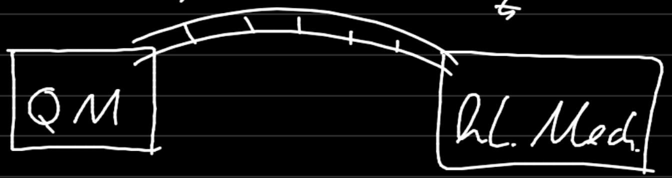
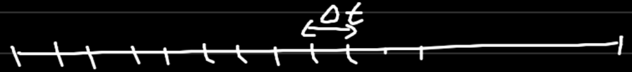
Wdhlg.: **Pfadintegral**

$$\int D[x]\, e^{iS[x]/\hbar}$$



$i\hbar\dot{\psi} = H\psi$

$\delta S[x_{cl}] = 0$



Propagator: $K(x_f, x_i, t) := \langle x_f | U_t | x_i \rangle$

$e^{iHt/\hbar}$; $|x\rangle \equiv |\psi_x\rangle$

Teilchen

$$H = \frac{\hat{p}^2}{2m} + U(\hat{x})$$

Pfadintegraldarst. des Prop.:

$$K(x_f, x_i, t) =$$

$$= \int D[x,p]\, e^{-\frac{i}{\hbar}\int_0^t d\tilde{t}\,(H(p,x) - p\dot{x})}$$

woher? : $K(x_f, x_i, t)$

$\int dx_n |x_n\rangle\langle x_n|$

$$= \langle x_f | \mathbb{1} U_{\Delta t} \cdots\cdots \mathbb{1} U_{\Delta t} \mathbb{1} \cdots | x_i \rangle$$

$$= \int dx_N \cdots \int dx_1 \langle x_f | \cdots\cdots \langle x_n | U_{\Delta t} | x_{n-1} \rangle \cdots | x_i \rangle$$

$$\mathbb{1} = \int \frac{dp_n}{2\pi\hbar} |p_n\rangle\langle p_n|$$

"Kurzzeitpropagator": $\Delta t \to 0$

$$\langle x_n | U_{\Delta t} | x_{n-1} \rangle = \int \frac{dp_n}{2\pi\hbar} \langle x_n | p_n \rangle \langle p_n | e^{-iH\Delta t/\hbar} | x_{n-1} \rangle$$

$$= \int \frac{dp_n}{2\pi\hbar} e^{i x_n \frac{p_n}{\hbar}} e^{-iH(p_n, x_{n-1})\frac{\Delta t}{\hbar}} e^{-i p_n x_{n-1}/\hbar}$$

$$= \int \frac{dp_n}{2\pi\hbar} e^{-\frac{i}{\hbar}\left\{ H(p_n, x_{n-1}) - p_n \underbrace{\frac{(x_n - x_{n-1})}{\Delta t}}_{\dot{x}_n} \right\}\Delta t}$$

Ausführung des Impulsint.

$$H(p, x) = \frac{p^2}{2m} + U(x)$$

$$\underbrace{\int \frac{dp}{2\pi\hbar} \left\{ e^{-\boxed{\frac{i\Delta t}{\hbar 2m}}^{a} p^2 + \boxed{i \dot{x}_n \frac{\Delta t}{\hbar}}^{b} p} \right\}} \quad e^{-\frac{i}{\hbar}\Delta t\, U(x_n)}$$

Gauß-Integral:

$$\int dp\, e^{-ap^2 + bp} = \sqrt{\frac{\pi}{a}}\, e^{b^2/4a}$$

$\rightarrow \langle x_n | U_{\Delta t} | x_{n-1} \rangle$

$$= \left(\frac{2\pi\hbar m}{i\Delta t}\right)^{1/2} \frac{1}{2\pi\hbar}\, e^{i \dot{x}_n^2 \left(\frac{\Delta t}{\hbar}\right)^2 \frac{\cancel{\hbar}\, \cancel{2} m}{\cancel{4}\, 2\, \cancel{\Delta t}}}$$

$$= \sqrt{\frac{m}{2\pi\hbar\Delta t\, i}} \cdot e^{i \frac{m}{2} \dot{x}_n^2 \frac{\Delta t}{\hbar}}$$

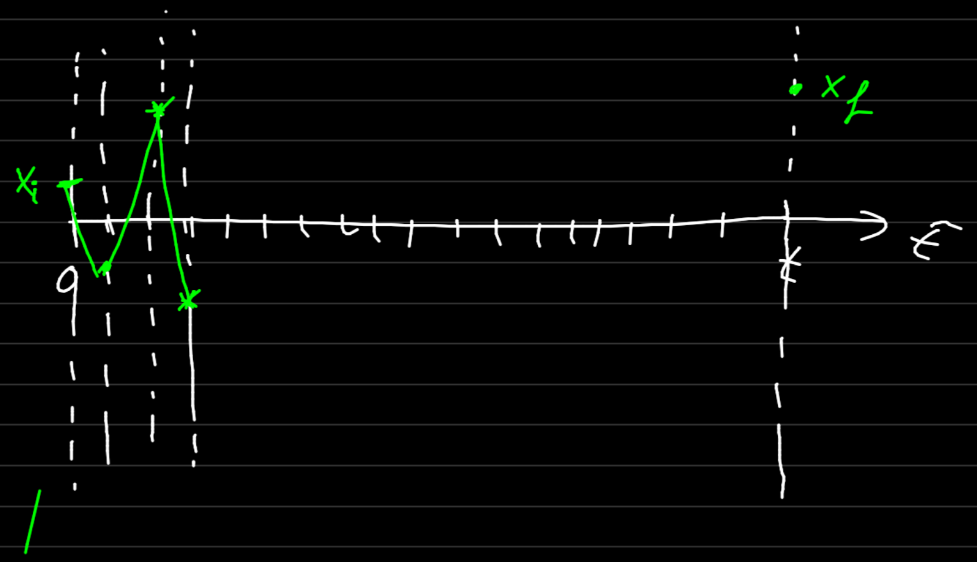
$$\rightarrow K(x_f, x_i, t) = \int D[x]\, e^{\frac{i}{\hbar} \int_0^t d\tilde{t}\, \underbrace{\frac{m}{2}(\dot{x}(\tilde{t}))^2 - U(x(\tilde{t}))}_{L(x(\tilde{t}), \dot{x}(\tilde{t}))}}$$

mit Wirkung $S[x] = \int_0^t d\tilde{t}\, L(x, \dot{x})$

$$\boxed{K(x_f, x_i, t) = \int D[x]\, e^{i S[x]/\hbar}}$$

$$\int D[x] = \left(\frac{mN}{2\pi\hbar i t}\right)^{N/2} \int dx_N \int dx_{N-1} \cdots \int dx_1$$

für $N \rightarrow \infty$!



aufgrund "sehr starker" Oszillation von $e^{iS[x]/\hbar}$

für nicht-stationäre Wege $\delta S[x] \neq 0$

$\longrightarrow$ nur signifikante Beiträge zum P.I. durch stationäre Wege

d.h. für Wege $x : [0,t] \to \mathbb{R}^d$, $t \longmapsto x(t)$

mit:

$$\delta S[x] = 0$$

$$\Leftrightarrow \quad \frac{\partial L(x(t),\dot{x}(t))}{\partial x} = \frac{d}{dt}\frac{\partial L(x(t),\dot{x}(t))}{\partial \dot{x}}$$

$$\left(\frac{\partial L}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \right)$$

d.h. Beitrag zum P.I. im wesent. durch klassische Pfade!

P.I. im semiklassischen Näherung:

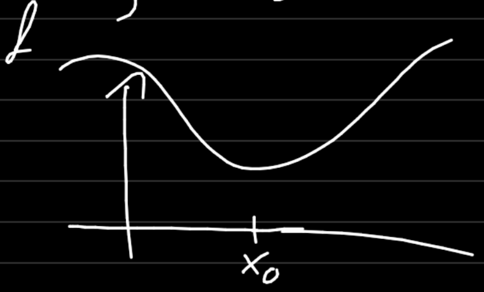
$$\boxed{K(x_f, x_i, t) = \sum_{x_{kl}} \alpha[x_{kl}]\, e^{iS[x_{kl}]\frac{1}{\hbar}}}$$

von $x_i \to x_f$ in Zeit t

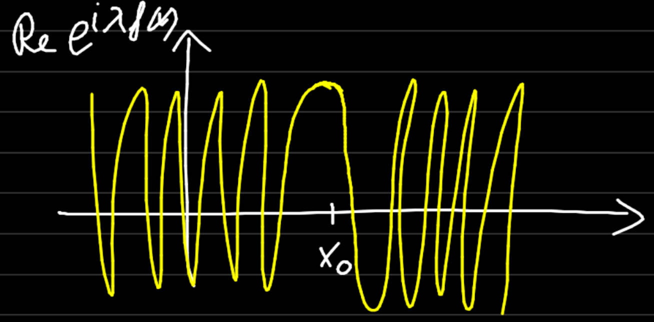
„Näherung der stat. Phase“: $\lambda = \frac{1}{\hbar}$

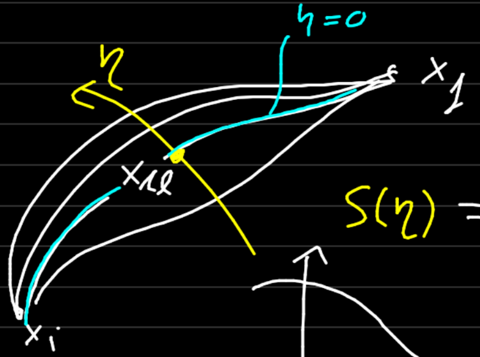
$\lambda \to \infty$!

$$\int dx\, g(x)\, e^{i\lambda f(x)} = g(x_0)\sqrt{\frac{2\pi i}{\lambda f''(x_0)}}\, e^{i f(x_0)} + \mathcal{O}\left(1/\lambda\right)$$



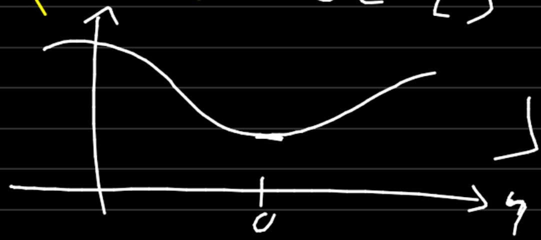
$$\rightarrow f(x) = f(x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2$$



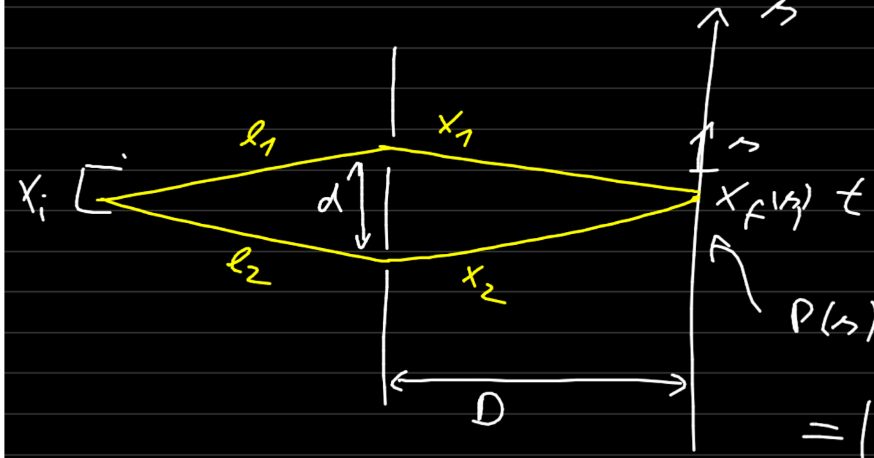


$X_0 = X_{cl}$

$S(\eta) = S[x_\eta]$



Doppelspalt



$$P(s) = |\underbrace{K(x_f(s), x_i, t)}|^2 = |K(s)|^2$$

$$l_2 = l_1 + \delta l$$

$$\boxed{\delta l = \frac{d}{D} \cdot s}$$

$$K(s) = \alpha\left(e^{iS[x_1]/\hbar} + e^{iS[x_2]/\hbar}\right)$$

→ Geschw. betrag: $v_1 = l_1/t$

$v_2 = l_2/t \approx v_1$

$$\Rightarrow S[x_1] = \int_0^t d\tilde{t} \left(\frac{m}{2}|\dot{x}(\tilde{t})|^2 - \overset{=0}{\cancel{V(x(\tilde{t}))}}\right)$$

$$S[x_1] = \frac{m}{2} v_1^2 t = \frac{m}{2}\frac{l_1^2}{t}$$

$$S[x_2] = \frac{m}{2} l_2^2/t$$

$$K(s) = \tilde{\alpha}\left(1 + e^{i\underbrace{(S[x_2] - S[x_1])}_{\Delta S} 1/\hbar}\right)$$

$$\Delta S = \frac{m}{2}\frac{1}{t}\left((l_1 + \delta l)^2 - l_1^2\right) \overset{\delta l \ll l_1 \approx l_2}{=}$$

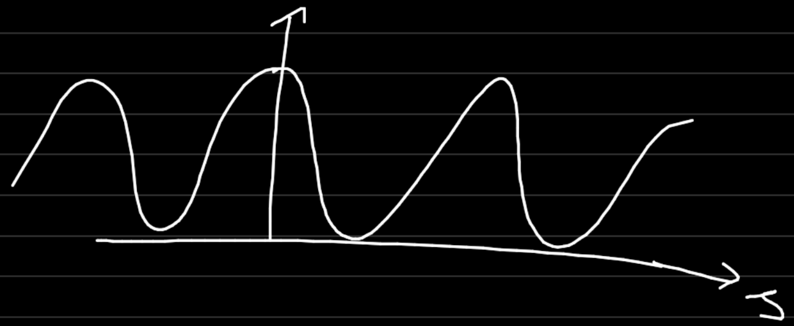
$$= \frac{m l_1}{t}\cdot \delta l = \underbrace{m v_1}_{p!}\cdot \delta l = 2\pi\hbar\,\frac{\delta l}{\lambda}$$

$$p = \hbar k = \frac{\hbar 2\pi}{\lambda}$$

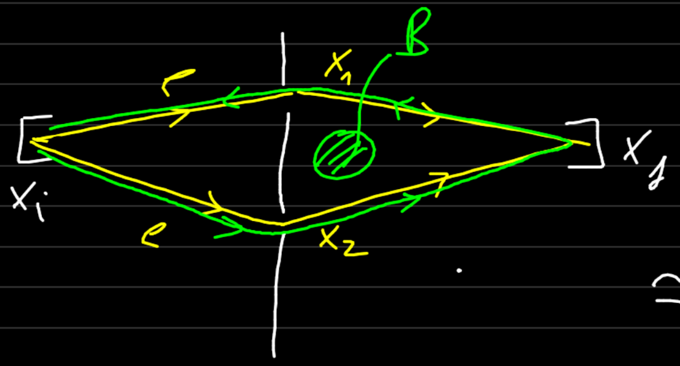
$$\rightarrow K(s) = \tilde{\alpha}\left(1 + e^{i2\pi\delta l/\lambda}\right)$$

$$\rightarrow P(s) = \tilde{\tilde{\alpha}} \cos^2\left(\frac{\pi\,\delta l}{\lambda}\right)$$

$$= \tilde{\alpha} \cos^2\left(\frac{\pi}{\lambda}\frac{d}{D}\cdot s\right)$$



2) Aharonov-Bohm-Effekt



$$P(B) = ?$$

$$\simeq \tilde{\tilde{\lambda}} \cos\left(\vartheta + 2\pi \frac{\phi_B}{\phi_e}\right)$$

$$\phi = \frac{h}{e} \; !$$

$$K(x_i, x_f, t) = \tilde{\lambda}\left(1 + e^{i \Delta S/\hbar}\right)$$

Lagrange-Fkt eines Teilchens im Magnetfeld:

$$L(\vec{x}, \dot{\vec{x}}) = \frac{m}{2}\dot{\vec{x}}^2 + e\,\dot{\vec{x}}\cdot\vec{A}(x)$$

Vektorpotential $\vec{A}(\vec{x})$

$$\to \quad \vec{B} \overset{!}{=} \mathrm{rot}\,\vec{A}$$

$$S[x_1] = \underbrace{S_0}_{\nearrow B=0}[x_1] + e \underbrace{\int_0^t \vec{A}(\vec{x}(\tilde{t}))\cdot\dot{\vec{x}}(\tilde{t})\,d\tilde{t}}_{\equiv \int_{x_1} \vec{A}\,d\vec{l}}$$

$$S[x_2] = S_0[x_2] + e\int_{x_2} \vec{A}\,d\vec{l}$$

$$\rightarrow \Delta S = S[x_2] - S[x_1]$$

$$= \Delta S_0 + e\int_{x_2} \vec{A}\, d\vec{l} - e\int_{x_1} \vec{A}\, d\vec{l}$$

$$= \Delta S_0 + e\oint_{\tau = \partial F} \vec{A}\, d\vec{l} = \Delta S_0 + e\int_F \overbrace{\mathrm{rot}\,\vec{A}}^{\vec{B}}\, d\vec{f}$$

$$\boxed{\Delta S = \Delta S_0 + e\,\Phi(B)} \quad \text{mit } \Phi(B) = \int_F \vec{B}\, d\vec{f}$$

$$K(B) = \alpha\left(1 + e^{i\Delta S_0/\hbar + i\frac{e}{\hbar}\Phi(B)}\right)$$

$$\rightarrow P(S) = \tilde{\alpha}\cos^2\left(\frac{\Delta S_0}{2\hbar} + \frac{e}{2\hbar}\phi(B)\right)$$