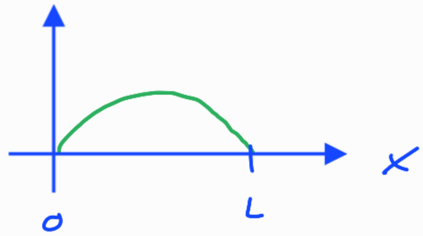
Lösungshinweise Blatt 10

38)

$$H_0 = p^2/2m\,, \quad 0 \le x \le L \;\rightsquigarrow\; \psi_0(x) = \sqrt{\tfrac{2}{L}}\,\sin k_0 x$$

$$E_0 = \hbar^2 k_0^2/2m \qquad \text{mit} \quad k_0 = \pi/L\,,$$



$$H = H_0 - e\mathcal{E}x$$

$$\rightsquigarrow\; E_0^{(1)} = E_0 - \langle\psi_0|\,e\mathcal{E}x\,|\psi_0\rangle$$

$$= E_0 - e\mathcal{E}\,\underbrace{\langle x\rangle_{\psi_0}}_{=L/2} = E_0 - e\mathcal{E}L/2$$

39)

$$V(t) = -xq\mathcal{E} = -xqA\,\frac{e^{-t^2/\tau^2}}{\sqrt{\pi}\,\tau}$$

$$\rightsquigarrow\; P_{0n} = \frac{1}{\hbar^2}\left|\int_{-\infty}^{\infty} dt\,\langle n|V(t)|0\rangle\, e^{i\left(\frac{E_n-E_0}{\hbar}\right)t}\right|^2, \quad n \neq 0$$

$$= \frac{q^2A^2\ell^2}{\hbar^2}\,\Big|\underbrace{\langle n|\,\overset{(a^\dagger+a)/\sqrt{2}}{\overset{\shortparallel}{x/\ell}}\,|0\rangle}_{\frac{1}{\sqrt{2}}\delta_{n,1}}\Big|^2\,\Big|\underbrace{\int\frac{dt}{\sqrt{\pi}\,\tau}\,e^{-t^2/\tau^2+i\omega n t}}_{e^{-\tau^2\omega^2n^2/4}}\Big|^2$$

$$= \frac{q^2A^2\ell^2}{2\hbar^2}\,\underline{e^{-\tau^2\omega^2/2}}\cdot\delta_{n,1}\,, \qquad \ell^2 = \hbar/m\omega$$

(*)

(*) : $\tau \gg 1/\omega \rightsquigarrow$ kein Übergang!

40) Entwickle allg. norm. $|\psi\rangle$ in Energieeigenzust. $|0\rangle, |1\rangle, |2\rangle, \dots$ zu Energien $E_0 < E_1 \leq E_2 \leq \dots$:

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle, \quad \sum_n |c_n|^2 = 1$$

$$\rightsquigarrow \langle\psi|H|\psi\rangle = \sum_{n,m} c_n^* c_m \underbrace{\langle n|H|m\rangle}_{= E_m \delta_{n,m}}$$

$$= \sum_n |c_n|^2 E_n$$

$$= \underbrace{\sum_n |c_n|^2}_{=1} E_0 + \sum_n |c_n|^2 \underbrace{(E_n - E_0)}_{>0} \geq E_0 ;$$

Falls $|\psi\rangle = |0\rangle$ offenbar $\langle\psi|H|\psi\rangle = E_0$;

Falls $|\psi\rangle \neq |0\rangle$ gibt es m_0 so mit $c_{m_0} \neq 0$,

$$\rightsquigarrow \langle\psi|H|\psi\rangle \geq E_0 + \underbrace{|c_{m_0}|^2 (E_{m_0} - E_0)}_{>0} \underset{\neq}{\geq} E_0 .$$

b) $E(\alpha) = \langle \psi | p^2/_{2m} + c\, x | \psi \rangle$

$$= 4\alpha^3 \int_0^\infty dx \; x e^{-\alpha x} \left(-\frac{\hbar^2}{2m} \partial_x^2 + c x \right) x e^{-\alpha x}$$

$$= 4\alpha^3 \int_0^\infty dx \; x e^{-\alpha x} \left(-\frac{\hbar^2}{2m} \alpha^2 x + \frac{\hbar^2}{m} \alpha + c x^2 \right) e^{-\alpha x}$$

$$= 4\alpha^3 \Big\{ -\frac{\hbar^2 \alpha^2}{2m} \underbrace{\int_0^\infty dx \; x^2 e^{-2\alpha x}}_{= \frac{2}{8\alpha^3}} + \frac{\hbar^2}{m}\alpha \underbrace{\int_0^\infty dx \; x e^{-2\alpha x}}_{= \frac{1}{4\alpha^2}} + c \underbrace{\int_0^\infty dx \; x^3 e^{-2\alpha x}}_{= \frac{6}{16\alpha^4}} \Big\}$$

$$\rightarrow E(\alpha) = \frac{\hbar^2}{2m} \alpha^2 + \frac{3c}{2} \frac{1}{\alpha}$$

$$0 \overset{!}{=} \frac{dE}{d\alpha}(\alpha) = \frac{\hbar^2}{m} \alpha - \frac{3c}{2} \frac{1}{\alpha^2}$$

$$\rightarrow \alpha_0 = \left(\frac{3mc}{2\hbar^2} \right)^{1/3}$$

$$\rightarrow E_0 \simeq E(\alpha_0) = \frac{9}{4} \left(\frac{2c^2 \hbar^2}{3m} \right)^{1/3} .$$