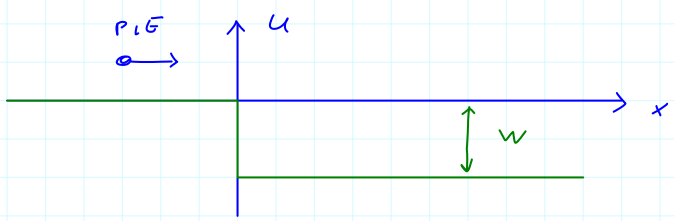
$$= \int \frac{dk}{2\pi} \, e^{-i\frac{\hbar (k+k_0)^2}{2m}t} \, e^{i(k+k_0)x} \, \tilde{\psi}(k)$$

$$= e^{-i\left(\hbar k_0^2 t/2m - k_0 x\right)} \int \frac{dk}{2\pi} \, e^{-i\frac{\hbar k^2}{2m}t} \, e^{-i\frac{\hbar k_0}{m}kt} \, e^{ikx} \, \tilde{\psi}(k)$$

$$= e^{-i(\dots)} \underbrace{\int \frac{dk}{2\pi} \, e^{-i\frac{\hbar k^2}{2m}t} \, e^{ik\left(x - \frac{\hbar k_0}{m}t\right)} \, \tilde{\psi}(k)}_{= \psi\left(x - \frac{\hbar k_0}{m}t, \, t\right)}$$

$$\rightarrow \quad |\chi(x,t)| = \left|\psi\left(x - \frac{p_0}{m}t, \, t\right)\right| \qquad \blacksquare$$

33 a)



Streuansatz:
$$\phi(x) = \begin{cases} e^{ikx} + r e^{-ikx} & : x < 0 \\ t \, e^{i\tilde{k}x} & : x \geq 0 \end{cases} \; ; \qquad \begin{aligned} k &= \sqrt{2mE}/\hbar \\ \tilde{k} &= \sqrt{2m(E+W)}/\hbar \end{aligned}$$

Stetigkeit von ϕ, ϕ' in $x = 0$:

$$\left.\begin{aligned} 1 + r &= t \\ k(1-r) &= \tilde{k} t \end{aligned}\right\} \rightsquigarrow \; k(1-r) = \tilde{k}(1+r)$$

$$\rightarrow \quad r = \frac{k - \tilde{k}}{k + \tilde{k}} = \frac{1 - \tilde{k}/k}{1 + \tilde{k}/k} = \frac{1 - \sqrt{1 + \frac{W}{E}}}{1 + \sqrt{1 + \frac{W}{E}}} \, ,$$

$$\rightarrow \quad R = |r|^2 = \left| \frac{1 - \sqrt{1 + w/E}}{1 + \sqrt{1 + w/E}} \right|^2 \underset{w \to \infty}{\longrightarrow} 1 .$$

b) $$w/E = 80 \rightarrow \sqrt{1 + w/E} = 9 \rightarrow R = \left| \frac{8}{10} \right|^2 = \underline{0.64}$$

34) Streuansatz:

$$\phi(x) = \begin{cases} e^{ikx} + r\, e^{-ikx} & : x < 0 \\ t\, e^{ikx} & : x \geq 0 \end{cases}, \quad k = \sqrt{2mE}/\hbar$$

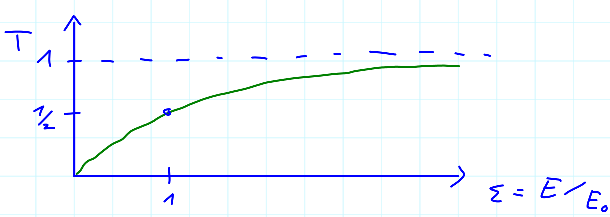
- ϕ stetig in $x=0$ $\rightarrow$ $1 + r = t$ $(*)$
- ϕ' springt in $x=0$ $\leadsto$ $$\underbrace{\delta\phi'}_{= ik(t - 1 + r)} = \int_{-\varepsilon}^{\varepsilon} \phi''\, dx = \frac{2mu}{\hbar^2} \underbrace{\phi(0)}_{= t}$$

d.h. $$t - 1 + r = -i \frac{2mu}{k\hbar^2} \cdot t$$

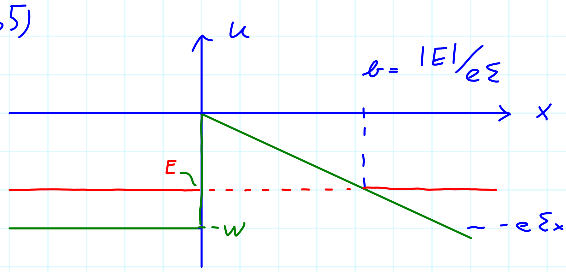
$$\overset{(*)}{\longrightarrow} \quad t - 1 = -i \frac{mu}{k\hbar^2} t \rightarrow t = \frac{1}{1 + i\frac{mu}{k\hbar^2}}$$

$$\leadsto T = |t|^2 = \left(1 + \left(\frac{mu}{k\hbar^2}\right)^2\right)^{-1} \underset{k = \sqrt{2mE}/\hbar}{=} \left(1 + \frac{mu^2}{2\hbar^2 E}\right)^{-1} \underset{E_0 = \frac{mu^2}{2\hbar^2}}{=} \left(1 + E_0/E\right)^{-1}$$

$$\rightarrow T(E_0) = 1/2 \quad \text{und} \quad T(\varepsilon) = \frac{1}{1 + 1/\varepsilon} = \frac{\varepsilon}{\varepsilon + 1}$$



35)



$$\frac{1}{\hbar} \int_0^b \left(8m(U(x) - E)\right)^{1/2} dx = \frac{\sqrt{8m}}{\hbar} \int_0^b \left(|E| - e\varepsilon x\right)^{1/2} dx$$

$$= -\frac{\sqrt{8m}}{\hbar} \frac{2}{3e\varepsilon} \left(|E| - e\varepsilon x\right)^{3/2} \Big|_0^b = \frac{4}{3} \frac{\sqrt{2m}}{\hbar} \frac{|E|^{3/2}}{e\varepsilon}$$

$$\rightarrow \quad I(E,\mathcal{E}) \simeq I_0 \exp\left(-\frac{4}{3}\frac{\sqrt{2m}}{\hbar}\frac{|E|^{3/2}}{e\,\mathcal{E}}\right)$$

$\rightsquigarrow$ signifikanter Tunnelstrom bei Feldstärke

$$\mathcal{E}_0 \overset{!}{=} \frac{4}{3}\frac{\sqrt{2m_e}}{\hbar}\frac{|E|^{3/2}}{e} \simeq 10^{10}\ V/m \quad ; \qquad (m_e = 10^{-30}\,\mathrm{kg})$$