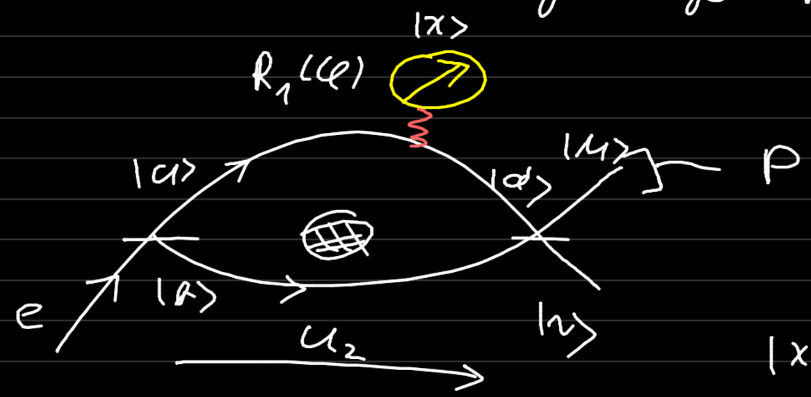
Dekohärenz im A.-B.-Interferometer

durch Umgebungs-Spin



$|\chi'\rangle = R_1(\varphi)|\chi\rangle$

$U_2 : |a\rangle|\chi\rangle \mapsto e^{i\vartheta}|a'\rangle|\chi'\rangle$

$\quad\; |b\rangle|\chi\rangle \longmapsto |b'\rangle|\chi\rangle$

$\vartheta = 2\pi\phi/\phi_0$

$\rightarrow U_2 : \underbrace{(|a\rangle + |b\rangle)|\chi\rangle}_{\text{separablen Zust.}} \longrightarrow \underbrace{e^{i\vartheta}|a'\rangle|\chi'\rangle + |b'\rangle|\chi\rangle}_{\text{verschränkten Zust.}}$

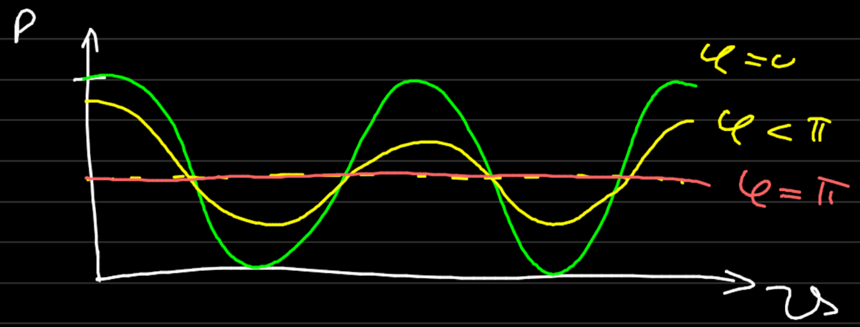
$$\rightarrow \boxed{P = \frac{1}{2} + \frac{1}{2}\,\mathrm{Re}\, e^{i\vartheta}\underbrace{\langle\chi|\chi'\rangle}_{= \langle\chi|R_1(\varphi)|\chi\rangle}}$$

$R_1(\varphi) = \cos\frac{\varphi}{2}\,\mathbb{1} - i\sin\frac{\varphi}{2}\,\sigma_1$

① : $|x\rangle = |\uparrow\rangle \,, \quad \varphi$

$$\rightarrow \quad P = \frac{1}{2} + \frac{1}{2}\,\mathrm{Re}\, e^{i\vartheta} \cdot \cos\varphi/2$$

$$= \frac{1}{2} + \frac{1}{2}\cos\vartheta \cdot \cos\varphi/2$$

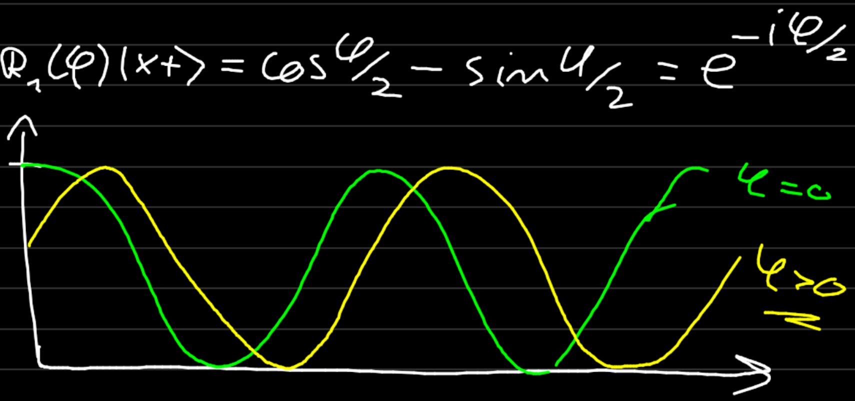


② $|x\rangle = \dfrac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} = |x+\rangle \,, \quad \varphi = \pi$

$$P = \frac{1}{2} + \frac{1}{2}\,\mathrm{Re}\, e^{i\vartheta} \underbrace{e^{-i\varphi/2}}_{= \langle x' | x' \rangle} = \frac{1}{2} + \frac{1}{2}\cos\left(\vartheta - \frac{\varphi}{2}\right)$$

NR:

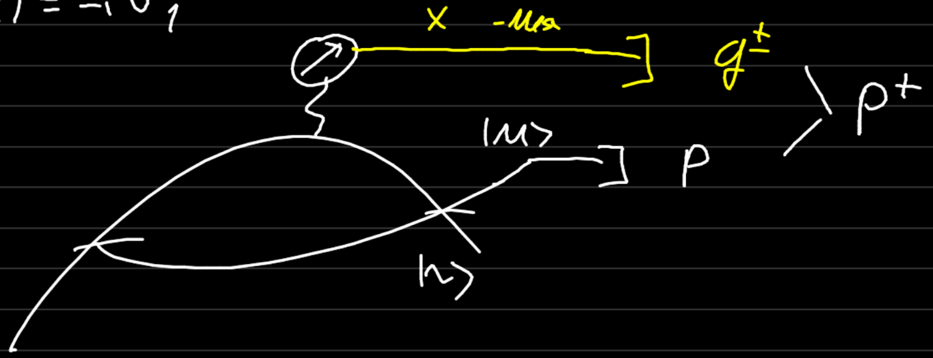
$$\langle x+ | R_1(\varphi) | x+ \rangle = \cos\varphi/2 - \sin\varphi/2 = e^{-i\varphi/2}$$



③ $|x\rangle = |\uparrow\rangle_1$ + $|x\pm\rangle$ Messung des Uhr-Spins!

$\varphi = \pi$

$R_1(\varphi) = -i\sigma_1$



$$p^+ = |\langle u|\langle x+|\psi'\rangle|^2$$

$$|\psi'\rangle = \frac{1}{2}\left(e^{i\vartheta}\otimes(-i\sigma_1) + 1\right)|u\rangle|\uparrow\rangle + \frac{1}{2}(\dots\dots)|v\rangle|\uparrow\rangle$$

$$\to p^+ = \frac{1}{4}\left|\frac{-ie^{i\vartheta}+1}{\sqrt{2}}\right|^2$$

$$= \frac{1}{8}\Big(1+1+2\,\mathrm{Re}\underbrace{(-ie^{i\vartheta})}_{\sin\vartheta}\Big)$$

$$= \frac{1}{4} + \frac{1}{4}\sin\vartheta$$

$$p^- = |\langle u|\langle x-|\psi'\rangle|^2 = \frac{1}{4} - \frac{1}{4}\sin\vartheta$$

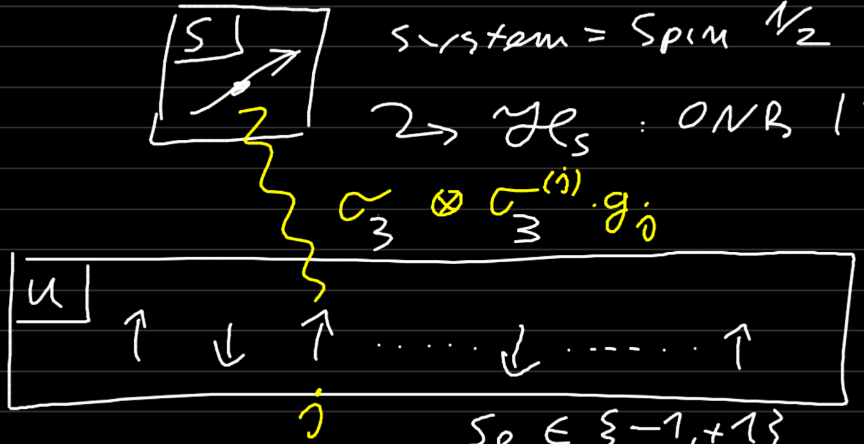
$$p = p^+ + p^- = \frac{1}{2} \;!$$

Fazit: Superposition nicht "zerstört", sondern in System-Umgebungs-Verschränkung "versteckt"!

Spin-Modell (Zurek ~1985 ...)

$$|\vec{s}\rangle = |+1,-1,-1,\dots,1\rangle$$

$$= |\uparrow\rangle\otimes|\downarrow\rangle\otimes|\downarrow\rangle \cdots \otimes|\uparrow\rangle$$



System = Spin $1/2$

$\leadsto \mathcal{H}_S$: ONB $|\uparrow\rangle, |\downarrow\rangle$

$\sigma_3 \otimes \sigma_3^{(j)} \cdot g_j$

n Spin $1/2$

$s_\ell \in \{-1,+1\}$

$$|s_\ell\rangle = \begin{cases} |\uparrow\rangle & : s_\ell = 1 \\ |\downarrow\rangle & : s_\ell = -1 \end{cases}$$

$\mathcal{H}_U$: ONB $\{|\vec{s}\rangle\}$ $\vec{s} \in \{-1,+1\}^n$

$\dim \mathcal{H}_U = 2^n$

Dynamik: $H_S = 0 \; ; \; H_U = 0$

$\rightarrow$ W.W. $$V = \sum_{j=1}^{n} g_j \, \sigma_z \otimes \sigma_z^{(j)}$$

$$= \sigma_z \otimes \underbrace{\sum_{j=1}^{n} g_j \, \sigma_z^{(j)}}_{=: A} \; ;$$

Kopplungskonst.en

$g_j \in [0, g]$

$\leadsto$ gleichverteilt, zufällig gewählt.

Was wollen wir zeigen?

System-S.-P.:

$$|x+\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \quad \text{(S)}$$

$\xrightarrow{1/2} |\uparrow\rangle$

$\xrightarrow{1/2} |\downarrow\rangle$

jew. mit Wk $1/2$ (G)

Superposition $\qquad$ **Gemisch**

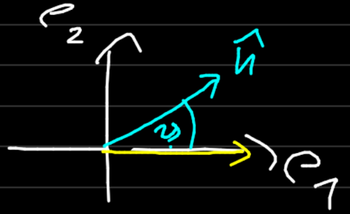
Γ **Dichte-Operator-Formalismus**

$$\rho_S = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \longrightarrow \rho_G = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

zur Unterscheidung von (S) und (G)

rotiere $R_3(-\vartheta)$, messe in X+ Richtung

$\hat{=}$ Spin Messung in Richtung $\hat{n}$



$$\hat{n} = R_3(\vartheta)\,|X+\rangle$$

Rechnung: $|\Psi_0\rangle_{SU} = |\psi_S\rangle|\varphi_0\rangle_U$

$$|\psi_S\rangle = |X+\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$$

$$|\varphi_0\rangle_U = \sum_{\vec{\zeta}\in\{-1,+1\}^n} c_{\vec{\zeta}}\,|\vec{\zeta}\rangle \quad , c_{\vec{\zeta}} \in \mathbb{C},$$

Normierung: $\sum_{\vec{\zeta}} |c_{\vec{\zeta}}|^2 \overset{!}{=} 1$

$$|\Psi_0\rangle \xrightarrow{U(t)} |\Psi(t)\rangle\,;$$

$$U(t):\ |\uparrow\rangle|\varphi_0\rangle = e^{-i\,\sigma_z\otimes A t/\hbar}\,|\uparrow\rangle|\varphi_0\rangle$$

$$= |\uparrow\rangle\cdot e^{-iAt/\hbar}|\varphi_0\rangle$$

$$|\downarrow\rangle|\varphi_0\rangle = |\downarrow\rangle\, e^{+iAt/\hbar}\,|\varphi_0\rangle$$

Zustände $|\uparrow\rangle, |\downarrow\rangle$ stabil

unter WW. $V = \sigma_z\otimes A$!

("pointer states", "Zeiger-Zust.")

$$t=0$$

$$\left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)\otimes|\varphi_0\rangle$$

$$\xrightarrow{U(t)} |\Psi(t)\rangle = \left\{ \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle\, e^{-iAt/\hbar}|\varphi_0\rangle + |\downarrow\rangle\, e^{+iAt/\hbar}|\varphi_0\rangle\Big) \right.$$

$\Rightarrow$ $|\hat{n},+\rangle$ Messung Position mit

Wkt: $P(\vartheta) = \Big\langle\, |\hat{n}\rangle\langle\hat{n}|\otimes \mathbb{1}_u \Big\rangle_{\Psi(t)}$

$$\to P(\vartheta) = \frac{1}{2} + \frac{1}{2}\,\mathrm{Re}\, e^{-i\vartheta} \underbrace{\langle\varphi_0| e^{2iAt/\hbar}|\varphi_0\rangle}_{=:\, M(t)}$$

Berechnung des Matrixelements:

$$M = \sum_{\vec s\,\vec s\,'} \langle \vec s\,'|\, r^*_{\vec s\,'}\, e^{2i\sum_j g_j \sigma_3^{(j)} t/\hbar}\, r_{\vec s}\, |\vec s\rangle$$

$$= \sum_{\vec s} |r_{\vec s}|^2 e^{2i \sum_j g_j s_j t/\hbar}$$

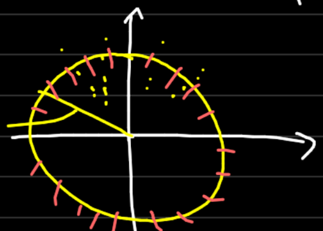
ω_j

$$M(t) = \sum_{\vec s} |r_{\vec s}|^2 e^{\sum_j s_j \omega_j t}$$

mit $\omega_j = 2 g_j/\hbar$

$$r_{\vec s} = \frac{1}{\sqrt{2^n}} \cdot e^{i\varphi_{\vec s}} = \frac{1}{\sqrt{2^n}}$$

$\mathbb{C}$



$$M(t) = \prod_{j=1}^{n} \left(\frac{e^{+i\omega_j t} + e^{-i\omega_j t}}{2} \right)$$

$\rightarrow M(t) = \prod_{j=1}^{n} \cos(\omega_j t)$

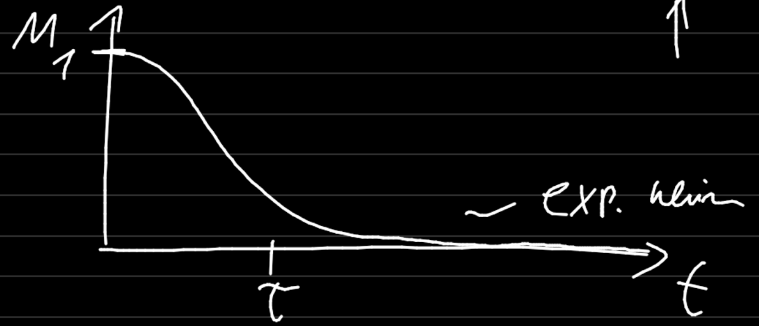
Kurzzeitlimes: $t \ll \frac{1}{\omega_j} \leq \frac{\hbar}{2g}$

$$M(t) = \prod_j \left(1 - \frac{(\omega_j t)^2}{2}\right)$$

$$= \prod_j e^{-(\omega_j t)^2/2} = e^{-\frac{1}{2} t^2 \cdot \sum_j \omega_j^2}$$

d.h. $M(t) = e^{-\frac{1}{2} t^2/\tau^2}$

$$\tau = \sqrt{\frac{1}{\sum_j \omega_j^2}} \rightarrow \tau = \sqrt{\frac{3}{n}} \frac{\hbar}{2g}$$



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