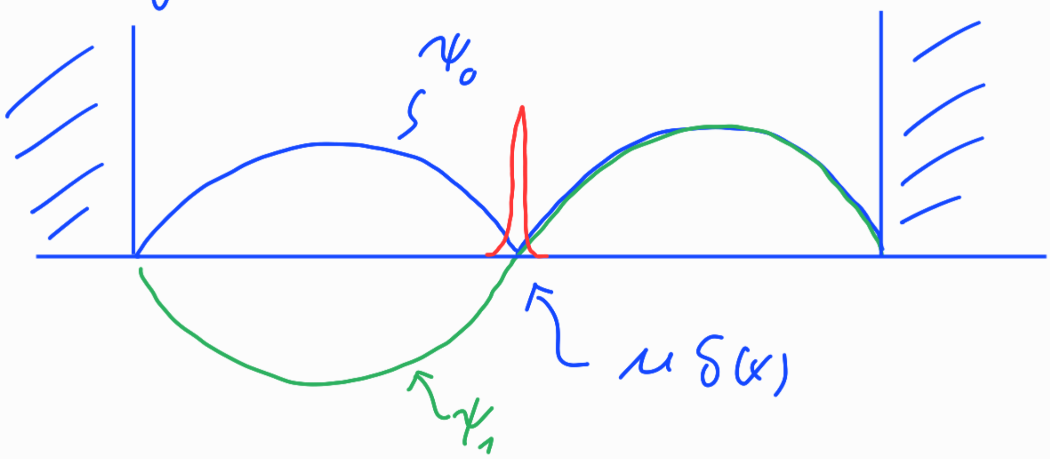
Lösungshinweise Blatt 6

19 a) Grundzustand symmetrisch, kein Nullstelle;

1. angeр. Zustand antisymmetrisch, eine Nullst.



$$\frac{\Delta E}{E_0} = \frac{E_1 - E_0}{E_1} \ll 1$$

für u hinreichend groß

19 b) Teilchen im rechten Kasten

$$\hat{=} \quad |\psi_+\rangle = \left(|\psi_0\rangle + |\psi_1\rangle\right)/\sqrt{2}$$

Superposition zweier Zustände

mit $\Delta E \neq 0$

$\rightarrow$ Oszillation mit $\omega = \frac{1}{2}\frac{\Delta E}{\hbar}$

Vorlsg.: $|\psi_+\rangle \xrightarrow{t} |\psi(t)\rangle = \frac{e^{i\Omega t}}{\sqrt{2}}\left(\cos\frac{\omega t}{2}|\psi_+\rangle + \sin\frac{\omega t}{2}|\psi_-\rangle\right)$

20)

a) $H(t) = \frac{\varepsilon}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + u\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cos\omega t + u\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\omega t$

$$= \begin{pmatrix} \varepsilon/2 & u e^{-i\omega t} \\ u e^{i\omega t} & -\varepsilon/2 \end{pmatrix}$$

b)

S.Gl.: $i\hbar \frac{d}{dt}\begin{pmatrix} a_+ e^{-i\omega t/2} \\ a_- e^{+i\omega t/2} \end{pmatrix} = \begin{pmatrix} \varepsilon/2 & u e^{-i\omega t} \\ u e^{+i\omega t} & -\varepsilon/2 \end{pmatrix}\begin{pmatrix} a_+ e^{-i\omega t/2} \\ a_- e^{i\omega t/2} \end{pmatrix}$

$$\rightarrow \quad i\hbar \dot{a}_+ + \frac{\hbar\omega}{2} a_+ = \frac{\varepsilon}{2} a_+ + u a_-$$

$$i\hbar \dot{a}_- - \frac{\hbar\omega}{2} a_- = -\frac{\varepsilon}{2} a_- + u a_+$$

d.h. $\frac{d}{dt}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = -\frac{i}{\hbar}\begin{pmatrix} \frac{\varepsilon-\hbar\omega}{2} & u \\ u & -\frac{\varepsilon-\hbar\omega}{2} \end{pmatrix}\begin{pmatrix} a_+ \\ a_- \end{pmatrix}$

c)

$\varepsilon = \hbar\omega \quad \rightsquigarrow \quad \frac{d}{dt}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \underbrace{-\frac{iu}{\hbar}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{=:\tilde{H}}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} \qquad (*)$

$$\rightarrow \quad \frac{d^2}{dt^2}\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \tilde{H}^2\begin{pmatrix} a_+ \\ a_- \end{pmatrix} = -\underbrace{\frac{u^2}{\hbar^2}}_{=:\tilde{\omega}^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

$$\rightarrow \quad a_+(t) = \alpha\cos(\tilde{\omega} t) + \beta\sin(\tilde{\omega} t)$$

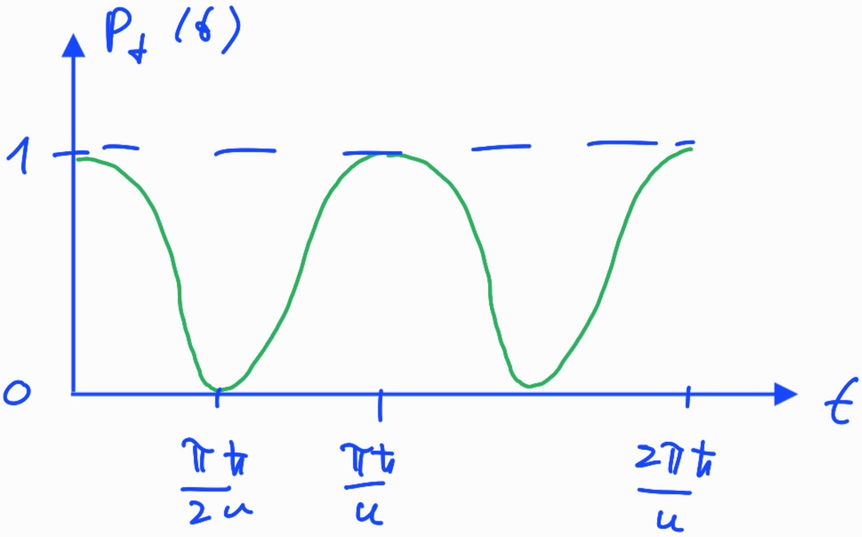
$$\xrightarrow{(*)} \quad a_-(t) = i\beta\cos(\tilde{\omega} t) - i\alpha\sin(\tilde{\omega} t)$$

$$|\psi(0)\rangle = |z+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} a_+(0) \\ a_-(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad \begin{pmatrix} a_+(t) \\ a_-(t) \end{pmatrix} = \begin{pmatrix} \cos \tilde{\omega} t \\ -i \sin \tilde{\omega} t \end{pmatrix}$$

$$\rightarrow \quad \psi(t) = \begin{pmatrix} \cos \tilde{\omega} t \; e^{-i\omega t/2} \\ -i \sin \tilde{\omega} t \; e^{i\omega t/2} \end{pmatrix}$$

$$\rightarrow \quad P_+(t) = |\langle z+|\psi(t)\rangle|^2 = (\cos \tilde{\omega} t)^2 \, , \quad \tilde{\omega} = \frac{|u|}{\hbar}$$



21) $$j = \frac{\hbar}{m} \operatorname{Im} \psi^* \frac{\partial}{\partial x} \psi$$

$$\rightarrow \quad j_2(x) = \frac{\hbar}{m} \operatorname{Im} e^{-ikx} t^* (ik) t e^{ikx}$$

$$= \frac{\hbar k}{m} |t|^2 = \frac{p}{m} |t|^2$$

mit $r = |r| e^{i\alpha}$:

$$j_1(x) = \frac{\hbar}{m} \operatorname{Im}\left((e^{-ikx} + r^* e^{ikx})(ik e^{ikx} - ikr e^{-ikx}) \right)$$

$$= \frac{\hbar}{m} \operatorname{Im}\Big(ik \Big\{ 1 - |r|^2 + |r| \underbrace{(e^{i(2kx-\alpha)} - e^{-i(2kx-\alpha)})}_{= 2i\sin(2kx-\alpha)} \Big\} \Big)$$

$$= \frac{\hbar k}{m}(1 - |r|^2) = \frac{p}{m}(1 - |r|^2) .$$

22)

- Stetigkeit von $\psi(x)$ in $x=0$: $\quad 1 + r \overset{!}{=} t \quad (*)$
- $\int_{-\varepsilon}^{\varepsilon} dx \left(\psi''(x) = \frac{2m}{\hbar^2}(u\,\delta(x) - E) \right)$

$$\rightarrow \quad \underbrace{\psi'(0+) - \psi'(0-)}_{= ik(t - 1 + r)} \overset{!}{=} \frac{2mu}{\hbar^2} \underbrace{\psi(0)}_{= t}$$

$$ik(t - 1 + r) \overset{!}{=} \frac{2mu}{\hbar^2} t$$

$(*)$

$$t - 1 = -i \frac{mu}{k\hbar^2} t$$

$$\hookrightarrow \quad \frac{1}{t} = 1 + ic, \quad c = mu/k\hbar^2$$

$$\rightarrow \quad r = t - 1 = \frac{1}{1+i\kappa} - 1 = \frac{-i\kappa}{1+i\kappa} = \frac{-1}{1-i/\kappa}$$

$\rightarrow$ Reflexionswkt. $R = |r|^2 = \dfrac{1}{1+\kappa^{-2}}$

$$= \left(1 + \left(\frac{k\hbar^2}{m\kappa}\right)^2\right)^{-1}$$

$$\lim_{|\kappa|\to\infty} R = 1 \quad \checkmark$$

$$\lim_{|\kappa|\to 0} R = 0 \quad \checkmark$$

$$\lim_{\hbar\to 0} R = 1 \quad \checkmark$$

$$\lim_{k\to\infty} R = 0 \quad \checkmark$$