Lösungshinweise Blatt 1

2 a) $$|\langle \varphi_{x+}, \varphi_{y+}\rangle|^2 = \left|\tfrac{1}{2}\langle \varphi_{z+}+\varphi_{z-}, \varphi_{z+}+i\varphi_{z-}\rangle\right|^2$$
$$= \left|\tfrac{1}{2}(1+i)\right|^2 = 1/2 \quad ,$$

$$|\langle \varphi_{z+}, \varphi_{y-}\rangle|^2 = \left|\tfrac{1}{\sqrt{2}}\langle \varphi_{z+}, \varphi_{z+}-i\varphi_{z-}\rangle\right|^2 = 1/2 \, .$$

b) bzgl. ONB $(\varphi_{z+}, \varphi_{z-})$ ist $\psi = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\1+i\end{pmatrix}$

$$\to \quad |\psi|^2 = \tfrac{1}{6}\left(|2|^2 + |1+i|^2\right) = 1$$

c) μ_y - Messung ergibt $+\mu_0$ mit Wkt.

$$P_{y+} = |\langle \varphi_{y+}, \psi\rangle|^2 = \frac{1}{2\cdot 6}\left|\left\langle \begin{pmatrix}1\\i\end{pmatrix}, \begin{pmatrix}2\\1+i\end{pmatrix}\right\rangle\right|^2$$
$$= \tfrac{1}{12}\,|2-i+1|^2 = 5/6$$

$$P_{y-} = 1 - P_{y+} = 1/6$$

$$\to \quad \langle \mu_y \rangle_\psi = \mu_0\,(P_{y+} - P_{y-}) = \tfrac{2}{3}\,\mu_0$$

3) $$\frac{1}{\sqrt{2}}(\varphi_{z+} + \varphi_{z-}) \overset{!}{=} \varphi_{x+}$$

$\to$ μ_x - Messung am Quelle-A-Atom ergibt $+\mu_0$ mit Wkt. $P^A_{x+} = 1$.

μ_x - Messung an Quelle - B - Atom ergibt

$+\mu_0$ mit Wkt.

$$P^B_{x+} = \underset{=}{\frac{1}{2}} \left| \langle \varphi_{x+}, \varphi_{z+} \rangle \right|^2 + \underset{=}{\frac{1}{2}} \left| \langle \varphi_{x+}, \varphi_{z-} \rangle \right|^2 = \underline{\underline{1/2}}$$

Wkt. für $\psi =$ Wkt. für $\psi =$

wegen $P^B_{x+} \neq P^A_{x+}$ kann Quelle durch

μ_x - Messungen an (hinreichend vielen)

Atomen identifiziert werden.

d.h.:

Superposition $\neq$ Gemisch !

4 a) Rechnungen in komp. bzg. ONB $(\varphi_{z+}, \varphi_{z-})$:

$$\varphi_{z+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varphi_{z-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\rightarrow \quad P_{z+} = \varphi_{z+} \cdot \varphi_{z+}^{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot (1, 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_{z-} = \varphi_{z-} \cdot \varphi_{z-}^{+} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot (0, 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow \quad \sigma_3 = \mu_z / \mu_0 = P_{z+} - P_{z-} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\varphi_{x\pm} = 1/\sqrt{2} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$$

$$\rightarrow \quad P_{\varphi_{x\pm}} = \varphi_{x\pm} \circ \varphi_{x\pm}^{+} = \frac{1}{2} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} (1, \pm 1) = \frac{1}{2} \begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$

$$\rightarrow \quad \sigma_1 = \mu_x / \mu_0 = P_{\varphi_{x+}} - P_{\varphi_{x-}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\varphi_{y\pm} = 1/\sqrt{2} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$\rightarrow \quad P_{\varphi_{y\pm}} = \frac{1}{2} \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \circ (1, \mp i) = \frac{1}{2} \begin{pmatrix} 1 & \mp i \\ \pm i & 1 \end{pmatrix}$$

$$\rightarrow \quad \sigma_2 = \mu_y / \mu_0 = P_{\varphi_{y+}} - P_{\varphi_{y-}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

4b)

$$\sigma_1 \varphi_{x+} \overset{a)}{=} (P_{\varphi_{x+}} - P_{\varphi_{x-}}) \varphi_{x+} = \varphi_{x+}$$

$$\sigma_1 \varphi_{x-} = (P_{\varphi_{x+}} - P_{\varphi_{x-}}) \varphi_{x-} = - \varphi_{x-} .$$

$\rightarrow$ σ_1 besitzt EWe ± 1 zu EVen $\varphi_{x\pm}$

analog: σ_2 " " " " " $\varphi_{y\pm}$

σ_3 " " " " " $\varphi_{z\pm}$

5) offenbar $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \equiv \mathbb{1}$

$$\rightarrow \quad \sigma_j^{2\ell} = \left(\sigma_j^2\right)^\ell = \mathbb{1}$$

$$\sigma_j^{2\ell+1} = \sigma_j^{2\ell}\,\sigma_j = \sigma_j$$

$$\rightarrow \quad e^{i\sigma_j \varphi} = \underbrace{\sum_{\ell=0}^{\infty} \frac{1}{(2\ell)!}(i\varphi)^{2\ell}}_{=\cos\varphi} \underbrace{\sigma_j^{2\ell}}_{=\mathbb{1}} + \underbrace{\sum_{\ell=0}^{\infty} \frac{1}{(2\ell+1)!}(i\varphi)^{2\ell+1}}_{=i\sin\varphi} \underbrace{\sigma_j^{2\ell+1}}_{=\sigma_j}$$