Lösungshinweise Blatt 12

46a) • $[\underbrace{J_3, J_1 \pm i J_2}_{= J_\pm}] = \underbrace{[J_3, J_1]}_{= i\hbar J_2} \pm i \underbrace{[J_3, J_2]}_{= -i\hbar J_1}$

$$= \pm \hbar (J_1 \pm i J_2) = \pm \hbar J_\pm$$

• $[J^2, J_\ell] = \sum_k [J_k^2, J_\ell] =$

$$= \sum_k J_k [J_k, J_\ell] + [J_k, J_\ell] J_k$$

$$= \sum_{k,m} i\hbar\, \varepsilon_{k\ell m} (J_k J_m + J_m J_k)$$

$$= \sum_{k,m} i\hbar\, \varepsilon_{k\ell m} (J_k J_m - J_k J_m) = 0$$

$\rightarrow [J^2, J_1 \pm i J_2] = 0$.

46b) • $J^2 J_\pm |a, b\rangle \overset{[J^2, J_\pm] = 0}{=} J_\pm J^2 |a, b\rangle = J_\pm a |a, b\rangle$

$$= a J_\pm |a, b\rangle$$

• $J_3 J_\pm |a, b\rangle \overset{[J_3, J_\pm] = \pm\hbar J_\pm}{=} (J_\pm J_3 \pm \hbar J_\pm) |a, b\rangle$

$$= (J_\pm b \pm \hbar J_\pm) |a, b\rangle = (b \pm \hbar) J_\pm |a, b\rangle$$

46c) als Eigenvektoren zu <u>unterschiedlichen</u> Eigenwerten $b \neq b + \hbar$ sind $|a, b\rangle$ und $J_+ |a, b\rangle$ <u>orthogonal</u>

$$\rightarrow \quad 0 \overset{!}{=} \langle a, b | J_+ | a, b\rangle =$$

$$= \underbrace{\langle J_1 \rangle_{|ab\rangle}}_{\in \mathbb{R}} + i \underbrace{\langle J_2 \rangle_{|ab\rangle}}_{\in \mathbb{R}}$$

$$\rightarrow \quad \langle J_1 \rangle_{|ab\rangle} = \langle J_2 \rangle_{|ab\rangle} = 0 .$$

47) $$\langle J^2 \rangle_{|j,m\rangle} = \hbar^2 j(j+1)$$

$$\langle J_3 \rangle_{|j,m\rangle} = \hbar m$$

$$\langle J_1 \rangle_{|j,m\rangle} = 0 \quad (\text{nach } 46c)$$

$$\langle J_1^2 \rangle_{|j,m\rangle} \overset{!}{=} \langle J_2^2 \rangle_{|j,m\rangle} = \frac{1}{2} \langle J^2 - J_3^2 \rangle_{|j,m\rangle}$$

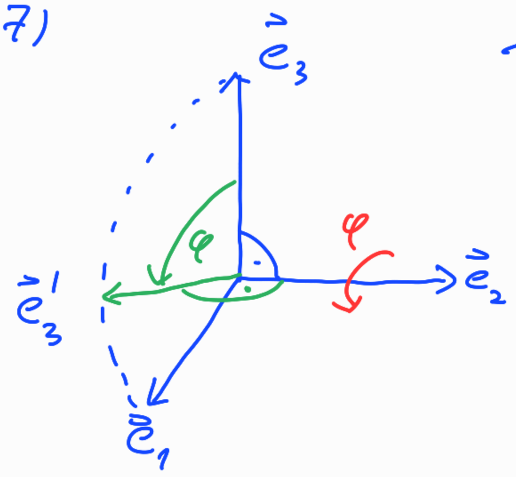
Symmetrie!

$$= \frac{\hbar^2}{2} \left(j(j+1) - m^2 \right),$$

$$j \in \tfrac{1}{2}\mathbb{N} = \{ 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \dots \}$$

$$\hookrightarrow m \in \{ -j, -j+1, \dots, j-1, j \} .$$

47)



$$\rightsquigarrow \quad |z+'\rangle = \mathcal{U}_{sp}(R_{2,\varphi})$$

$$= e^{-i\sigma_2 \frac{\varphi}{2}}$$

$$= \mathbb{1}\cos\frac{\varphi}{2} - i\sigma_2 \frac{\varphi}{2}$$

$$= \begin{pmatrix} \cos\varphi/2 & -\sin\varphi/2 \\ \sin\varphi/2 & \cos\varphi/2 \end{pmatrix}$$

$$\rightsquigarrow \quad P'_+ = |\langle z+'|z+\rangle|^2 = |\langle z+|\,\mathcal{U}_{sp}(R_{2,\varphi/2})\,|z+\rangle|^2$$

$$= \cos^2\varphi/2 = \cos^2\pi/6 = 3/4\,.$$

$$\varphi = 60^\circ = \pi/3$$

49)

$$U(t) = e^{-iHt/\hbar} = e^{i\vec\mu\cdot\vec B\, t/\hbar}$$

$$= \exp\Big(i\, \underbrace{\frac{geB}{2mc}}_{\omega}\, \vec n\cdot\frac{\cancel{\hbar}}{2}\vec\sigma\; t/\cancel{\hbar}\Big)$$

$$= e^{i\,\vec n\cdot\vec\sigma\,\frac{\omega t}{2}} \stackrel{!}{=} \mathcal{U}_{spin}(R_{-\vec n},\, \omega t)$$

Rotation um $-\vec n$ mit Winkelfr. ω.