Wdhlg.: Obs. $A = \sum_\ell a_\ell |\chi_\ell\rangle\langle\chi_\ell|$

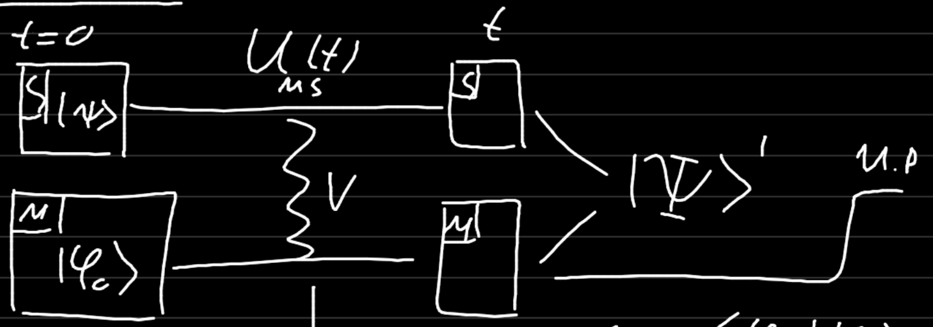
Messung von A an Syst. im Zust.

$$|\psi\rangle = \sum_\ell c_\ell |\chi_\ell\rangle$$

∘ direkt:

$$\boxed{S \quad |\psi\rangle} \xrightarrow{\text{M.P.}} P_\ell^{(0)} = |\langle\chi_\ell|\psi\rangle|^2 = |c_\ell|^2$$

∘ indirekt:



$$\delta_{\ell\ell'} = \langle\varphi_\ell|\varphi_{\ell'}\rangle$$

$$|\varphi_0\rangle|\chi_\ell\rangle \xrightarrow{U_{MS}(t)} |\varphi_\ell\rangle|\chi'_\ell\rangle \quad !$$

(von Neumann

Linearität der QM $\Longrightarrow$

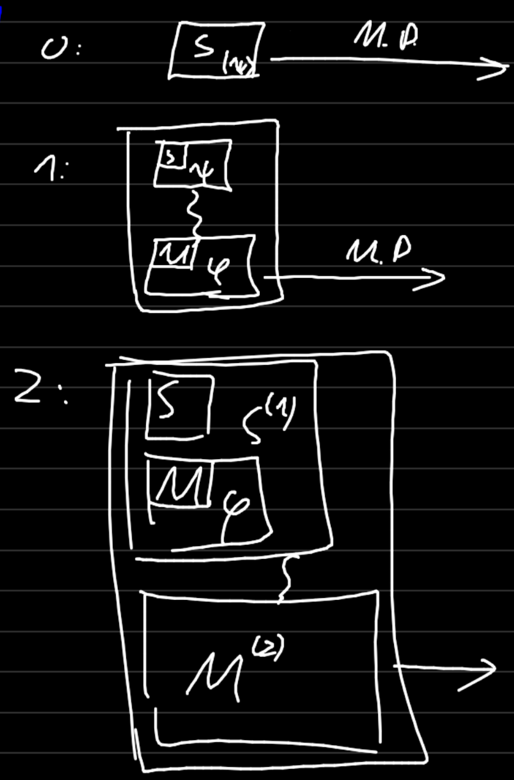
$$|\varphi_0\rangle|\psi\rangle \longrightarrow \sum_\ell c_\ell |\varphi_\ell\rangle|\chi_\ell\rangle = |\Psi'\rangle$$

M. P.

→ Messung: $|\varphi_\ell\rangle\langle\varphi_\ell| \otimes \mathbb{1}_S$ $\overset{!}{=}$

$$P_\ell^{(1)} = \left\langle |\varphi_\ell\rangle\langle\varphi_\ell| \otimes \mathbb{1}_S \right\rangle_{\Psi'} = |c_\ell|^2 = P_\ell^{(0)}$$

→ Iteration der Messungen:



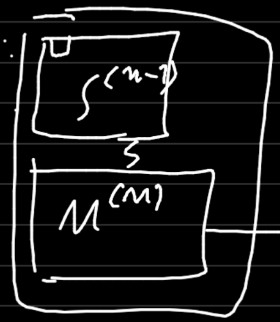
0: $\xrightarrow{M.P.}$ $P_\ell^{(0)}$

1: $\xrightarrow{M.P.}$ $P_\ell^{(1)} \overset{!}{=} P_\ell^{(0)}$

2: $\longrightarrow$ $P_\ell^{(2)} = P_\ell^{(1)}$

...

n:

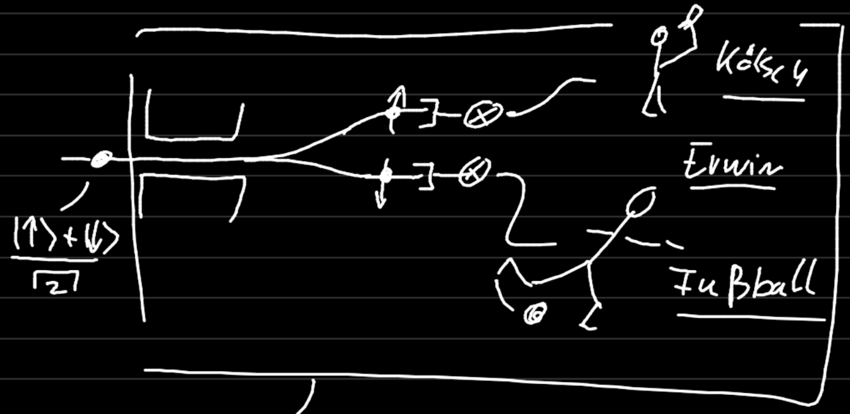


$$P_\ell^{(n)} \overset{!}{=} P_\ell^{(n-1)}$$

→ Messwerten sind unabhängig von Ordnung der Messung!

- <u>Gedankenexperiment</u>:

Schrödingers → "<u>Biertrinken</u>"



$M^{(10)}$

<u>QM</u>: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|K\rangle + |\downarrow\rangle|F\rangle\right)$

seit 1935!

- Bellschen Ungleichungen (1994)

II + Experimente (seit ~1970
~1974
⋮
~2015)

zeigen

1) Widerlegung der Bell. Ugln.

2) Übereinstimmung mit QM!

Zeh, Joos, C. Kiefer,
Giulini, Stamatescu.

Dekohärenz (H.D. Zeh. ~1968)

I

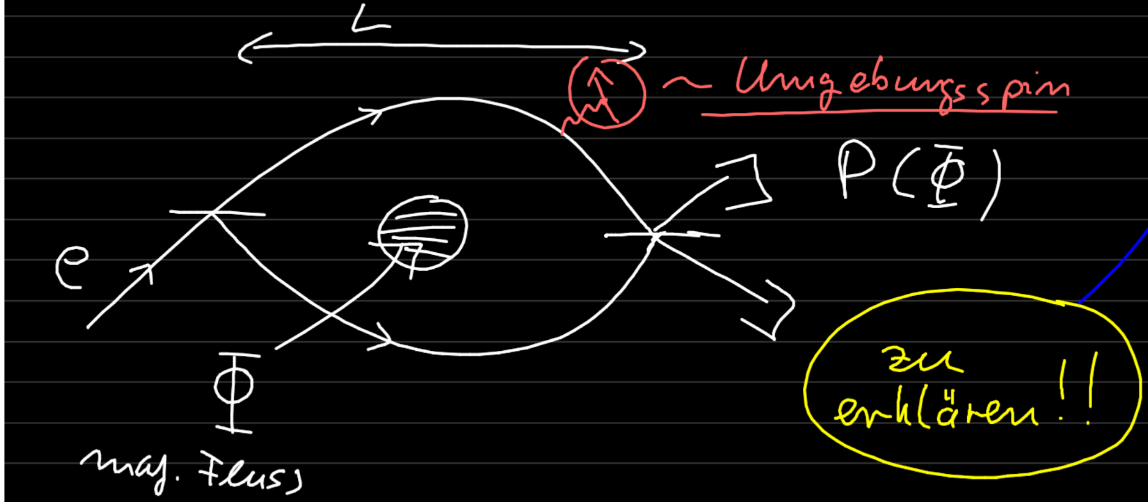
~1996)

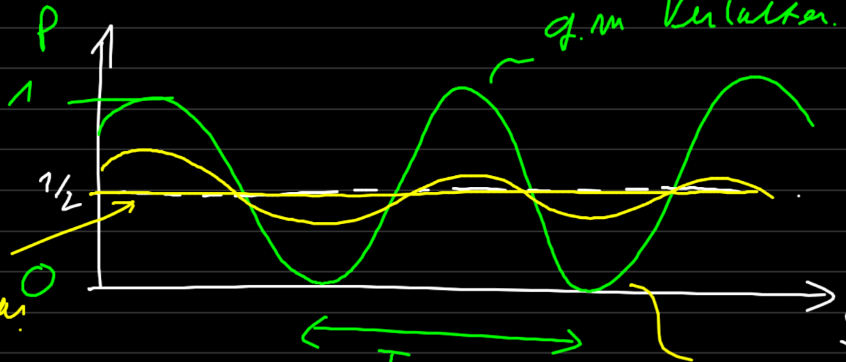
klassisches Verhalten
in (großen!) Quantensystemen
erklärbar durch quantenmechanische
Verschränkung!

„Decoherence or the Appearance of a
classig world in QM"

Dekohärenz

1) <u>Bsp</u>: Aharonov-Bohm-Interferometer



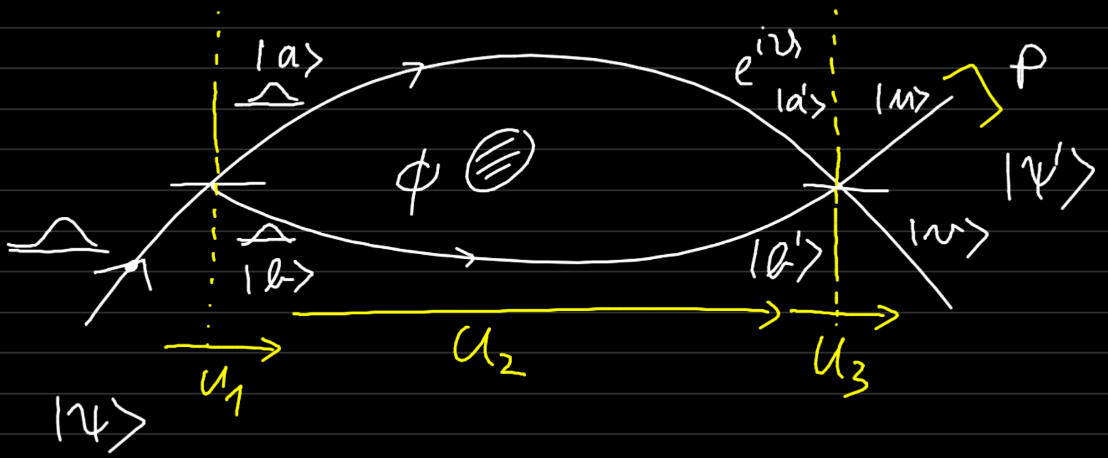


$\Phi_0 = h/e$!

$$P = \cos^2\left(\frac{\pi \Phi}{\Phi_0}\right)$$

Dynamik

I: ohne Umgebungsspin:



$$U_1 : \quad |\psi\rangle \longmapsto \frac{1}{\sqrt{2}}\left(|a\rangle + |b\rangle\right)$$

$$U_2 : \quad |a\rangle \longmapsto e^{i\vartheta}\,|a'\rangle \qquad : \; \vartheta = \frac{2\pi\Phi}{\phi_0}$$

$$|b\rangle \longmapsto |b'\rangle$$

$$U_3 : \quad |a'\rangle \longmapsto \frac{1}{\sqrt{2}}\left(|u\rangle + |v\rangle\right) \qquad \perp!$$

$$!\perp \qquad |b'\rangle \longmapsto \frac{1}{\sqrt{2}}\left(|u\rangle - |v\rangle\right)$$

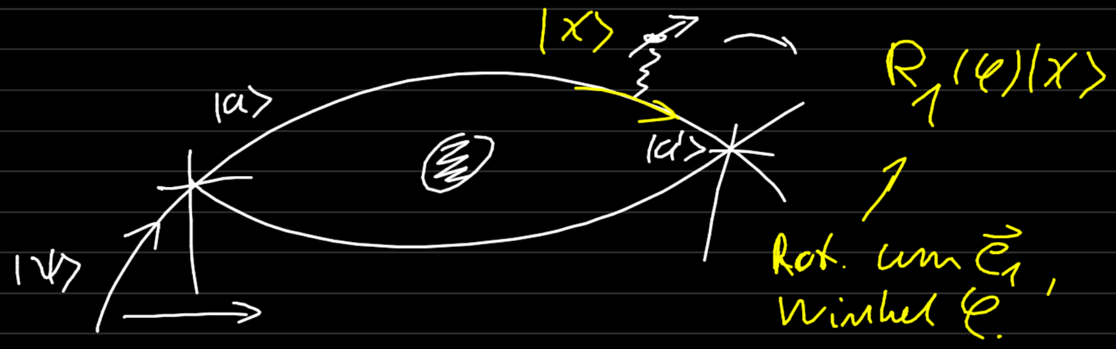
$$|\psi\rangle \longrightarrow |\psi'\rangle = U_3 U_2 U_1 |\psi\rangle$$

$$= U_3 \frac{1}{\sqrt{2}}\left(e^{i\vartheta}|a'\rangle + |b'\rangle\right)$$

$$= \underline{\underline{\frac{1}{2}\left(e^{i\vartheta}+1\right)|u\rangle}} + \frac{1}{2}\left(e^{i\vartheta}-1\right)|v\rangle$$

$$\rightsquigarrow P = \left|\langle u|\psi'\rangle\right|^2 = \cos^2(\vartheta/2) \; !$$

II mit WW mit Umgebungsspin.



$\rightarrow U_1: \ |\psi\rangle|\chi\rangle \longrightarrow \frac{1}{\sqrt{2}}(|a\rangle+|b\rangle)|\chi\rangle$

$U_2: \ |a\rangle|\chi\rangle \longmapsto e^{i\vartheta}|a'\rangle R_1(\varphi)|\chi\rangle$

$= (e^{i\vartheta}\otimes R_1(\varphi))|a'\rangle|\chi\rangle$

$|b\rangle|\chi\rangle \longmapsto |a\rangle|\chi\rangle$

$U_3: \ |a'\rangle|\chi\rangle \longrightarrow \frac{1}{\sqrt{2}}(|u\rangle+|v\rangle)|\chi\rangle$

$|b'\rangle|\chi\rangle \longrightarrow \frac{1}{\sqrt{2}}(|u\rangle-|v\rangle)|\chi\rangle$

$|\Psi\rangle \longrightarrow |\Psi'\rangle = U_3 U_2 U_1 |\psi\rangle|\chi\rangle$

$= \underline{\frac{1}{2}(e^{i\vartheta}\otimes R_1(\varphi)+1)|u\rangle|\chi\rangle}$

$+ \frac{1}{2}(e^{i\vartheta}\otimes R_1(\varphi)-1)|v\rangle|\chi\rangle$

Messung: $\underline{|u\rangle\langle u| \otimes \mathbb{1}_S}$

$$\rightarrow P = \left\langle |u\rangle\langle u| \otimes \mathbb{1}_S \right\rangle_{\Psi}$$

$$= \frac{1}{4} \langle u|\langle x| \left(e^{i\vartheta} \otimes R_1(\varphi) + 1 \right)^{+} |u\rangle\langle u| \otimes \mathbb{1}_S \left(e^{i\vartheta} \otimes R_1(\varphi) + 1 \right) |u\rangle|x\rangle$$

$$\langle u|\langle x|\, 1 \left(|u\rangle\langle u| \otimes \mathbb{1}_S \right) e^{i\vartheta} \otimes R_1(\varphi) \big) |u\rangle|x\rangle$$

$$= \frac{1}{4}\Big(1 + 1 + e^{-i\vartheta} \langle x| R_1^{+}(\varphi)|x\rangle + e^{+i\vartheta} \langle x| R_1(\varphi)|x\rangle \Big)$$

$$\boxed{P = \frac{1}{2} + \frac{1}{2} \mathrm{Re}\left(e^{i\vartheta} \langle x| R_1(\varphi)|x\rangle \right)}$$

z.B.:

(i) $\varphi = 0$, $|x\rangle$ beliebig.

$\Rightarrow R_1(0) = \mathbb{1} \rightarrow P = \frac{1}{2}(1 + \cos\vartheta) = \cos^2(\vartheta/2)$ ✓

$$\boxed{\text{(iii)} \quad \varphi > 0 \;, \quad |\chi\rangle = |\uparrow\rangle \equiv |z+\rangle}$$

$$R_1(\varphi) = e^{-i S_1 \cdot \varphi/\hbar} \underset{\substack{\uparrow \\ S_1 = \frac{\hbar}{2}\sigma_1}}{=} e^{-i\sigma_1 \cdot \varphi/2}$$

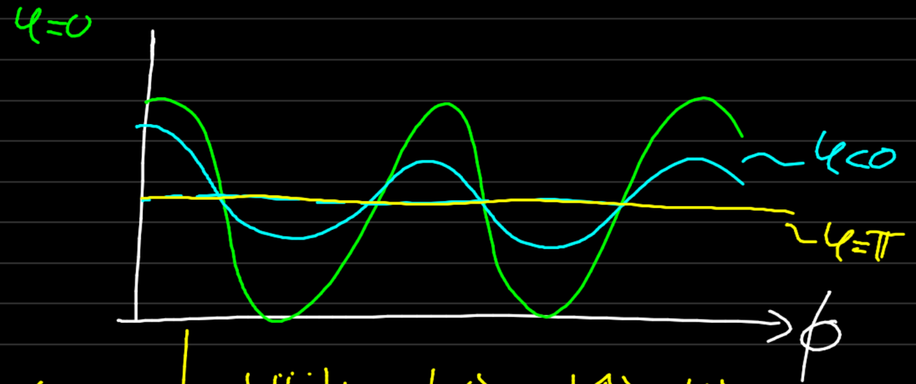
$$= \cos(\varphi/2)\,\hat{1} - i \sin(\varphi/2)\,\hat{\sigma}_1$$

$$\rightarrow \langle \chi | R_1(\varphi) | \chi \rangle = \langle \uparrow | \ldots | \uparrow \rangle$$

$$= \cos(\varphi/2)$$

(iv): $|\chi\rangle = |\uparrow\rangle$ $\longrightarrow$ x+ Messg!

$$\rightsquigarrow \quad P = \frac{1}{2} + \frac{1}{2}\cos(\varphi/2)\cdot\cos(\vartheta)$$



$\varphi = \pi$

(iii): $|\chi\rangle = \dfrac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$

$\Rightarrow P = ?$

[redacted]