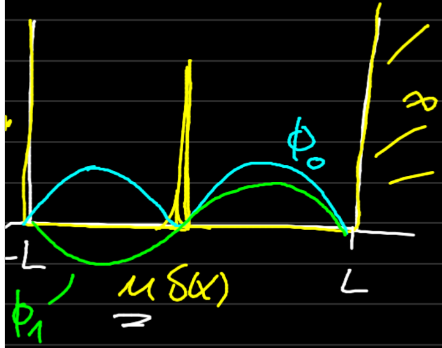
Wdhlg.: Teilchen im "Doppelkasten"



($\simeq$ Doppelmuldenpot. $\cong$ NH_3-Molekül)

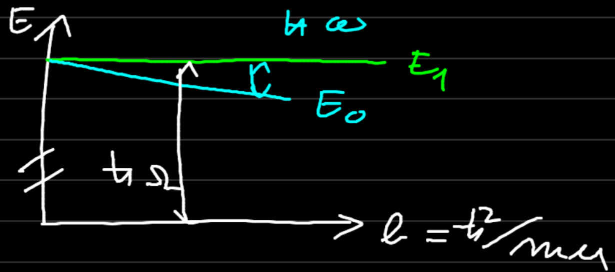
$\rightarrow\ \omega = 24\ \mathrm{GHz}$

(($\hbar\Omega \simeq 0.1\ \mathrm{eV}$ $\rightarrow\ \Omega = 10\ \mathrm{THz}$))

$$\rightarrow\ E_1 = \frac{\hbar^2\pi^2}{2m}\frac{1}{L}\ ,\quad \phi_1(x) = c_1 \sin\left(\frac{\pi}{L}x\right)$$

$$E_0 = E_1 \frac{1}{(1+\ell/L)^2}\ ,\quad \phi_0(x) = c_0 \sin\left(\frac{\pi}{L+\ell}(|x|+\ell)\right)$$

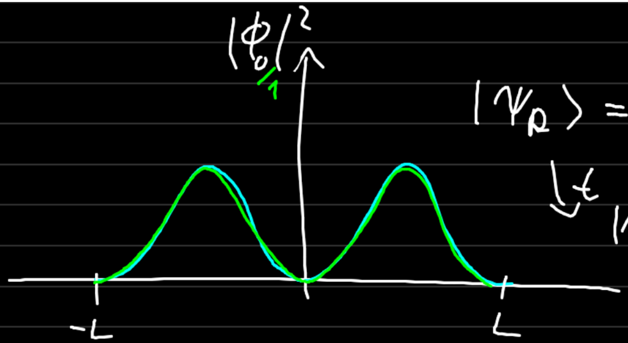
$$= E_1 - 2E_1\,\ell/L$$



$$\hbar\Omega := \frac{E_1+E_0}{2} \approx E_1, E_0$$

$$\hbar\omega := E_1 - E_0$$

$$\boxed{\omega \ll \Omega}$$



$$|\psi_R\rangle = \frac{1}{\sqrt{2}}\left(|\phi_0\rangle + |\phi_1\rangle\right)$$

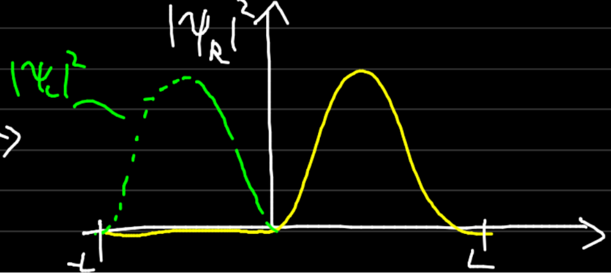
$$\downarrow t$$

$$|\psi(t)\rangle = U_t\,|\psi_R\rangle = \frac{1}{\sqrt{2}}\, e^{-i(\Omega-\frac{\omega}{2})t}\,|\phi_0\rangle + \frac{1}{\sqrt{2}}\, e^{-i(\Omega+\frac{\omega}{2})t}\,|\phi_1\rangle$$

$$|\psi_{R/L}\rangle = \frac{1}{\sqrt{2}}\left(|\phi_0\rangle \pm |\phi_1\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}_B$$

$$|\phi_{0/1}\rangle = \frac{1}{\sqrt{2}}\left(|\psi_R\rangle \pm |\psi_L\rangle\right)$$

$\rightsquigarrow$



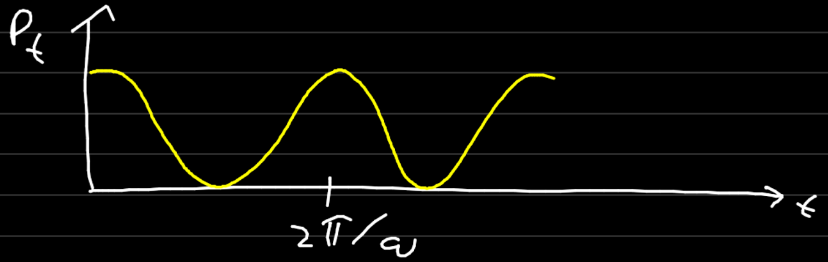
$$B = \left(|\phi_0\rangle, |\phi_1\rangle\right)$$

$$|\psi_0\rangle \equiv |\psi_R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\to\ |\psi(t)\rangle = \frac{e^{-i\Omega t}}{\sqrt{2}}\begin{pmatrix}e^{i\omega t/2}\\ e^{-i\omega t/2}\end{pmatrix}$$

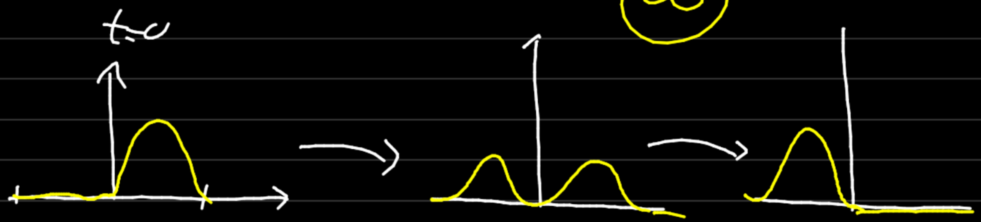
$\to$ Wkt, dass nach Zeit $t>0$ Teilchen im Zust. $|\psi_R\rangle$

$$P_t = \left|\langle\psi_R|\psi(t)\rangle\right|^2 = \left|\frac{1}{2}\left\langle\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}e^{i\omega t/2}\\ e^{-i\omega t/2}\end{pmatrix}\right\rangle\right|^2 = \cos^2\left(\frac{\omega t}{2}\right)$$

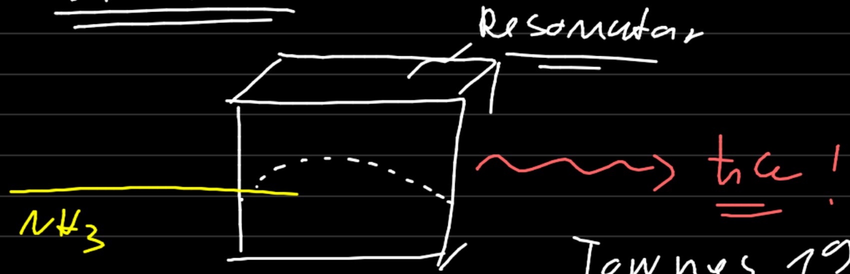


$$|\psi(x,t)|^2 = \begin{cases} \frac{1}{2}|\psi_R(x)|^2\left(1+\cos\left(\frac{\omega t}{2}\right)\right) & : x>0 \\ \frac{1}{2}|\psi_L(x)|^2\left(1-\cos\frac{\omega t}{2}\right) & : x\leq 0 \end{cases}$$

ω

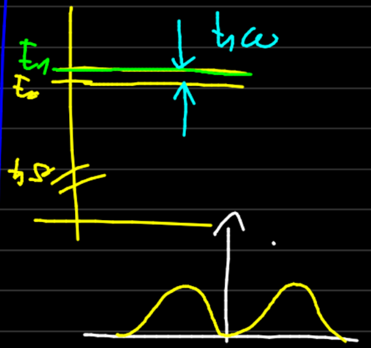


MASER



im Zust. $|\phi_1\rangle$

Townes 1953



Microwave
Aplification by
Stimmulated
Emission of
Radiation

QM im Pfadintegralformalismus

↑ Feynman, Dirac

Vorteile:
- Verbindung zur klass. Mechanik
  - ↳ • Anschauung!
  - • effiziente Analyse „semiklassischer" Phänomene
- • keine Operatoren

Nachteil: "viele" Integrale!

hier: Teilchen im Raum

$$\rightarrow H = \underbrace{\frac{|\hat{\vec{p}}|^2}{2m}} + U(\hat{\vec{x}})$$

$$= T(\hat{\vec{p}}) + U(\hat{\vec{x}})$$

↳ Propagator

$$\boxed{K(\vec{x}, \vec{x}_0, t) := \langle \varphi_{\vec{x}} | \underset{\substack{\parallel \\ e^{-iHt/\hbar}}}{U_t} | \varphi_{\vec{x}_0} \rangle}$$

ab sofort: $|\varphi_{\vec{x}}\rangle =: |\vec{x}\rangle$, $|\varphi_{\vec{x}_0}\rangle = |\vec{x}_0\rangle$

$$\Rightarrow \boxed{K(\vec{x},\vec{x}_0,t) = \langle\vec{x}|U_t|\vec{x}_0\rangle}$$

$$|\psi(0)\rangle \xrightarrow{t} |\psi(t)\rangle$$

$$\downarrow \qquad\qquad \parallel$$

$$\psi_0(\vec{x}_0) = \langle\vec{x}_0|\psi(0)\rangle \qquad \psi(x,t)$$

$$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle = \langle\vec{x}|U_t|\psi(0)\rangle$$

$$= \langle\vec{x}|U_t \, \mathbb{1} \, |\psi(0)\rangle$$

$$\mathbb{1} = \int d\vec{x}_0 \, |\vec{x}_0\rangle\langle\vec{x}_0| = \int dx_0 \, \overbrace{|\varphi_{\vec{x}_0}\rangle\langle\varphi_{\vec{x}_0}|}^{\mathbb{1}}$$

$$= \int d\vec{x}_0 \, \underbrace{\langle\vec{x}|U_t|\vec{x}_0\rangle}_{K(\vec{x},\vec{x}_0,t)} \underbrace{\langle\vec{x}_0|\psi(0)\rangle}_{\psi_0(\vec{x}_0)}$$

$$\boxed{\psi(x,t) = \int d\vec{x}_0 \, K(\vec{x},\vec{x}_0,t) \, \psi_0(\vec{x}_0)}$$

Darstellung des Propagators durch Pfadintegral (1D!)

benötigen: Ortseigenzust. $|x\rangle$;

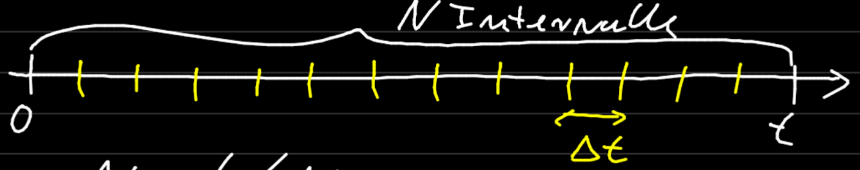
↳ Vollst. $\mathbb{1} = \int dx\, |x\rangle\langle x|$

Impulseigenzust. $|p\rangle$, $(= |\chi_p\rangle)$

Vollst.: $\mathbb{1} = \int \frac{dp}{2\pi\hbar} |p\rangle\langle p|$

$\rightsquigarrow$ $\langle x|p\rangle = \langle \varphi_x|\chi_p\rangle = e^{ipx/\hbar}$.

$$K(x_f, x_i, t) = \langle x_f| U_t |x_i\rangle$$



$\Delta t = t/N$ $\qquad N \to \infty$

$$K(x_f, x_i, t) = \langle x_f| \underbrace{U_{\Delta t} U_{\Delta t} U_{\Delta t} \cdots U_{\Delta t}}_{N} |x_i\rangle$$

$$= \langle x_f| \mathbb{1}\, U_{\Delta t}\, \mathbb{1}\, U_{\Delta t}\, \mathbb{1} \cdots \mathbb{1}\, U_{\Delta t} |x_i\rangle$$

x_N, p_N $\qquad x_n, p_n$ $\qquad x_1, p_1$

mit $\mathbb{1} = \mathbb{1}_x \cdot \mathbb{1}_p = \int dx\, |x\rangle\langle x| \int \frac{dp}{2\pi\hbar} |p\rangle\langle p|$

$$= \int_{\mathbb{R}^2} \frac{dx\,dp}{2\pi\hbar}\, |x\rangle\langle x|p\rangle\langle p|$$

$$= \int \frac{dx\,dp}{2\pi\hbar}\, |x\rangle\langle p|\, e^{ipx/\hbar}$$

folgt:

$$K(x_f,x_i,t) = \underbrace{\int \frac{dx_N\,dp_N}{2\pi\hbar} \cdots\cdots \int \frac{dx_1\,dp_1}{2\pi\hbar}}_{N} \cdot \, (*)$$

$(*)$ $\langle x_f | x_N\rangle \langle p_N | U_{\Delta t} | x_{N-1}\rangle e^{ip_N x_N/\hbar} \langle p_{N-1}| \cdots$

$$\cdots\; \boxed{\langle p_n | U_{\Delta t} | x_{n-1}\rangle e^{ip_n x_n/\hbar}} \;\cdots$$

$$\cdots\; \langle p_1 | U_{\Delta t} | x_i \rangle e^{ip_1 x_1/\hbar}$$

$\to$ im Grenzfall $\Delta t \to 0$ $(N \to \infty)$:

$$= \langle p_n | \underbrace{e^{-\frac{i}{\hbar}(T(\hat p)+U(\hat x))\Delta t}}_{\shortparallel} | x_{n-1}\rangle e^{ip_n x_n/\hbar}$$

$$= \langle p_n | 1 - \frac{i}{\hbar}(T(\hat p)+U(\hat x))\Delta t | x_{n-1}\rangle e^{ip_n x_n/\hbar} + O(\Delta t^2)$$

$$= \langle p_n | \overbrace{1 - \frac{i}{\hbar}\Delta t \left(T(p_n) + U(x_{n-1}) \right)}^{\in \mathbb{C}\,!} | x_{n-1} \rangle \, e^{i p_n x_n/\hbar}$$

$$+ \mathcal{O}(\Delta t^2)$$

$$= \underbrace{\langle p_n | x_{n-1} \rangle}_{e^{-i p_n x_{n-1}/\hbar}} \, e^{-i \frac{\Delta t}{\hbar} \left(T(p_n) + U(x_{n-1}) \right)} \, e^{i p_n x_n/\hbar}$$

$$= \boxed{e^{-i \frac{\Delta t}{\hbar} \left\{ T(p_n) + U(x_{n-1}) - p_n \underbrace{\frac{x_n - x_{n-1}}{\Delta t}}_{=: \dot{x}_n} \right\}}}$$

$$K(x_f, x_i, t) =$$

$$\rightarrow \int \frac{dx_N \, dp_N}{2\pi\hbar} \cdots \int \frac{dx_1 \, dp_1}{2\pi\hbar} \cdot$$

$$\exp\left(- \frac{i}{\hbar} \sum_{n=1}^{N} \Delta t \left(\overbrace{T(p_n) + U(x_{n-1})}^{H} - p_n \dot{x}_n \right) \right)$$

Kontinuumslimes: $N \rightarrow \infty$

$$\sum_{i=1}^{N} \Delta t \, \cdots = \int_0^t d\tilde{t} \, \cdots$$

[redacted]

$$p_n \longrightarrow p(\tilde t)$$

$$x_{n-1} \longrightarrow x(\tilde t)$$

$$\dot x_n \longrightarrow \dot x(\tilde t)$$

$$\int \mathcal{D}[x,p] \ldots := \int \frac{dx_N\, dp_N}{2\pi\hbar} \cdots\cdots \int \frac{dx_1\, dp_1}{2\pi\hbar}$$

$$\rightarrow \boxed{K(x_f, x_i, t) = \int \mathcal{D}[x,p] \cdot e^{-\frac{i}{\hbar}\int_0^t d\tilde t\, \underbrace{H(p(\tilde t), x(\tilde t)) - p(\tilde t)\,\dot x(\tilde t)}_{\stackrel{!}{\approx} -L(x(t),\, \dot x(t))}}}$$

$$\rightarrow \; +\frac{i}{\hbar}\underbrace{\int_0^t L(x,\dot x)\,dt}_{S[x]} = +\frac{i}{\hbar} S[x]$$