WdHlg.: • QM im Heisenbergbild

$$\langle A \rangle_{\psi(t)} = \langle \psi_0 | \underbrace{U^\dagger(t)\, A\, U(t)}_{=: A_H(t)} | \psi_0 \rangle = \langle A_H(t) \rangle_{\psi_0}$$

$$\rightarrow \dot{A}_H(t) = \frac{i}{\hbar} [H, A_H(t)] \,, \quad A_H(0) = A$$

• zeitabhängige Störungstheorie

$$H = H_0 + V(t)$$

$|n\rangle, E_n$: „einfach" „klein"

im Wechselwirkungsbild

• $|\psi(t)\rangle_I := U_0^\dagger(t) |\psi(t)\rangle = e^{+\frac{i}{\hbar} H_0 t} |\psi(t)\rangle$

• $A_I(t) = U_0^\dagger(t)\, A\, U_0(t)$

$$\rightarrow |\dot{\psi}(t)\rangle_I = -\frac{i}{\hbar} V_I(t) |\psi(t)\rangle_I$$

Iteration:

$\rightarrow$

$$|\psi(t)\rangle_I = \Big\{ \mathbb{1} - \frac{i}{\hbar}\int_0^t dt'\, V_I(t') - \frac{1}{\hbar^2}\int_0^t dt' \int_0^{t'} dt''\, V_I(t')\, V_I(t'') + \frac{i}{\hbar^3}\int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt'''\, V_I \cdots V_I \cdots V_I \cdots + \ldots + \ldots \Big\} |\psi(0)\rangle$$

$\to$ Übergangswkt.: $|n\rangle \xrightarrow{V(t)} |m\rangle$

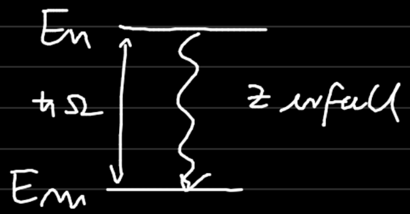
$$P_{mn}^{(1)}(t) = \frac{1}{\hbar^2}\left| \int_0^t dt'\, e^{\frac{i(E_m - E_n)t'}{\hbar}} \langle m|V(t)|n\rangle \right|^2$$

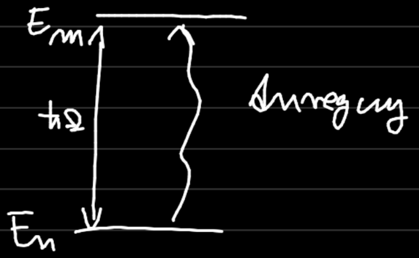
Übergangsrate: $\Gamma_{mn} = \lim\limits_{t\to\infty} \frac{P_{mn}(t)}{t}$

für $V(t) = \hat{u}\cos(\Omega t)$: $\Omega > 0$

$$\Gamma_{mn}^{(1)} = \frac{\pi}{\hbar}\frac{|u_{mn}|^2}{2}\Big\{ \delta(E_m - E_n + \hbar\Omega) + \delta(E_m - E_n - \hbar\Omega) \Big\}$$

$E_n > E_m$ | $E_n < E_m$





Γ genauer: quantenmech. el. mg. Feld —

Moden: $\vec{k}$, $\omega_k = c|\vec{k}|$

Harmonischer Oszillator: $H_{\vec{k}} = \hbar\omega_{\vec{k}}\left(a^+_{\vec{k}} a_{\vec{k}} + \frac{1}{2}\right)$

$a^+_{\vec{k}} / a_{\vec{k}}$ | Erzeugen / Vernichten
| Photon —

$\rightarrow$ u.w. $\sim i\hat{X}\cdot\underbrace{(a^+_{\vec{k}} - a_{\vec{k}})}$

$|n\rangle \longrightarrow |m\rangle|\ldots(N_{\vec{k}}-1)\ldots\rangle$

$|\ldots\ldots N_{\vec{k}} \ldots\rangle$

Γ genauer: AQM, QFT;

klass. E.D. $\rightarrow$ kl. Feldtheorie

$\Omega = 0$: $V = \hat{u}$

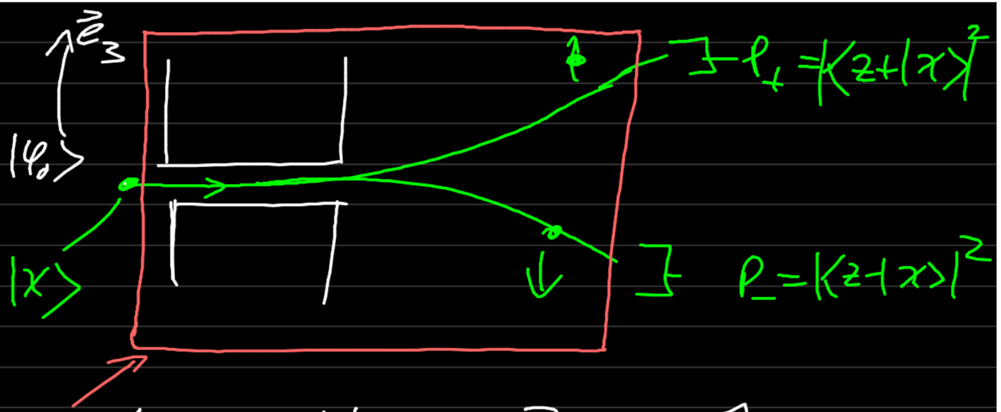
$$\Gamma^{(1)}_{mn} = \frac{\pi}{\hbar}|u_{mn}|^2 \cdot \delta(E_m - E_n)$$

Quantenmechanik der Messung

QM universell gültig ?!

→ quantenmechanische Beschreibung der Messung !?

z.B.: Spin-Messung im Stern-Gerlach-Experiment



Spin-Messgerät = Zeiger ≙ Bahn des Atoms !

→ zeigt: Spin-Zustand !

Ortszustand des Atoms

$$|\varphi_0\rangle \longrightarrow \text{Wellenfkt}$$

$$\varphi_0(\vec{x}) = \langle \vec{x} | \varphi_0 \rangle \, ;$$

Spin-Zustand des Atoms:

$$|\chi\rangle_S = \chi_+ |z+\rangle + \chi_- |z-\rangle \equiv \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}$$

→ gemeinsamer Gesamtzustand:

$$|\varphi_0\rangle \otimes |\chi\rangle_S = |\varphi_0\rangle \otimes \left(\chi_+ |z+\rangle + \chi_- |z-\rangle \right)$$

→ Spinor-Darstellung.

$$\Psi(\vec{x}) = \varphi_0(\vec{x}) \cdot \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix}$$

$$= \begin{pmatrix} \varphi_0(\vec{x})\, \chi_+ \\ \varphi_0(\vec{x})\, \chi_- \end{pmatrix}$$

↳ allgemein:

$$\Psi(\vec{x}) = \begin{pmatrix} \psi_+(\vec{x}) \\ \psi_-(\vec{x}) \end{pmatrix}$$

$\hat{=}$ $\quad |\psi\rangle_\sigma = |\psi_+\rangle \otimes |z+\rangle + |\psi_-\rangle \otimes |z-\rangle$

$|\varphi_0\rangle \otimes |x\rangle = |\psi_0\rangle$

<u>Normierung</u>: $\quad \|\psi_+\|^2 + \|\psi_-\|^2 = 1$ ✓

<u>Dynamik</u>:

<u>Orts</u>: $\quad H_\sigma = |\vec{p}|^2 / 2m \quad ,$

<u>Spin</u>: $\quad H_S = -\mu_0 \, \vec{B}(\hat{\vec{x}}) \cdot \hat{\vec{S}} \quad , \quad \hat{\vec{S}} = \frac{\hbar}{2} \vec{\sigma}$

$\mu_0 = \frac{e}{m_e c} \; ;$

$\vec{B}(0,0,x_3) = B(x_3)\,\vec{e}_3 = \left(B_0 + \frac{\partial B}{\partial x_3} \cdot x_3\right) \vec{e}_3$

$$\to H_S = -\mu_0 B_0 \hat{S}_3 - \mu_0 \frac{\partial B}{\partial x_3} \cdot \hat{x} \hat{S}_3$$

$$\to H_{0S} = \underbrace{\frac{|\vec{p}|^2}{2m} - \mu_0 B_0 \hat{S}_3}_{H_0} \underbrace{\overbrace{- \mu_0 \frac{\partial B}{\partial x_3}}^{\alpha} \hat{x}_3 \hat{S}_3}_{V}$$

Vereinfachung: $\underline{\underline{B_0 = 0}}$, $\alpha = -\mu_0 \frac{\partial B}{\partial x_3}$

$$H = \underbrace{\frac{|\vec{p}|^2}{2m}}_{H_0} + \underbrace{\alpha \hat{x}_3 \hat{S}_3}_{V}$$

Anfangszustand:

$$|\Psi_0\rangle = |\varphi_0\rangle |\chi\rangle$$

$$\langle \vec{X} \rangle_{\varphi_0} = 0 \; ;$$

$$\langle P_1 \rangle_{\varphi_0} = m v_0 \; , \quad \langle P_2 \rangle_{\varphi_0} = 0 = \langle P_3 \rangle_{\varphi_0}$$

$$[H, P_1] = 0, \quad [H, P_2] = 0, \quad [H, \hat{S}_3] = 0$$

d.h. P_1, P_2, S_3 Erhaltungsgrößen,

$$|\dot{\psi}_I(t)\rangle = -\frac{i}{\hbar} V_I(t) \, |\psi_I(t)\rangle$$

mit $V_I(t) = e^{iH_0 t/\hbar} \, \alpha X_3 S_3 \, e^{-iH_0 t/\hbar}$

$$H_0 = \frac{1}{2m}\left(\underline{\underline{P_1^2}} + \underline{\underline{P_2^2}} + P_3^2\right) - \mu_0 B S_3$$

Γ $\hat{X}_3 \equiv \hat{X}_3 \otimes \mathbb{1}_S$, $\hat{S}_3 = \mathbb{1}_O \otimes \hat{S}_3$ $\quad \begin{matrix} \hat{X}\hat{S}_3 \\ \hat{S}\hat{X} \end{matrix} = \begin{matrix} (X_3 \otimes \mathbb{1}_S)(\mathbb{1}_O \otimes \hat{S}) \\ (\mathbb{1}_O \otimes S)(X_3 \otimes \mathbb{1}_S) \end{matrix}$ $(X_3 \cdot \mathbb{1}_O) \otimes (\mathbb{1}_S \hat{S}) = (\mathbb{1}_O \cdot X_3) \otimes (S \, \mathbb{1})$

$$V_I(t) = \underbrace{\alpha \, e^{iP_3^2 t/2m\hbar} \, X_3 \, e^{-iP_3^2 t/2m\hbar}}_{} \cdot S_3$$

$$\left\{ \begin{aligned} e^{A} B e^{-A} &= e^{[A, \ldots]} B \\ &= B + [A, B] + \frac{1}{2!}[A, [A, B]] \\ &\quad + \ldots \end{aligned} \right.$$

$$V_I(t) = \alpha S_3 \left(X_3 + \frac{i t}{2m\hbar} [P_3^2, X_3] + \ldots \right)$$

$$= \alpha S_3 \left(X_3 + \frac{i t}{2m\hbar} \big(2 P_3 \underbrace{[P_3, X_3]}_{= -i\hbar}\big) + 0 \right)$$

$$V_I(t) = \alpha S_3\left(\hat{X}_3 + \frac{\hat{P}_3}{m}t\right) \checkmark$$

$$|\dot{\psi}_I(t)\rangle = -\frac{i}{\hbar}\alpha S_3\left(\hat{X}_3 + \frac{\hat{P}_3}{m}t\right)|\psi_I(t)\rangle$$

$$|\psi_I(t)\rangle \overset{!}{=} e^{i(\cdots)}\, e^{-\frac{i}{\hbar}\alpha S_3\left(\hat{X}_3 t + \frac{\hat{P}_3}{2m}t^2\right)}|\psi_I(0)\rangle$$

$-[P_3, X_3] = i\hbar$

$$= e^{i(\cdots)}\, e^{-\frac{i}{\hbar}\alpha S_3 \boxed{\hat{X}_3} t}\, e^{-\frac{i}{\hbar}\alpha S_3 \boxed{\hat{P}_3}\frac{t^2}{2m}}|\psi_I(0)\rangle$$

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$$

- $e^{-ia\hat{P}_3/\hbar} \overset{!}{=} T_3(a) \checkmark$
- $e^{+iq\hat{X}_3/\hbar} = \tilde{T}_3(q) \checkmark$

$a \otimes b \neq v \in \mathcal{H}_1 \otimes \mathcal{H}_2$

$A \otimes B$

$$\rightarrow |\psi_I(t)\rangle = \tilde{T}_3(-\alpha\hat{S}_3 t)\,\hat{T}_3\left(\frac{\alpha S_3 t^2}{2m}\right)|\psi_0\rangle$$

$T(a\hat{S}_3)|x\rangle \otimes T(a\hat{S}_3)$

$T(a\hat{S}_3)\,|x\rangle \otimes |z\pm\rangle$

$= T\left(\pm\frac{a\hbar}{2}\right)|x\rangle|z\pm\rangle = |x \pm \frac{a\hbar}{2}\rangle|z\pm\rangle$

$$|\varphi_+(t)\rangle \otimes |z+\rangle + |\varphi_-(t)\rangle |z-\rangle \triangleq \Psi(t) = \begin{pmatrix}\varphi_+(t)\\ \varphi_-(t)\end{pmatrix}$$

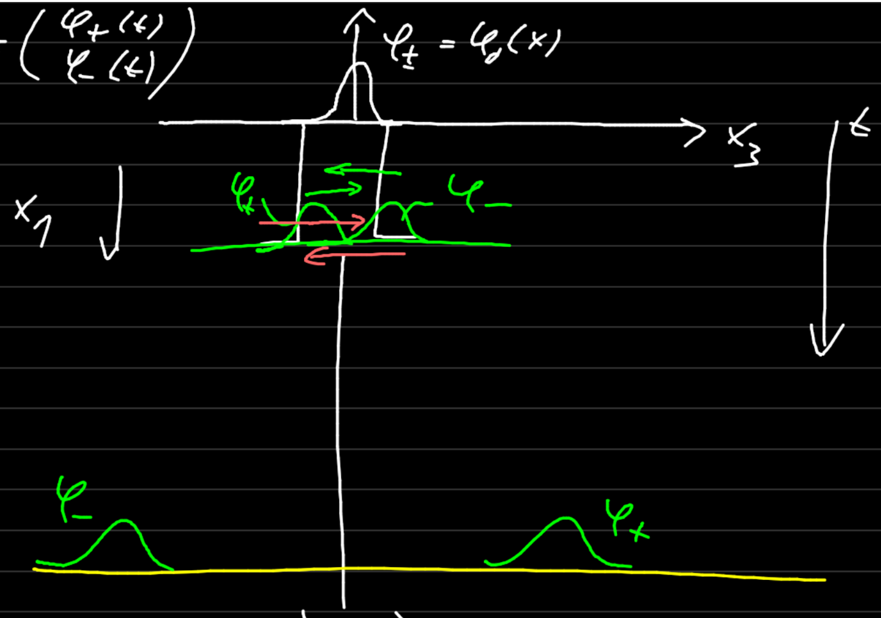
$$|\psi(t)\rangle = e^{-iH_0 t/2m} |\psi_I(t)\rangle$$

$$= e^{-iH_0 t/\hbar}\, \tilde{T}_3(\alpha S_3 t)\, T_3\left(\frac{\alpha S_3 t^2}{2m}\right) |\psi_0\rangle$$

$$t < \tau = L/v_0$$

$t > \tau$:

$$|\psi(t)\rangle = e^{-iH_0 t/\hbar}\, \hat{T}_3(\alpha S_3 \tau)\, T_3\left(\frac{\alpha S_3 \tau^2}{2m}\right) |\psi_0\rangle$$



$$\vec{B}\cdot\vec{S}$$